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공학석사학위논문

# Resale Royalty Optimization for Maximal Non-Fungible Token (NFT) Creator Profit

대체불가능토큰(NFT) 제작자 이익 극대화를 위한 재판매  
로열티 최적화

2022 년 8 월

서울대학교 대학원  
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임 성 아

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이 논문을 공학석사 학위논문으로 제출함

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## Abstract

# Resale Royalty Optimization for Maximal Non-Fungible Token (NFT) Creator Profit

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Non-fungible tokens (NFTs) are financial, transferrable securities on a blockchain with unique identification codes and metadata that distinguish them from one other. One of the distinctive characteristics of NFTs is the blockchain-based ability to distribute the royalties from resales to the creators. This study analyzes the effect of NFT's resale royalty on the NFT creator's total profit, which includes the primary sale revenue and the secondary sales royalty income, when the NFT is traded in repeated English auctions. Resale royalty can affect NFT buyers' resale strategies, the bubble of NFT bid prices, and the number of NFT resales, and considering all these aspects, this study finds the optimal resale royalty rate to maximize the NFT creator's expected profit. The model used in this study is formulated as a Stackelberg game to explore the optimal strategies of an NFT creator and buyers. The numerical results show that the optimal resale royalty rate that yields the maximum creator profit varies depending on the time duration considered by the NFT creator, the short-term and the long-term expected value increase rates of the NFT, and the

auction winner's buyer type. This study considers various types of NFT buyers in auctions including collectors and speculators, and when collectors and speculators compete, it is often recommended for the NFT creator to set an appropriate resale royalty rate so that a royalty-sensitive speculator can win. Since speculators tend to resell more often than collectors, the creator can earn a larger secondary sale income when a speculator wins, which in turn yields larger expected profit.

**Keywords:** NFT(Non-fungible token), resale royalty, English auction, Stackelberg game

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# Chapter 1

## Introduction

### 1.1 Research Background

Non-fungible tokens (NFTs) are unique digital items whose transaction details have been verified through a blockchain. A blockchain allows reliable transactions between network participants without the need of a third-party credit institution, enabling decentralized and horizontal business in digital infrastructure. The first known NFT was minted in 2014 by Kevin McCoy and Anil Dash, but NFTs have not received mainstream attention until 2021.

There are some unique characteristics of NFTs that differentiate them from other general products. First, NFTs have a high potential for future profit due to its non-fungibility. Second, there is no external control agency such as the government or banks in blockchains, so NFT transactions are made only by the power of demand. Third, NFTs have high risks of fraud, hacking, or loss of accounts. Fourth, combined with metaverse, digital wallets, and non-bankable assets, the market and scope of NFT applications can grow enormously. And lastly, NFTs are programmed to automatically distribute resale royalty to the NFT creator whenever an NFT is resold. In particular, Fortnow and Terry [1] pointed out that NFT's blockchain-based resale royalty is one of the most revolutionary features of NFT.

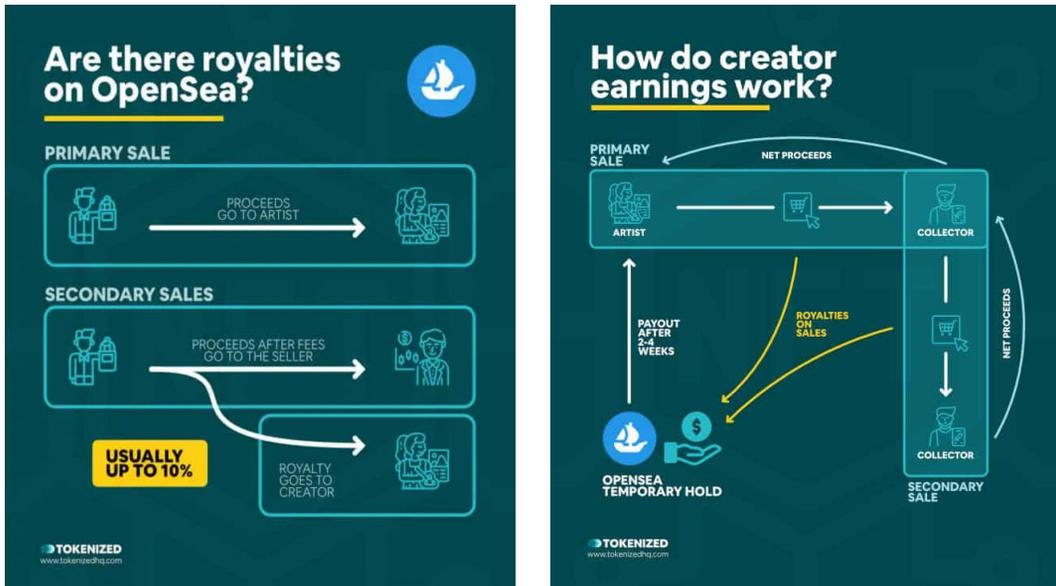
According to *U.S. Copyright Office*, resale royalty is the right to provide artists with an opportunity to benefit from the increased value of their works over time by granting them a percentage of the proceeds from the resale of their original works of art [2]. *Droit de suite*, the world's first art resale royalty, was introduced in France in 1920s.

However, many countries including Korea, the United States, China, Switzerland, and Canada still have not introduced the art resale royalty, and the existing art resale royalty has many limitations. The countries that have applied the resale royalty right follow different regulations from each other, so the resale royalty is not applied when it is traded oversea or traded by foreigners. Not only that, Schten [3] stressed that it is difficult to manage and track all resale transactions, and Filer [4] mentioned that resale royalty is often evaded by reselling the artwork to a dummy corporation.

On the other hand, resale royalty of NFT is guaranteed due to blockchain contracts called *smart contracts*, providing a new phase in artist continuous funding right[5]. Szabo [6] first introduced the concept of smart contracts, which combines computer protocols with user interfaces to execute contract terms. Nofer, et al. [7] insisted that smart contracts began to gain attention of its usability and efficacy with the advent of blockchain technology. Smart contracts are created whenever an NFT is minted and the distributed smart contracts cannot be modified. Every time smart contracts are deployed, a gas fee may be paid, but this study does not consider the fee because there are some networks such as Polygon that do not incur a gas fee.

*Tokenized*, a site that provides digital asset information, explained how OpenSea, the largest NFT open marketplace in the world, distributes resale royalties to the NFT creators [8]. According to *Tokenized*, an NFT creator earns proceeds from pri-

mary sale immediately, and whenever a NFT is resold, the creator receives his or her royalties 2-4 weeks after the transaction. The process of resale royalty distribution is summarized in Figure 1.1.



(a) NFT resale royalty on OpenSea

(b) Creator income in OpenSea

Figure 1.1: Resale royalty on OpenSea

From: Christian Heidorn 2022, *Explained: How OpenSea Royalties Really Work*, TOKENIZED, 9 May 2022, <<https://tokenizedhq.com/opensea-royalties/>>

Though there is no universally optimal royalty percentage, *Tokenized* also offered resale royalty rate suggestion, which is summarized in Figure 1.2. A high percentage of resale royalty could yield a large portion of profit from secondary sales, but may discourage resale. 5%, average resale royalty, is considered reasonable and fair, and many successful NFTs set as such. A low percentage of resale royalty can be compensated for frequent resales.



Figure 1.2: Resale royalty suggestion

From: Christian Heidorn 2022, *Explained: How OpenSea Royalties Really Work*, TOKENIZED, 9 May 2022, <<https://tokenizedhq.com/opensea-royalties/>>

Most of the existing studies on NFT deal with the applications of NFT or relation with other products such as cryptocurrencies rather than focusing on the unique characteristics of NFT. Furthermore, most of the previous studies analyzed the effect of resale royalty are experimental studies or use oversimplified models[9], [10], [11], [12]. On the other hand, this thesis focuses on the blockchain-based resale royalty, one of the innovative features of NFTs, and analyzes the economic effect of resale royalty on the NFT creator's profit using a mathematical model.

## 1.2 Research Motivation and Contribution

The research motivation and main contributions of the thesis are as follow.

- (a) Unlike previous experimental studies, this study mathematically models the economic effect of NFT resale royalty on the creator's profit, and finds the optimal resale royalty rate that maximizes the expected profit.
- (b) The thesis considers multiple related factors including the time duration considered by the creator, the short-term and the long-term expected increase rates of the NFT, and the auction winner's buyer type.
- (c) This study models the NFT resale royalty optimization process as a Stackelberg game.
- (d) This study considers various types of buyers including collectors and speculators.
- (e) The model used in this study is based on a time duration rather than a period-unit, which eliminates the limitation to the number of resales.
- (f) With the development of a blockchain, it is expected that blockchain-based resale royalty may appear in some other fields or products in the near future, and this study can be utilized to investigate the economic effects of resale royalty for those fields.

### **1.3 Organization of the Thesis**

The thesis is organized as follows. Chapter 2 reviews the related literature including auction with resale in section 2.1, NFTs in section 2.2, and resale royalty in section 2.3. Chapter 3 presents the model of resale royalty optimization. More specifically, the problem description in section 3.1, Stackelberg game modeling in section 3.2, mathematical modeling in section 3.3, NFT English auction results in section 3.4, and the subgame perfect equilibrium of the problem in section 3.5. In Chapter 4, numerical results are shown with experiment environment in section 4.1. The numerical results are presented in section 4.2, and main implications of the results in section 4.3. Finally, Chapter 5 concludes the thesis with future research directions.

# Chapter 2

## Literature Review

### 2.1 Auction with Resale

There are many previous works relating to auction with resale. Myerson [13] analyzed the optimal auction structure, but did not consider the possibility of resale. Zhoucheng Zheng [14] considered the possibility of resale in Myerson's work, but conditions such as uniform distribution are needed when there are more than three bidders. Mylovanov and Tröger [15] showed that optimal auction with resale exists even when there are more than three bidders without the assumption of uniform distribution. In addition, Gupta and Lebrun [16] studied auction with resale, but considered only two bidders. These previous studies assumed inter-bidder resale or only few bidders, whereas this thesis assumes multiple new buyers participated in repeated auctions.

Some previous studies on speculator bidders [17], [18], [19], [20] assumed that speculators have no value in the auction item. However, the model of this study assumes that even speculators appreciate the value of NFT such that the speculators have motivation to participate in the auction.

Pagnozzi [21] found that when resale is allowed, then speculators participate in the auction to earn return profit, and the entry of a speculator in the auction raises

the seller's expected revenue. From this insight, the model of the thesis considers various types of buyers, such as speculator and collector types.

Hafalir and Krishna [22] studied the first and the second-price auctions with independent private values with resale. Hafalir and Krishna [23] extended their study of [22], and analyzed the revenue and efficiency of first-price auctions with resale. However, in the studies of [22], [23], the winner makes a take-it-or-leave-it offer to the loser, whereas this thesis assumes that NFT is always traded through an English auction.

## 2.2 Non-Fungible Token

NFT was first created in 2014, but global interest in it began in 2021 [24]. Rather than focusing on the unique characteristics of NFT, many existing studies of NFT deal with the applications of NFT in other fields, relationships with other products such as cryptocurrencies, and asset pricing.

Bao and Roubaud [25] analyzed 13 papers on NFT in their review paper. Among the 13 papers, [26], [27], [28], [29], [30], [31], [32], [33] are on asset pricing of NFT, [34], [35], [36], [5] on business , and [37], [26] on financial risk [37], [26].

Chen, et al. [38] approached the repurchase process of NFT securities as a Stackelberg game to explore the equilibrium prices. Ante [39], Aharon and Demir [31], Ko, et al. [32] and Borri, et al. [40] investigated the interrelationships between NFTs and cryptocurrencies in their studies. Ante [39] showed that Bitcoin shock can trigger the increase in NFT sales, and implicated that cryptocurrency affects the growth and development of the NFT market, but not vice versa. However, Aharon and Demir [31] suggested that NFTs are independent of common assets classes and even from Ethereum, and that NFTs may have diversification benefits in portfolio investment. Ko, et al. [32] concluded that the inclusion of NFTs can have diversification effect. Borri, et al. [40] showed that NFTs have low exposures to other cryptocurrency factors and traditional asset market factors.

Nadini, et al. [41] predicted NFT resales by using simple machine learning algorithms. Pelechrinis, et al. [42] and von Wachter [43] detected an abnormal NFT transactions called wash trading. Wang, et al. [44], Chalmers, et al. [36], Raimondi, et al. [45] concluded that NFTs have an intrinsic value, but the value of NFTs is undermined by speculation, fraud, inadequate regulation, and large bubbles. Ito, et

al. [46] predicted the bubble of NFT using data, and concluded that NFTs in general have a small bubble. Maouchi [26] compared bubbles of NFT to that of cryptocurrencies and found that bubbles occur more frequently in cryptocurrencies but have a larger volume in NFT.

## 2.3 Resale Royalty

Resale royalty is the right to claim a share of the proceeds from resale. Since NFT's resale royalty does not require an intermediary and is fully guaranteed, van Haften-Schick and Whitaker [5] insisted that "resale royalty of NFT can change the structure of contracting, and that NFTs have introduced a shift in resale market norms which may have a profound financial impact for artists"[5]. Duffy [47] also argued that "NFT royalties could transform resale royalties, effecting a positive development within the intensifying digitalization of the present-day art world"[47]. In fact, many artists have long been in need of resale royalty because their works are often recognized after the first sale.

*Droit de suite* is the world's first art resale royalty introduced in France. Thus, under *droit de suite*, French fine artists have the right to receive royalty from the proceeds of any resale of their work, and *droit de suite* has influenced many other countries [47]. Studies on the need for resale royalty legislation have been done several times, especially in the United States. Reddy [48], Goetzl [49], Wirsching [50], and Mione [51] insisted the need for resale royalty in the US.

Previous studies [9], [10], [11], [12] on the effect of art resale royalty on the artist profit are experimental studies or use oversimplified models. According to Stanford [9], resale royalty may reduce creator's profit because the price of artwork will increase due to the resale royalty, and artworks without resale royalty will become more popular, which results in lower demand for artworks with resale royalty. However, [9] did not consider the potential of value increase in artworks, or the cases where resale royalty is applied without exceptions can exist, for example, resale royalty on blockchain platform. Only three factors, the time discount factor, the target

price, and the current price of artworks are considered in [9].

Wang [10] considered two types of artists: a price setter and a price taker. A price setter is a popular artist who can set the price, and a price taker is like a novice artist who accepts the offered price. The results of Wang's work showed that resale royalty increases the profits of any type of artists, but decreases the social welfare and consumer surplus. Wang considered 2-periods so resale can happen at most once, whereas this thesis impose no specific limit to the number of resales.

