Master's Thesis of Jaehong Lim

Contest Mechanism Design with Budget-Constraint Agents

예산제약 하에서의 Contest Mechanism에 대한 연구

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Abstract

In this paper, I analyzed how inserting of budget constraint assumption makes difference with the results of previous studies on contest mechanism design problems such as optimal contest mechanism design and affirmative action design. When there are some budget-constrained agents, mechanism designers have to think over not only the best way to balance gains (desirable allocation) and losses (wasteful cost) from the contest but also the best way to balance allocation of budget-constrained agents and budget-free agents. Compared to assortative matching mechanism, random matching mechanism and coarse matching mechanism are more likely to balance allocation of budget-constrained agents and budget-free agents well. Since random matching mechanism and coarse matching mechanism make participants waste less resources in signaling, they relieve some problems caused by budget constraints. When affirmative action is implemented, therefore, random matching mechanism and coarse matching mechanism allow agents with budget constraint problems to win for open seats. Consequently, more minority people are expected to achieve school seats under random or coarse matching mechanism.

Keywords: contest mechanism, affirmative action, inequality
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Contents

1 Introduction

Contest mechanism, also known as a tournament mechanism, allocates prizes to agents depending on each agents' relative performance on the contest. A contest is a widely implemented mechanism in diverse areas of real-life including school admissions and job recruiting, because contest mechanism allows schools and employers to get information from candidates and choose their most favorable people. In the contest, participants exert efforts and reveal how competent they are. Schools or employers observe every candidate's performance on the contest and choose whom they would admit or recruit. Performance that agents achieve in the contest works as a signal revealing how competent they are.

One unsatisfying characteristic of a contest is that it usually accompanies a wasteful cost. Participants of the contest need to spend a resource to make their performance. The input for performance may improve their productivity, but it may not. As many papers analyzing signaling models assume, when signaling activity is not productive, the only merit of the signal is revealing and conveying private information of candidates. Sometimes, the waste of signaling costs is too much, so the revelation of information can be inefficient. For this reason, some research, including Hoppe et al. (2009), Condorelli (2012), Olszewski and Siegel (2019), Kleiner et al. (2021), try to find optimal contest mechanism well balancing gains and losses.

An additional problem, which is closely related to the first problem, arises when some agents are faced with a constrained choice problem. If agents are faced some constraints that hinder them from choosing their desired effort level, the economy cannot attain efficient allocation. In addition to the wasteful signaling cost, a biased matching allocation can cause economic inefficiency. Affirmative action is the representative example to overcome an allocation problem. The primary purpose of affirmative action is to provide additional opportunities to include racially and sexually underrepresented groups.

Nowadays, economic polarization has emerged as a major economic problem. Underprivileged people are faced with serious budget constraint problems, so they are easily losing educational and career opportunities. If budget constraints of competent agents hamper their desirable signaling investment, relatively less competent but affluent agents will take seats. As Fernandez and Gali (1999) has shown, the effect of budget constraint on matching allocation is clear; both willingness to pay and ability to pay signaling costs are necessary to be allocated to agents' desirable school.

In this paper, I analyzed how inserting of budget constraint assumption makes difference with the results of previous studies on contest mechanism design problems such as optimal contest mechanism design and affirmative action design. In general, research on optimal contest mechanism design is closely related to the first shortcoming of contest mechanism, so they are looking for the best way to balance gains from desirable matching outcome and losses from wasteful cost. Additionally, a mechanism designer has to think over the best way to balance allocation of budget-constrained agents and budget-free agents. If changing mechanism treats majority group and minority group differently, budget constraint assumption of agents must be critical in optimal contest mechanism design analysis. Affirmative action, the policy of improving educational and job opportunities of underrepresented groups, is a well-known solution to correct a biased contest allocation. After conducting an optimal mechanism design approach with I found a few interesting results.

According to the results of this study, when some agents in the economy are budget-constrained, random matching mechanism and coarse matching mechanism are better at balancing budget-constrained agents and budget-free agents. Since random matching mechanism and coarse matching mechanism make participants waste less resources in signaling, they relieve some problems caused by budget constraints. When affirmative action is implemented, assortative random matching does not allow agents with budget constraint problems to win open seats. In comparison, random matching mechanism and coarse matching mechanism allow them to. Therefore, more minority people are expected to achieve school seats under random or coarse matching mechanism.

Michael Sandel, the professor of political philosophy at Harvard University, has insisted on drawing lots for Harvard entry in his book "The Tyranny of Merit". (Sandel (2020)) His advocate is closely related to the findings of this paper. Although this paper does not reach to finding conditions under which random or coarse matching mechanism performs better than the assortative matching mechanism, it confirms that such scenario is well explained theoretically.

