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The Role of the Buyout Price in the Internet Auction

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Abstract

We consider an Internet auction with the “buyout price” as an optional feature. Under *IID* assumption and risk-neutral buyers, we show that the expected revenue to the seller from the Internet auction with the buyout price is larger than the expected revenue from the Internet auction without the buyout price. Moreover, given the uniform distribution for reservation values, the revenue gain of employing the buyout price option is getting smaller as the number of potential buyers becomes larger.

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1. Introduction

Internet auctions have recently attracted the attention of both practitioners and academic researchers. Internet auctions have several advantages over traditional auctions. They overcome the

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geographic limitation of traditional auctions. Any buyers all over the world can participate in the auction staying at their houses. Moreover, Internet auctions provide temporal benefits to buyers by allowing them to bid at any time within the designated ending date(Lucking-Reiley, 2000).

Internet auctions are equipped with several unique institutional features that will raise many interesting research issues for marketing researchers. We study one of these institutional features, namely “buyout price.” Most of auction sites now employ this option even though Yahoo calls it “Buy Price” and eBay calls it “Buy It Now.” The buyout price is the price at which a seller is willing to close their auction immediately and sell the item. If a bidder offers a bid equal to the auction’s buyout price, that bidder automatically wins the auction(Yahoo, 2003). Even though Lucking-Reiley(2000) mentioned that the buyout price is a feature not yet tackled by auction theory, he conjectured that the buyout option might benefit buyer and seller by bringing the auction to an early end. The buyer can win the bidding with certainty, but she may have to pay more. The seller may be able to sell the item early with acceptable price, but gives up the possibility of even higher price. Consistent with Lucking-Reiley’s conjecture, Sun(2001) has shown that the buyout option should be introduced when the(inventory) holding cost is high enough.

Considering an Internet auction with the buyout price as an optional feature, we provide an alternative explanation why the seller wants to employ the buyout price option. Under *IID* assumption and risk-neutral buyers, the expected revenue to the seller from the Internet auction with the buyout price is larger than the expected revenue from the Internet auction without the buyout price. In addition, given the uniform distribution for reservation values, the revenue gain of employing the buyout price option is getting smaller as the number of potential buyers becomes larger.

2. The model

Consider a single-object ascending English Internet auction. A seller expects n potential buyers where the value of the object to buyer i is v_i. All the buyers are assumed to be risk-neutral. Hence, if a buyer with v_i wins the bid by the bidding price of b, her gain becomes $v_i - b$. We also assume that v_i's are independent and identically distributed, drawn from the common cumulative distribution function $F(v)$ with support $[\underline{v}, \overline{v}]$. That is, $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$. We also assume that $F(v)$ is strictly increasing and differentiable over the interval $[v, \overline{v}]$. First presented by Vickery(1961), this *IID* assumption is frequently employed in the auction literature(Riley and Samuelson, 1981; Bulow and Roberts, 1989; Wang, 1993). This assumption practically means that each buyer knows her/his own value (v_i) but the seller and the other buyers are uncertain about this value. And all the buyers and the seller share common knowledge that everybody views the buyers' values as independent draws from a common distribution $F(v)$.

2.1. Internet auction without the buyout price option

Let us first review a case of traditional English auction without the buyout price option. The results in this section are identical to the results derived by Riley and Samuelson(1981). Given that each buyer will bid her true reservation price or value, the buyer with the reservation value v will win the bid when all others bid below v. Given *IID* assumption, the probability that a randomly chosen individual has the value below v is $F(v)$ and the probability that all $(n-1)$ others have the values below v is $[F(v)]^{n-1} = F^{n-1}(v)$. Noticing that $F^{n-1}(v)$ is simply the cumulative distribution function of the maximum among $(n-1)$ reservation values, its probability density function is $\partial F^{n-1}(v)/\partial v = (n-1)f(v)F^{n-2}(v)$. Because she should pay as much as this maximum when she wins the bid, her expected payment will be

$$P(v) = \int_{\underline{v}}^{v} x(n-1)f(x)F^{n-2}(x)dx = vF^{n-1}(v) - \int_{\underline{v}}^{v} F^{n-1}(x)dx \tag{1}$$