Sollow [11] considered 2-period model where the artist decides the optimal amount of artwork production where resale royalty exists in order to maximize the profit. However, in this thesis, only one NFT is sold throughout the model, and the model is based on time selling horizon other than the period-unit.

Tunç, et al. [52] studied the effect of NFT resale royalty on a creator's profit by using sales data. The results of Tunç, et al. (2021) implicated that the average price of initial sales was significantly lowered due to resale royalty, but the average secondary sales price was increased considerably.

Most previous studies on the effect of resale royalty did not consider some important factors such as the time duration considered by the NFT creator, NFT expected value increase rate, or various types of buyers. On the other hand, this thesis considers such crucial factors, and shows that the optimal resale royalty varies depending on those factors.

# Chapter 3

## System Model

### 3.1 Problem Description

This study assumes that NFT is traded in an English auction, where bidders increase their bids until only one bidder remains [53]. It is a loss if bidders bid at higher prices than their perceived auction item values, so bidders only bid up to their perceived values of items.

This thesis considers various types of buyers in an NFT English auction. In the first phase of the auction model, only two types of buyers who are highly likely to be the winner of auction are considered: one is called a collector type who has a high collection value, and the other is called a speculator type who has a high expected resale profit. Collectors are more interested in possessing the NFT than reselling it, whereas speculators are more interested in NFT resales than collection. In the second phase, more than two types are considered, where each type is given with distinctive coefficients of a collection value and a resale threshold.

Resale royalty can affect buyers' resale strategies and bidding prices, which may determine the NFT auction winner and the number of resales. The goal of this study is to find the optimal resale royalty rate that maximizes the creator's profit considering all these related factors.

### 3.2 Stackelberg Game Modeling

The system model of the study is illustrated in Figure 3.1. This study approaches resale royalty optimization problem as a Stackelberg game. A Stackelberg game is a sequential game where a leader takes an action first and then followers observe the leader’s action and make their own best strategies. Hence, an NFT creator, the leader, decides the resale royalty rate that maximizes his or her total expected profit, and NFT buyers, the followers of the game, observe the resale royalty rate and decide their resale strategies to maximize their resale utilities.

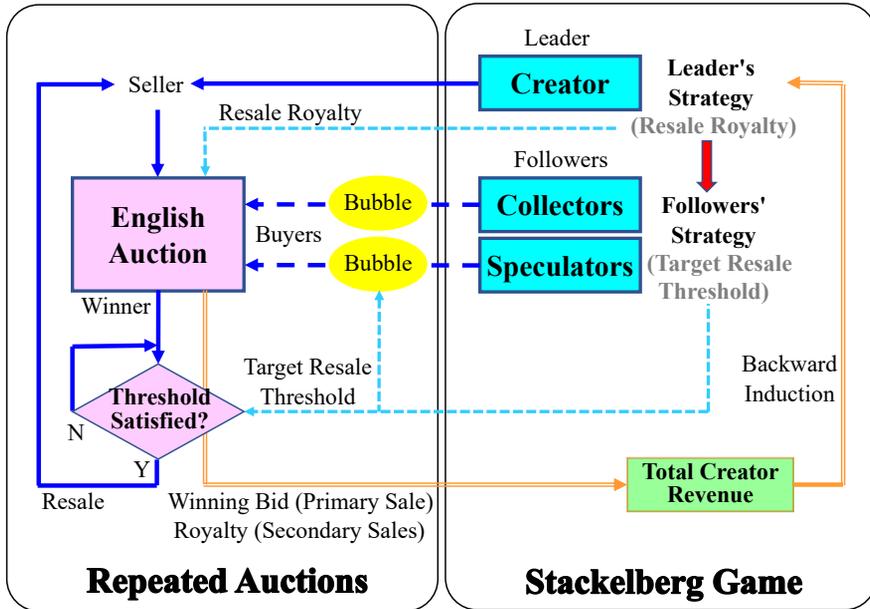


Figure 3.1: Game model of the system

The Stackelberg game of the system model is defined as  $\langle G, S, U \rangle$ , and the components of the game are as follows.

*Players*

Game players set  $G = \{0, 1, \dots, m\}$  consists of element 0 corresponding to the NFT creator and  $m$  types of NFT buyers  $1, \dots, m$ . It is assumed that there is only one NFT creator with just one NFT, and multiple buyer types.

***Strategies***

$S_0 = [0, \alpha_{\max}]$  is the set of resale royalties from which the NFT creator can choose.

$S_i, (i \in G/\{0\})$  is the set of buyers' target resale yield with  $S_i = \{x \in \mathbb{R} : x \geq \gamma_{i\min}\}$ .

***Utilities***

$u_0: Z \rightarrow \mathbb{R}$  is the expected utility of the NFT creator which includes the initial sale and the resale royalties from resale.  $u_i: Z \rightarrow \mathbb{R}$  is the expected utility of the buyer type  $i$ , calculated by subtracting the bid paid from the NFT personal commodity value with calculating the probability of winning.

The leader of the Stackelberg game determines the optimal resale royalty that maximize his or her expected profit which is calculated using backward induction considering the initial sales revenue, resale royalty income, the number of resales, and the auction winner's buyer type.

**Definition 3.1** (Backward induction). *Backward induction is the method of finding the best strategies of each participants by inferring from the result of the game to the beginning of the game [54].*

**Definition 3.2** (Subgame perfect equilibrium). *The best combination of strategies in all partial games is called subgame perfect equilibrium[54].*

**Theorem 3.3.** *The optimal strategies obtained by using backward induction of Stackelberg game is subgame perfect equilibrium[54].*

In this study, the creator’s revenue according to the resale royalty rate is illustrated in Figure 3.2. The creator considers a long period of time to earn his or her profit, and the creator decides the resale royalty rate considering the number of resales and buyers’ strategies to maximize the profit.

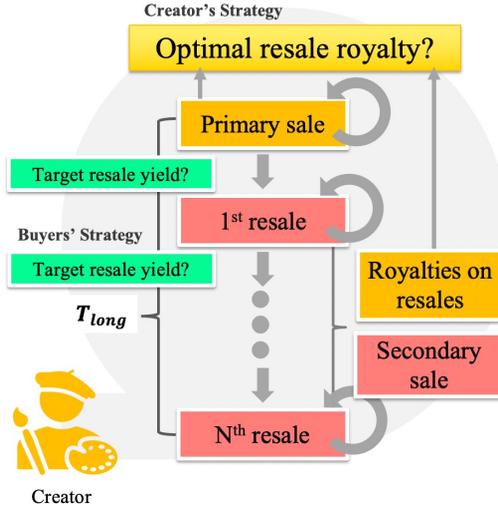


Figure 3.2: NFT creator profit with resale royalty

## Repeated English Auctions

In this study, the repeated English auctions and the Stackelberg game are closely related. This study assumes that a large number of new buyers participate in each auction. In the first auction, the NFT creator is the seller, and the creator gets the winning bid as a primary sale revenue, and in secondary sales auctions, the previous winning bidder becomes the seller if he or she wants to resell. In this model, NFT resale occurs when the previous winning bidder’s target resale yield is met and the creator earns the resale royalty.

The followers’ resale strategies affect the bubble in NFT price, and the bidder

who has the highest bubble in his or her perceived NFT value becomes the winner. The winner's resale propensity determines the number of resale occurrences and the resale royalty income. Hence, the creator, who is the leader of the Stackelberg game, needs to determine the optimal resale royalty rate that maximizes his or her expected profit by considering all these related factors using backward induction.

### 3.3 Mathematical Modeling

The definitions of terms used in this study are shown in Table 3.1, and the decision variables and input parameters are summarized in Table 3.2.

Table 3.1: Term definitions

Term	Definition
Collector	Buyer type interested in NFT collection than resale
Speculator	Buyer type interested in NFT resale profit than collection
Expected increase rate	Estimated maximum increase value rate of NFT
Commodity value	Total value within NFT
Collection value	The value of owning an NFT
Resale value	Monetary value of expected resale profit
Resale threshold	Threshold at which a buyer considers NFT resale
Target resale yield	Buyer's target resale return based on the paid bid

Table 3.2: Decision variables and input parameters

Decision variable	$\alpha$	Resale royalty set by the creator, $\alpha \in [0, \alpha_{\max}]$
Index	$t$	Index of time, $t \in [0, T_{\text{long}}]$
	$i, j$	Index of buyer types, ( $i \neq j$ )
	$n$	Index of resale numbers, $n \in [0, N]$
Input parameter	$T_{\text{long}}$	Duration considered by the creator
	$T_{\text{short}}$	Duration considered by the buyers
	$f_{\text{long}}$	Probability density function of long-term increase rate $\mathbf{R}_{\text{long}}$
	$f_{\text{short}}$	Probability density function of short-term increase rate $\mathbf{R}_{\text{short}}$
	$\alpha_{\max}$	Maximum resale royalty rate on NFT platform
	$\beta$	Transaction fee
	$\theta_i$	Density of buyer type $i$ , $\theta_i \in (0, 1)$
	$\lambda_i$	NFT collection value coefficient of buyer type $i$
	$\gamma_{i\text{min}}$	Resale threshold of buyer type $i$

An NFT creator does not pay any additional investment after minting the NFT, so this model assumes that an NFT creator considers a profit in a long period of time, denoted as  $T_{\text{long}}$ . On the other hand, NFT buyers invest their capital to purchase NFT, so it is assumed that NFT buyers consider a relatively shorter-term period than the creator, denoted as  $T_{\text{short}} (\ll T_{\text{long}})$ .

A point in time is denoted as  $t \in [0, T_{\text{long}}]$ , and the starting point as  $t_0$  ( $t_0 = 0$ ), and the  $n$ th resale point as  $t_n \in (0, T_{\text{long}}], n = 1, 2, \dots, N$ , if resale occurs  $N$  times during the whole selling horizon  $[0, T_{\text{long}}]$ .

Since an NFT is considered as a commodity with some intrinsic value, this model assumes that an NFT has a collection value, just like any other collectible items. This model assumes that each buyer type has a different NFT collection value and the collection value coefficient of buyer type  $i$  is denoted as  $\lambda_i$ .

The common NFT collection value at  $t \in [0, T_{\text{long}}]$  is denoted as  $\tilde{v}(t)$ , and it is assumed that  $\tilde{v}(t)$  is a continuous function for  $t \geq 0$ . The time discount factor is denoted as  $d(t)$  with the assumption that it is a non-increasing continuous function for  $t \geq 0$ , and the common collection value converted to the current value considering the time discount factor is denoted as  $v(t) = \tilde{v}(t)d(t)$ , where  $v(t)$  is a continuous function for  $t \in [0, T_{\text{long}}]$ . Thus, the personal collection value of buyer type  $i$  can be written as:  $\lambda_i v(t)$ .

## Expected Value Increase Rate of NFT

*Long-term expected increase rate* (LEIR), denoted as  $R_{\text{long}}$ , is defined as the value increase ratio of NFT's expected maximum value in  $[0, T_{\text{long}}]$  to  $v(0)$  and expressed

as follows.

$$R_{\text{long}} = \max_{t \in [0, T_{\text{long}}]} \frac{v(t)}{v(0)} \quad (3.1)$$

Since it is difficult to predict the future maximum value increase rate of NFT accurately, this model assumes that LEIR is a random variable denoted as  $\mathbf{R}_{\text{long}}$ .

The NFT creator chooses resale royalty rate that maximizes his or her expected profit by predicting  $\mathbf{R}_{\text{long}}$ . The probability density function (PDF) of  $\mathbf{R}_{\text{long}}$ , denoted as  $f_{\text{long}}$ , satisfies the following.

$$\int_{x=1}^{\infty} f_{\text{long}}(x) dx = 1 \quad (3.2)$$

*Short-term expected increase rate* (SEIR), denoted as  $R_{\text{short}}$ , is defined as the ratio between the expected maximum value increase rate during  $[t, t + T_{\text{short}}]$  compared to  $v(t)$ , and written as follows.

$$R_{\text{short}} = \max_{t \in [t, t + T_{\text{short}}]} \frac{v(t)}{v(t)} \quad (3.3)$$

This model assumes that SEIR is also a random variable denoted as  $\mathbf{R}_{\text{short}}$ . NFT buyers choose their resale strategies taking PDF of  $\mathbf{R}_{\text{short}}$  into account, and this model assumes that the PDF of  $\mathbf{R}_{\text{short}}$ , denoted as  $f_{\text{short}}$ , is unchanged regardless of time and satisfies the following.

$$\int_{x=1}^{\infty} f_{\text{short}}(x) dx = 1 \quad (3.4)$$

## Resale Occurrences

This model assumes that the bid price of buyer type  $i$ , denoted as  $P_i(t)$ , is

proportional to the common NFT collection value.

$$P_i(t) \propto v(t) \tag{3.5}$$

The resale royalty set by the creator is denoted as  $\alpha$ . An open NFT marketplace has resale royalty maximum limit, denoted as  $\alpha_{\max}$ . Since this model is based on a marketplace called OpenSea, resale royalty can be set from 0 to 10% in increments of 0.1% and transaction fee, denoted as  $\beta$ , is 2.5% of the sale price.

$$\begin{aligned} \alpha &\in [0, 10\%] \\ \beta &= 2.5\% \end{aligned} \tag{3.6}$$

In this model, it is assumed that NFT buyers have their individual collection values and resale thresholds, and if the expected resale revenue is not above the buyer's resale threshold, then the buyer does not resell and continues to keep the NFT. The resale threshold of buyer type  $i$ , denoted as  $\gamma_{i\min}$ , is the minimum profit from resale compared to the purchase price. Resale thresholds can vary depending on the buyer type. This model assumes that buyer chooses the target resale yield that maximizes the buyer's expected resale profit, and the target resale yield is denoted as  $\gamma(\alpha) \geq \gamma_{i\min}$ .

If  $(n - 1)$ th auction winner  $i$ 's target resale yield is given as  $\hat{\gamma}$ , i.e.,  $\gamma_i(\alpha) = \hat{\gamma} \geq \gamma_{i\min}$ , then  $n$ th resale occurs at  $t_n \in (t_{n-1}, t_{n-1} + T_{\text{short}}]$  when the following equation is satisfied.

$$P(t_{n-1}) \frac{1 + \hat{\gamma}}{1 - \alpha - \beta} \leq P(t_n) \tag{3.7}$$

Since  $T_{\text{short}}$  is assumed as a short period of time, the time discount factor during

$T_{\text{short}}$  is ignored in this model.

$$d(t + T_{\text{short}}) \approx d(t) \quad (3.8)$$

Furthermore, this model assumes that the collection value coefficient of  $(n - 1)$ th resale auction winner remains as  $\lambda_i v(t_{n-1})$  during  $[t_{n-1}, t_{n-1} + T_{\text{short}}]$ .

