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Master's Thesis of Jooa Park

# All-angle elastic meta-mirror with perfect mode selectivity

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Jooa Park

# All-angle elastic meta-mirror with perfect mode selectivity

Yoon Young Kim

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Jooa Park

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Seoul National University  
Mechanical Engineering Major

Jooa Park

Confirming the master's thesis written by  
Jooa Park  
February 2023

Chair	<u>Do-Nyun Kim</u>	(Seal)
Vice Chair	<u>Yoon Young Kim</u>	(Seal)
Examiner	<u>Jinkyu Yang</u>	(Seal)

## **Abstract**

# **All-angle elastic meta-mirror with perfect mode selectivity**

Jooa Park

Department of Mechanical Engineering

The Graduate School

Seoul National University

In general, when elastic waves are obliquely incident at a stress-free solid boundary, both longitudinal and transverse waves are reflected with determined reflectance.

This paper proposes an elastic meta-mirror that reflects only the selected wave mode regardless of the incidence angle and wave modes. In particular, the waves are purely reflected in the same mode as the incident mode. We realize the meta-mirror by the anisotropic elastic metamaterials. Unlike isotropic materials whose properties are determined by two parameters, the use of anisotropic materials in which all six stiffness constants can exist independently enables us to realize the desired performance. To establish the conditions that the material properties of the elastic meta mirror must have, a reflection-type scattering matrix named “J matrix” is newly proposed. Also, the elastic meta-mirror was designed by utilizing three different slit shapes that are easy to manufacture.

We verify the proposed meta-mirror by theoretical and numerical investigations. Numerical results show that when longitudinal waves are incident at the meta-mirror, only the longitudinal waves are

reflected. The same results show for the transverse wave incidence. It can be a promising strategy to address the fundamental complexity and difficulty of controlling elastic waves in solid boundaries.

**Keyword:** Elastic waves, stress-free solid boundary, elastic meta-mirror, anisotropic elastic metamaterials, perfect mode selectivity, incidence angle, wave mode.

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# Chapter 1. Introduction

## 1.1. Background of the study

Recently, wave control technology using metamaterials has been widely used in lenses, antennas, medical ultrasounds, and non-destructive testing. Energy can be stored and dissipated by using technology to control the direction and intensity of reflected waves, which is very important for ultrasonic applications.

Elastic waves moving through a solid have longitudinal and transverse waves, unlike electromagnetic and sound waves, which only have transverse or longitudinal waves. Given that elastic waves have two different wave modes, when the elastic waves are encountered on a stress-free boundary of a solid, two different wave modes are reflected simultaneously as shown in Figure 1.1 (a). In the case of aluminum, for example, the reflectance at the stress-free boundary when the longitudinal wave is incident and the transverse wave is incident is presented in Figure 1.2(a) and (b), respectively. Figure 1.2(a) illustrates that only the reflectance from longitudinal wave to longitudinal wave exists both when the longitudinal wave is incident normally and when it is incident at  $90^\circ$ . There is no reflection of transverse waves. In the case of incidence at  $60^\circ$  and  $77.24^\circ$ , only the reflectance from longitudinal to transverse waves exists [1–3]. Additionally, it can be demonstrated that incident angles other than those mentioned above cause longitudinal and transverse waves to always be reflected simultaneously. As shown in Figure 1.2(b), like longitudinal waves, when the transverse wave is normally incident, only transverse wave reflectance exists, and the longitudinal wave does not reflect. The angle at which the mode is converted can be calculated, and accordingly, only longitudinal reflection exists at that angle [1–4]. In transverse waves, a critical angle exists uniquely. When a transverse wave is an incident beyond this critical angle, it is always reflected only as a transverse wave. In all other cases, longitudinal and transverse waves are always

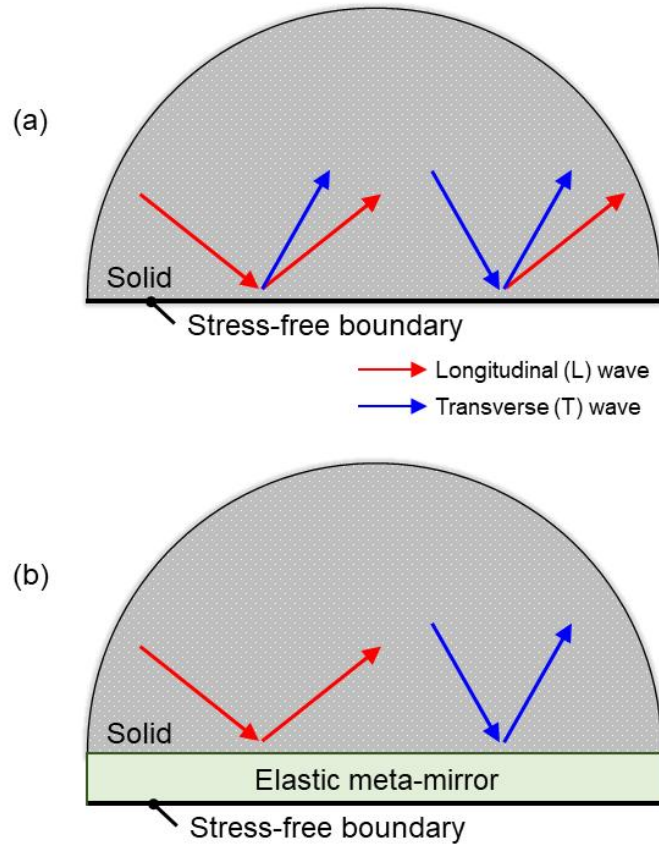
reflected simultaneously. However, there is a fundamental challenge in controlling the reflected wave at the stress-free boundary because two different wave modes are always reflected.

Previous research has been done to manipulate the reflected angle or the wave modes, such as by solely reflecting the desired wave, to get around this limitation [5–20]. For instance, when the longitudinal wave is obliquely incident on a solid boundary using metasurfaces based on phase gradients and critical angles, only the transverse wave can be purely reflected [5–15]. It is challenging to simultaneously manage longitudinal and transverse waves due to controlling them simultaneously. Only the longitudinal wave can be purely retro-reflected when utilizing a device that combines a corner reflector and a mode conversion anisotropic metamaterial when the longitudinal wave impinges on the solid barrier in an oblique manner [16, 17]. They are constrained by the finite operable incidence angle and wave mode of elastic waves, though. By using a metagrating, when the longitudinal wave is normally incident, we can control the reflected wave that we design [18–20]. However, it can be possible to the intended incidence angle.

