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공학석사학위논문

Optimal Price and Wage in a Food Delivery  
Platform with Multiple Types of Customers and  
Couriers

다유형의 고객과 배달원이 존재하는 음식배달 플랫폼에서의  
최적 가격과 임금에 대한 연구

2023 년 2 월

서울대학교 대학원

산업공학과

서 유 석

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이 논문을 공학석사 학위논문으로 제출함

2022 년 12 월 5 일

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## Abstract

# Optimal Price and Wage in a Food Delivery Platform with Multiple Types of Customers and Couriers

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This study developed a food delivery platform comprising multiple types of customers and couriers to derive optimal service price and wage per service and achieve profit maximization. The developed platform faced price- and time-sensitive customers with heterogeneous delivery distances and earning-sensitive couriers with heterogeneous transportation modes. Furthermore, we formulated the model using multi-class multi-pool system and incorporated multiple types and differing service times upon class-pool set. To solve this complex problem, we proposed an approximation algorithm to derive optimal values. Finally, extensive numerical experiments were conducted, and practical managerial insights were driven.

**Keywords:** food delivery services, endogenous supply and demand, multi-class multi-pool systems, queuing models, convex programming

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# Chapter 1

## Introduction

### 1.1 Problem Description

Owing to the development of mobile technology and the advent of on-demand service platforms, people all over the world can enjoy various services anytime instantly. Such dynamicity is due to the continuous coordination from on-demand platforms comprising self-scheduled service providers (or freelancers) and arbitrarily incoming service requests from customers. These on-demand services include meal delivery (e.g., Doordash, UberEats, Deliveroo (United Kingdom), and Baemin (South Korea)), grocery delivery (e.g., Instacart, Amazon Fresh, and B-Mart (South Korea)), goods delivery (e.g., Uber Rush and B-Mart (South Korea)), and ride-hailing (e.g., Uber and Lyft). Owing to its temporal and spatial convenience, the popularity of on-demand services has multiplied in the past five years. In particular, food delivery services have experienced a major jump in sales and growth in market size due to COVID-19. The market value of the food delivery market tripled to \$150 billion in 2021, and continues to increase.<sup>1</sup> However, the uncertainty of supply and demand imposes operational difficulties on the platform. Therefore, the platform must match the ever-changing supply and demand to each other.

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<sup>1</sup><https://www.mckinsey.com/industries/technology-media-and-telecommunications/our-insights/ordering-in-the-rapid-evolution-of-food-delivery>

Self-scheduling service providers can freely choose whether to join the platform and provide services to cater to the demands. However, for their convenience, providers are not guaranteed a specific annual wage and their idle times are not compensated, making them earning-sensitive. Therefore, the supply depends on the wage per service and utilization, which in turn depends on the demand. Meanwhile, the demand itself is affected by the supply. Given the price- and time-sensitive nature of customers, the number of providers directly influences the waiting time, and therefore, the demand. Hence, supply and demand directly impact each other. Likewise, food delivery platforms comprise self-scheduled couriers and impatient customers. However, owing to the jointly interacting nature of supply and demand in on-demand service platforms, setting the appropriate price and wage level has always been challenging.

In South Korea, the food delivery market has grown 10 times within four years, reaching ₩ 26 trillion in 2021. However, compared to the growth in the market size, the courier numbers have failed to scale up in size within the same time span.<sup>23</sup> Such difference in growth has resulted in a courier shortage problem for the platform, which became exacerbated when the "one-order-per-delivery" policy was adopted. In this policy, a courier is mandated to deliver one order at a time to ensure fast deliveries; however, the courier shortage problem increased proportionally. To mitigate the shortage problem, the platform increased its expenditure on hiring couriers. This resulted in greater revenue through higher service levels; however, the surge in outsourced delivery expenditures led to a consistent operating loss. Moreover, increased expenditure on couriers increased the delivery price imposed on customers.