Resale royalty rate affects the strategies of the buyers, and the probability of resale occurrence according to the winner's resale strategy can be formulated since resale occurs when the target resale yield is met. The probability of resale occurrence according to the winner's resale strategy  $\gamma(\alpha)$  is denoted as  $\rho(\alpha, \gamma(\alpha))$ , or simply  $\rho(\alpha)$ .

If  $\mathbf{R}_{\text{short}}$  and  $\gamma_i(\alpha)$  are set as fixed values, i.e.,  $\mathbf{R}_{\text{short}} = \hat{F}_{\text{short}}$ ,  $\gamma_i(\alpha) = \hat{\gamma} \geq \gamma_{i\text{min}}$ , then  $n$ th resale occurs at time  $t_n \in (t_{n-1}, t_{n-1} + T_{\text{short}}]$  with bid price  $P(t_n)$  satisfying the following.

$$P(t_n) = P(t_{n-1}) \frac{1 + \hat{\gamma}}{1 - \alpha - \beta} \leq \hat{F}_{\text{short}} \quad (3.9)$$

$\rho(\alpha, \hat{\gamma})$  when given  $\mathbf{R}_{\text{short}} = \hat{F}_{\text{short}}$ ,  $\gamma(\alpha) = \hat{\gamma}$  is expressed as Equation (3.10) with the assumption of Equation (3.11).

$$\rho(\alpha, \hat{\gamma}) = \int_{\frac{1+\hat{\gamma}}{1-\alpha-\beta}}^{\infty} f_{\text{short}}(x) dx \quad (3.10)$$

$$\rho(\alpha, \gamma(\alpha)) < \frac{1}{1 + \gamma(\alpha)} \quad (3.11)$$

The *expected resale yield* considering resale occurrence probability is denoted as

$\Gamma(\alpha)$ , and when given fixed value of  $\hat{\gamma}$ ,  $\Gamma(\alpha)$  is written as the following equation.

$$\Gamma(\alpha) = \hat{\gamma} \times \rho(\alpha, \hat{\gamma}) \quad (3.12)$$

The buyer determines the resale target yield that generates the maximal expected resale profit, so  $\gamma(\alpha)$  is formulated as the following equation.

$$\gamma(\alpha) = \max \left\{ \gamma_{\min}, \arg \max_{\hat{\gamma}} \{ \hat{\gamma} \times \rho(\alpha, \hat{\gamma}) \} \right\} \quad (3.13)$$

### Values of NFT

This study considers many different types of values of NFT, shown in Figure 3.3. NFT buyers have different NFT collection values depending on their buyer types, and different expected resale profit values. In this model, the *commodity value* of NFT is defined as the sum of NFT collection value and the expected resale profit value.

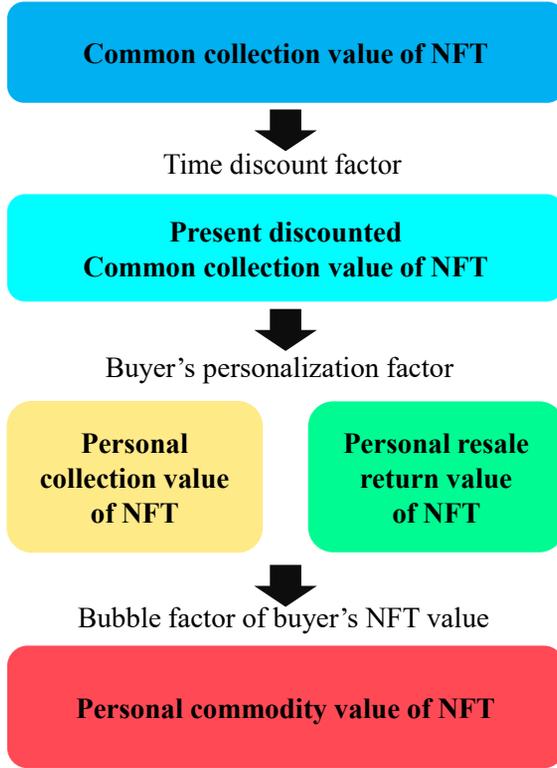


Figure 3.3: Different values of NFT

The NFT commodity value of buyer type  $i$  is denoted as  $p_i(t)$ .  $p_i(t)$  can be seen as the bubble in NFT because  $p_i(t)$  has an additional value other than the just common collection value. Therefore,  $p_i(t)$  can be expressed as

$$p_i(t) = b_i(\alpha)v(t) \quad (3.14)$$

where  $b_i(\alpha)$  is the bubble coefficient of the NFT.

$p_i(t)$  also equals to the sum of the NFT collection value and the expected resale

return value of buyer type  $i$  during  $[t, t + T_{\text{short}}]$ , so  $p_i(t)$  can be written as follows.

$$p_i(t) = b_i(\alpha)v(t) = (1 - \rho_i(\alpha))\lambda_i v(t) + \rho_i(\alpha)(1 + \gamma_i(\alpha))b_i(\alpha)v(t) \quad (3.15)$$

By organizing Equation (3.15),  $b_i(\alpha)$  can be obtained as follows.

$$b_i(\alpha) = \frac{\lambda_i(1 - \rho_i(\alpha))}{1 - \rho_i(\alpha)(1 + \gamma_i(\alpha))} \quad (3.16)$$

### 3.4 NFT English Auction Winner

This model considers various buyers types: the first phase considers only two types, and the second phase involves 8 types of buyers. This section starts with investigating two types of buyers in NFT English auctions.

This model considers collector and speculator buyer types assuming that there are only the two types in the auction. The collector type is referred to as  $c$  and the speculator as  $s$ . The resale threshold of buyer type  $i$  ( $i = c, s$ ) is denoted as  $\gamma_{i\min}$ , the collection value coefficient of  $i$  as  $\lambda_i$ , the resale occurrence probability as  $\rho_i(\alpha)$ , the NFT bubble coefficient as  $b_i(\alpha)$ , and  $p_i(t) = b_i(\alpha)v(t)$ .

Since a collector prefers to collecting the NFT, a collector does not resell unless there is a significant amount of profit. So collectors' collection value coefficient and resale threshold are greater than those of the speculators', and the relationships of the parameters can be summarized as follows.

$$\lambda_s < \lambda_c \tag{3.17}$$

$$\gamma_{s\min} < \gamma_{c\min} \tag{3.18}$$

$$p_c(t) = b_c(\alpha)v(t) \tag{3.19}$$

$$p_s(t) = b_s(\alpha)v(t) \tag{3.20}$$

In this model, the NFT creator becomes the seller of the first auction, and after that, the previous successful bidder becomes the seller. This study assumes that a large number of new buyers participate in every auction, and a buyer belongs to the collector type with probability of  $\theta_c \in (0, 1)$  and the speculator type with  $\theta_s (= 1 - \theta_c)$ . The buyer type probabilities are summarized in Table 3.3. If buyers

with the same buyer type compete, then it is assumed that the buyers have equal probability of winning the auction.

Table 3.3: Distribution of collector and speculator

Buyer Type	Probability
Collector	$\theta_c \in (0, 1)$
Speculator	$\theta_s = 1 - \theta_c \in (0, 1)$

The expected utility of the buyer type  $i(i = c, s)$  at point  $t$  is denoted as  $v_i(t)$ , and the utility of the seller as  $v_{\text{seller}}(t)$ .

### 3.4.1 2-Buyers 2-Types English Auction

This subsection examines the results of 2-buyers 2-types English auction. Since there are only two buyers, only three possible cases can be considered: the case where the two buyers belong to the collector type, the case where they belong to the speculator type, and the case where there are one collector and one speculator.

First, consider the case where a collector buyer type has a higher NFT commodity value than a speculator, i.e.,  $p_c(t) > p_s(t)$ . When there is one speculator and one collector, the collector wins with bidding price  $p_s(t)$  and has the utility of  $p_c(t) - p_s(t)$ . If there are two collectors, both bidders will bid up to  $p_c(t)$ , and the probability of winning the bid is the same for each bidder. If there are two speculators, they bid up to  $p_s(t)$  with the winning probability of 50%. The seller earns  $p_c(t)$  if there are two collectors, and  $p_s(t)$  otherwise. The expected utilities of the seller and each buyer type are summarized as follows.

$$\begin{aligned}
 v_c(t) &= \theta_c \times 0.5 \times (p_c(t) - p_c(t)) + (1 - \theta_c)(p_c(t) - p_s(t)) \\
 &= (1 - \theta_c)(p_c(t) - p_s(t)) \\
 v_s(t) &= \theta_c \times 0 + (1 - \theta_c) \times 0.5 \times (p_s(t) - p_s(t)) \\
 &= 0 \\
 v_{\text{seller}}(t) &= \theta_c^2 \times p_c(t) + ((1 - \theta_c)^2 + 2\theta_c(1 - \theta_c)) \times p_s(t) \\
 &= \theta_c^2 p_c(t) + (1 - \theta_c^2) p_s(t).
 \end{aligned} \tag{3.21}$$

When a speculator has higher commodity value, i.e.,  $p_s(t) > p_c(t)$ , the expected utilities of the seller and the two buyer types can be obtained in a similar way, and

the utilities are written as follows.

$$\begin{aligned}
v_c(t) &= \theta_c \times 0.5 \times (p_c(t) - p_c(t)) + (1 - \theta_c) \times 0 \\
&= 0 \\
v_s(t) &= \theta_c \times (p_s(t) - p_c(t)) + (1 - \theta_c) \times 0.5 \times (p_s(t) - p_s(t)) \\
&= \theta_c (p_s(t) - p_c(t)) \\
v_{\text{seller}}(t) &= (\theta_c^2 + 2\theta(1 - \theta_c)) \times p_c(t) + (1 - \theta_c)^2 \times p_s(t) \\
&= (2\theta_c - \theta_c^2)p_c(t) + (1 - \theta_c)^2 p_s(t).
\end{aligned} \tag{3.22}$$

When a collector and a speculator have the same commodity value, i.e.,  $p_c(t) = p_s(t)$ , the buyers bid up to the their NFT commodity value with the same probability of winning. The utilities in this case are summarized as follows.

$$\begin{aligned}
v_c(t) &= 0 \\
v_s(t) &= 0 \\
v_{\text{seller}}(t) &= p_c(t) = p_s(t).
\end{aligned} \tag{3.23}$$

### 3.4.2 $M$ -Buyers 2-Types English Auction

In this subsection, it is assumed that there are  $M(> 2)$  bidders in an English auction with the two buyer types. When a collector has a higher NFT commodity value, a buyer belonging to the collector type wins with bidding price  $p_s(t)$  if all other opponents are speculators. However, if there is at least one more collector, then bidders belonging to the collector type bid up to  $p_c(t)$  with the equal probability of winning. If all buyers in the auction are speculators, then a buyer gets the NFT with bidding price  $p_s(t)$  having winning probability of  $\frac{1}{M}$ . If there are two or more collectors, the seller gets  $p_c(t)$ , and  $p_s(t)$  otherwise. The utilities are summarized as follows.

$$\begin{aligned}
v_c(t) &= \sum_{M=1}^{M-1} M^{-1} C_M \theta_c^M (1 - \theta_c)^{M-1-M} \frac{1}{M+1} \times (p_c(t) - p_c(t)) \\
&\quad + (1 - \theta_c)^{M-1} \times (p_c(t) - p_s(t)) \\
&= (1 - \theta_c)^{M-1} \times (p_c(t) - p_s(t)) \\
v_s(t) &= (1 - (1 - \theta_c)^{M-1}) \times 0 + (1 - \theta_c)^{M-1} \times \frac{1}{M} \times (p_s(t) - p_s(t)) \quad (3.24) \\
&= 0 \\
v_{\text{seller}}(t) &= (1 - (1 - \theta_c)^M - M\theta_c(1 - \theta_c)^{M-1}) \times p_c(t) \\
&\quad + ((1 - \theta_c)^M + M\theta_c(1 - \theta_c)^{M-1}) \times p_s(t).
\end{aligned}$$

The case where a speculator has a higher NFT commodity value than a collector

is summarized as follows.

$$\begin{aligned}
v_c(t) &= \theta_c^{M-1} \times \frac{1}{M} \times (p_c(t) - p_c(t)) + (1 - \theta_c^{M-1}) \\
v_s(t) &= \theta_c^{M-1} \times (p_s(t) - p_c(t)) \\
&\quad + \sum_{M=1}^{M-1} {}_{M-1}C_M (1 - \theta_c)^M \theta_c^{M-1-M} \frac{1}{M+1} \times (p_s(t) - p_s(t)) \\
&= \theta_c^{M-1} (p_s(t) - p_c(t)) \\
v_{\text{seller}}(t) &= ({}_M C_M \theta_c^M + {}_M C_{M-1} \theta_c^{M-1} (1 - \theta_c)) p_c(t) \\
&\quad + (1 - {}_M C_M \theta_c^M - {}_M C_{M-1} \theta_c^{M-1} (1 - \theta_c)) p_s(t) \\
&= (\theta_c^M + M \theta_c^{M-1} (1 - \theta_c)) p_c(t) + (1 - \theta_c^M - M \theta_c^{M-1} (1 - \theta_c)) p_s(t).
\end{aligned} \tag{3.25}$$

The expected utilities when the NFT commodity values of the two types are the same is summarized as follows.

$$\begin{aligned}
v_c(t) &= 0 \\
v_s(t) &= 0 \\
v_{\text{seller}}(t) &= p_c(t) = p_s(t).
\end{aligned} \tag{3.26}$$

Suppose that the number of bidders in English auction is large enough, and the probability of belonging to the collector type,  $\theta_c$ , is not close to either 0 or 1, which infers that there are at least two bidders of each buyer type, i.e.,  $M\theta^{M-1} \approx 0$ ,  $M(1 - \theta)^{M-1} \approx 0$ . In this case, the buyer type with a higher commodity value wins.