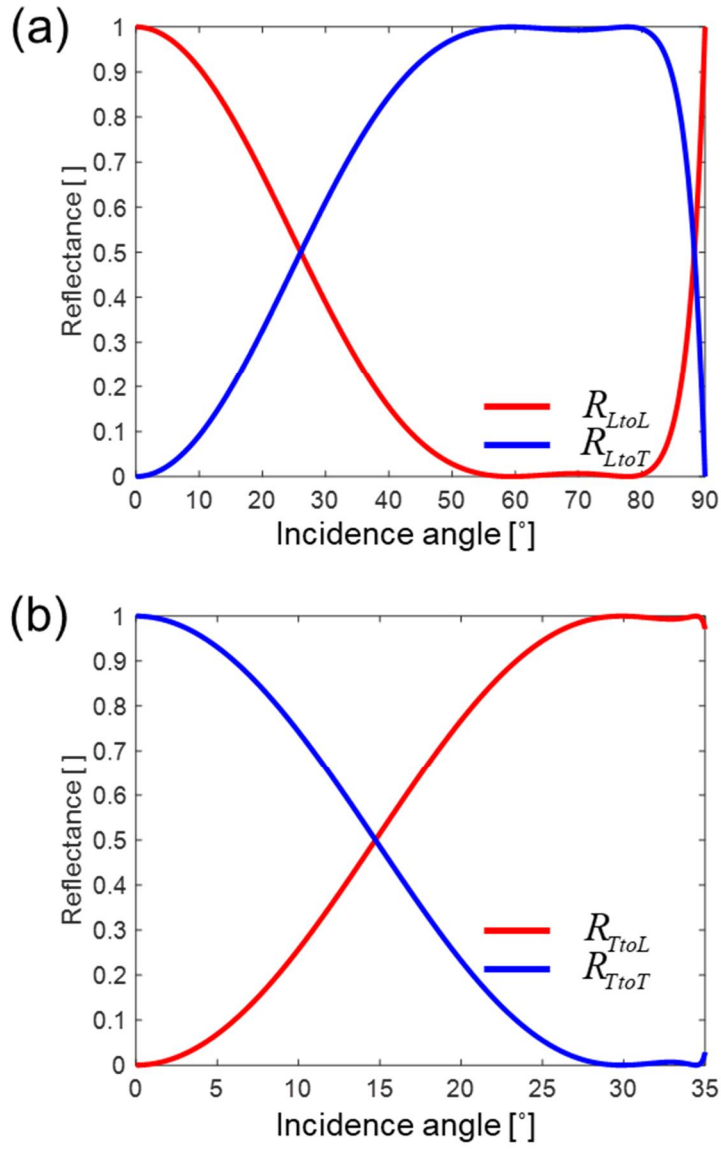


## 1.2. Purpose of Research

We suggest an elastic meta-mirror to get beyond the aforementioned restrictions, which exclusively reflects the chosen mode no matter the incidence angles or wave modes. In particular, just like in the incident mode, the elastic waves are purely mirrored in the wave mode. As shown in Figure 1.1(b), with only one elaborately designed elastic meta-mirror, the longitudinal wave is reflected when the longitudinal wave is incident, and the transverse wave is reflected when the transverse wave is incident. We newly propose a “J matrix”, which is compared to the Scattering matrix, to calculate the reflection coefficients and using it, optimize the effective properties of the meta-mirror [4]. We also design the microstructure of the meta-mirror and verify the performance by numerical investigations.



**Fig 1.1** Schematics for reflecting waves at a stress-free solid boundary (a) without and (b) with the proposed elastic meta-mirror



**Fig 2.2** The reflectance as a function of the incidence angle at a stress-free boundary (a) in the case of the incident longitudinal wave and (b) in the case of the incident transverse wave.

## Chapter 2. Design of the elastic meta-mirror

### 2.1. Optimized the elastic meta-mirror with effective properties

The anisotropy of the meta-mirror produces the wave phenomenon of the ideal mode of selective reflection. There are forward and backward quasi-longitudinal (QL+, QL-) waves as well as forward and backward quasi-shear (QS+, QS-) waves inside the meta-mirror. The combined four longitudinal-shear-linked waves explain the behavior of the meta-mirror. The wavenumber and polarization vector for each wave mode is obtained by resolving the Christoffel equation [1–4]. Generally, as shown in Figure 2.1 when the longitudinal wave of frequency  $f$  (angular frequency of  $\omega$ ) is incident at an anisotropic layer of thickness  $d$  and incidence angle of  $\theta_{inc}$ , two different wave modes are reflected. Let  $x = 0$  represent the anisotropic layer's end and  $d$  represent the interface between the incident medium and the layer. The velocity and the stress field at  $x = 0$  can be expressed as Equations (2.1) and (2.2) [4].

$$\begin{bmatrix} v_x \\ v_y \\ \sigma_{xx} \\ \sigma_{xy} \end{bmatrix}_{x=0^+} = \mathbf{M}_{ani} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \quad (2.1)$$

$$\begin{bmatrix} v_x \\ v_y \\ \sigma_{xx} \\ \sigma_{xy} \end{bmatrix}_{x=0^-} = \mathbf{M}_{iso} \begin{bmatrix} 1 \\ r_L \\ 0 \\ r_T \end{bmatrix} \quad (2.2)$$

