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3차원 원형 브레이드 유리 섬유 강화 복합 재료를 이용한 3점 굽힘 실험의 탄소성 해석

류한선 · 김지훈 · 이명규[1] · 김돈건 · 이형림 · 정관수† · 윤재륜 · 강태진
서울대학교 재료공학부, [1]Department of Materials Science and Engineering, Ohio State University, U.S.A
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Elastic-Plastic Analysis of Three Point Bending Tests for 3D Circular Braided Glass Fiber Reinforced Composites

Hansun Ryou, Ji Hoon Kim, Myoung-Gyu Lee[1], Dongun Kim, Hyung Rim Lee, Kwansoo Chung†, Jae Ryoun Youn and Tae Jin Kang
School of Materials Science and Engineering, Seoul National University, Seoul 151-744, Korea
[1]Department of Materials Science and Engineering, Ohio State University, U.S.A
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Abstract: In order to describe the mechanical behavior of highly anisotropic and asymmetric materials such as fiber reinforced composites, the elastic-plastic constitutive equations were used based on the recently developed yield criterion and hardening laws. As for the yield criterion, modified Drucker-Prager yield surface was used to represent the orthotropic and bi-modular (asymmetric) properties of composite materials, while the anisotropic evolution of back-stress was used to account for the hardening behavior. Experimental procedures to obtain the material parameters of the hardening laws and yield surface are presented for 3D braided glass fiber reinforced composites. For verification purposes, finite element simulation results based on the proposed constitutive laws have been compared with measurements for the three point bending tests.

Keywords: *3D circular braided glass fiber reinforced composites, modified Drucker-Prager yield criterion, bi-modular, elastic-plastic constitutive equations*

1. Introduction

Fiber reinforced composite materials are widely used for various industrial applications to take advantage of their good durability, light weight and good processibility. Recently, composite materials are applicable to various mechanical components that are in the shape of beams, plates and shells, which are subjected to various ranges of stresses, temperatures and loading conditions. Among those composite materials, laminated composite structures have been widely used where the in-plane properties are important. However, the laminate composite materials show low resistance to delamination through the thickness. To make up the through-thickness drawback of the laminated composites, three-dimensional braided composites have been recently developed. Contrary to the laminated composites, the 3D braided composites have better out-of-plane stiffness and strength with their 3D structures.

Many attempts have been made to characterize the mechanical properties of the 3D braided composites, but mainly based on the linear anisotropic elasticity [1-3]. Experimental studies under static loading however confirm that fiber reinforced composites show plastic behavior in addition to elastic behavior [4,5]. Although significant progresses have been made over the years, little work has been done to implement sophisticated material behavior for structural applications.

In general, fiber reinforced composites show strong directional difference (anisotropy) and also the different constitutive behavior between tension and compression (bi-modular property or asymmetry) [6]. The constitutive equation considered in this paper describes the elastic-plastic behavior with both the anisotropic and asymmetric properties under the plane stress condition. For the anisotropy of the fiber reinforced composites, both the initial anisotropic yielding and the anisotropic hardening have been considered here. As for the initial anisotropic yielding (and also for asymmetry), the isotropic Drucker-Prager yield criterion has been modified [7]. As for the anisotropic hardening, the anisotropic back stress evolution rules based on the kinematic hardening law have been utilized [8]. In the current work, the asymmetric as well as anisotropic properties of 3D braided glass fiber reinforced composite materials have been experimentally measured and applied for calculations. The constitutive laws have been

† Correspondence to Kwansoo Chung (kchung@snu.ac.kr)

incorporated into the general purpose finite element program ABAQUS/STANDARD using the user subroutine UMAT [9] and three point bending tests have been performed to verify the simulation results.

2. Constitutive Laws

In the elastic-plasticity theory, the strain increment is assumed linearly decoupled into elastic and plastic parts as

$$d\boldsymbol{\varepsilon} = d\boldsymbol{\varepsilon}^e + d\boldsymbol{\varepsilon}^p \tag{1}$$

where the Cauchy stress increment is related to the elastic strain increment by the elastic stiffness coefficient as

$$d\boldsymbol{\sigma} = \mathbf{C}^e_{\mathrm{T\ or\ C}} d\boldsymbol{\varepsilon}^e \tag{2}$$