Since $P(v)$ is the expected payment from an individual with the value v, the expected revenue from a randomly chosen buyer will be its expected value. Given that n buyers are expected to participate, the expected revenue to the seller will be

$$R_E = n\int_{\underline{v}}^{\overline{v}} P(v)f(v)dv = n\left[\int_{\underline{v}}^{\overline{v}} (vf(v) + F(v) - 1)F^{n-1}(v)dv\right] \tag{2}$$

2.2. Internet auction with the buyout price option

Now consider an auction with the buyout price option. The seller specifies the buyout price of z. The auction ends as soon as a buyer offers a bid equal to the buyout price. Note that a buyer will submit the buyout price if her reservation value is greater than the buyout price. Hence the probability that the first buyer ends the auction with the buyout price is $1 - F(z)$. Similarly, it is $F(z)[1 - F(z)]$ for the second buyer. The probability that the auction ends with the buyout price of z will be $[1 - F(z)][1 + F(z) + F^2(z) + \cdots + F^{n-1}(z)] = 1 - F^n(z)$. The seller earns z if the auction ends with the buyout price z. Therefore, the expected revenue to the seller from the buyout price becomes

$$[1 - F^n(z)]z \tag{3a}$$

If the reservation values for all n buyers are less than the buyout price z (and its probability is $F^n(z)$), then the auction will end by the traditional auction. Hence the expected revenue to the seller from successive bidding processes, contingent on all n buyers' reservation values less than z, will be

$$\begin{aligned} &F^n(z) \times n\int_{\underline{v}}^{z} (vg(v) + G(v) - 1)G^{n-1}(v)dv \\ &= n\int_{\underline{v}}^{z} (vf(v) + F(v) - F(z))F^{n-1}(v)dv \end{aligned} \tag{3b}$$

In equation (3b), $g(v) = f(v)/F(z)$ and $G(v) = F(v)/F(z)$.

The (total) expected revenue from the English auction with the buyout price of z is the sum of the quantity from the equation (3a) and the equation (3b). That is,

$$R_{EB}(z) = [1 - F^n(z)]z + n\int_{\underline{v}}^{z} (vf(v) + F(v) - F(z))F^{n-1}(v)dv \tag{4}$$

Proposition 1: Suppose that all the buyers are risk-neutral and the *IID* assumption holds. The expected revenue to the seller from the auction with the buyout price option is greater than the expected revenue from the auction without the buyout price.

Proof. As seen from equation (4), the expected revenue with the buyout price option depends on the buyout price z. Differentiating $R_{EB}(z)$ with respect to z,

$$\begin{aligned} R_{EB}(z)' = \partial R_{EB}(z)/\partial z &= 1 - F^n(z) - znF^{n-1}(z)f(z) + n[zf(z) \\ &\quad + F(z)]F^{n-1}(z) - \left[f(z)\int_{\underline{v}}^{z} F^{n-1}(v)dv + F(z)F^{n-1}(z)\right] \\ &= 1 - F^n(z) - nf(z)\int_{\underline{v}}^{z} F^{n-1}(v)dv \end{aligned} \tag{5}$$

Evaluating $R_{EB}(z)$ and $R_{EB}(z)'$ at $z = \underline{v}$ and $z = \overline{v}$ yields

$$\begin{aligned} &R_{EB}(\underline{v})' = 1 - F^n(\underline{v}) - nf(\underline{v})\int_{\underline{v}}^{\underline{v}} F^{n-1}(v)dv = 1 > 0 \\ &(\Theta F(v) = 0 \quad and \quad \int_{\underline{v}}^{\underline{v}} F^{n-1}(v)dv = 0) \end{aligned} \tag{6a}$$

$$\begin{aligned} &R_{EB}(\underline{v})' = 1 - F^n(\overline{v}) - nf(\overline{v})\int_{\underline{v}}^{\overline{v}} F^{n-1}(v)dv = -nf(\overline{v})\int_{\underline{v}}^{\overline{v}} F^{n-1}(v)dv < 0 \\ &(\Theta F(\overline{v}) = 1) \end{aligned} \tag{6b}$$

$$R_{EB}(\underline{v}) = [1 - F^n(\underline{v})\underline{v} + n\int_{\underline{v}}^{\underline{v}} (vf(v) + F(v) - F(\underline{v}))F^{n-1}(v)dv = \underline{v} \tag{6c}$$

$$\begin{aligned} R_{EB}(\overline{v}) &= [1 - F^n(\overline{v})]\overline{v} + n\int_{\underline{v}}^{\overline{v}} (vf(v) + F(v) - F(\overline{v}))F^{n-1}(v)dv \\ &= n\int_{\underline{v}}^{\overline{v}} (vf(v) + F(v) - 1)F^{n-1}(v)dv = R_E \end{aligned} \tag{6d}$$