2 Literature Review

The studies of optimal contest mechanism in a general setting are probably the most closely related to my research. I use the concept of "random rationing with a minimum standard mechanism", referring to a coarse mechanism, or a partially random matching mechanism of optimal contest mechanism design research. Hoppe et al. (2009), Condorelli (2012), Olszewski and Siegel (2019), Kleiner et al. (2021) show that the optimal mechanism can be a fully revealing mechanism, a fully pooling mechanism, or partially pooling mechanism, and analyze conditions that affect welfare maximizing contest mechanism. These papers explain that the agents' ability distribution and hazard rate of the distribution are detrimental in designing an optimal mechanism. In previous research, agents are heterogeneous in ability only, so there are no wealth inequality or budget constraint problem.

The research on reserve seat design is also very closely related to my paper. To treat a minority group favorably, minority reserve and majority cap policy are widely implemented. In this paper, I also consider optimal reserve seat design. Hafalir et al. (2013) compare the matching allocation with minority reserve affirmative action and the allocation with majority quota affirmative action. Dur et al. (2018) show that in the presence of reserve seat, not only the sizes of reserve seats but also precedence order for reserve seats critically affect allocation. According to their result, lowering the precedence order of reserve seats weakly increases minority group assignment at the school.

Fernandez and Gali (1999) study a budget constraint problem under the contest mechanism and conveys that assortative allocation can be damaged by budget constraint of some agents. It compares the performance of matching markets and tournaments and shows that tournament mechanism is superior when there are borrowing constrained agents. This paper shows that how a model assumes the relationship between agents' ability and wasteful cost can make a huge difference in equilibrium. The tournament mechanism makes a budget constraint problem less serious by alleviating a monetary burden for more competent agents.

The analysis of the contest mechanism must be conducted with an understanding of the signaling model. A proper signaling technology that reveals agents' ability is necessary for the contest mechanism to work well. In this paper, I use the signaling model of Spence (1978), and therefore agents are assumed to have a cost function that is negatively correlated to their ability.

3 The economy

3.1 Model

The economy consists of a unit measure of agents and a single number of school. Agents are characterized by each one's endowment of ability a and type of wealth. Each agent's ability a is assumed to be uniformly distributed on [0,1]. As many signaling models assume, the ability is privately given to agents, and there exists an incomplete information problem.

Agents who win at the contest will be admitted to the school. The total measure of seat is given as a constant number $q \in (0, 1)$, so only a limited number of agents can achieve the chance of education. An agent with ability a can generate an output $X(a)$ when admitted to school. An agent with positive ability has positive productivity, and an agent with higher ability benefits much more from education. (*i.e.,* $x(0) = 0,\ x'(a) \geq 0$, for all $a \in [0, 1]$) Furthermore, assume that education is essential to achieve positive productivity in this economy, so all losing agents generate 0.

School seats are allocated according to agents' performance level at each contest and the seat allocation rule (matching mechanism) of the contest. In contest, agents choose their performance level (or signaling score) v. The signaling cost depends on each agent's ability level a and his signaling level v. Let's assume that an agent with ability level a costs $c(a) \cdot v$ to send a signal level v. An agent with higher ability can deliver performance at a lower cost. Therefore, the signaling cost function $c(\cdot)$ is assumed to be a decreasing function in a. The resources expended by an agent in enhancing her signal are not productive. Consequently, the agent who has an a ability level and chooses to perform v obtains utility $u(v; a) = X(a) \cdot m(a, v) - c(a) \cdot v$, where $m \in [0, 1]$ denotes an indication function of school allocation.

Whether or not each agent can burden the signaling cost depends on his wealth type. There are two types of agents. $\mathcal{H}$ type of agents are not faced with a budget constraint, thus they can give any signals they want. On the other hand, $\mathcal{L}$ type agents are faced with a homogeneous budget constraint e which is an exogenously given constant number. In other words, $\mathcal{L}$ type agents can implement performance v that satisfies $c(a) \cdot v \leq e$. Among all agents, $l \in (0, 1)$ measure of agents belong to type $\mathcal{L}$, and $h \in (0, 1)$ measure of agents belong to type $\mathcal{H}$. All agents are either $\mathcal{H}$ type or $\mathcal{L}$ type, thus $l + h$ must be 1.

3.2 Benchmark Equilibrium

As a benchmark, the equilibrium under conventional decentralized contest mechanism will be analyzed in this subsection. Under the conventional decentralized contest mechanism, agents who rank top q quantile win at the contest.