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<sup>2</sup><https://www.joongang.co.kr/article/25050978>

<sup>3</sup><https://www.seoul.co.kr/news/newsView.php?id=20220204022014>

Increased price has dwindled customer demand, but this was further aggravated as social distancing policies of COVID-19 alleviated and customers had a lesser need to use delivery services. In contrast, couriers demanded higher wages per service to compensate for their low earnings owing to the "one-order-per-delivery" policy; furthermore, fuel costs increased.

Food delivery platforms worldwide share similar problems. In China, order surges occurred owing to COVID-19 related restrictions, and the courier capacity was insufficient to cover them.<sup>4</sup> In India, food delivery platforms experienced a supply-side crunch owing to rising fuel prices and inflation.<sup>5</sup> In the US and Europe, courier strikes have increased and they are demanding higher pay rates and better treatment.<sup>6 7</sup>

Therefore, food delivery platforms are experiencing a trade-off between price and wage. A platform must lower delivery prices to secure a large customer pool; however, the wage per service must be raised to ensure a large courier pool and fast deliveries. However, a platform cannot lower delivery prices and raise wages per service for obtaining profit. Therefore, a food delivery platform to determine a method to optimize its profit regarding the trade-off between price and wage.

In this study, we consider a food delivery platform with various types of customers and couriers. The customers are differentiated by the delivery distances, and couriers are discerned through their mode of transport. In practice, couriers

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<sup>4</sup><https://www.globaltimes.cn/page/202211/1280250.shtml>

<sup>5</sup><https://economictimes.indiatimes.com/tech/startups/how-shortage-of-gig-workers-is-affecting-services-of-food-delivery-quick-commerce-startups/articleshow/91475586.cms>

<sup>6</sup>[https://www.einnews.com/pr\\_news/602747180/gig-workers-go-on-3-day-thanksgiving-strike-over-poor-pay-and-lack-of-safety-by-gig-companies](https://www.einnews.com/pr_news/602747180/gig-workers-go-on-3-day-thanksgiving-strike-over-poor-pay-and-lack-of-safety-by-gig-companies)

<sup>7</sup><https://www.reuters.com/markets/europe/strikes-protests-europe-over-cost-living-pay-2022-11-24/>

are matched to customers based on the delivery distance and transportation mode. If there is an excess supply of couriers that can serve all demands, the platform wouldn't have had to make any demand routing decisions. However, owing to courier shortage, the platform needs to coordinate supply and demand through wages and prices and decide which order demand must be serviced by which type of courier. Therefore, the order allocating decisions of a platform must also be investigated.

This study aimed to solve the profit maximization problem of food delivery platforms involving multiple types of customers and couriers. Additionally, we examined how the price and wage decisions of the platform changed the dynamics within the players in the model. Particularly, the questions below are to be answered through this study:

1. What is the optimal size for each customer and courier type?
2. What is the optimal demand routing ratio of each customer type to each courier type?
3. What is the optimal service price and wage per service?

To address these problems, we incorporated the Multi-Class Multi-Pool (MCMP) system to portray the current delivery platform. This system assumes multiple types of customer classes and server pools, where the service time is different by each class-pool set. Fig 1.1 shows a schematic representation of the system. The customer class can be classified by delivery distance, and the server pool is divided by the transportation modes. This method portrays the current food delivery platform effectively, thereby improving the single-type customer and courier assumption