or, in the matrix form,

$$\begin{pmatrix} d\sigma_x \\ d\sigma_y \\ d\sigma_{xy} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix}_{\mathrm{T\ or\ C}} \begin{pmatrix} d\varepsilon_x^{\ e} \\ d\varepsilon_y^{\ e} \\ 2d\varepsilon_{xy}^{\ e} \end{pmatrix} \tag{3}$$

in the plane stress condition. Here, the subscripts 'x' and 'y' stand for the axial and transverse directions of fiber reinforced composites, respectively, while the subscripts 'T' and 'C' mean the material properties for the tension and compression behavior, respectively. The reduced stiffness components Q_{ij} are defined as

$$Q_{11} = \frac{E_x}{1-\nu_x\nu_y},\quad Q_{12}(=Q_{21}) = \frac{\nu_y E_x}{1-\nu_x\nu_y} = \frac{\nu_x E_y}{1-\nu_x\nu_y}$$

$$Q_{22} = \frac{E_y}{1-\nu_x\nu_y},\quad Q_{33} = G \tag{4}$$

where E_x, E_y, ν_x, ν_y and G are Young's moduli and Poisson's ratios in the axial and transverse directions and the shear modulus, respectively.

The modified Drucker-Prager yield criterion was developed to describe the anisotropy and asymmetry of composite materials during the plastic deformation [7]:

$$\begin{aligned}\Phi &= p[\hat{\sigma}_x^2 - \beta_{22}\hat{\sigma}_x\hat{\sigma}_y + \beta_{22}^2\hat{\sigma}_y^2 + 3\beta_{33}^2\hat{\sigma}_{xy}^2]^{1/2} \\ &\quad + q(\hat{\sigma}_x + \kappa\hat{\sigma}_x) - \bar{\sigma}_{iso} = 0\end{aligned} \tag{5}$$

while

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - \boldsymbol{\alpha} \tag{6}$$

Here, $\boldsymbol{\sigma}$ is the Cauchy stress, $\boldsymbol{\alpha}$ is the back stress, $\bar{\sigma}_{iso}$ is the size of the yield surface, p, q, β_{22}, κ and β_{33} are material constants characterizing the anisotropic and asymmetric behavior. The modified Drucker-Prager yield criterion can describe different values of tensile yield stresses in two directions (anisotropy) and different values of tensile and compressive yield stresses (asymmetry). Also, the shear yield stress can be given independently.

In the isotropic-kinematic hardening law, the effective quantities are defined considering the following modified plastic work equivalence principle; i.e.,

$$dw_{iso} = (\boldsymbol{\sigma} - \boldsymbol{\alpha}) \cdot d\boldsymbol{\varepsilon}^p = \bar{\sigma}_{iso} d\bar{\varepsilon} \tag{7}$$

where $d\bar{\varepsilon}$ is the effective plastic strain increment. In order to account for the directional difference in the hardening behavior of the fiber reinforced composites, the original Chaboche isotropic evolution rule [10] was modified to give the following anisotropic back stress evolution rule [8]: the anisotropic kinematic hardening,

$$d\boldsymbol{\alpha} = \boldsymbol{\Gamma}_1 \cdot \frac{(\boldsymbol{\sigma} - \boldsymbol{\alpha})}{\bar{\sigma}_{iso}} d\bar{\varepsilon} - \boldsymbol{\Gamma}_2 \cdot \boldsymbol{\alpha} d\bar{\varepsilon} \tag{8}$$

where $\boldsymbol{\Gamma}_1$ and $\boldsymbol{\Gamma}_2$ are the fourth order tensors representing the anisotropic hardening behavior.

The developed elastic-plastic constitutive law was implemented into the general purpose finite element program ABAQUS/STANDARD using the material user subroutine UMAT. To update the stress increment which involves solving a non-linear equation, the Newton-Raphson method based on the incremental deformation theory was utilized [11].

3. Experiments

The preform of 3D braided glass fiber reinforced composites was made by 3D braiding machine with 2014 carriers and 104 pistons and by 4 step cycle movements. The fabricated preform by the circular braiding process is shown in *Figure* 1. By using 3D circular braided composite preform fabricated, RTM (resin transfer molding) process was performed [12]. As a resin curing agent during RTM process, epoxy was used. After 10 hours' injection of the epoxy resin into RTM cast and curing in oven at 130 ^{o}C for 120 min, 3D circular braided composites were prepared.