If collectors' commodity value is higher, the utilities of participants in the auction

are written as follows.

$$\begin{aligned}
v_c(t) &= (1 - \theta_c)^{M-1} \times (p_c(t) - p_s(t)) \\
&\approx 0 \\
v_s(t) &= 0 \\
v_{\text{seller}}(t) &= (1 - (1 - \theta_c)^M - M\theta(1 - \theta_c)^{M-1}) \times p_c(t) \\
&\quad + ((1 - \theta_c)^M + M\theta_c(1 - \theta_c)^{M-1}) \times p_s(t) \\
&\approx p_c(t).
\end{aligned} \tag{3.27}$$

When the commodity value of the speculator type is higher, the utilities are written as follows.

$$\begin{aligned}
v_c(t) &= 0 \\
v_s(t) &= \theta^{M-1}(p_s(t) - p_c(t)) \\
&\approx 0 \\
v_{\text{seller}}(t) &= (\theta^M + M\theta^{M-1}(1 - \theta))p_c(t) + (1 - \theta^M - M\theta^{M-1}(1 - \theta))p_s(t) \\
&\approx p_s(t).
\end{aligned} \tag{3.28}$$

### 3.4.3 $m$ -Buyers $m$ -Types English Auction

In  $m$ -buyers  $m$ -types English auction, the buyer type with the highest NFT commodity value is denoted as  $w_1 \in \{1, 2, \dots, m\}$  and the type with the second largest commodity value as  $w_2 \in \{1, 2, \dots, m\}/w_1$ . Then the following can be obtained.

$$P(t) = p_{w_2}(t), \tag{3.29}$$

$$\gamma(\alpha) = \gamma_{w_1}(\alpha), \tag{3.30}$$

$$b(\alpha) = b_{w_2}(\alpha). \tag{3.31}$$

### 3.4.4 Auction Assumption

Unless otherwise stated, auctions in the rest of this study assume a large number of  $M$ -buyers, 2-types English auction. If the NFT commodity value of a buyer type  $i(= c, s)$  is higher than that of the type  $j \in \{c, s\}/i$ , then the following can be obtained.

$$P(t) = p_i(t), \tag{3.32}$$

$$\gamma(\alpha) = \gamma_i(\alpha), \tag{3.33}$$

$$b(\alpha) = b_i(\alpha). \tag{3.34}$$

### 3.4.5 Short-Term Expected Increase Rate

The NFT commodity value of the buyer type  $i$  is affected by the short-term expected increase rate and the resale royalty set by the creator, and in this subsection, the auction results according to the SEIR is examined.

If the SEIR is low, the resale occurrence probability and the expected resale value are low as well, i.e.,  $\rho_c(\alpha) \approx 0, \rho_s(\alpha) \approx 0$ , resulting in the bubble coefficient almost the same size as the collection value coefficient. In other words, the collector type gets the NFT. The personal bubble coefficients and personal commodity values when the SEIR is low are summarized as follows.

$$b_i(\alpha) \approx \lambda_i \quad (3.35)$$

$$p_i(t) = b_i(\alpha)v(t) \approx \lambda_i v(t), \quad (i = c, s). \quad (3.36)$$

When the SEIR is not very low, the commodity value of collectors might be still close to the collection value whereas the expected resale return value of speculators' may increase. Speculator can expect some resale profit, and the winner is determined by the resale royalty rate. The case where the SEIR is not very low is summarized as follows.

$$b_c(\alpha) \approx \lambda_c, \quad (3.37)$$

$$b_s(\alpha) \gg \lambda_s, \quad (3.38)$$

$$p_c(t) = b_c(\alpha)v(t) \approx \lambda_c v(t), \quad (3.39)$$

$$p_s(t) = b_s(\alpha)v(t) \gg \lambda_s v(t) \quad (3.40)$$

If the SEIR is high enough, then even the collector type is interested in resale return, and  $\gamma_c > \gamma_{\text{cmin}}$ . Therefore, a collector wins in the auction because a collector has higher NFT collection value along with expected resale return value as well.

$$p_c(t) = b_c(\alpha)v(t) = \frac{\lambda_c}{\lambda_s}b_s(\alpha)v(t) = \frac{\lambda_s}{\lambda_c}p_s(t) > p_s(t) \quad (3.41)$$

### 3.5 Subgame Perfect Equilibrium

The initial sale price of the NFT, denoted as  $P(t_0)(t_0 = 0)$ , can be written with the bubble coefficient as follows:  $P(0) = b(\alpha)v(0)$ . The first *resale* price is formulated as the following equation.

$$P(t_1) = b(\alpha)v(t_1) = b(\alpha)v(0) \left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^1 \quad (3.42)$$

Similarly, the  $n$ th resale price of NFT is as follows:

$$P(t_n) = b(\alpha)v(0) \left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^n \quad (3.43)$$

The number of resales during  $[0, T_{\text{long}}]$  is denoted as  $N(\alpha, \hat{F}_{\text{long}})$  when  $\mathbf{R}_{\text{long}}$  is given as a deterministic value of  $\hat{F}_{\text{long}}$ . Resale occurrences based on the system model are illustrated in Figure 3.4.

Using the intermediate value theorem, there exists an integer  $n$  that satisfies the following:  $P(t_n) = b(\alpha)v(0) \left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^n \leq b(\alpha)v(0)\hat{F}_{\text{long}}$ . Then,  $N(\alpha, \hat{F}_{\text{long}})$  is expressed as the following equation.

$$N(\alpha, \hat{F}_{\text{long}}) = \left\lfloor \frac{\log(\hat{F}_{\text{long}})}{\log(1 + \gamma(\alpha)) - \log(1 - \alpha - \beta)} \right\rfloor \quad (3.44)$$

The *expected number of resales* is the total number of resales considering  $f_{\text{long}}$  during  $[0, T_{\text{long}}]$ , denoted as  $E[N(\alpha)]$ .

$$E[N(\alpha)] = \int_1^{\infty} \frac{\log x}{\log(1 + \gamma(\alpha)) - \log(1 - \alpha - \beta)} \times f_{\text{long}}(x) dx \quad (3.45)$$

In this model, resale occurs when the target resale yield of the previous auction winner is satisfied, so the number of resales is affected by the long-term expected increase rate.

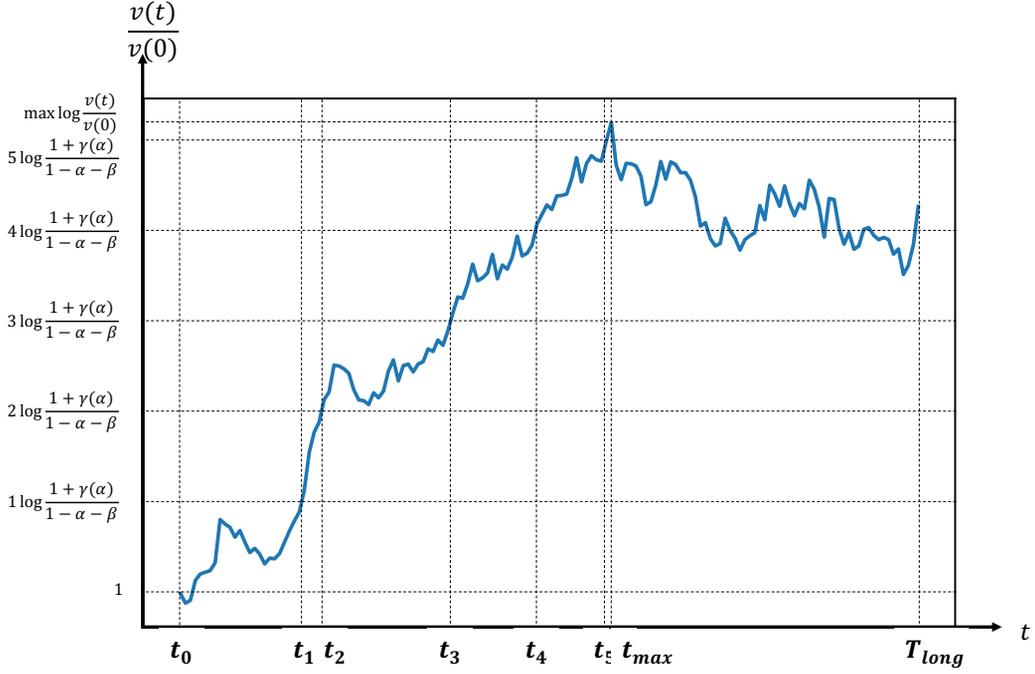


Figure 3.4: Resale occurrence

An NFT creator's total revenue is affected by resale royalty and the long-term expected increase rate. If the LEIR is given as a deterministic value of  $\hat{F}_{\text{long}}$ , then the number of resale occurrences,  $N(\alpha, \hat{F}_{\text{long}})$ , can be determined using Equation (3.44). The creator's total revenue, denoted as  $Revenue(\alpha, \hat{F}_{\text{long}})$ , is formulated as

the following equation.

$$\begin{aligned}
Revenue(\alpha, \hat{F}_{\text{long}}) &= (1 - \beta)b(\alpha)v(0) + \alpha \sum_{n=1}^{N(\alpha, \hat{F}_{\text{long}})} b(\alpha)v(0) \left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^n \\
&= (1 - \beta)b(\alpha)v(0) - \alpha b(\alpha)v(0) \\
&\quad + \alpha b(\alpha)v(0) \sum_{n=0}^{N(\alpha, \hat{F}_{\text{long}})} \left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^n \\
&= (1 - \alpha - \beta)b(\alpha)v(0) + \alpha b(\alpha)v(0) \frac{\left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right)^{N(\alpha, \hat{F}_{\text{long}}) + 1} - 1}{\left( \frac{1 + \gamma(\alpha)}{1 - \alpha - \beta} \right) - 1}
\end{aligned} \tag{3.46}$$

The creator's expected revenue during  $[0, T_{\text{long}}]$  considering  $f_{\text{long}}$ , denoted as  $E[Revenue(\alpha)]$ , is written as follows.

$$E[Revenue(\alpha)] = \int_1^{\infty} Revenue(\alpha, x) \times f_{\text{long}}(x) dx \tag{3.47}$$

### Subgame Perfect Equilibrium

The creator determines the optimal resale royalty rate, denoted as  $\alpha_{\text{opt}}$ , for the maximal expected profit by using backward induction.

$$\alpha_{\text{opt}} = \arg \max_{\alpha} E[Revenue(\alpha)] \tag{3.48}$$

Buyers observe the creator's optimal strategy,  $\alpha_{\text{opt}}$ , and by substituting  $\alpha$  with  $\alpha_{\text{opt}}$  in Equation (3.13), the optimal target resale yield, denoted as  $\gamma(\alpha_{\text{opt}})$ , can be obtained.

**Theorem 3.4** (Subgame Perfect Equilibrium).  $(\alpha_{\text{opt}}, \gamma(\alpha_{\text{opt}}))$  is subgame perfect

equilibrium of the Stackelberg game.

### Continuous Value of Resale Royalty

In this model, resale royalty can be set from  $[0, 10\%]$  with  $0.1\%$  units and thus there are 101 choices. However, if the creator can set the resale royalty rate from continuous numbers, then optimal resale royalty may not exist.

**Remark 3.1** (Optimality existence). *If resale royalty rate can be chosen from continuous level, then the optimal resale royalty that yields the creator's maximal profit may not exist.*

*Proof.* Figure 3.5 presents the creator's expected profit according to the resale royalty with  $F_{\text{short}}$  given as uniform distribution of  $[1, 1.7]$ . In this case, the optimal resale royalty rate is a rate that is little bit smaller than  $6.8\%$ , with  $F_{\text{long}}$  of uniform distributions of  $[1, 4]$ ,  $[1, 6]$ , or  $[1, 8]$ . However, there is no such value that is *little bit* smaller than  $6.8\%$  if resale royalty rate can be set from continuous level.  $\square$

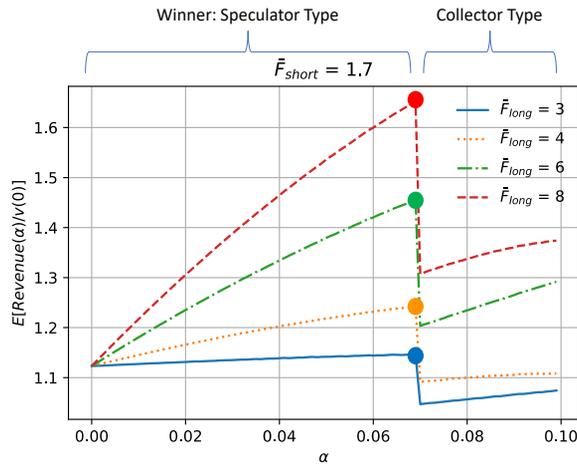


Figure 3.5: Proof figure of optimality nonexistence

## Chapter 4

### Numerical Results

This chapter examines the impact of various input parameters on the optimal decisions of the NFT creator and the buyers, and discusses the implications from numerical results. Due to the complexity of the problem, no closed-form royalty solutions to the equilibrium strategies might be obtained. Thus, this study resorts to the numerical experiments with multiple settings to obtain underlying insights of the model.

In the following sections, this thesis first deals with the input parameters of buyers' resale thresholds, coefficients of the NFT collection value, and PDFs of the short-term and the long-term expected increase rates in section 4.1. Then, section 4.2 proceeds to obtain some numerical results of  $M$ -buyers 2-types and 8-buyers 8-types NFT English auctions, and section 4.3 addresses the main indications of the numerical results.

## 4.1 Experiment Environment

This study follows the regulations of OpenSea, which implies that the NFT creator can set the resale royalty from  $[0, 10\%]$  in units of 0.1%, and the NFT marketplace receives 2.5% of the sale price as a transaction fee.

The following Table 4.1 organizes the parameters of the two buyer types in the NFT auction, the collector and the speculator. The resale threshold and coefficient of NFT collection value are given as  $(\gamma_{c\min} = 0.75$  for the collector type,  $\lambda_c = 1)$ , and  $(\gamma_{s\min} = 0.10, \lambda_s = 0.8)$  for the speculator type.

Table 4.1: Parameters of collector and speculator

Buyer type	Input parameters
Collector	$\gamma_{c\min} = 0.75, \lambda_c = 1.0$
Speculator	$\gamma_{s\min} = 0.10, \lambda_s = 0.8$

This section also shows some numerical results of the NFT English auction when every bidder belongs to a different buyer type. More specifically, this thesis considers 8 buyers with 8 different buyer types in the auction. Table 4.2 summarizes the input parameters of 8 buyer types' resale thresholds and the NFT collection value coefficients. The most speculative out of the 8 buyers is written as Type1, and the most collector type as Type8.