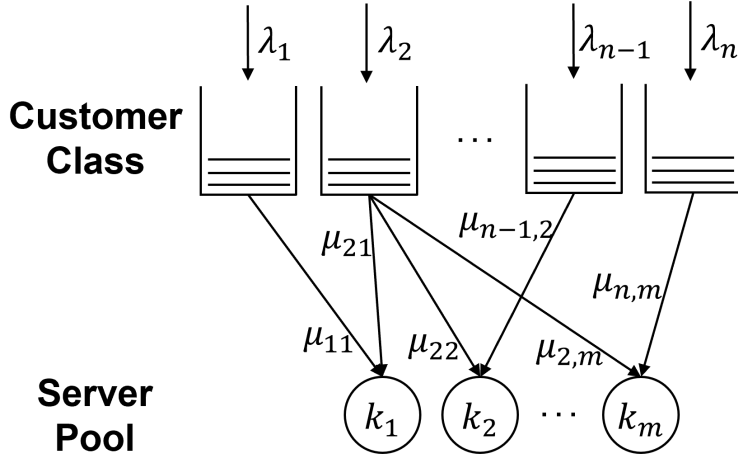


Figure 1.1: Schematic representation of multi-class multi-pool system

from previous literature. However, the problem dimension and complexity increase significantly with the increase in the types of customers and couriers.

To solve such an important and difficult problem, we propose an approximation algorithm that successfully derives an optimal solution regardless of the problem size. Moreover, we construct a two-class two-pool model as a base model to analyze the optimal price and wage that maximize the profit. Then, extensive numerical experiments were conducted to acquire managerial insights. Furthermore, real data from a Chinese online delivery platform were used in our model to check the validity of the model, and realistic answers were obtained.

## 1.2 Research Motivation and Contribution

Very little research has been conducted to solve the pricing problem while incorporating the unique characteristics of customers and couriers. In practice, delivery distances tend to vary widely and adequate couriers are matched based on the dis-

tances and transportation. Therefore, studies that only consider a single type of customer and courier may have some drawbacks in portraying reality. Therefore, models that project the actual delivery service platform are necessary to derive practical managerial insights.

This study makes three main contributions. First, we applied MCMP and modeled a profit maximization problem for a food delivery platform comprising multiple types of customers and couriers. Second, we suggested an approximation algorithm to solve this computationally burdensome problem and obtained optimal value for price, wage, and demand routing ratio. Finally, we obtained practical managerial insights that reflect the distance characteristics of customers and the transportation characteristics of couriers.

### **1.3 Organization of the Thesis**

This paper is organized as follows. Chapter 2 provides a brief overview of previous related literature. Chapter 3 introduces the modeling framework and Chapter 4 analyzes the model and shows the approximation algorithm that was used to solve the model. Chapter 5 states the results and main insights obtained from the numerical experiments. Finally, Chapter 6 summarizes this study and presents future research directions.

## Chapter 2

### Literature Review

This study combines two streams of literature : "on-demand platform" and "multiple types of customers and service providers". Through integration, we could model a more realistic framework that portrays the actual delivery platform. Moreover, analyzing the model enabled us to acquire some practical managerial insights.

#### 2.1 On-demand Platforms

The recent growth of various on-demand service platforms has evoked considerable research interest in determining internal operational issues. Our paper is relevant to coordinating self-scheduled service providers and highly variable demands. Cachon et al. [6] analyzed various contracts in ride-hailing and showed that near-optimal profits could be achieved through surge pricing. Bimpikis et al. [5] focused on ride-hailing platforms that have services upon networks of locations and investigated spatial price discrimination. Guda and Subramanian [11] studied a two-location setting where workers may move between them. Furthermore, they concentrated on the benefits of strategic pricing in the short term. Gurvich et al. [12] adopted a newsvendor-like model to examine the cost of self-scheduling capacity. Besbes et al. [4] proposed a two-dimensional framework that matched price-sensitive customers to variable supply units in a given geographical area for a revenue-maximizing plat-

form. In particular, our problem setting shares a lot in common with Taylor [19] and Bai et al. [3], both of which solved a profit maximization problem of an on-demand platform that has price- and time-sensitive customers and earning-sensitive service providers by coordinating supply and demand with price and wage. Moreover, they also incorporated queuing to reflect the waiting time on the rational behavior of customers. However, the studies above used a single type of customer and courier in their model. In contrast, this study integrated different delivery distances of customers and transportation modes of couriers in our model using MCMP system, which not only helped our model to capture actual delivery platforms more vividly but also enabled us to present more practical managerial insights.