The tensile and compressive tests of 3D braided composites were carried out by the standard procedures, ASTM D3039-

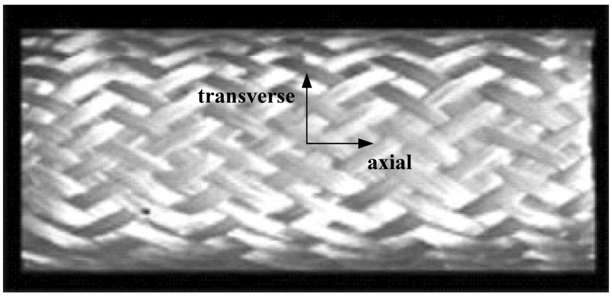

Figure 1. The 3D circular braided glass fiber preform.

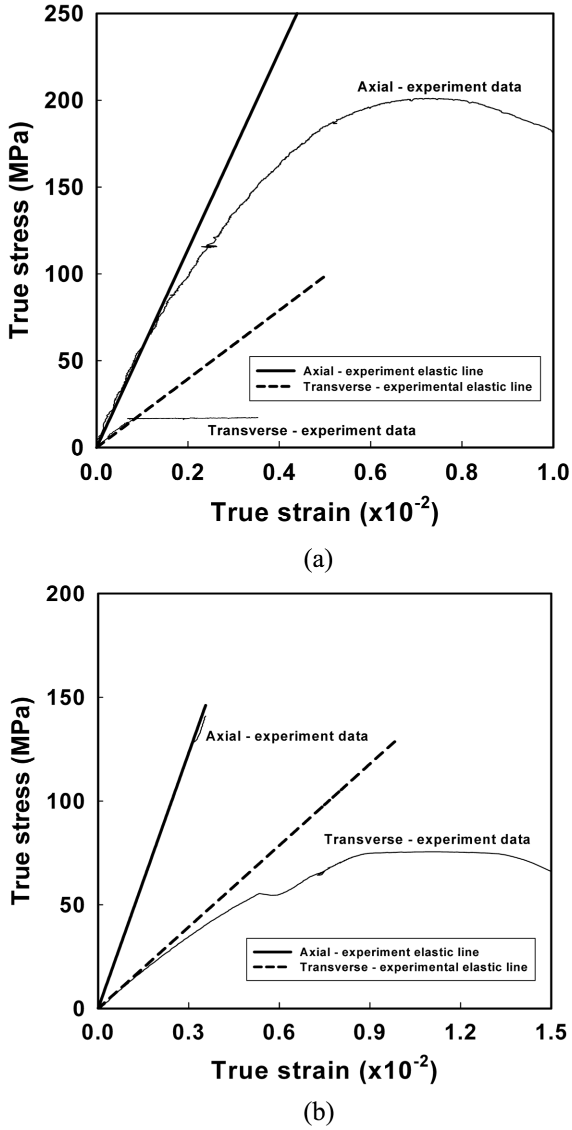

Figure 2. Experimental results of (a) tensile test and (b) compression test.

76 and ASTM D3410-87 using the 10-ton tensile and compression test machine Instron 8516 system. The measured true stress-strain curves of tension and compression tests are shown in *Figure* 2. The figure shows that the linear regions exist in the early strain ranges but slight non-linear behavior is also shown beyond the linear regions. Beyond the measurable strain regions for the compression tests, the specimens were buckled due to the thin specimen geometry.

From the tensile initial yield stresses σ_x^T and σ_y^T in the axial and transverse directions and the compressive initial yield stresses σ_x^C and σ_y^C in the axial and transverse directions, the four parameters p, q, β_{22} and κ in Eq. (5) were determined. Then, the parameter β_{33} was determined from shear yield stress. The tri-component yield surface of the current material is shown in *Figure* 3.

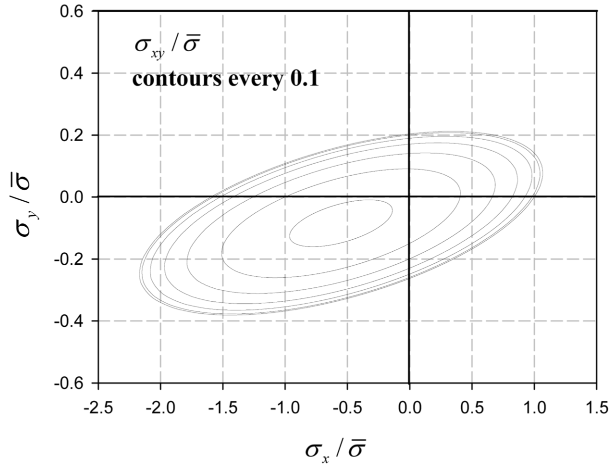

Figure 3. The yield surface of the 3D braided glass fiber reinforced composite.