From the equation (6a) and (6b), we conclude that the expected revenue function, $R_{EB}(z)$, has a concave region within $[\underline{v}, \overline{v}]$ and there exists z^* that will maximize $R_{EB}(z)$. From the equation (6c), the expected revenue is $\overline{v}$ that is the lowest for the seller to be able to get. On the other hands, the equation (6d) tells us that the expected revenue with the buyout price option becomes identical to the expected revenue from the auction without the buyout price if the buyout price is set at $z = \overline{v}$. Hence, from (6a) to (6d), there exist z^* within $[\underline{v}, \overline{v}]$ that will maximize $R_{EB}(z)$ and $R_{EB}(z^*) > R_{EB}(\overline{v})$. That is, the expected revenue to the seller from the auction with the buyout price option is greater than the expected revenue from the auction without the buyout price. QED

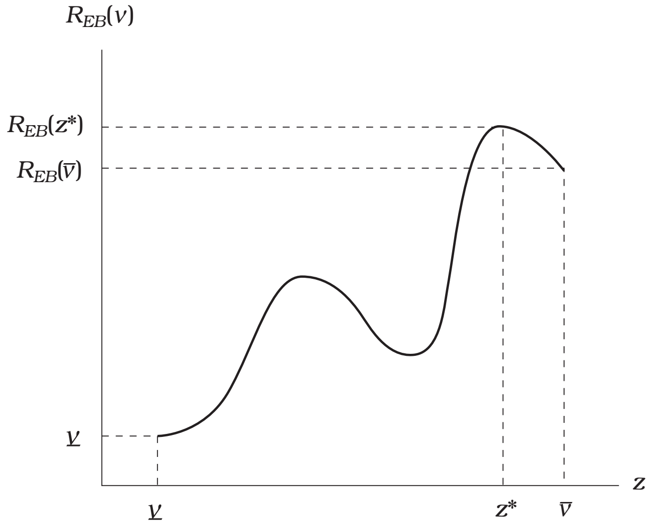

Figure 1. The expected revenue curve given the buyout price *z*.

Based on the results from the equation (6a) through (6d), we draw an approximate shape of the expected revenue curve given the buyout price z in Figure 1. Here we clearly see $R_{EB}(z^*) > R_{EB}(\bar{v}) = R_E$. Finally, let z^* maximize the expected revenue with the buyout price option, $R_{EB}(z)$. Subtracting R_E from $REB(z^*)$ yield

$$\begin{aligned}\Delta &= R_{EB}(z^*) - R_E \\ &= [1 - F^n(z^*)]z^* + n\int_{\underline{v}}^{z^*}(vf(v) + F(v) - F(z^*))F^{n-1}(v)dv \\ &\quad - n\int_{\underline{v}}^{\bar{v}}(vf(v) + F(v) - 1)F^{n-1}(v)dv = [1 - F^n(z^*)]z^* \\ &\quad + n(1 - F(z^*))\int_{\underline{v}}^{z^*}F^{n-1}(v)dv + n\int_{z^*}^{\bar{v}}(1 - vf(v) - F(v))F^{n-1}(v)dv \qquad (7)\end{aligned}$$

3. A Parametric Example: Case of Uniform Distribution

For an illustrative example, we now assume that $f(v) \sim U(0, 1)$. We still employ the same assumptions such as n risk-neutral buyers and *IID* as before.

Consider an auction without buyout price. With uniform distribution, the probability that a randomly chosen individual has the value below v is $F(v) = vI_{[0,1]}(v)$ and the probability that all $(n-1)$ others have the values below v is simply $[F(v)]^{n-1} = F^{n-1}(v) = v^{n-1}$. And its probability density function is $\partial F^{n-1}(v)/\partial v = (n-1)v^{n-2}$. Hence her expected payment will be $P(v) = \int_0^v x(n-1)x^{n-2}dx = \frac{n-1}{n}v^n$. And given that n buyers are expected to participate, the expected revenue to the seller will be