## **2.2 Multiple Types of Customers and Service Providers**

Owing to the intrinsic nature of the market, customers are generally segmented, and the type and skills of servers vary to fulfill the heterogeneous customer. To account for such nature, there has been a large stream of literature dealing with multiple customers and service providers and is closely related to our study. Studies in this field focus on networks that involve multiple job types with different service times depending on the server types. Armony and Haviv [2] concentrated on a uniform pricing problem under competition between two firms where customers had different delay sensitivities. Ni et al. [16] studied the optimal price and service speed in customer-intensive services comprising heterogeneous customers in terms of intensity. Zhou et al. [21] examined an optimal uniform pricing problem regarding two classes of customers with different valuations about the service and different sensitivity to waiting. Zhong et al. [20] formulated a model that captures heterogeneous customers with



different congestion sensitivities and derived optimal strategies. Furthermore, they compared the model with the one that presumes customers are homogeneous and analyzed the need for classifying customers for the platform. de Véricourt and Zhou [8] used a Markov decision process to determine the optimal demand routing policy that minimizes the average total time of call resolution under various customer classes. Argon and Ziya [1] focused on a priority assignment policy that minimized long-run average waiting cost under two types of customers. Mehrotra et al. [15] considered a call center with various call types and heterogeneous agents and explored call routing strategies that would maximize resolution and minimize waiting time. Nourbakhsh and Turner [17] modeled a waiting time minimization problem with multiple job and server types. Various dynamic routing policies were implemented and compared with a static policy. However, these studies assumed that the number of servers is fixed and tried to control the demand side through pricing or routing policies. In contrast, our model successfully incorporates the endogenous dependency of courier capacity on wage so that actual supply movements in an on-demand platform are well captured.

# Chapter 3

## Problem

### 3.1 Problem Definition

This study determines the optimal service prices and wages per service regarding multiple types of customers and couriers to maximize profit for a food delivery platform. However, the setting of the food delivery industry may be further extended to other on-demand platforms or situations where customers and service providers each have multiple characteristics that can be grouped. The setting was fixed to a certain industry to illustrate our problem precisely.

Here, the platform coordinates randomly arriving customers with couriers using service price  $p$  and wage per service  $w$  that are pre-committed. However, the values may vary across different customer and courier types, and across different periods. We assume that customers are price- and time-sensitive so that a customer would decide whether to enter the platform depending on the announced service price and delivery time. Couriers are assumed as wage-sensitive so a courier can decide its entry upon the platform based on the announced wage per service. Therefore, we assume that all orders could be met by any couriers that are within the distance that can be traveled during food preparation time.

Additionally, to incorporate the multiple characteristics innate to customers and

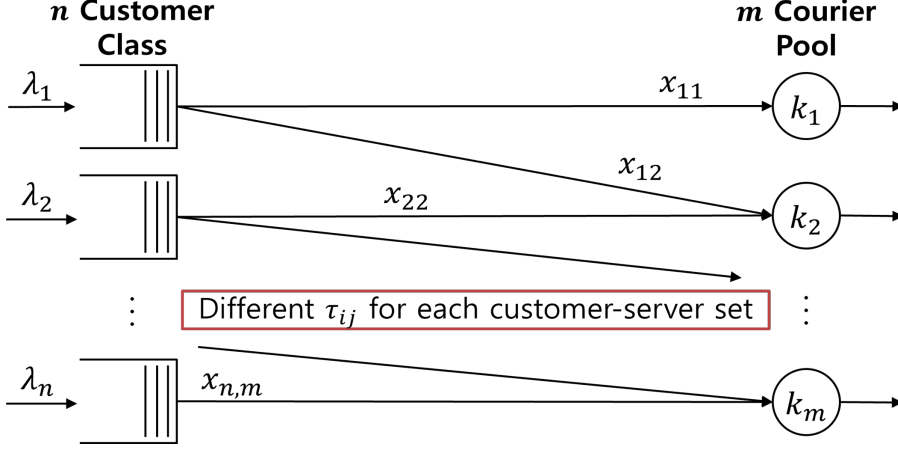


Figure 3.1: Schematic representation of food delivery platform

couriers, we considered this problem as a multi-class multi-pool system. A schematic representation of the system is shown in Fig 3.1. The customer classes and courier pools were classified based on the delivery distance  $i \in I$  and transportation modes  $j \in J$ , respectively. We assume that the demand of each customer class and the service time of each class-pool set follows  $Poisson(\lambda_i)$  and  $exp(\frac{1}{\tau_{ij}})$ , respectively.