The equilibrium consists of each agent's choice of performance and equilibrium cut-off score. The equilibrium consists of each agent's choice of performance and equilibrium cut-off score. However, we can fully characterize the equilibrium by specifying the cut-off score in this economy. Since there is only one school, all winners will give the exact level of performance with the cut-off score level, while all losers will not spend any of their resources. To characterize the equilibrium cut-off score, define $V(a)$ as

$$V(a) \equiv \frac{X(a)}{c(a)}.$$

$V(a)$ is the signal level that makes agents' gain from education and cost for signaling be equivalent when their ability level is a. (*i.e.* $X(a) = c(a) \cdot V(a)$) For any $v \geq V(a)$, $X(a) - (a) \cdot V(a) = 0 \geq X(a) - c(a) \cdot v$. Therefore, $V(a)$ is the maximum performance level that agents with ability level a would be willing to make if they can win at the tournament.

For $\mathcal{L}$ type agents, both willingness to pay and ability to pay matter in making a signaling choice. Agents of $\mathcal{L}$ type may not afford $c(a) \cdot V(a)$, so similarly define $V^e(a)$ as

$$V^e(a) = \frac{\min\{X(a), e\}}{c(a)}.$$

$V^e(a)$ is the maximum performance level of $\mathcal{L}$ agents that they are willing to make and can afford to make at the same time if they can win at the contest.

Let $v^0 \in R_+$ be the equilibrium cut-off score. $\mathcal{H}$ type agents with ability level a will invest if the equilibrium cut-off score is lower than $V(a)$, and $\mathcal{L}$ type agents with ability level a will invest if the equilibrium cut-off score is lower than $V^e(a)$. Let a_H be the ability level that $V(a_H) = v^0$, and a_L be the ability level that $V^e(a_L) = v^0$. Since both $V(a)$ and $V^e(a)$ is weakly increasing in a, $V(a) \geq V(a_H)$ for all $a \geq a_H$ and $V^e(a) \geq V(a_L)$ for all $a \geq a_L$.[1] Therefore, a_H and a_L are the lowest level of ability for each type to win this benchmark

[1] $V'(a) = \frac{X'(a)c(a) - X(a)c'(a)}{c(a)^2} \geq 0$.

$V^{e\prime}(a)$ is either $\frac{X'(a)c(a) - X(a)c'(a)}{c(a)^2}$ or $-\frac{ec'(a)}{c(a)^2}$, and non-negative in both cases.

contest with the equilibrium threshold v^0. Finally, we need the matching clearing condition; the measure of winners in contest with the cut-off level v must be equal to the measure of school seat q. At the equilibrium, $1 - a_H$ measure of $\mathcal{H}$ type agents and $1 - a_L$ measure of $\mathcal{L}$ type agents will be admitted to school. To clear all seats, $l \times (1 - a_L) + h \times (1 - a_H) = q$.

Therefore, the equilibrium condition in this benchmark model is

$$\begin{cases} V(a_H) = V^e(a_L) \\ l \times (1 - a_L) + h \times (1 - a_H) = q \end{cases} . \tag{3.1}$$

and the equilibrium cut-off performance level v^0 equals $V(a_H) = V^e(a_L)$.

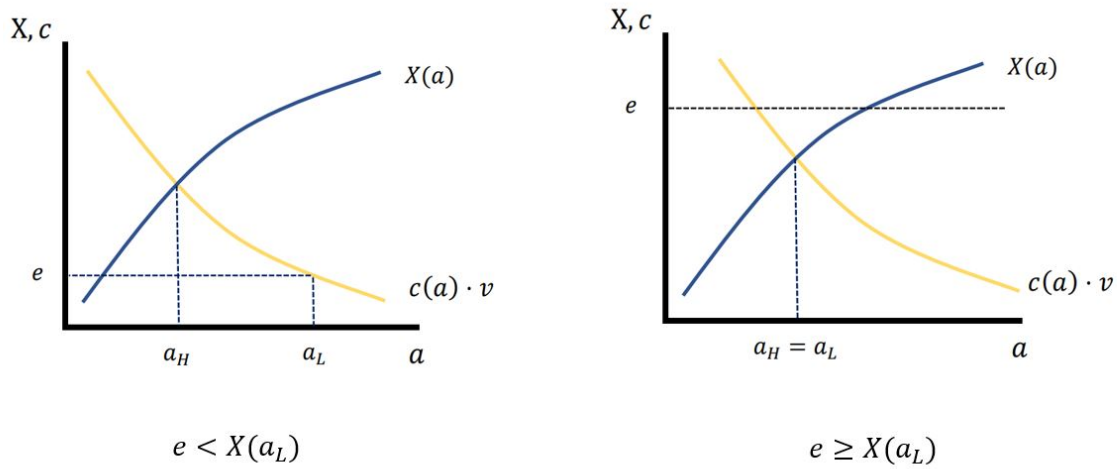

Figure 3.1: Benchmark Equilibrium

If budget constraint level e is higher than $X(1-q)$, all agents in this economy can choose their optimal signalling action, free from budget constraint. If not, however, less measure of $\mathcal{L}$ type agents and higher measure of $\mathcal{H}$ type agents are allocated to the school seat. This result corresponds to the result of Fernandez and Gali (1997). The social welfare is equal to integration of all agents' utility, and