$$UR_E = n\int_0^1 \frac{n-1}{n}v^n dv = \frac{n-1}{n+1} \tag{8a}$$

Next, consider an auction with the buyout price option. Here the probability that the auction ends with the buyout price of z will be $1 - F^n(z) = (1 - z^n)$. The seller earns z if the auction ends with the buyout price z. Therefore, the expected revenue to the seller from the buyout price becomes $[1 - F^n(z)]z = (1 - z^n)z$. On the other hands, if the reservation values for all n buyers are less than the buyout price z, then the auction proceeds in traditional way. Hence the expected revenue to the seller from this traditional auction, contingent on all n buyers' reservation values less than z, will be

$$n\int_0^z (v + v - z)v^{n-1}dv = \frac{n-1}{n+1}z^{n+1} \tag{8b}$$

The (total) expected revenue from an auction with the buyout price of z is the sum of $(1 - z^n)z$ and the quantity from the equation (8b). That is,

$$UR_{EB}(z) = (1 - z^n)z + \frac{n-1}{n+1}z^{n+1} \tag{9}$$

In order to find the optimal buyout price z^* maximizing $UR_{EB}(z)$, we differentiate the equation (9) with respect to z. $UR_{EB}(z)' = 1 - (n+1)z^n + (n-1)z^n = 0$. Hence, $z^* = (1/2)^{1/n}$. Substituting this optimal z^* into the equation (9), we have the optimal expected revenue with the optimal buyout price z^*.

$$UR_{EB}(z^*) = (1/2)^{1/n}\left[\frac{n}{n+1}\right] \tag{10}$$

Subtracting UR_E from $UR_{EB}(z^*)$ yields

$$\begin{aligned}\Delta(n) = UR_{EB}(z^*) - RU_E &= (1/2)^{1/n}\left[\frac{n}{n+1}\right] - \left[\frac{n-1}{n+1}\right] \\ &= [1 - n(1-2^{-1/n})]/(n+1)\end{aligned} \tag{11}$$

$\Delta(n) > 0$ for all $n > 1$ since $1 - n(1 - 2^{-1/n}) > 0$ for all $n > 1$. Hence, the expected revenue to the seller from an auction with the buyout price option is greater than the expected revenue from an auction without the buyout price.

Proposition 2: Suppose that all the buyers are risk-neutral and the *IID* assumption holds. Given the uniform distribution for reservation values, the revenue gain of an auction with the buyout price option over an auction without the buyout price is getting smaller as the number of potential buyers n becomes larger.

Proof. In order to evaluate the revenue gain by providing buyout price option, we observe how $\Delta(n)$ changes as the number of potential buyers changes. Differentiating $\Delta(n)$ with respect to n, we have

$$\partial\Delta(n)/\partial n = \frac{2^{-1/n}[(1+1/n)\log 2 + 1] - 2}{(n+1)^2} > 0, \quad \forall n > 1$$

since $\log[1 + (1 + 1/n)\log 2] > (1 + 1/n)\log 2$ or $\log\{2^{-1/n}[(1 + 1/n)\log 2 + 1]\} > \log 2$ for all $n > 1$.

$\Delta(n)$ decreases as n increases. Or the revenue gain of an auction with the buyout price option over an auction without the buyout price is getting smaller as the number of potential buyers n becomes larger. QED

Finally, it is interesting to observe that the expected revenue with the buyout price option becomes identical to the expected revenue without the buyout option or $\lim_{n\to\infty}\Delta(n) = 0$. This result makes sense intuitively since the optimal buyout price $z^* = (1/2)^{1/n}$ goes to 1 as n goes to infinity.

4. Discussion and concluding remarks

A casual conversation with managers in Internet auction sites indicates that there are still other reasons to employ the buyout price option. For example, the buyout price option plays a role in reducing the probability of bid retraction. A few buyers often retract successful bids even though the Internet auction sites explicitly prohibit the retraction of bids. It is difficult to prevent from bid retractions completely because buyers are supposed to pay money after they win the bids. The bid retraction ratio is said to be much lower among buyers submitting the buyout prices. Empirical studies are required to see which explanations are better in justifying the buyout price.

Our paper has several limitations. First, it is not trivial for managers to find the optimal buyout price in practice. It requires for managers to know the distribution of reservation values. Second, our assumptions may not be very realistic in the Internet auction. Bajari and Hortaçsu(2000) claim that common value models are more appropriate in the Internet auction.

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