In order to represent the anisotropic hardening behavior using the anisotropic kinematic hardening law shown in Eq. (8), the three stress-strain curves were considered after the initiation of plastic deformation: the x, y direction tension and the pure shear test curves. Using the material parameters obtained from the measured test data, the tension and compression test data were re-calculated using the kinematic hardening law.

Even though the anisotropic kinematic hardening law shown in Eq. (8) was used to represent anisotropic hardening for the composites in this work, the isotropic hardening calculation was also performed for comparison purposes, using the following hardening law:

$$\bar{\sigma}_{iso} = \bar{\sigma}_0 + a(1-\exp(-b\bar{\varepsilon})) \tag{9}$$

where $\bar{\sigma}_0$ is the initial yield stress in the reference state, a and b are material parameters.

4. Application

In order to validate the prediction capability of the developed constitutive equations including the anisotropic/asymmetric yield criterion and anisotropic hardening law, comparisons of simulations and experiments were performed for the three point bending tests using the 3D braided glass fiber reinforced composites. In particular, in order to verify the anisotropic and asymmetric properties of the uni-axial tension and compression behavior, the three point bending tests were performed along the axial and transverse directions, respectively, since these two tests mainly involve the uni-axial tension and compression behavior in each direction. The experimental procedure was guided by ASTM 790-02 and the schematic view of the test with dimensions is shown in *Figure* 4. The tests were performed using the MTS machine with 0.5-ton capacity at a constant crosshead speed, 1 mm/min.

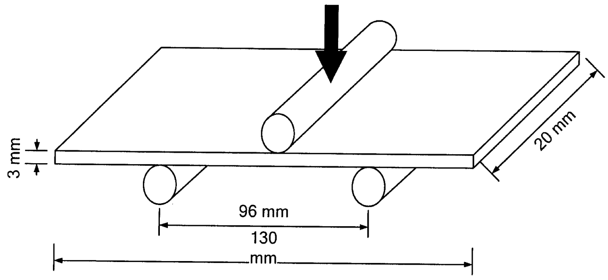

Figure 4. A schematic of the three point bending test.

As for the finite element formulations, half of the specimen was considered, applying the symmetric boundary condition. The friction coefficient between the tools and the composite specimen was assumed constant with 0.15. The four-node reduced shell elements (S4R) with one integration point were employed.

The calculated load-displacement curves of the three point bending tests were compared with experiments. To clarify the effectiveness of the developed constitutive equations, the calculations using simpler constitutive equations were also compared. In *Figure* 5, the results using orthotropic elastic constitutive equations are compared for the three cases: (A) with tensile elastic constants, (B) with compressive elastic constants and (C) with bi-modular elastic constants. The curve of Case (C) is better than the other two in the linear regions but all three cases fail to represent the nonlinear load-displacement curve. The bi-modular elastic constants were used in the following comparisons.

In *Figure* 6, the results using elastic-plastic constitutive equations with two different hardening laws are compared: (D) with the isotropic hardening law as shown in Eq. (9) and (E) with the anisotropic hardening law developed here. Case (E) shows good agreements both in the axial and transverse directions since Case (E) properly accounts for the hardening difference in the axial and transverse directions as well as