$$WF = l \cdot \int_{a_L}^{1} \{X(a) - c(a) \cdot V^e(a_L)\} da + h \cdot \int_{a_H}^{1} \{X(a) - c(a) \cdot V(a_H)\} da \tag{3.2}$$

One thing to note is that budget constraint does not always reduce social welfare. The existence of budget constraint makes competition less fierce and it leads every agent to spend less resources. The total welfare effect of a budget constraint highly depends on second derivatives of $X(\cdot)$, $c(\cdot)$, and $V(\cdot)$, and the size of constant numbers l, h, and e.

3.3 Reserve Seat Design

To give minority students higher chances to attend school, affirmative action is widely implemented. In this economy, $\mathcal{L}$ type could be though of a minority group, and we will briefly see how to comprehend affirmative action policy that treats $\mathcal{L}$ type agents favorably. According to Hafalir et al. (2013), when all students and all schools have the same priority, as is in this economy, the matching allocation with minority reserve affirmative action and the allocation with majority quota affirmative action are equivalent.[2] Therefore, without loss of generality, it is enough to analyze a minority reserve affirmative action design problem.

From the result of benchmark equilibrium, we have observed that at least $1-a_L$ measure of $\mathcal{L}$ type agents can be admitted to school without any affirmative action. Therefore, we can say that the reserve seats whose measure is less than $1-a_L$ are not effective. Effective reserve seats are going to be the subject of reserve seat design analysis. In addition to that, since the purpose of reserve seat is to correct biased allocation, thus the case where the minority group is exceedingly favored will be excluded: the measure of reserve seats must be smaller than $l \times q$.[3]

Dur et al. (2018) shows that in the presence of reserve seat, the effect of the precedence order is comparable to the effect of adjusting reserve sizes. They analyzed the case where all agents' performance are already fixed and the only problem that designer cares is matching allocation. According to their result, lowering the precedence order of reserve seats weakly increases minority group assignment at the school. Because minority students with high

[2] Proposition 2 in Hafalir et. al. (2013) is analysis for common preferences case.

[3] If $e \geq X(a_L)$, the measure of reserve seats must be larger than $l \times q$ for any effective setting of reserve seat. Therefore, it is assumed that $e < X(1-q)$ from this section.

enough performance are eligible for both open and reserve seat, decreasing the precedence of reserve seats increases competition for open seats and decreases competition for reserve seats.

However, I found that precedence order of reserve seats is insignificant under the conventional contest mechanism. In this model where agents choose their performance level, agents can choose which seats they will be allocated to, and this makes a difference in result.

Proposition 1 *Under the conventional contest mechanism, regardless of the precedence order of reserve seats, effective reserve seats design will make*

i) only $\mathcal{H}$ type agents are allocated to the open seats. In other words, $\mathcal{L}$ type agents compete for reserve seats, while $\mathcal{H}$ type agents compete for open seats.

ii) the equilibrium cut-off signaling level is always higher in open seats than reserve seats.

An intuition for Proposition 1 is that any $\mathcal{L}$ type agents who are eligible for open seats must be eligible for reserve seats, and they are always better to win at the competition for reserve seats which is less competitive. Since agents choose their performance level at the contest with costly effort, high ability agents in minority group choose not to struggle to win for a more competitive seats. This eliminates the effect of precedence order in reserve design.

Therefore, under conventional contest mechanism, reserve seat design problem narrows down to decision of a reserve size. If the size of reserve seat is s, the cut-off signal level for $\mathcal{L}$ type agents will be $V^e(1-s)$ and the cut-off signal level for $\mathcal{H}$ type agents will be $V(1-(q-l\cdot s)/h)$ by the result of proposition 1. The optimal reserve size can be achieved by solving the following maximization problem.

$$\begin{aligned}\underset{s}{\operatorname{argmax}}\ & l\cdot\int_{1-s}^{1}\{X(a)-c(a)\cdot V^e(1-s)\}da \\ & + h\cdot\int_{1-(q-l\cdot s)/h}^{1}\{X(a)-c(a)\cdot V(1-(q-l\cdot s)/h)\}da\end{aligned}$$