with

$$\begin{aligned}
\mathbf{M}_{i,11} &= -j\omega P_x^a, \mathbf{M}_{i,12} = -j\omega P_x^b, \mathbf{M}_{i,13} = -j\omega P_x^c, \mathbf{M}_{i,14} = -j\omega P_x^d, \\
\mathbf{M}_{i,21} &= -j\omega P_y^a, \mathbf{M}_{i,22} = -j\omega P_y^b, \mathbf{M}_{i,23} = -j\omega P_y^c, \mathbf{M}_{i,24} = -j\omega P_y^d, \\
\mathbf{M}_{i,31} &= jk_x^a (C_{11}^i P_x^a + C_{16}^i P_y^a) + j\sigma (C_{12}^i P_y^a + C_{16}^i P_x^a), \\
\mathbf{M}_{i,32} &= jk_x^b (C_{11}^i P_x^b + C_{16}^i P_y^b) + j\sigma (C_{12}^i P_y^b + C_{16}^i P_x^b), \\
\mathbf{M}_{i,33} &= jk_x^c (C_{11}^i P_x^c + C_{16}^i P_y^c) + j\sigma (C_{12}^i P_y^c + C_{16}^i P_x^c), \\
\mathbf{M}_{i,34} &= jk_x^d (C_{11}^i P_x^d + C_{16}^i P_y^d) + j\sigma (C_{12}^i P_y^d + C_{16}^i P_x^d), \\
\mathbf{M}_{i,41} &= jk_x^a (C_{16}^i P_x^a + C_{66}^i P_y^a) + j\sigma (C_{26}^i P_y^a + C_{66}^i P_x^a), \\
\mathbf{M}_{i,42} &= jk_x^b (C_{16}^i P_x^b + C_{66}^i P_y^b) + j\sigma (C_{26}^i P_y^b + C_{66}^i P_x^b), \\
\mathbf{M}_{i,43} &= jk_x^c (C_{16}^i P_x^c + C_{66}^i P_y^c) + j\sigma (C_{26}^i P_y^c + C_{66}^i P_x^c), \\
\mathbf{M}_{i,44} &= jk_x^d (C_{16}^i P_x^d + C_{66}^i P_y^d) + j\sigma (C_{26}^i P_y^d + C_{66}^i P_x^d).
\end{aligned}$$

where  $k_x$  depicts the x-directional wavenumber,  $\sigma$  depicts the y-directional wavenumber of the incident wave, and  $C_{ij}$  ( $i, j = 1, 2, 6$ ) depicts the stiffness constant. If the subscript  $i$  equals to ani, a, b, c, and d represent  $QL^+$ ,  $QL^-$ ,  $QS^+$ , and  $QS^-$  wave modes respectively. If the subscript  $i$  equals isotropic materials, a, b, c, and d represent the incident and reflected longitudinal ( $L^+$ ,  $L^-$ ) and the incident and reflected transverse ( $T^+$ ,  $T^-$ ) wave modes respectively. A, B, C, and D depict the displacement amplitudes of  $QL^+$ ,  $QL^-$ ,  $QS^+$ , and  $QS^-$  waves.  $r_L$  and  $r_T$  denote the displacement amplitudes of  $L^-$  and  $S^-$  waves. The velocity and the stress field at  $x = d$  can be expressed as Equation (2.3).

$$\begin{bmatrix} v_x \\ v_y \\ \sigma_{xx} \\ \sigma_{xy} \end{bmatrix}_{x=d^-} = \mathbf{M}_{ani} \mathbf{N}_{x=d} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \quad (2.3)$$

$$\text{with } \mathbf{N}_{x=d} = \begin{bmatrix} e^{jk_x^{QL^+} d} & 0 & 0 & 0 \\ 0 & e^{jk_x^{QL^-} d} & 0 & 0 \\ 0 & 0 & e^{jk_x^{QS^+} d} & 0 \\ 0 & 0 & 0 & e^{jk_x^{QS^-} d} \end{bmatrix}$$

Due to the field continuity at  $x = 0$ , Eq. (2.4) is obtained.

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \mathbf{M}_{ani}^{-1} \mathbf{M}_{iso} \begin{bmatrix} 1 \\ r_L \\ 0 \\ r_T \end{bmatrix} \quad (2.4)$$

Substituting Equation (2.4) into Eq. (2.3), we get the J matrix that we newly propose in this paper to calculate the reflection coefficients.

$$\begin{bmatrix} v_x \\ v_y \\ \sigma_{xx} \\ \sigma_{xy} \end{bmatrix}_{x=d^-} = \mathbf{M}_{ani} \mathbf{N}_{x=d} \mathbf{M}_{ani}^{-1} \mathbf{M}_{iso} \begin{bmatrix} 1 \\ r_L \\ 0 \\ r_T \end{bmatrix} = \mathbf{J} \begin{bmatrix} 1 \\ r_L \\ 0 \\ r_T \end{bmatrix} \quad (2.5)$$

From Equation (2.5), we can calculate the reflection coefficients:

$$r_{LL} = \frac{J_{31}J_{44} - J_{34}J_{41}}{J_{34}J_{42} - J_{32}J_{44}}, r_{LT} = \frac{J_{32}J_{41} - J_{31}J_{42}}{J_{34}J_{42} - J_{32}J_{44}} \quad (2.6)$$

where  $r_{LL}$  denotes the reflection coefficients of the longitudinal wave while  $r_{LT}$  denotes the reflection coefficients of the transverse wave. If the aforementioned procedure is used in the process of a transverse wave incidence, we can obtain the reflection coefficients as Equation (2.7).

$$r_{TL} = \frac{J_{33}J_{44} - J_{34}J_{43}}{J_{34}J_{42} - J_{34}J_{42}}, r_{TT} = \frac{J_{32}J_{43} - J_{33}J_{42}}{J_{34}J_{42} - J_{34}J_{42}} \quad (2.7)$$

where  $r_{TL}$  denotes the reflection coefficients of the longitudinal wave while  $r_{TT}$  denotes the reflection coefficients of the transverse wave. Combining the power intensity equation (Equation (2.8)) and Equation (2.7), the reflectance can be obtained as Equation (2.9).

$$P = \frac{1}{2} \text{real}(\sigma_{xx} \times \text{conj}(v_x) + \sigma_{xy} \times \text{conj}(v_y)) \quad (2.8)$$

$$R_{LL} = |r_{LL}|^2, R_{LT} = \frac{\sqrt{C_{66}} \cos \theta_S}{\sqrt{C_{11}} \cos \theta_L} |r_{LT}|^2, R_{TT} = |r_{TT}|^2, R_{TL} = \frac{\sqrt{C_{11}} \cos \theta_L}{\sqrt{C_{66}} \cos \theta_T} |r_{TL}|^2 \quad (2.9)$$

where  $\theta_L$  denotes the angle at which the longitudinal wave is reflected and  $\theta_S$  denotes the angle at which the transverse wave is reflected.