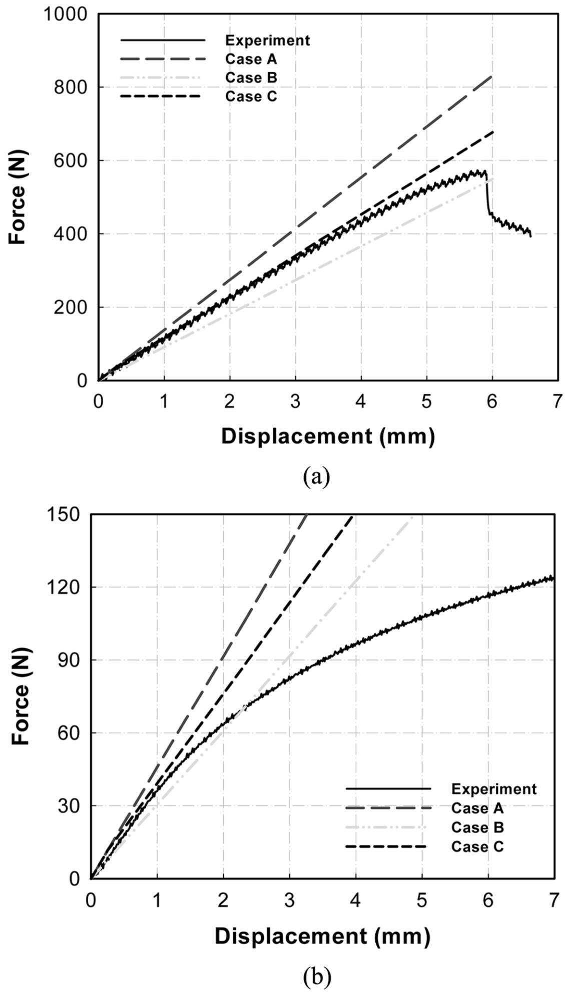

Figure 5. The load-displacement curves experimentally obtained and calculated from the different elastic constants for the specimens aligned in the (a) axial and (b) transverse directions.

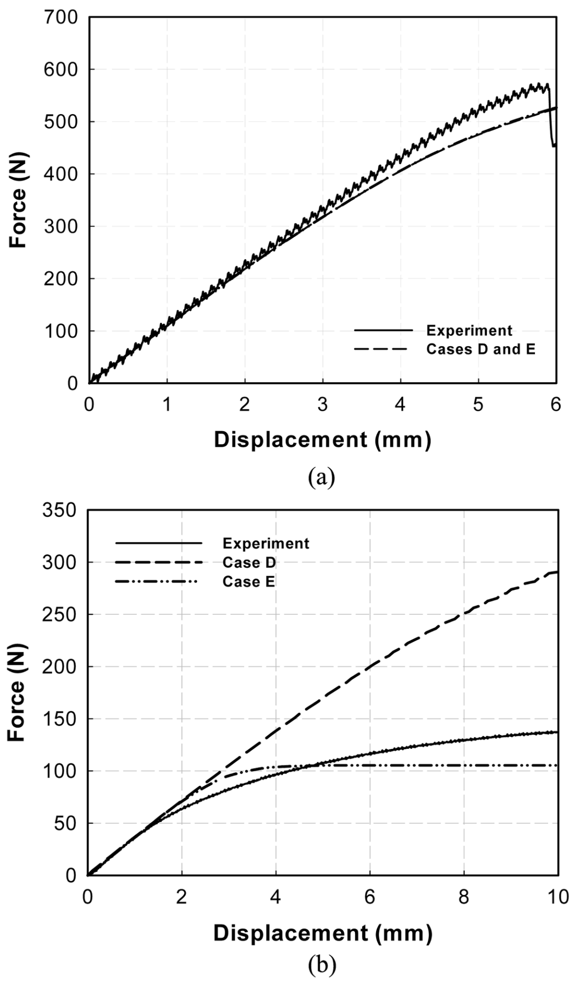

Figure 6. The load-displacement curves experimentally obtained and calculated using the isotropic hardening and anisotropic kinematic hardening laws for the specimens aligned in the (a) axial and (b) transverse directions.

asymmetry, while Case (D) does not accounts for the transverse hardening.

The comparison of the simulations and experiments verified that the elastic-plastic constitutive equations developed here well describe the mechanical behavior of the composite materials, especially the asymmetry and the anisotropic hardening in the uni-axial behavior, which were also confirmed essential to properly describe the behavior of fiber reinforced composites.

5. Conclusions

In order to describe the mechanical behavior of anisotropic and asymmetric materials such as fiber-reinforced composites, the elastic-plastic constitutive equations were developed. As for the yield criterion, the modified Drucker-Prager yield criterion was used to represent the anisotropic and asymmetric properties of fiber-reinforced composites, while the anisotropic evolution law of back-stress was applied to account for the anisotropic hardening behavior. Three point bending tests were carried out using 3D braided glass fiber reinforced composites. Finite element simulation results showed good agreements with experiments, especially for the elastic-plastic constitutive equation with proper description of the anisotropic hardening behavior.

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