Example *Consider the simplest case where $X(a) = a$ and $c(a) = 1$. When positive measure of $\mathcal{L}$ type agents can be admitted to school without reserve seat, it is optimal not to implement any affirmative action policy. On the other hands, if competition is not working properly due to harsh budget constraint on $\mathcal{L}$ type agents so only $\mathcal{H}$ agents win without affirmative action, positive measure of reserve seat always improves social welfare. The measure of $\mathcal{L}$ and $\mathcal{H}$ type l and q decide an exact size of reserve seats.*[4]

4 Minimum Standard Mechanism

The ambiguous welfare effect of signaling happens because revealing private information is usually costly. To efficiently allocate resources, the mechanism requires agents to reveal more information, but this must accompany a higher signaling cost. Hoppe et al. (2009), Condorelli (2012), Olszewski and Siegel (2019), and Kleiner et al. (2021) studied what is the optimal contest mechanism to induce proper level of signal. These papers explain that optimal mechanism can be an assortative matching, random matching, or partially random matching. How effectively information is revealed is important in finding optimal mechanism, and these papers conclude that agents' ability distribution and hazard rate of the distribution are detrimental factors in designing optimal mechanism.

Similarly, we can apply this result under budget-constrained agents model. In this research, I begin by introducing "random rationing with a minimum standard mechanism" (hereafter, minimum standard mechanism). Under the minimum standard mechanism, agents with a bid higher than minimum standard are pooled and randomly allocated to school. With a single number of school, minimum standard mechanism is is similar to coarse matching mechanism, or hybrid matching mechanism in which agents with a bid in a given quantile are randomly allocated to prizes in the same quantile. However, the minimum standard mechanism can be a assortative matching mechanism or fully random matching mechanism,

[4]The first order condition for the optimal reserve size problem is $l \times [(1-s) - e - \frac{q-l\cdot s}{h}] = 0$.

depending on where the minimum standard level is set; when the minimum standard level is 0 (or $1 - q$), the matching allocation will be fully randomized (or perfectly assortative).

This section analyzed an additional implication of optimal mechanism design study under budget-constrained agents model. Changing mechanism treats majority group agents and minority group agents differently. Additionally, This mechanism is primarily a kind of coarse matching mechanism, or hybrid matching mechanism in which agents with a bid in a given quantile are randomly allocated to prizes in the same quantile.

In this paper, $V(\cdot)$ explains how costly to reveal agents' private information, and plays similar role to the ability distribution in previous research. The minimum standard mechanism is a more general form of mechanism than contest mechanism, and it is clear that we can achieve Pareto improvement by finding optimal minimum level. It is a more versatile mechanism in that designer can decide minimum levels for open seats and reserve seats in addition to the measure of both seats. By looking for the best minimum level of signaling to balance matching output and signaling costs in less extreme ways, the social welfare must be improved.

What I want to focus on is the an additional implication of optimal mechanism design study under budget-constrained agents model; properties that make coarse matching mechanism or random mechanism more advantageous. Firstly, lowering minimum level of signal mitigates budget constraint problem, and it can be more favorable to minority group agents. Secondly, when we implement affirmative action, the minimum standard mechanism allows $\mathcal{L}$ type agents to be allocated to open seats. By implementing the minimum level mechanism, we expect the policy effect of affirmative action is strengthened.

Consider the case where only open seats exist first. Under the minimum standard mechanism, a designer can directly setting the threshold level (or minimum level). Determining a minimum standard directly is equivalent to deciding how much agents are pooled and randomly allocated to school, so assumes a designer chooses the measure of pooled agents. Among all $\mathcal{L}$ type and $\mathcal{H}$ type agents, s^o measure of agents are selected and randomly matched to $q^0 = q$ measure of school seats. By definition, s^o must be more than q^o. For a given s^o and q^o, the threshold signal level v^o must be satisfy $\frac{q}{s}X(a_H^o) - c(a_H^o) \cdot v^0 = 0$, where a_H^o denotes the lowest ability level of $\mathcal{H}$ type pooled agents. Similarly, $\min\{\frac{q}{s}X(a_L^o), e\} - c(a_L^o) \cdot v^o = 0$, where a_L^o denotes the lowest ability level of $\mathcal{L}$ type pooled agents. Therefore, the equilibrium with a given s^o and q^o is

$$\begin{cases} \frac{q^o}{s^o}\frac{X(a_H^o)}{c(a_H^o)} = \frac{\min\{\frac{q^o}{s^o}X(a_L^o),e\}}{c(a_L^o)} \\ l \times (1 - a_L^o) + h \times (1 - a_H^o) = s^o \end{cases}. \tag{4.1}$$

and social welfare is

$$WF = l \cdot \int_{a_L^o}^{1} \frac{q^o}{s^o}X(a) - c(a) \cdot \frac{\min\{\frac{q^o}{s^o}X(a_L^o), e\}}{c(a_L^o)}da + \frac{q^o}{s^o} \cdot h \cdot \int_{a_H^o}^{1} X(a) - c(a)\frac{X(a_H^o)}{c(a_H^o)}da. \tag{4.2}$$

Proposition 2 *(Allocation under Minimum Standard Mechanism)*
If $V(\cdot)$ function is concave, lowering minimum standards for open seats is weakly more favorable to minority group.