The reflectance is composed of the J matrix components, and the J matrix components are composed of the effective properties ( $\rho$  and  $C_{ij}$ ). So if they are well controlled, the anisotropic layer can be designed with the desired reflectance. In particular, since the elastic meta-mirror purely reflects only the same mode as the incident wave mode for all the incidence angles, the reflectance to other modes than the incident mode should be zero. The conditions for the elastic meta-mirror to be satisfied are as follows:

$$R_{LT} = 0 \quad (\text{L-to-L perfect reflect}) \quad (2.10)$$

$$R_{TL} = 0 \quad (\text{T-to-T perfect reflect}) \quad (2.11)$$

For a case with a background material of aluminum under a plane stress condition, the width of the meta-mirror  $d = 20$  mm, and a designed frequency of 100 kHz, we optimized the meta-mirror based on the conditions above, Equation (10) and (11). Following was our strategy for optimization. First, we extracted the reflectance for three or more target incidence angles of the meta-mirror with any effective properties. Second, the optimization proceeded in minimizing the objective function, which is the sum of the reflectance ( $R_{LT}$ ,  $R_{TL}$ ) to the different modes from the incident mode for the target incidence angle. Finally, we can get the effective properties of the meta-mirror as shown in Table 2.1.

Figure 2.2 displays the reflectance as a function of the incident

angle. The incidence longitudinal (transverse) wave is depicted in Figure 2.2 as the longitudinal (transverse) wave that is reflected 100 % of all the incident angles, indicating that the optimized effective properties have effectively achieved the perfect mode selectivity for the reflectance.

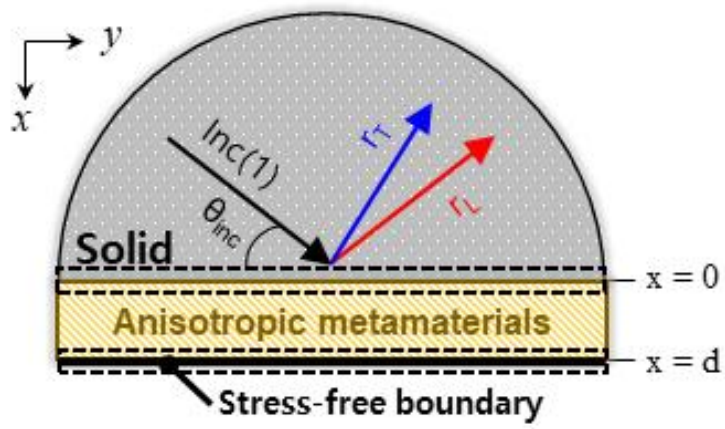


## 2.2. Metamaterial design and numerical verification

We were able to realize the meta-mirror in the previous part thanks to its useful characteristics. It is, however, challenging to employ realistically because it does not exist. Therefore, a metamaterial was used, which achieves novel wave phenomena with a periodic arrangement of artificial microstructures.

Here, we set the background material as aluminum and the designed frequency as 100 kHz. The unit structure of the meta-mirror consists of three different slit shapes, which have nine geometric parameters, defined as  $L_x$ ,  $L_y$ ,  $l_1$ ,  $r_1$ ,  $l_2$ ,  $r_2$ ,  $l_3$ ,  $r_3$ , and  $w$  in Figure 2.3(a). It is easy to manufacture with a very simple structure, and it is possible to control the degree of freedom by simply increasing the number of slits, so this geometry is utilized. They are optimized to minimize the reflectance of the wave mode different from the incidence wave mode. By using the method of moving asymptotes, the optimization results were as shown in Table 2.2. Numerical simulations utilizing the finite element analysis-based module of COMSOL Multiphysics were carried out to confirm the performance of the optimized geometry shown in Figure 2.3(b). As shown in Figure 2.4, under the plane wave assumption, the reflected longitudinal and transverse fields both exist when the longitudinal wave is incident without the meta-mirror. While, with the meta-mirror, only the reflected longitudinal fields exist as shown in Figure 2.5, suggesting the designed metamaterial successfully realizes the perfect mode selectivity for the reflectance. The transverse wave incidence exhibits the same results as in Figures 2.6 and 2.7, and they were carried out at an angle below the critical angle. For the finite-length source, we also carried out a numerical analysis to assess the feasibility. With the elastic meta-mirror, the only wave reflected is the longitudinal wave for the wave incident at  $30^\circ$ . Also, for the transverse wave incident at  $20^\circ$ , only the reflected transverse wave exists. Further, the reflection was computed as the function of the incidence angle, presented in Figure 2.8. The results show that the wave mode same as the incidence wave mode is

reflected in more than 99% of all the incidence angles and wave modes. Finally, the reflectance according to the incidence angle and frequency was obtained in Figure 2.9. A white solid line is displayed for the component that has at least 90% reflectance in the same mode as the incident mode. It has a reflectance of 100% in all directions for the designed frequency of 100 kHz and a reflectance of 90% or greater in all directions for the neighboring frequency range of 97 kHz to 104 kHz.