Table 4.2: Parameters of eight different buyer types

Buyer type	Input parameters
Type1	$\gamma_{1\min} = 0.65, \lambda_1 = 0.25$
Type2	$\gamma_{2\min} = 0.70, \lambda_2 = 0.50$
Type3	$\gamma_{3\min} = 0.75, \lambda_3 = 0.75$
Type4	$\gamma_{4\min} = 0.80, \lambda_4 = 1.00$
Type5	$\gamma_{5\min} = 0.85, \lambda_5 = 1.25$
Type6	$\gamma_{6\min} = 0.90, \lambda_6 = 1.50$
Type7	$\gamma_{7\min} = 0.95, \lambda_7 = 1.75$
Type8	$\gamma_{8\min} = 1.00, \lambda_8 = 2.00$

The PDFs of the short-term and the long-term expected increase random variable rates are given as uniform distributions of  $[1, \bar{F}_{\text{short}}]$  and  $[1, \bar{F}_{\text{long}}]$ , respectively, and these settings are summarized in the following table.

Table 4.3: Probability density function

Random variable	Probability density function
SEIR	Uniform distribution of $[1, \bar{F}_{\text{short}}]$
LEIR	Uniform distribution of $[1, \bar{F}_{\text{long}}]$

## 4.2 Numerical Results

This section examines the impact of various input parameters on the optimal strategies of the creator and the buyers. Given that the PDF of the short-term expected increase random variable rate is the uniform distribution of  $[1, \bar{F}_{\text{short}}]$ , the target resale yield of a buyer can be obtained as follows.

$$\gamma(\alpha) = \max(\gamma_{\min}, \arg \max_{\hat{\gamma}} \hat{\gamma} \frac{\bar{F}_{\text{short}} - \frac{1+\hat{\gamma}}{1-\alpha-\beta}}{\bar{F}_{\text{short}} - 1}) \quad (4.1)$$

$\arg \max_{\hat{\gamma}} \hat{\gamma} \frac{\bar{F}_{\text{short}} - \frac{1+\hat{\gamma}}{1-\alpha-\beta}}{\bar{F}_{\text{short}} - 1}$  of Equation (4.1) can be obtained by the following equations.

$$\frac{d}{d\hat{\gamma}} \left( \hat{\gamma} (\bar{F}_{\text{short}} (1 - \alpha - \beta) - 1) - \hat{\gamma}^2 \right) = 0, \quad (4.2)$$

$$\hat{\gamma} = \frac{\bar{F}_{\text{short}} (1 - \alpha - \beta) - 1}{2}. \quad (4.3)$$

From the above equations, the target resale yield of an NFT buyer is expressed as the following equation:

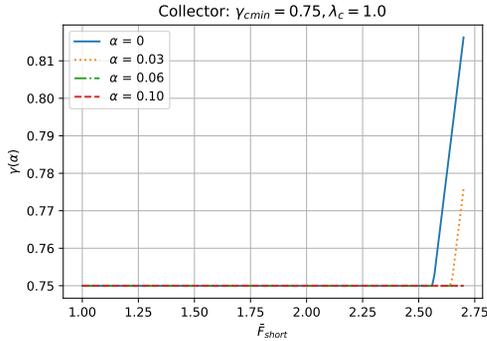
$$\gamma(\alpha) = \begin{cases} \gamma_{\min} & \text{if } \bar{F}_{\text{short}} \leq \frac{1+2\gamma_{\min}}{1-\alpha-\beta}, \\ \frac{\bar{F}_{\text{short}}(1-\alpha-\beta)-1}{2} & \text{otherwise.} \end{cases} \quad (4.4)$$

## 4.2.1 Results of *M*-Buyers 2-Types NFT English Auction

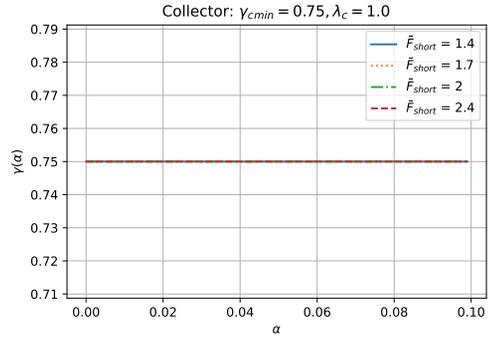
### Target Resale Yield

This subsection shows some results of NFT English auctions where speculators and collectors participate. Figure 4.1 shows numerical results of collector type's target resale yield.

A collector has a higher resale threshold than that of a speculator, i.e.,  $\gamma_{cmin} > \gamma_{smin}$ , so a collector tends to keep and not to resell the NFT unless the collector can expect a huge resale profit due to its high potential for value increase. The target resale yield of a collector is affected by the resale threshold of the collector, the short-term expected increase rate of NFT, and the resale royalty rate. It can be observed Figure 4.1 (b) that collector type's target resale yield is the same as the resale threshold unless there is a high expected increase value of NFT in the near future.



(a) Target resale yield based on SEIR



(b) Target resale yield based on resale royalty

Figure 4.1: Collector's target resale yield

Since a speculator type has a lower resale threshold and a lower collection value of NFT, the speculator tends to seek profits in the secondary market even when the

expected resale return is not large. Thus, a slight change in the short-term expected increase rate may affect the speculator's resale strategy. The resale target yield of the speculator according to the resale royalty and the SEIR is illustrated in Figure 4.2.

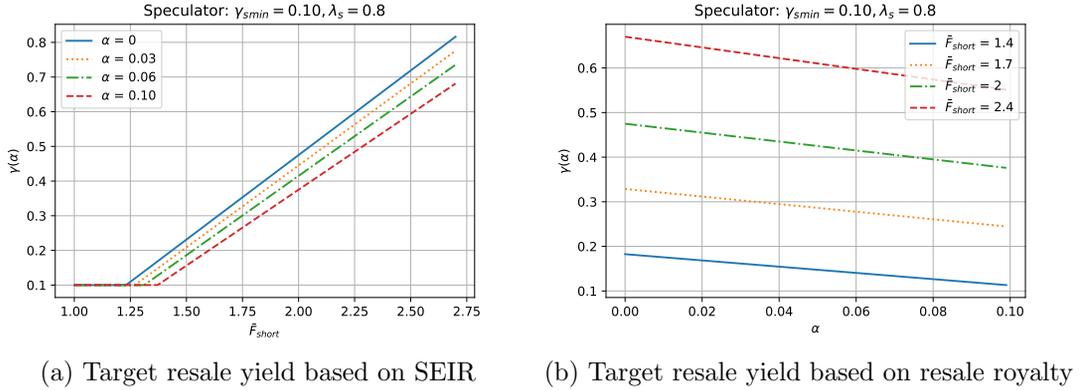
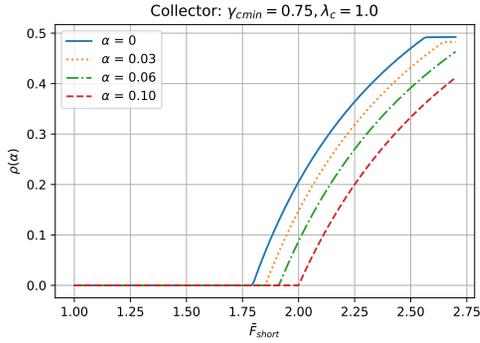


Figure 4.2: Speculator's target resale yield

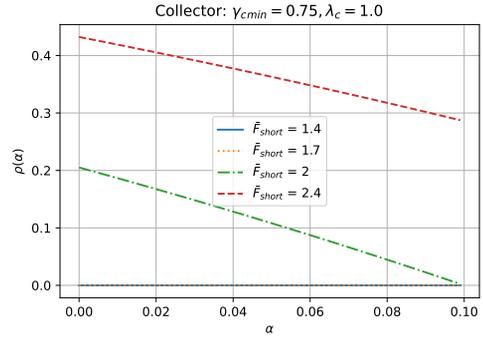
### Probability of Resale Occurrence

When  $\gamma_i(\alpha)$ , the target resale yield of a buyer of type  $i$ , is determined, then  $\rho_i(\alpha)$  ( $i = c, s$ ), probability of resale occurrence, can be obtained from Equation (3.10).  $\rho_i(\alpha)$  increases as the resale royalty decreases and as  $\bar{F}_{\text{short}}$  increases. If  $\gamma_{i\text{min}}$ , buyer type  $i$ 's resale threshold, is high and  $\bar{F}_{\text{short}}$  is low, resale may not occur, i.e.,  $\rho_i(\alpha) = \int_{\frac{1+\gamma_{i\text{min}}}{1-\alpha-\beta}}^{\infty} f_{\text{short}}(x) dx \approx 0$ , ( $i = c, s$ ).

Collectors do not resell if  $\bar{F}_{\text{short}}$  is not large enough. When  $\bar{F}_{\text{short}}$  is significantly high, the smaller the resale royalty is, the larger the possibility of the resale occurrence is. Numerical results of resale probability of resale with collector's target resale yield are shown in Figure 4.3.



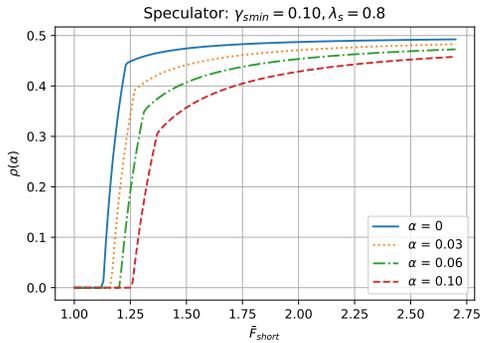
(a) Resale probability based on SEIR



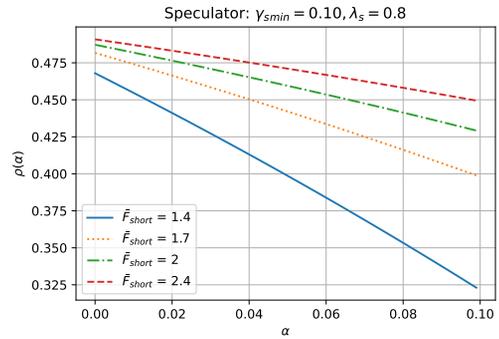
(b) Resale probability based on resale royalty

Figure 4.3: Resale occurrence probability with collector's resale strategy

For the speculator type, even a slight increase in  $\bar{F}_{\text{short}}$  could cause resale. The larger  $F_{\text{short}}$  is and the smaller the resale royalty is, the larger the probability of resale occurrence is. Numerical results of probability of resale occurrence with speculator type's resale strategy can be found in Figure 4.4.



(a) Resale probability based on SEIR



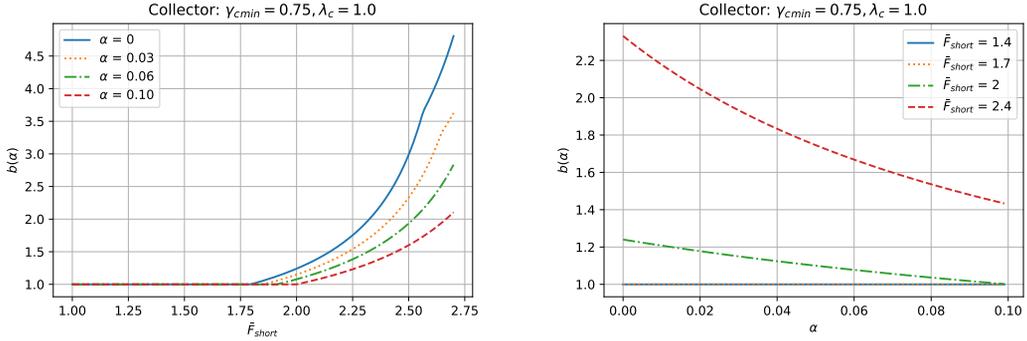
(b) Resale probability based on resale royalty

Figure 4.4: Resale occurrence probability with speculator's resale strategy

## Bubble Coefficient of NFT Commodity Value

Section 3.4 have examined that winning bid in  $M$ -buyers 2-types English auction is the same as the NFT commodity value of the buyer type with a higher NFT

bubble coefficient. The coefficient of NFT bubble increases as the buyer's expected resale profit value increases. Therefore,  $b_c(\alpha)$ , the bubble coefficient of the collector, increases as resale royalty decreases and as  $\bar{F}_{\text{short}}$  increases. Numerical results of the NFT bubble coefficient of collector type can be found in Figure 4.5.

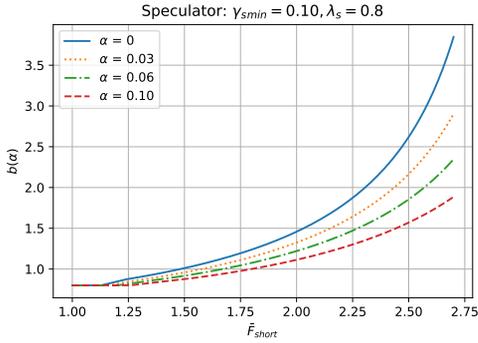


(a) Bubble coefficient based on SEIR

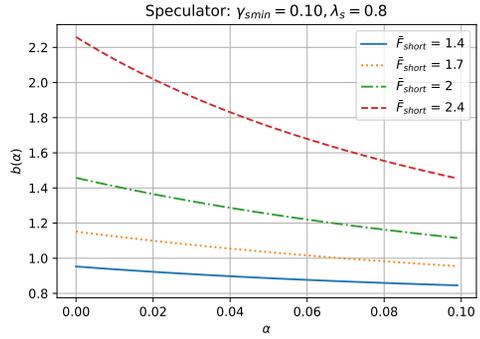
(b) Bubble coefficient based on resale royalty

Figure 4.5: NFT bubble coefficient of collector

When  $\bar{F}_{\text{short}}$  is low so that resale may not happen in the near future, then a speculator has a smaller NFT commodity value because the speculator cannot expect resale return profit and a collector has a higher NFT collection value. However, as  $\bar{F}_{\text{short}}$  increases, the expected resale profit value of the speculator type increases, which in turn enlarges the NFT bubble of the speculator. With a low rate of resale royalty, buyers earn more from the resale, so the lower the resale royalty is, the higher the expected resale profit value is. Although the bubble in NFT bid price of the collector type is also influenced by  $\bar{F}_{\text{short}}$  and the resale royalty, the speculator is much more sensitive to those factors. Numerical results of NFT bubble coefficient of the speculator type are shown in Figure 4.6.