In addition to balancing matching output and signaling costs in less extreme ways, coarse matching has an advantage in balancing matching allocation of $\mathcal{H}$ type agents and $\mathcal{L}$ type agents. The conventional contest mechanism aims at selecting most competent agents at the expense of large waste of resources, but it fails to achieve its goal because of some agents' budget constraint problem. Although coarse matching loses desirable matching outcome within same types, it can achieve more desirable matching outcome between two different types of wealth under a concavity condition of the $V(\cdot)$ function. Therefore, coarse matching is more likely to be superior when we consider budget constraint problem in designing contest mechanism.[1]

Now, consider the case where positive measure of reserve seats exist. Let v_r be the signaling threshold for reserve seats and v_o be the signaling threshold for open seats. Under the conventional contest mechanism, v_r is always smaller than v_o. Under coarse matching mechanism, however, the opposite case is possible. Let q^o and q^r be the measure of allocated agents for open seats and reserve seats, and s^o and s^r be the measure of pooled agents for open seats and reserve seats. By definition, $s_i \geq q_i$ for $i = o, r$ and $q_o + q_r = q$.

Proposition 3 *(Minimum Standard Mechanism and Reserve Seats Design)*
i) Both $v_r > v_o$ and $v_o > v_r$ cases are possible. In both cases, the most competent group of agents are those who are eligible for both seats, and they always chooses signal level $\max\{v_r, v_o\}$.
ii) For a given (v_r, v_o, q_r, q_o), lowering the precedence order of reserve seats weakly increases allocation of $\mathcal{L}$ type agents.

Under the conventional contest mechanism, $\mathcal{L}$ type agents do not want to compete for open seats. Under the minimum level mechanism, however, they might want to achieve chances to randomly allocated to open seats at a lower cost. If the minimum threshold for open seats are low enough, $\mathcal{L}$ agents are willing to cost to win for open seats for sure. As an extreme case, if the minimum signaling level for open seats is set to zero, all agents can freely achieve an additional chance of school allocation. Interestingly, under the minimum level mechanism, there might be some agents who take part in competition for open seats only. As a competition for open seats becomes less intense and a competition for reserve seats becomes more fierce, $\mathcal{L}$ type agents are more likely to eligible only for open seats. This observation is closely related to the result of *Proposition 2*, in that the change of minimum threshold affects the school allocation between $\mathcal{L}$ type and $\mathcal{H}$ type agents.

The minimum standard mechanism engenders a possibility that minority agents also

[1] Concavity condition in proposition 2 is a very strong sufficient condition for lowering minimum standards to be favorable to minority group. Therefore, in reality, it may be true under much weaker environment.

can win at the contest for open seats, so precedence order of reserve seats now becomes significant. Similar to Dur et al. (2018), some minority agents can be eligible for both open seats and reserve seats. Therefore, lowering the precedence order of reserve seats weakly increases minority agents' allocation to school seats.

5 Conclusion

In this paper, I constructed a model that agents compete for homogeneous prize (higher education) in contest, while some participants of contest are faced with budget constraint. Applying affirmative action design analysis and optimal contest mechanism design analysis, this paper has shown a few new findings regarding contest mechanism design under budget-constraint situation.

If agents are assumed to have a quasi-linear utility function and they choose their optimal signaling level, the optimal signaling for budget-constrained agents makes them ineligible for open seats under conventional contest mechanism. Therefore, reserve seats are the only chances for minority group. Under random matching mechanism and coarse mechanism, however, minority agents can be allocated to open seats. Lowering minimum level of open seats weakly favors minority group under some conditions. If some minority agents are eligible both for open seats and reserve seats, lowering precedence order of reserve seats is more favorable to the minority group.

The work of this paper is to discover a fragmented characteristic. I am looking forward to the more specific characterizations about optimal contest mechanism problem under budget-constraint situation.