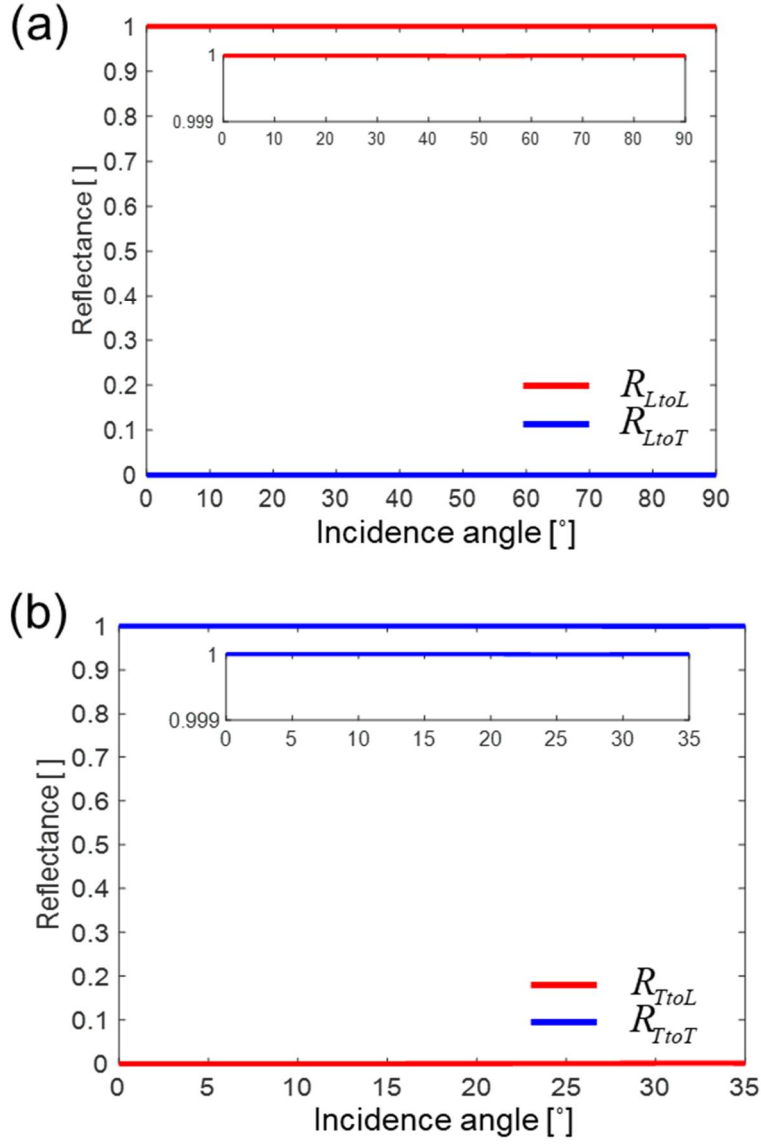
**Fig 2.1** Schematic of wave reflection at a stress-free boundary with an arbitrary anisotropic metamaterial when the wave of frequency  $f$  (angular frequency of  $\omega$ ) is incident at an anisotropic layer of thickness  $d$  and incidence angle of  $\theta_{inc}$ ,

**Table 2.1** Optimized results of the effective properties of the elastic meta-mirror

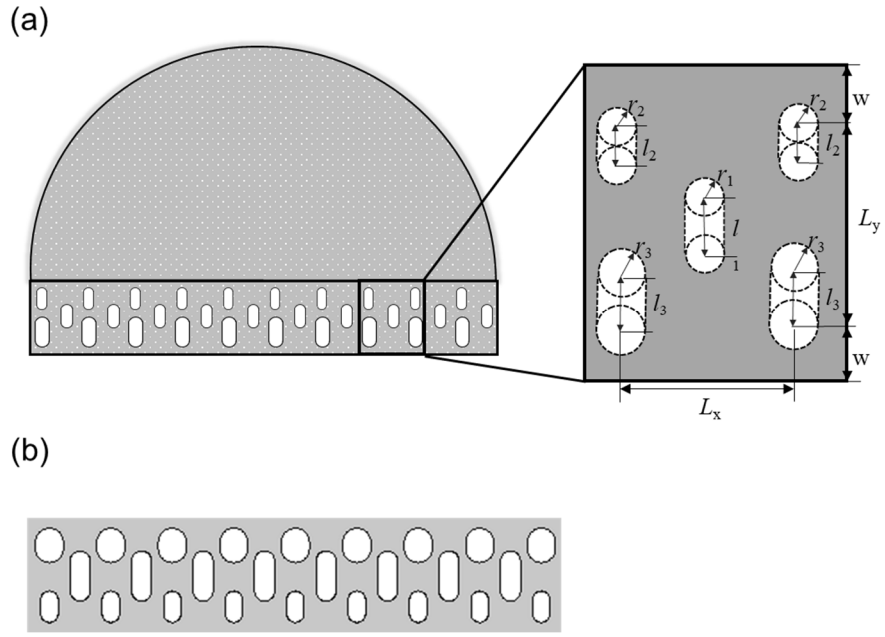
$\rho$ [kg/m <sup>3</sup> ]	2535.5
$C_{11}$ [GPa]	111.74
$C_{12}$ [GPa]	-26.656
$C_{16}$ [GPa]	0
$C_{22}$ [GPa]	16.526
$C_{26}$ [GPa]	0
$C_{66}$ [GPa]	26.656

**Table 2.2** Optimized results of the geometry of the elastic meta-mirror

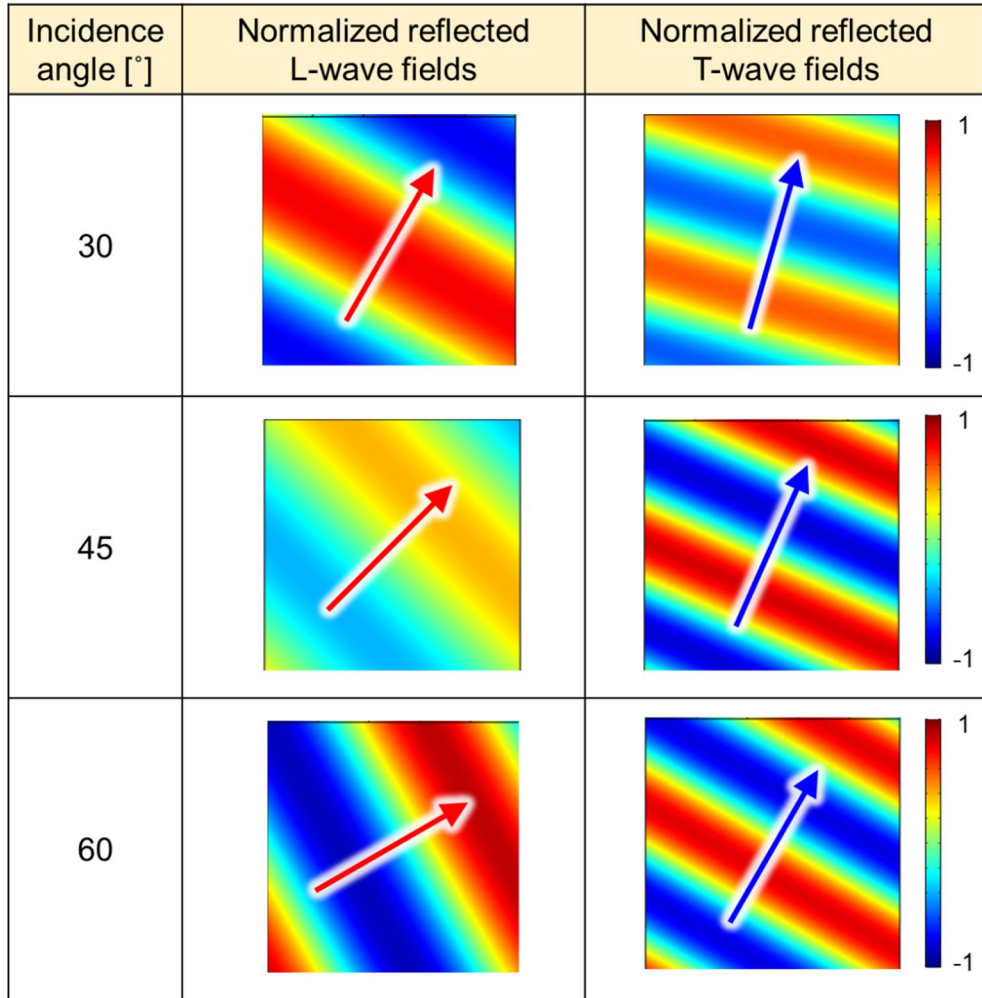
$L_x$ [mm]	10
$L_y$ [mm]	10
$l_1$ [mm]	4.97
$r_1$ [mm]	1.60
$l_2$ [mm]	1.07
$r_2$ [mm]	2.24
$l_3$ [mm]	2.38
$r_3$ [mm]	1.43
$w$ [mm]	5