(a) Bubble coefficient based on SEIR



(b) Bubble coefficient based on resale royalty

Figure 4.6: NFT bubble coefficient of speculator

## NFT Commodity Value

In NFT English auction with two buyer types, the buyer type with greater NFT bubble coefficient wins with the bid price same as the type's NFT commodity value, i.e.,  $P_i(t) = p_i(t) = b_i(\alpha)v(t)$ , ( $i = c, s$ ). Figure 4.7 shows the comparison of NFT bubble coefficients between the collector and speculator types.

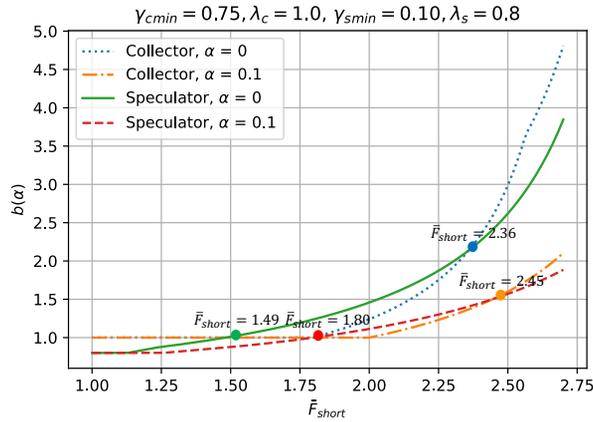


Figure 4.7: NFT bubble comparison between collector and speculator

## NFT Auction Winner's Buyer Type

The winner's buyer type based on  $\bar{F}_{\text{short}}$  and the resale royalty is summarized in Table 4.4. In the table, it can be observed that collector always wins regardless of the resale royalty rate until  $\bar{F}_{\text{short}} = 1.48$ . In  $1.49 \leq \bar{F}_{\text{short}} < 1.80$ , the resale royalty rate determines the auction winner's buyer type because a speculator's expected resale profit value can exceed a collector's collection value in NFT according to the resale royalty rate. When given  $1.8 \leq \bar{F}_{\text{short}} < 2.36$ , a speculator has a higher expected resale value than the collector's collection value regardless of  $\alpha$ , so a speculator wins invariably. With  $2.36 \leq \bar{F}_{\text{short}} < 2.45$ , a collector's expected resale return profit increases if the resale royalty rate is low, so the royalty rate affect the auction winner's type. When  $\bar{F}_{\text{short}} > 2.45$ , a collector behaves like a speculator, expecting a high resale return profit regardless of  $\alpha$ , so the collector type wins.

Table 4.4: Short-term expected increase rate and auction winner

	Collector Type $\rightarrow$ Speculator Type	Speculator Type $\rightarrow$ Collector Type
$\alpha = 0$	$\bar{F}_{\text{short}} = 1.49$	$\bar{F}_{\text{short}} = 2.36$
$\alpha = 0.1$	$\bar{F}_{\text{short}} = 1.80$	$\bar{F}_{\text{short}} = 2.45$

Referring to the results of Table 4.4, the rest of this section investigates the following five cases of  $\bar{F}_{\text{short}}$  to obtain some meaningful insights of the impact of resale royalty on the number of resales and the creator's expected profit.

- [1]  $\bar{F}_{\text{short}} = 1.4$ : the collector always wins regardless of resale royalty
- [2]  $\bar{F}_{\text{short}} = 1.7$ : the winner's buyer type is determined by resale royalty
- [3]  $\bar{F}_{\text{short}} = 2.0$ : the speculator type always wins regardless of resale royalty
- [4]  $\bar{F}_{\text{short}} = 2.4$ : the winner's buyer type is determined by resale royalty
- [5]  $\bar{F}_{\text{short}} = 2.5$ : the collector always wins regardless of resale royalty

## Number of Resale

The number of resale occurrences can be obtained by substituting  $\hat{F}_{\text{long}} = \frac{1+\bar{F}_{\text{long}}}{2}$ , the average value of  $[1, \bar{F}_{\text{long}}]$ , in Equation (3.44). Numerical results of the number of resales for the five cases are shown in Figure 4.8. In the figure, it can be observed that the number of resales is generally larger if the resale royalty rate is low or the long-term expected increase rate is high.

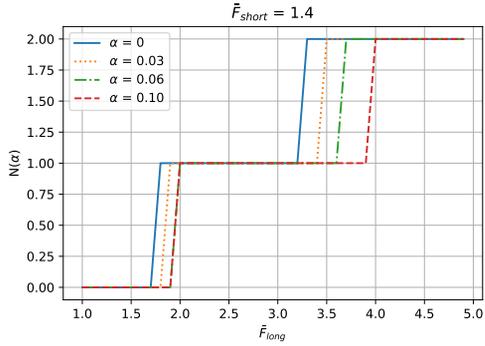
When  $\bar{F}_{\text{short}} = 1.4$ , a collector always wins, so resales occur only few times even with the high long-term expected increase rate or with no resale royalty.

When  $\bar{F}_{\text{short}} = 1.7$ , the resale royalty rate determines the winner's buyer type. If the creator sets the royalty rate high, then the collector wins, resulting in only few resales. However, when the resale royalty rate is low such as 0%, 3%, or 6%, speculators can win, causing frequent resales compared to the case of a high royalty rate.

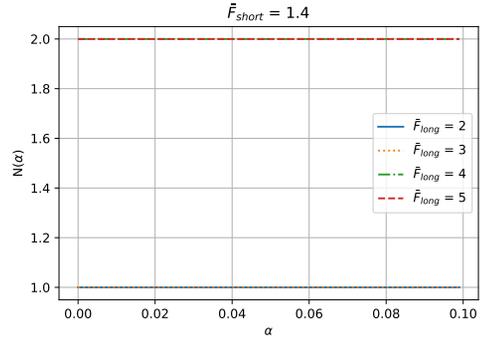
With  $\bar{F}_{\text{short}} = 2.0$ , a speculator wins regardless of the resale royalty.

When given  $\bar{F}_{\text{short}} = 2.4$ , the winner's buyer type is determined by the resale royalty rate, and it can be observed in Figure 4.8 (g) and (h) that the number of resales is affected by the resale royalty rate and the long-term expected increase rate.

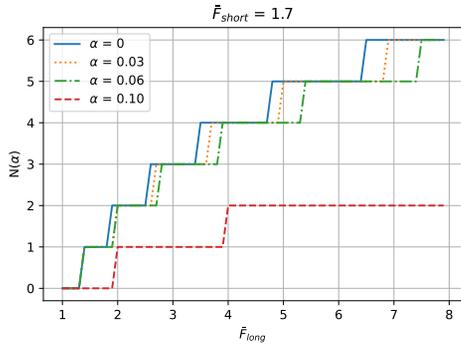
When  $\bar{F}_{\text{short}} = 2.5$ , a collector always wins regardless of the resale royalty.



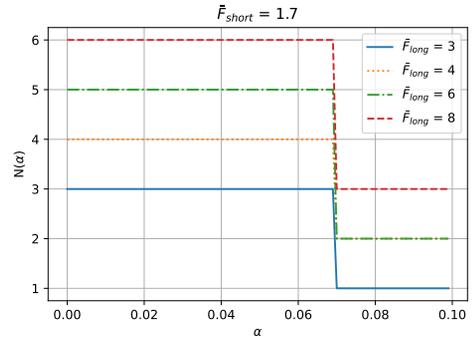
(a) Resales based on SEIR ( $\bar{F}_{short} = 1.4$ )



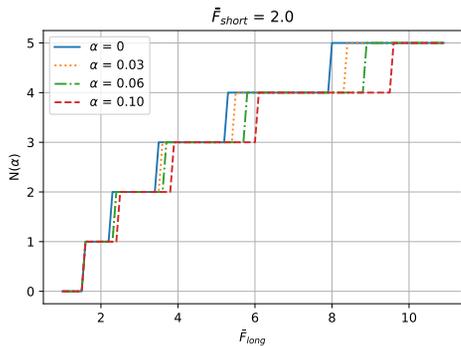
(b) Resales based on resale royalty ( $\bar{F}_{short} = 1.4$ )



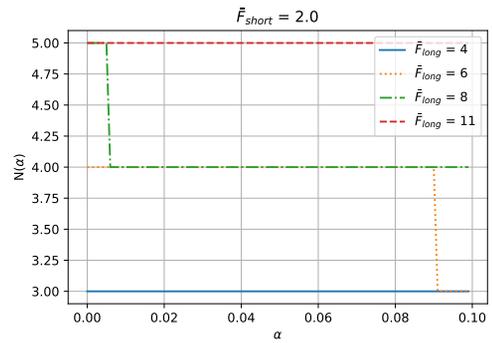
(c) Resales based on SEIR ( $\bar{F}_{short} = 1.7$ )



(d) Resales based on resale royalty ( $\bar{F}_{short} = 1.7$ )

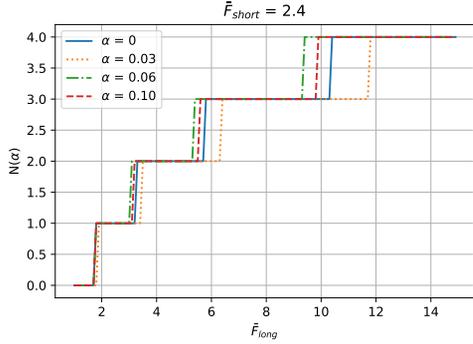


(e) Resales based on SEIR ( $\bar{F}_{short} = 2.0$ )

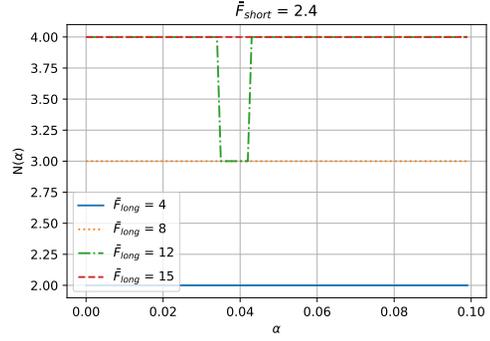


(f) Resales based on resale royalty ( $\bar{F}_{short} = 2.0$ )

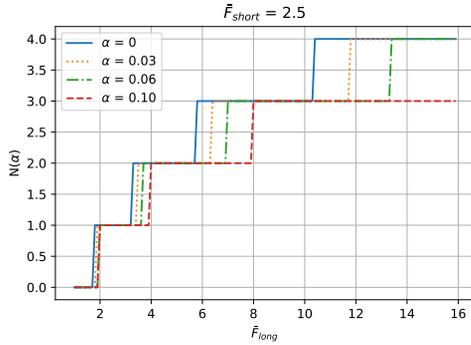
Figure 4.8: Number of resales



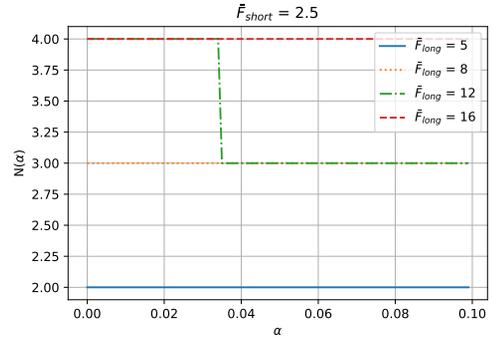
(g) Resale numbers based on SEIR  
( $\bar{F}_{short} = 2.4$ )



(h) Resale numbers based on resale royalty  
( $\bar{F}_{short} = 2.4$ )



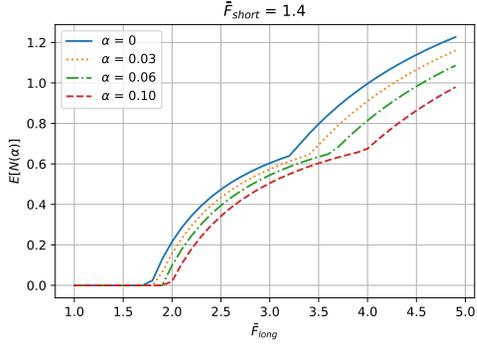
(i) Resale numbers based on SEIR  
( $\bar{F}_{short} = 2.5$ )



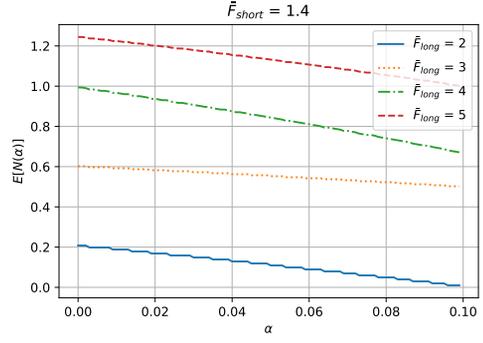
(j) Resale numbers based on resale royalty  
( $\bar{F}_{short} = 2.5$ )

## Expected Number of Resale

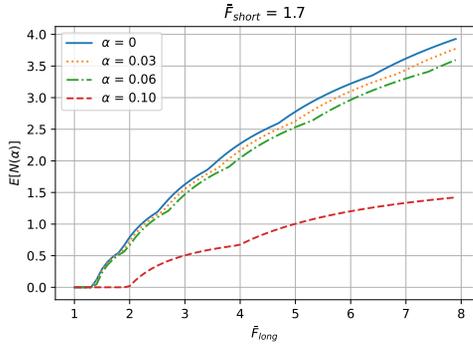
The expected number of resale occurrences during  $[0, T_{long}]$  can be obtained from Equation (3.45). Results of the expected number of resales are shown in Figure 4.9. When the resale royalty determines the winner's buyer type, such as the cases of  $\bar{F}_{short} = 1.7$  and  $\bar{F}_{short} = 2.4$ , it can be observed that discontinuous points may exist because the auction winner's buyer type can be determined by the resale royalty rate.