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Appendix

Proof for proposition 1

At equilibrium,

i) The cut-off level for open seats cannot be smaller than cut-off level for reserve seats. If not, winners for reserve seats would abandon his seat and go for open seats with lower cost.

ii) It is not possible for the threshold levels for both seats to be same and so that positive measure of $\mathcal{L}$ type agents are allocated to open seats. The only case where the two threshold levels are equivalent is that the threshold level is $V(a_H) = V^e(a_L)$ by market clearing condition. In this case, the equilibrium allocation is exactly equivalent with that of benchmark equilibrium, which is excluded by the effective reserve assumption.

iii) No $\mathcal{L}$ type agent win for open seats. Since the threshold signal level for reserve seats is strictly below the threshold signal level for open seats, it is less costly to compete for reserve seats.

The threshold signal level of reserve seats is at most $V^e(a_L)$, because we assume that the measure of reserve seats must be smaller than $l \times q$. If the cut-off performance level is smaller than $V^e(a_L) = V(a_H)$, the total measure of agents who are willing to give performance larger than threshold level will be higher than q. This is contradiction.

Proof for proposition 2

To begin with, let s_e be the s that satisfies $\frac{q^o}{s}X(1-s) = e$. Then, s_e always exists in $(q^o, 1)$. $\frac{q^o}{s}X(1-s)$ is decreasing in s. From the fact that $X(1-q^o) > e$ and $\frac{q^o}{s}X(1-s) = 0$ when $s = 1$, there always exists a $s_e \in (q^o, 1)$ that satisfies $\frac{q^0}{s_e}X(1-s_e) = e$ for a given q^0.

i) For $s^o \in (s_e, 1)$, $\min\{\frac{q^o}{s^o}X(a^o_L), e\} = \frac{q^o}{s^o}X(a^o_L)$, and $a^o_H = a^o_L = 1 - s^o$. Therefore, when s_e is in $(s_e, 1)$, $\frac{d}{ds}a^o_H = \frac{d}{ds}a^o_L$. If the minimum threshold is low enough, $\mathcal{L}$ type agents are equally treated to $\mathcal{H}$ type agents.

For the next step, what I need to show is $\frac{d}{ds}a^o_L < \frac{d}{ds}a^o_H$ for $\forall s^o \in (q, s_e)$. For $s \in (0, 1-s_e)$,

$\min\{\frac{q^o}{s^o}X(a_L^o), e\} = e$. By the equilibrium condition, $\frac{q^o}{s^o}V(a_H^o) = V^e(a_L^o)$, and

$$q^o \cdot V'(a_H) \cdot a_H'(s^o) = V^e(a_L) + s^o \cdot V^{e\prime}(a_L) \cdot a_L'(s^o). \tag{5.1}$$

From the second equilibrium condition,

$$a_L'(s^o) = \frac{-1 - h \cdot a_H'(s^o)}{l} \tag{5.2}$$

By substituting $a_L'(s)$ in equation (5) using equation (6), we can get

$$a_H'(s^o)\left(1 + \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} \cdot \frac{h}{l}\right) = \frac{1}{q^o} \cdot \frac{V^e(a_L)}{V'(a_H)} - \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} \cdot \frac{1}{l}, \tag{5.3}$$

and

$$(a_H'(s^o) + 1)\left(1 + \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} \cdot \frac{h}{l}\right) = \frac{1}{q^o} \cdot \frac{V^e(a_L)}{V'(a_H)} - \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} + 1 \tag{5.4}$$

From equation (6), $a_H'(s^o) - a_L'(a^o) \geq 0$ is equivalent to $a_H'(s^o) + 1 \geq 0$. On left-hand side of the equation (8), $\left(1 + \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} \cdot \frac{h}{l}\right)$ is always positive. Therefore, if right-hand side of the equation (8) is positive for all $s^o \in (0, s_e)$, then we can conclude that increasing reserve seat treats minority group more favorably.

$$\frac{1}{q^o} \cdot \frac{V^e(a_L)}{V'(a_H)} - \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} + 1 = \frac{s^o}{q^o} \cdot V'(a_H) \cdot \left(\frac{1}{s^o} \cdot V^e(a_L) - V^{e\prime}(a_L) + \frac{q^o}{s^o}V'(a_H)\right). \tag{5.5}$$

If s^o is in (o, s_e), $V^{e\prime}(a_L) \leq \frac{q^o}{s^o}V'(a_L)$. Finally if $V(\cdot)$ is concave, $V'(a_H) \geq V'(a_L)$. Therefore, if $V(\cdot)$ is concave,

$$(a_H'(s^o) + 1)\left(1 + \frac{s^o}{q^o} \cdot \frac{V^{e\prime}(a_L)}{V'(a_H)} \cdot \frac{h}{l}\right) \geq \frac{s^o}{q^o} \cdot V'(a_H) \cdot \left(\frac{1}{s^o} \cdot V^e(a_L) - \frac{q^o}{s^o}V'(a_L) + \frac{q^o}{s^o}V'(a_H)\right) > 0$$