**Fig 2.2** The reflectance as a function of the incidence angle with the elastic meta-mirror (a) in the case of the incident longitudinal wave and (b) in the case of the incident transverse wave

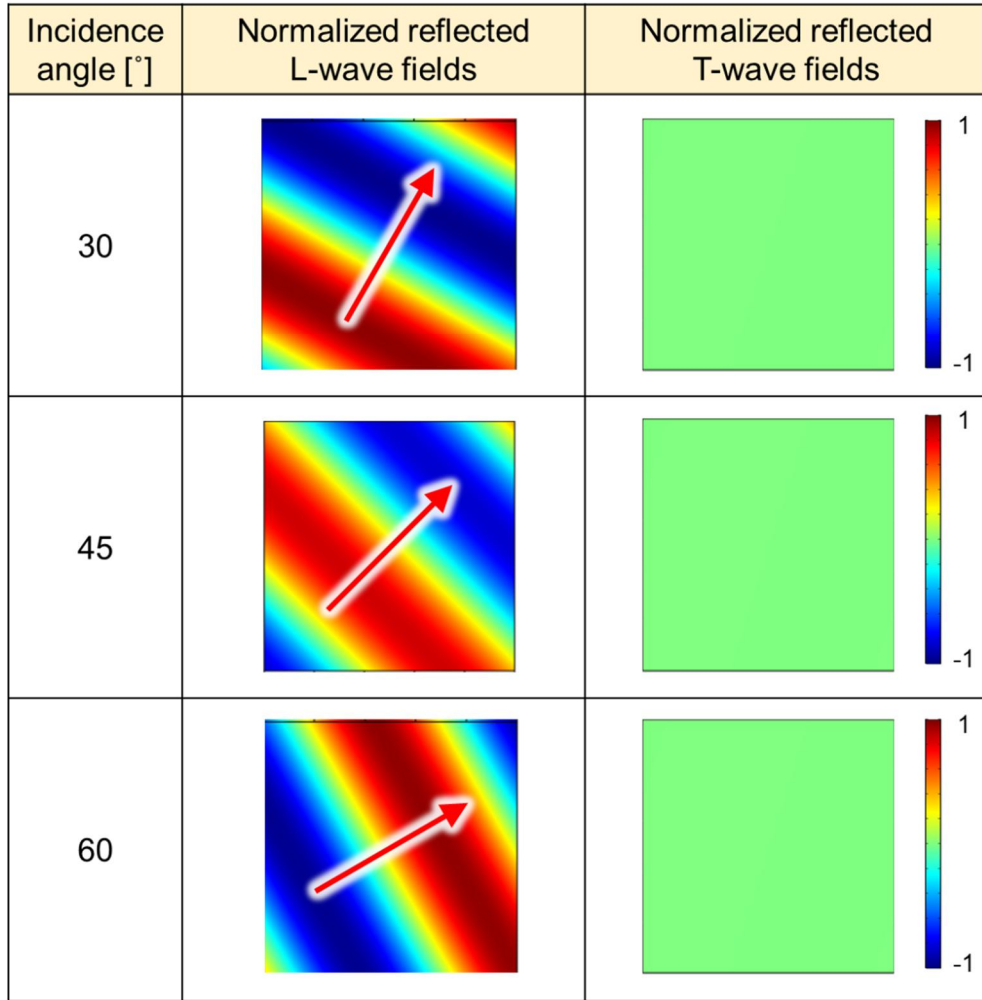


**Fig 2.3** (a) Geometrical parameters of the elastic meta-mirror.  
 (b) The optimization results

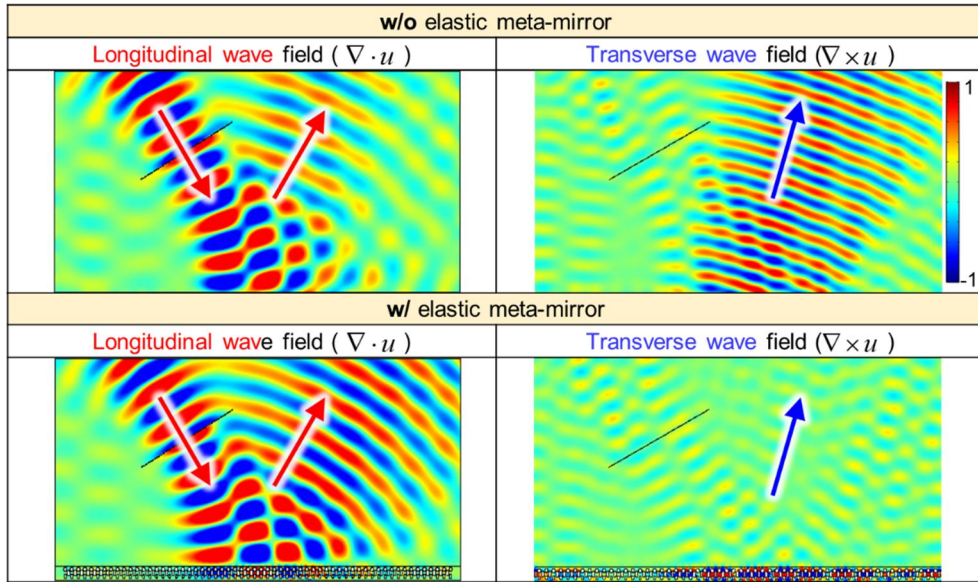


**Fig 2.4.** Normalized reflected longitudinal and transverse wave fields with the elastic meta-mirror. The case for the incident longitudinal wave at angles of  $30^\circ$  ,  $45^\circ$  , and  $60^\circ$  and a frequency of 100 kHz.