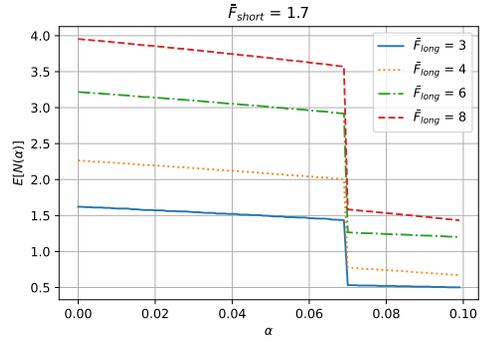
(a) Expected number of resales based on SEIR ( $\bar{F}_{short} = 1.4$ )



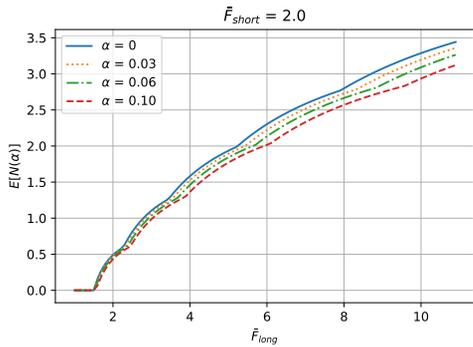
(b) Expected number of resales based on resale royalty ( $\bar{F}_{short} = 1.4$ )



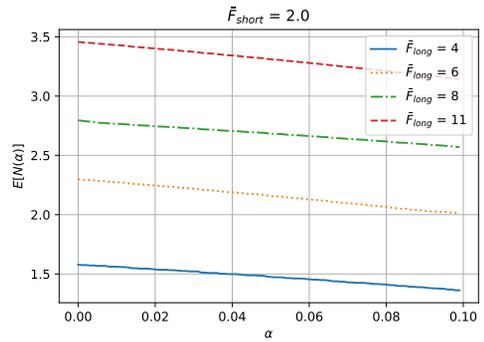
(c) Expected number of resales based on SEIR ( $\bar{F}_{short} = 1.7$ )



(d) Expected number of resales based on resale royalty ( $\bar{F}_{short} = 1.7$ )

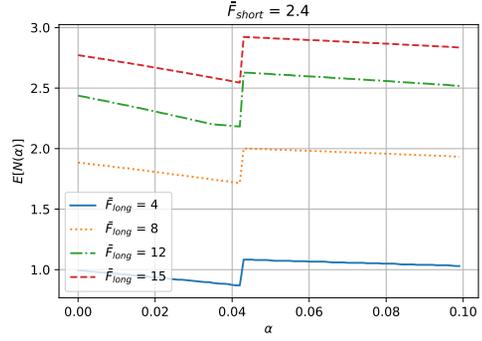
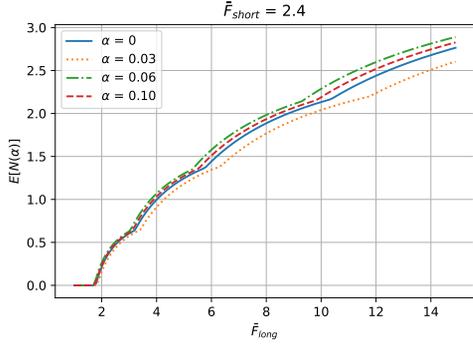


(e) Expected number of resales based on SEIR ( $\bar{F}_{short} = 2.0$ )

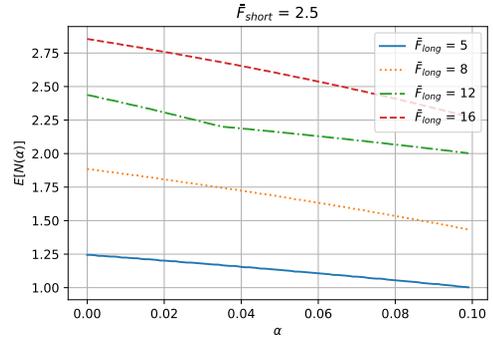
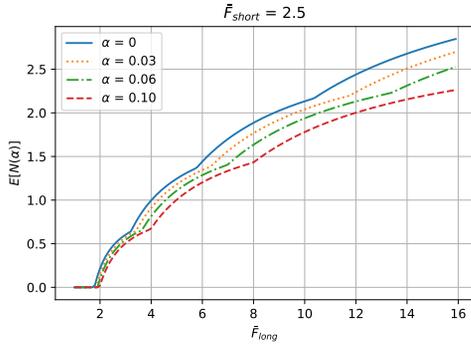


(f) Expected number of resales based on resale royalty ( $\bar{F}_{short} = 2.0$ )

Figure 4.9: Expected number of resales



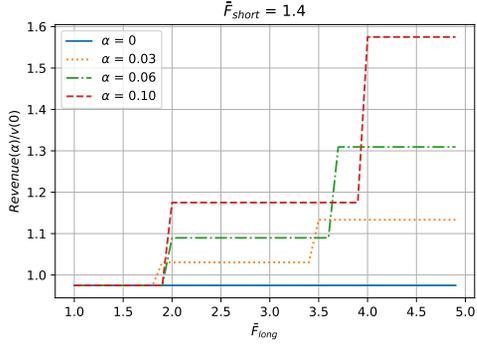
(g) Expected number of resales based on SEIR ( $\bar{F}_{short} = 2.4$ ) (h) Expected number of resales based on resale royalty ( $\bar{F}_{short} = 2.4$ )



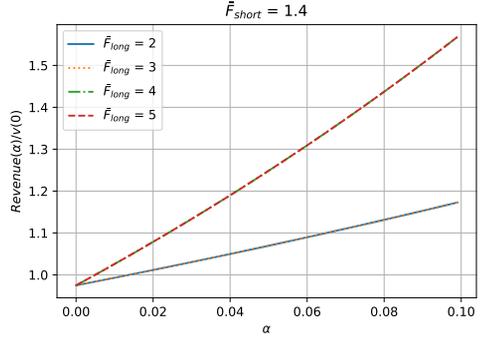
(i) Expected number of resales based on SEIR ( $\bar{F}_{short} = 2.5$ ) (j) Expected number of resales based on resale royalty ( $\bar{F}_{short} = 2.5$ )

## NFT Creator Profit

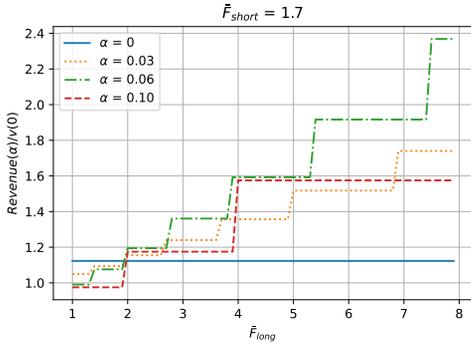
The NFT creator's profit can be obtained by Equation (3.46). The larger  $\bar{F}_{long}$  is, the greater the number of resale occurrences is, which in turn increases the creator's secondary sales revenue and ultimately the total profit of the creator  $Revenue(\alpha, \hat{F}_{long})$ . Numerical results of NFT creator profit are shown in Figure 4.10.



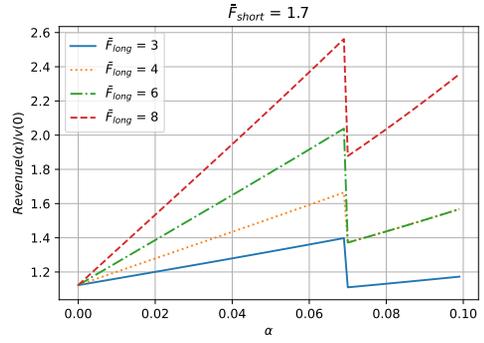
(a) Creator profit based on SEIR ( $\bar{F}_{\text{short}} = 1.4$ )



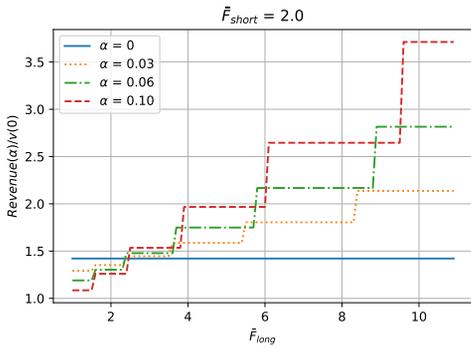
(b) Creator profit based on resale royalty ( $\bar{F}_{\text{short}} = 1.4$ )



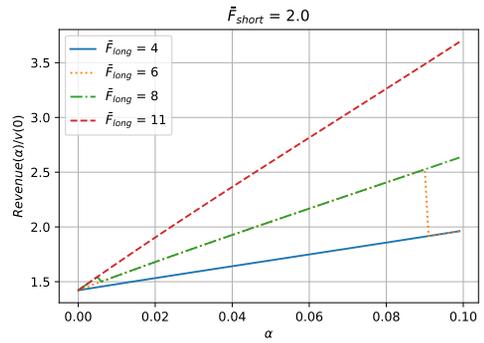
(c) Creator profit based on SEIR ( $\bar{F}_{\text{short}} = 1.7$ )



(d) Creator profit based on resale royalty ( $\bar{F}_{\text{short}} = 1.7$ )

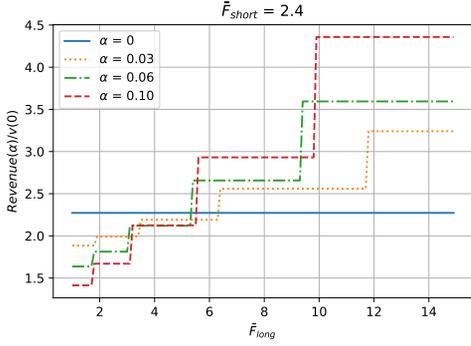


(e) Creator profit based on SEIR ( $\bar{F}_{\text{short}} = 2.0$ )

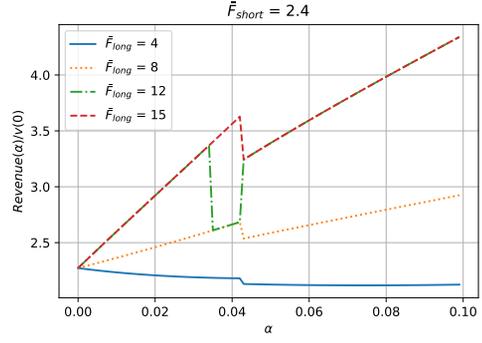


(f) Creator profit based on resale royalty ( $\bar{F}_{\text{short}} = 2.0$ )

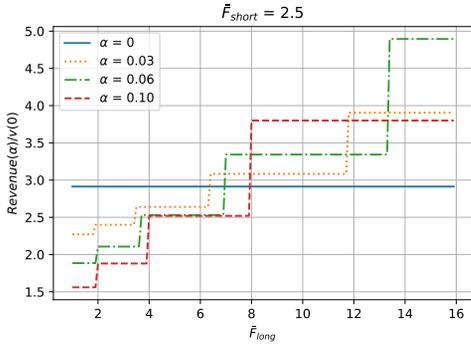
Figure 4.10: Creator profit ( $M$ -buyers 2-types auction)



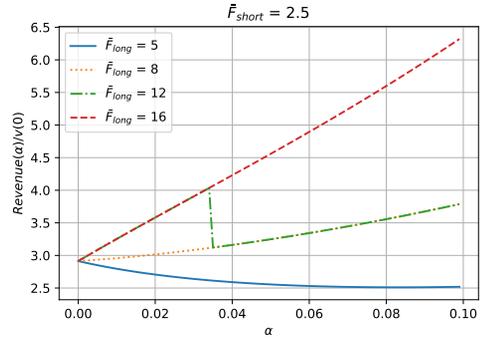
(g) Creator profit based on SEIR ( $\bar{F}_{\text{short}} = 2.4$ )



(h) Creator profit based on resale royalty ( $\bar{F}_{\text{short}} = 2.4$ )



(i) Creator profit based on SEIR ( $\bar{F}_{\text{short}} = 2.5$ )



(j) Creator profit based on resale royalty ( $\bar{F}_{\text{short}} = 2.5$ )

## Optimal Resale Royalty for the Maximal Expected Creator Profit

The goal of the thesis is to find the optimal resale royalty to maximize the total expected creator profit earned over the selling horizon. Figure 4.11 illustrates the optimal resale royalty represented as a dot with given  $\bar{F}_{\text{short}}$  and  $\bar{F}_{\text{long}}$ . The numerical results show that the optimal resale royalty rate varies depending on the various related factors such as  $\bar{F}_{\text{short}}$  or  $\bar{F}_{\text{long}}$ , and that there is no closed-form royalty solution to the equilibrium strategy.

When the size of  $\bar{F}_{\text{long}}$  is similar to that of  $\bar{F}_{\text{short}}$ , such as the case where  $\bar{F}_{\text{long}} = 2.0$  and  $\bar{F}_{\text{short}} = 1.4$ , it is recommended for the creator to set the resale royalty

low. In fact, for a such case, the optimal resale royalty rate is 5%. However, with relatively high long-term expected increase rate, i.e.,  $3.0 \leq \bar{F}_{\text{long}} \leq 5.0$ , the optimal resale royalty is 10%, as shown in Figure 4.11.

When the resale royalty determines the buyer type of the auction winner, for example,  $\bar{F}_{\text{short}}$  is 1.7, the creator should determine the appropriate royalty rate so that a royalty-sensitive speculator can win to yield the maximal expected profit. It can be observed in Figure 4.11 (b) that when a collector wins, the expected profit of the creator is relatively small compared to the case in which a speculator wins, because resale occurs rarely, resulting in a small resale royalty income and a small total expected profit.

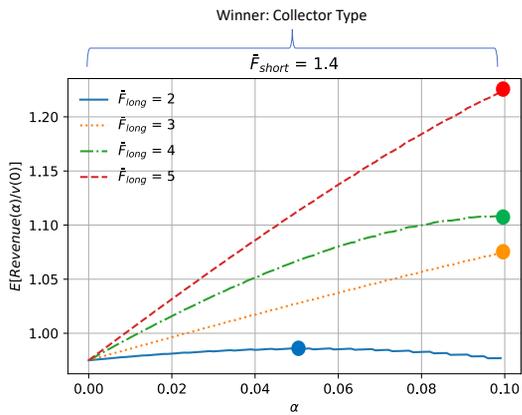
When the speculator type always wins regardless of the resale royalty rate such as the case of  $\bar{F}_{\text{short}} = 2.0$ , it is best to set the royalty rate as 10%, except for the case when  $\bar{F}_{\text{long}} = 4$ . However, if  $\bar{F}_{\text{long}}$  is not large compared to  $\bar{F}_{\text{short}}$ , resale is less likely to occur and the creator should earn the most from the primary sale by setting the resale royalty rate as 0%.

When  $\bar{F}_{\text{short}} = 2.4$ , both collectors and speculators react sensitively to the resale royalty rate. At this time, lowering the resale royalty increases a chance that a collector wins because a collector also considers resale return if the resale royalty is low. So, in this case, the creator needs to set an appropriate rate so that speculator type can win.