Proof for proposition 3

i) Without loss of generality, assume that $v_r > v_o$, which is a less trivial case. It is less trivial in that there is no agent who is eligible only for reserve seat. In this case, agents eligible for reserve seats are also eligible for open seats. Let $a_{L,r}$ be the lowest ability level that gives a signal v_r. Regardless of the precedence order, the expected payoff of agents with ability level $a_{L,r}$ is $\{\frac{q_o}{s_o} + \left(1 - \frac{q_o}{s_o}\right) \cdot \frac{q_r}{s_r}\} \cdot X(a_{L,r}) - c(a_{L,r}) \cdot v_r$. By an incentive compatibility condition, $\{\frac{q_o}{s_o}+\left(1-\frac{q_o}{s_o}\right)\cdot\frac{q_r}{s_r}\}\cdot X(a_{L,r})-c(a_{L,r})\cdot v_r \geq \frac{q_o}{s_o}\cdot X(a_{L,r})-c(a_{L,r})\cdot v_o$, and $\left(1-\frac{q_o}{s_o}\right)\cdot\frac{q_r}{s_r}\cdot X(a_{L,r})-c(a_{L,r})\cdot(v_r-v_o)\geq 0$. Since $X(a)$ increases in a and $c(a)$ decreases in a, all agents with ability levels higher than $a_{L,r}$ are always choose to give the signal level v_r. The same logic can be applied to the $v_o > v_r$ case. Therefore, agents who are eligible for both seats are agents at the top.

ii) It is enough to analyze the allocation outcome of open seats, because we can observe less $\mathcal{H}$ type agents are allocated to open seats. Similarly defines $a_{L,o}$ and $a_{H,o}$. If $v_o > v_r$, regardless of precedence order, $a_{L,o}$ is decided at $\frac{\min\{\left(\frac{q_o}{s_o}+\left(1-\frac{q_o}{s_o}\right)\cdot\frac{q_r}{s_r}\right)\cdot X(a_{L,o}),e\}}{c(a_{L,o})}=v_o$ and $a_{H,o}$ is decided by the equation $\frac{q_o}{s_o}\cdot V(a_{H,o})=v_o$. What makes differences is the group of agents who take part in the contest for open seats. If open seats preserve, all $\mathcal{H}$ type agents and $\mathcal{L}$ agents who are eligible for v_o take part in contest for open seats. The equilibrium condition is

$$\begin{cases}\frac{\min\{\left(\frac{q_o}{s_o}+\left(1-\frac{q_o}{s_o}\right)\cdot\frac{q_r}{s_r}\right)\cdot X(a_{L,o}),e\}}{c(a_{L,o})}=\frac{q_o}{s_o}\cdot V(a_{H,o})=v_o\\ l\cdot a_{L,o}+h\cdot a_{H,o}=1-s_o.\end{cases}$$

If reserve seats preserve, however, $\mathcal{L}$ type agents who take part in contest for open seats are those who are eligible for v_o and remained after contest for reserve seats. Consequently, the equilibrium condition is as follows.

$$\begin{cases}\frac{\min\{\left(\frac{q_o}{s_o}+\left(1-\frac{q_o}{s_o}\right)\cdot\frac{q_r}{s_r}\right)\cdot X(a_{L,o}),e\}}{c(a_{L,o})}=\frac{q_o}{s_o}\cdot V(a_{H,o})=v_o\\ l\cdot(1-a_{L,o})\cdot\left(1-\frac{q_r}{s_r}\right)+h\cdot(1-a_{H,o})=s_o\end{cases}$$

Similar logic can be applied to the $v_r > v_o$ cases. If open seats preserve, equilibrium condition is

$$\begin{cases}\frac{\min\{\frac{q_o}{s_o}\cdot X(a_{L,o}),e\}}{c(a_{L,o})}=\frac{q_o}{s_o}\cdot V(a_{H,o})=v_o\\ l\cdot a_{L,o}+h\cdot a_{H,o}=1-s_o.\end{cases}$$

If reserve seats preserve, however, the equilibrium condition is

$$\begin{cases}\frac{\min\{\frac{q_o}{s_o}\cdot X(a_{L,o}),e\}}{c(a_{L,o})}=\frac{q_o}{s_o}\cdot V(a_{H,o})=v_o\\ l\cdot a_{L,o}+h\cdot a_{H,o}=1-s_o-l\cdot q_r.\end{cases}$$

For both cases, $a_{H,o}$ must be smaller when reserve seats preserve, which means more $\mathcal{H}$ type agents are allocated to school.