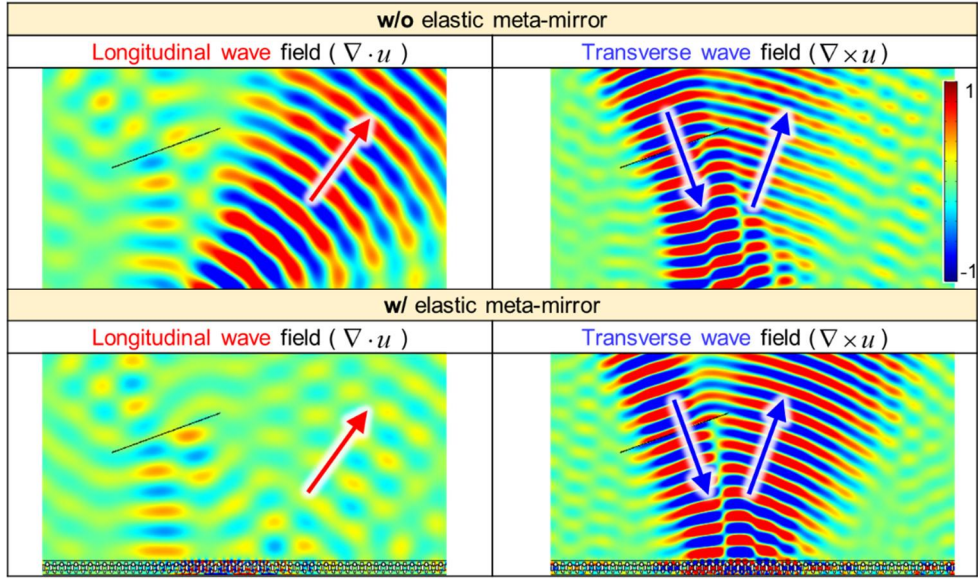




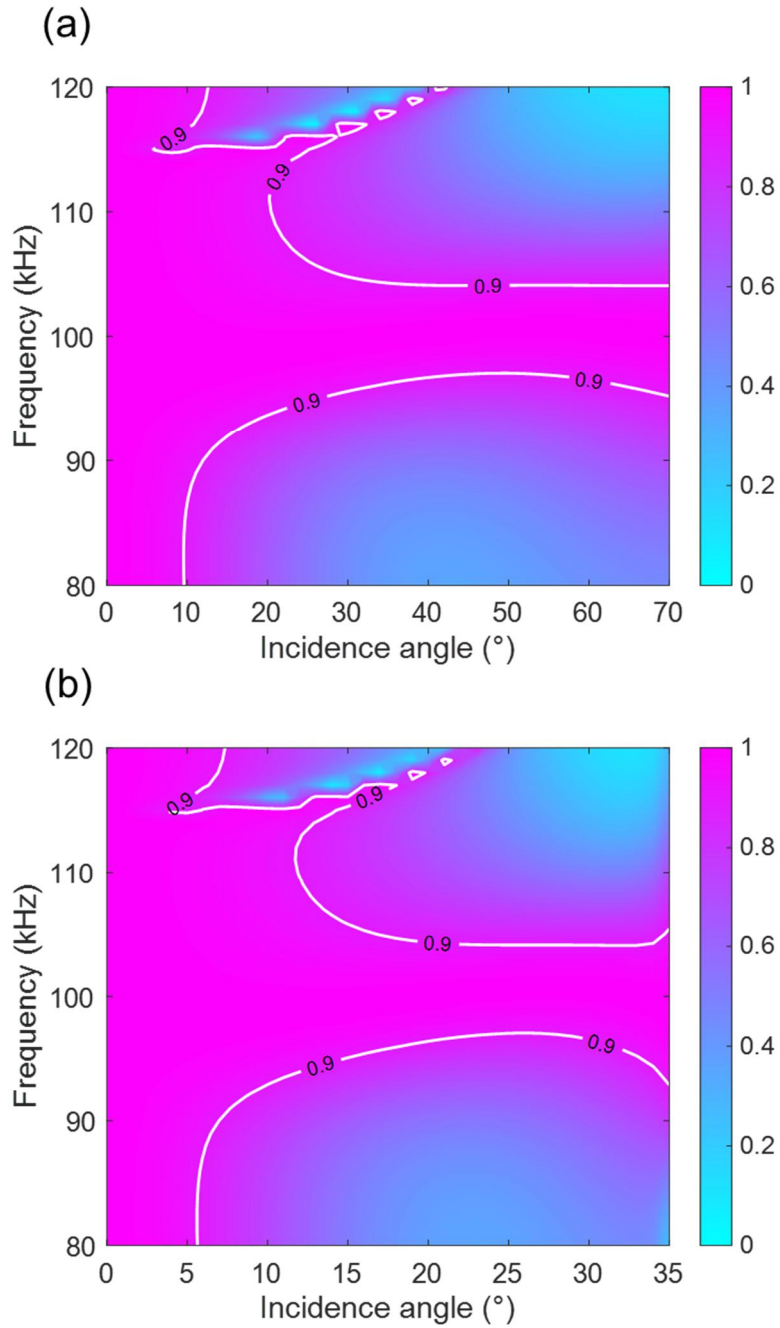
**Fig 2.5.** Normalized reflected longitudinal and transverse wave fields without the elastic meta-mirror. The case for the incident longitudinal wave at angles of  $30^\circ$  ,  $45^\circ$  , and  $60^\circ$  and a frequency of 100 kHz.



**Fig 2.6.** Normalized reflected longitudinal and transverse wave fields for the case of the incident longitudinal wave at the incidence angle of  $30^\circ$ , a frequency of 100 kHz. The length of the source is 3 times the wavelength.



**Fig 2.7.** Normalized reflected longitudinal and transverse wave fields for the case of the incident longitudinal wave at the incidence angle of  $20^\circ$ , a frequency of 100 kHz. The length of the source is 3 times the wavelength.