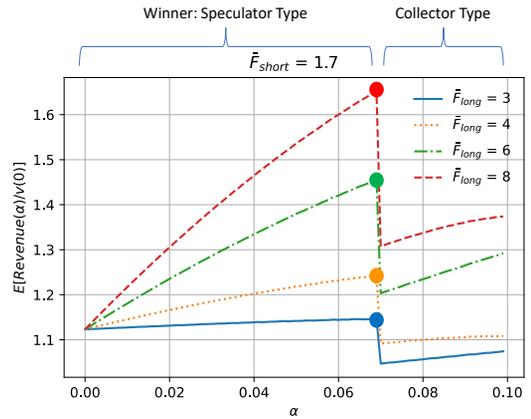
When  $\bar{F}_{\text{short}}$  is very high, i.e.,  $\bar{F}_{\text{short}} = 2.5$ , then even collectors expect high resale return regardless of the resale royalty rate.

From the numerical results of the optimal resale royalty rate, it can be concluded that resale royalty can affect the creator's profit significantly, especially when the

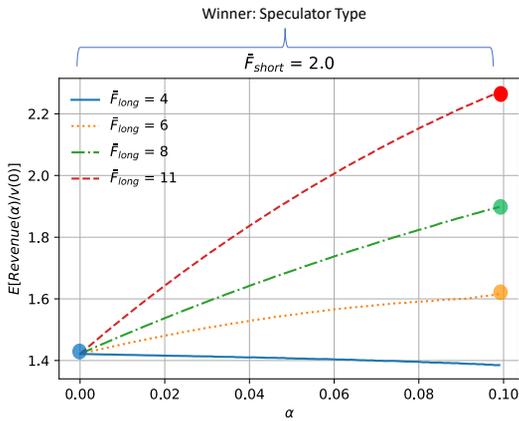
various types of buyers compete and the winner's buyer type is determined according to the royalty rate. Thus, the creator needs to set the optimal percentage of royalty to maximize one's expected profit considering various related factors.



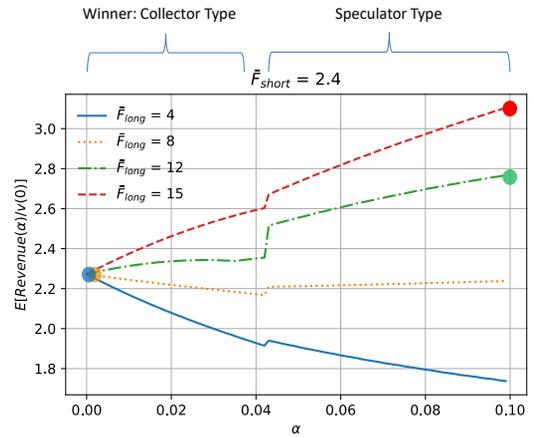
(a) Expected creator profit ( $\bar{F}_{short} = 1.4$ )



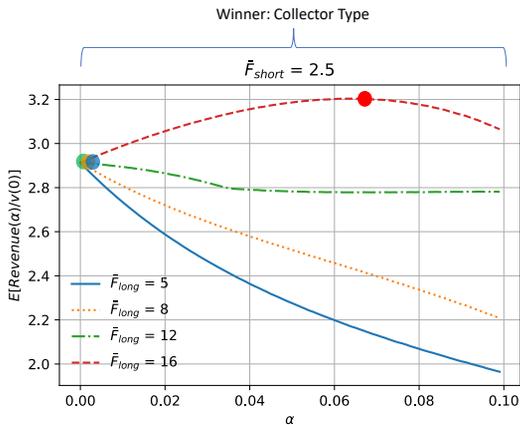
(b) Expected creator profit ( $\bar{F}_{short} = 1.7$ )



(c) Expected creator profit ( $\bar{F}_{short} = 2.0$ )



(d) Expected creator profit ( $\bar{F}_{short} = 2.4$ )



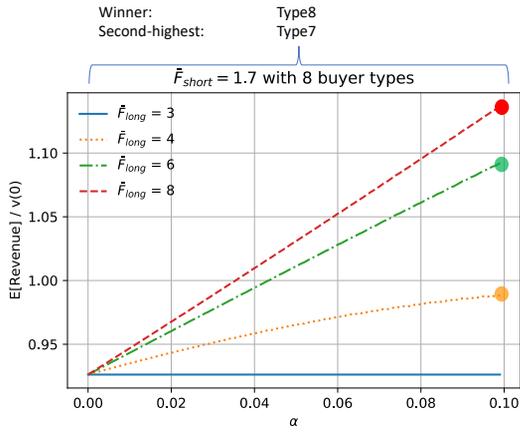
(e) Expected creator profit ( $\bar{F}_{short} = 2.5$ )

Figure 4.11: Optimal resale royalty ( $M$ -buyers 2-types auction)

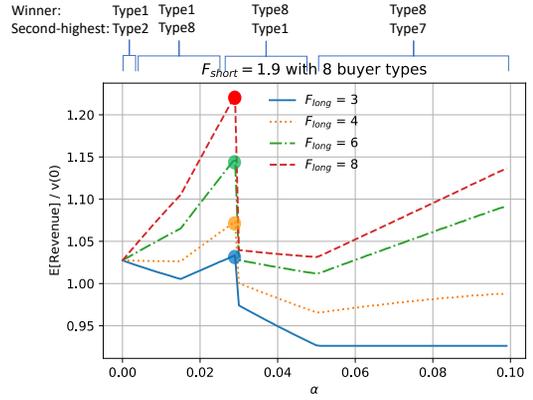
## 4.2.2 Results of 8-Buyers 8-Types NFT English Auction

This subsection investigates numerical results of an NFT English auction where 8 buyers belonging to all different buyer types participate. Each buyer has different NFT collection value  $\lambda$  and resale threshold  $\gamma_{\min}$ , and the input parameters of 8 buyer types are summarized in Table 4.2. The optimal resale royalty in 8 buyers 8 buyer types NFT English auction can be observed in Figure 4.12.

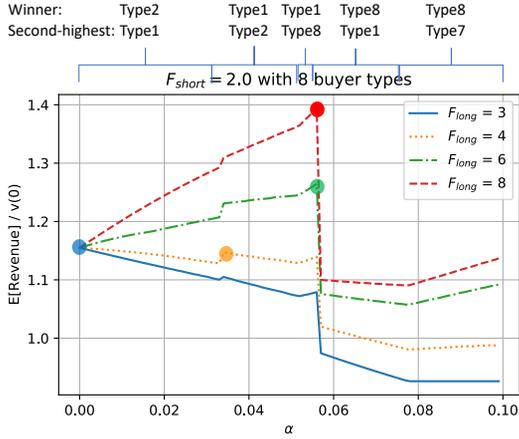
It can be shown in Figure 4.12 that especially when there are various buyer types, it is important for the creator to set an appropriate resale royalty.



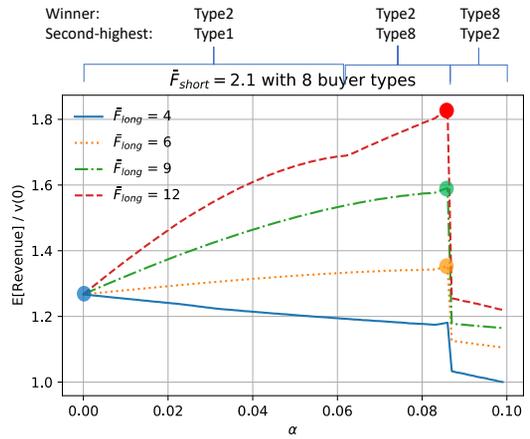
(a) Expected creator profit ( $\bar{F}_{short} = 1.7$ )



(b) Expected creator profit ( $\bar{F}_{short} = 1.9$ )



(c) Expected creator profit ( $\bar{F}_{short} = 2.0$ )



(d) Expected creator profit ( $\bar{F}_{short} = 2.1$ )

Figure 4.12: Optimal resale royalty (8-buyers 8-types auction)

### 4.3 Discussion

This section discusses the main implications of the model from the numerical results in section 4.2. More specifically, this study first investigates the effect of resale royalty on the target resale yield of an NFT buyer, the price bubble in NFT, and the number of resales, then examines some cases when it is best to set the resale royalty rate as the two extremes 0%, 10%, and some other value between them.

#### Target Resale Yield

If the value of NFT is expected to rise in the near future and an NFT buyer is willing to resell the NFT if there is a significant resale profit, then the target resale yield of the buyer is affected by the short-term expected increase value rate of NFT and the resale royalty rate. A buyer expects a higher profit from the NFT resale as the SEIR increases and the resale royalty decreases. However, if an NFT buyer has a high collection value in NFT, then the resale threshold of the buyer remains as the minimum resale threshold, regardless of the value of the short-term expected increase rate or the resale royalty.

#### Price Bubble in NFT

Ito, et al. (2022) empirically found that NFTs in general have a small bubble but did not explain why they have the price bubble in the first place [46]. The numerical results of this study showed that the NFT price bubble becomes larger if a buyer has a higher collection value than the commonly regarded collection value, or if a buyer expects a high resale return profit according to the short-term expected increase rate.

## Number of NFT Resales

Since the NFT creator receives the royalties from resale, it is appeared that the number of resale occurrences has a huge impact on the creator's profit. Resale occurrence is affected by the short-term (SEIR) and the long-term increase rates (LEIR) of the NFT. If the difference between the SEIR and the LEIR is large, then the number of resales increases. In addition, a low resale royalty rate can lead to frequent resales. When a collector acquires the NFT, there is a possibility that resale may not occur at all because the minimum resale threshold NFT may be high. Resales may not occur if buyers do not value resale profits in the distant future.

$$\hat{t} \gg t_0,$$

$$d(\hat{t}) \approx 0,$$

$$v(\hat{t}) = d(\hat{t})\tilde{v}(\hat{t}) \approx 0.$$

## Optimal Resale Royalty Rate: $\alpha = 0\%$

There are some cases when the NFT creator cannot benefit from royalties because resale is unlikely to happen. These cases include the following: (1) the case where the SEIR or the LEIR is low so buyers cannot expect a resale profit (2) the case where the SEIR is similar to the LEIR (3) the case where the auction winner's resale threshold is very high (4) the case where the time duration considered by the creator and the buyers are similar, i.e.,  $T_{\text{long}} \approx T_{\text{short}}$ . In such cases, it is recommended for the creator to set the resale royalty rate as zero, in order to maximize the initial sale revenue.

**Optimal Resale Royalty Rate:  $\alpha = \alpha_{\max}$** 

High resale royalty rate discourages resale occurrences. However, there are cases where the creator needs to set the resale royalty rate as high as possible. The cases where the creator can earn the most by setting the resale royalty as the maximum are as follows: (1) the case where the difference between the SEIR and the LEIR is large (2) the case where speculators always win regardless of the resale royalty rate (3) the case where the expected increase rate of NFT in the near future is very high.

**Optimal Resale Royalty Rate:  $\alpha \in (0, \alpha_{\max})$** 

There are many cases where it is important to set an appropriate resale royalty rate in the middle of  $(0, \alpha_{\max})$  to maximize the creator's profit. One important case is that different types of buyers compete and the winning buyer type depends on the resale royalty. It is also important for the creator to set an appropriate rate even when one buyer type always wins regardless of the resale royalty rate. Such cases can be found in Figure 4.11 (a) and (e), where the collector type always wins regardless of the long-term expected increase rate or the resale royalty rate.

## Chapter 5

### Conclusion

This thesis investigates the effect of resale royalty on the expected profit of an NFT creator. This study models the NFT resale royalty optimization process as a Stackelberg game, and finds the subgame perfect equilibria under several settings. The numerical results of the study show that the optimal resale royalty can vary depending on the time duration considered by the creator, the auction winner's buyer type, or the short-term and the long-term expected increase value rates of NFT. The results of the study imply that the price bubbles in NFT increase when the buyers can expect some significant resale returns. This study concludes that by considering various types of buyers in NFT English auctions, it is important to set an appropriate resale royalty rate so that a particular buyer type can win to maximize the expected profit of the creator.

Further studies may be conducted to generalize the model of the study in the future. In this study, it is assumed that the probability density function (PDF) of the short-term expected increase rate of NFT is always the same regardless of time, but the PDF changes with time can be considered. In addition, since the utility function or PDFs of the short-term and long-term expected increase rates are in the form of simple functions, some other PDFs that reflect the volatility of the NFT

market can be considered.

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## 국문초록

대체불가능토큰(NFT)은 블록체인을 통해 거래 내역이 검증된 고유한 디지털 아이টে을 의미한다. 일반 상품과 차별화되는 NFT 특징 중 하나는 NFT 제작자는 NFT의 블록체인 기반 계약으로 인해 NFT가 재판매될 때마다 재판매 로열티를 지급받는다라는 점이다. 본 논문은 NFT가 영국식 경매로 거래될 때, NFT의 재판매 로열티가 NFT 제작자의 총 기대 이익에 끼치는 영향을 분석한다. 본 연구는 재판매 로열티가 NFT 구매자의 재판매 전략, NFT 경매 낙찰가의 거품, NFT 재판매 횟수 등에 영향을 끼칠 수 있음을 고려하며, 제작자의 총 기대 이익을 극대화하는 최적의 재판매 로열티를 찾는다. 본 연구는 NFT 제작자와 NFT 구매자들이 각자의 최적의 전략을 세우는 과정을 Stackelberg 게임으로 접근한다. 본 연구에서는 NFT 제작자가 고려하는 기간, NFT의 단기 및 장기 예상가치상승률, NFT 낙찰자의 구매자 유형 등에 따라 최대의 이익을 내는 재판매 로열티가 달라지는 양상을 수리적 결과를 통해 설명한다. 본 논문은 수집가와 투기꾼 유형을 포함한 다양한 구매자 유형을 고려하며, 특히 수집가와 투기꾼 유형이 경합을 벌이는 경우에는 제작자의 기대 이익을 극대화하기 위해서는 투기꾼 유형이 낙찰되도록 재판매 로열티를 설정하는 것이 필요함을 보인다. 투기꾼 유형이 낙찰되어야 재판매가 많이 일어나 재판매 로열티 수익이 높아져 제작자의 총 기대 이익이 극대화되므로, NFT 제작자는 재판매 로열티에 민감한 투기꾼이 경매의 승자가 되도록 적절한 재판매 로열티를 설정하는 것이 중요할 수 있다.

**주요어:** 대체불가능토큰(NFT), 재판매 로열티, 영국식 경매, 슈타켈버그 게임

**학번:** 2020-23694