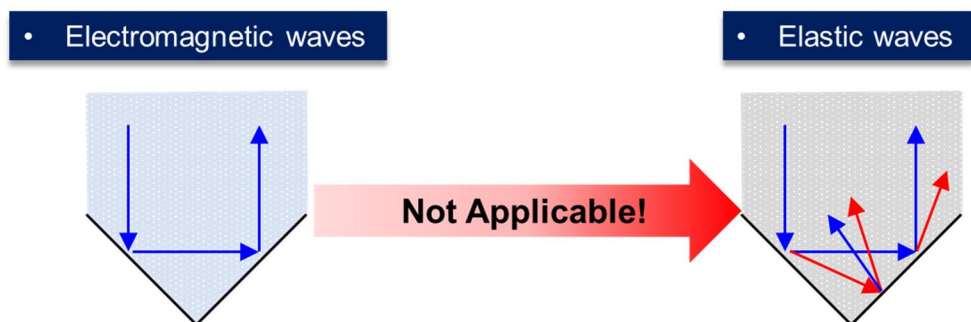
**Fig 2.8.** The reflectance as a function of the incidence angle and frequency (a) for the case of the incident longitudinal wave and (b) for the case of the incident transverse wave. The white line denotes the reflectance of 90%.

## Chapter 3. Conclusion

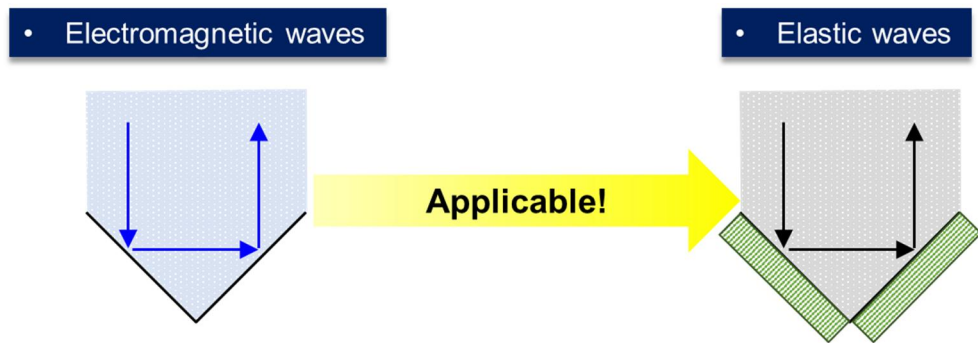
In this paper, we proposed an elastic meta-mirror with perfect mode selectivity for all incidence angles and wave modes. According to the simulation results, waves that are incident at an angle at the suggested elastic meta-mirror are only reflected in the same wave mode as waves that are incident. The effective characteristics of the meta-mirror were optimized by minimizing the reflectance in a mode other than the incident mode by calculating reflectance based on the J matrix technique. Numerical results validated the performance of the meta-mirror we designed using three different slit-shaped holes as artificial microstructures. Although we concentrated on the reflection in the same mode as the incident mode here, the present method may allow for the engineering of an elastic metamaterial with anisotropy for reflection in a different mode from the incident mode. Additionally, if more slits are employed, the degree of freedom can be increased, allowing for an expansion of the frequency range where the performance of the meta mirror is implemented.

The suggested elastic meta-mirror might be used to achieve elastic wave retroreflection. Elastic waves have two wave modes, whereas electromagnetic waves only have one, according to the different types of waves. This makes it challenging to regulate these two waves in the same manner. For example, as shown in Figure 3.1, the simplest way to implement retroreflection in electromagnetic waves is to use corner reflectors. However, it is not practical to employ corner reflectors for elastic waves since there is a basic problem controlling elastic waves in solid boundaries. In other words, we cannot apply wave manipulating technology to electromagnetic waves as it is. However, the retroreflection of elastic waves can be implemented with the suggested elastic meta-mirror by using only corner reflectors, allowing for the continued use of electromagnetic wave technology as shown in Figure 3.2. Therefore, the elastic meta-mirror can be a promising strategy to resolve the fundamental difficulty of controlling elastic waves at the stress-free boundary.





**Fig 3.1.** The applicable limitation due to the multimodality of the elastic waves.



**Fig 3.2.** Overcoming limitations by using an elastic meta-mirror to apply electromagnetic wave technology as it is





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## Abstract

# 완벽한 모드 선택성을 갖는 전방위 탄성 메타 거울

박 주 아

기계공학부

서울대학교 대학원

일반적으로 고체의 자유 응력 경계에 탄성파가 비스듬히 입사하면 종파와 횡파가 모두 반사된다.

본 논문에서는 입사각과 파동 모드에 관계없이 선택된 파동 모드만을 반사하는 탄성 메타 거울을 제안한다. 특히, 입사 파동 모드와 동일한 파동 모드만 순수하게 반사하는 탄성 메타 거울을 다루었다. 본 연구에서 제안한 탄성 메타 거울은 이방성 탄성 메타물질을 통해 구현될 수 있다. 두 가지 매개변수에 의해 강성 계수가 결정되는 등방성 물질과 달리 6개의 강성 계수가 모두 독립적으로 존재할 수 있는 이방성 물질을 활용하는 것이 핵심 아이디어다. 본 논문에서는 탄성 메타 거울의 물성이 만족해야 할 조건을 정립하기 위해 J 행렬이고 정의한 반사형 산란 행렬을 새롭게 제안하였다. 또한 제작이 용이한 3가지 슬릿 모양의 구멍을 활용하여 탄성 메타 거울을 설계하였다.

본 논문에서는 이론 및 수치 분석을 통해 제안된 메타 거울을 검증하였다. 해석 결과는 탄성 메타 거울에 종파가 입사하여 종파만 완전히 반사됨을 보여준다. 횡파가 입사하는 경우에 대해서도 동일한 결과가 나타나 는 것을 확인할 수 있다. 본 논문에서 제안하는 탄성 메타 거울은 자유 응력 경계에서 탄성과 제어의 근본적인 어려움을 해결하는데 크게 기여할 것으로 기대된다.

**주요어** : 탄성파, 자유 응력 경계, 탄성 메타 거울, 이방성 탄성 메타물질, 선택된 파동 모드, 입사각, 파동 모드.

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