Owing to the different types innate to customers and couriers, the demand of each customer class is allocated to each courier pool, and the allocated demand flow is denoted as  $x_{ij}$ . Depending on the problem setting, there may be constraints on feasible class-pool sets. Additionally, as each customer class can be served by multiple courier pools, its demand routing probability is driven by the ratio of optimal flow for each class-pool set. Therefore, demand routing probability of customer class 1 is  $\frac{x_{11}}{x_{11}+x_{12}}$  in Fig 3.1.

Table 3.1: Model Notation

---

<u>Indices and Sets</u>	
$i \in I$	Index for customer distance types; set of all customer classes
$j \in J$	Index for transportation modes of couriers; set of all courier pools
<u>Parameters</u>	
$c$	cost of waiting per unit time for customers
$\bar{\lambda}_i$	maximum potential demand rate of customer class $i$
$\bar{k}_j$	maximum number of potential courier pool $j$
$\tau_{ij}$	service time of courier pool $j$ serving customer class $i$
<u>Variables</u>	
$p_i$	price of service for customer class $i$
$W_i$	expected waiting time of customer class $i$
$\lambda_i$	demand rate of participating customer class $i$
$x_{ij}$	demand rate of customer class $i$ routed to courier pool $j$
$v_i$	value of service for a customer class $i$ . $v_i \sim F_{v_i}(\cdot)$
$w_j$	wage per service that courier pool $j$ get paid
$k_j$	number of participating couriers
$o_j$	opportunity cost per unit time. $o_j \sim G_{o_j}(\cdot)$
$r_j$	workload of courier pool $j$ . $\sum_{i \in I} \tau_{ij} x_{ij}$

---

### 3.1.1 Realized Customer Demand $\lambda_i$ and Price $p_i$

Each customer class has its maximum potential demand rate of  $\bar{\lambda}_i$  at a certain period. Each customer can decide whether to enter the platform by comparing their service valuation  $v_i$  with the service price and waiting time. The customer's value  $v_i$  follows a certain cumulative distribution function  $F_{v_i}(\cdot)$ , where  $F_{v_i}(\cdot)$  strictly increases.

For each customer, the utility of value subtracted from cost is gained at every service granted, which is expressed through the utility function as:

$$U_d(v_i) = v_i - (p_i + cW_i), \quad (3.1)$$

where  $c$  is the cost of waiting per unit time for customers and  $W_i$  is the waiting time for delivery in the case of customer class  $i$ . Assuming the customers enter the platform when their utility is non-negative, the realized demand of the platform may be represented as:

$$\lambda_i = \sum_{j \in J} x_{ij} = \text{Prob}\{U_d(v_i) \geq 0\} \cdot \bar{\lambda}_i = \{1 - F_{v_i}(p_i + cW_i)\} \cdot \bar{\lambda}_i, \quad (3.2)$$

where  $x_{ij}$  is the amount of demand of customer class  $i$  served by courier pool  $j$ . Therefore,  $\lambda_i$  could be expressed by the sum of  $x_{ij}$  upon  $j$ . Using (3.2) and the fact that  $v_i \sim F_{v_i}(\cdot)$ , the price  $p_i$  can be expressed as:

$$p_i = F_{v_i}^{-1} \left( 1 - \frac{\lambda_i}{\bar{\lambda}_i} \right) - cW_i = F_{v_i}^{-1} \left( 1 - \frac{\sum_{j \in J} x_{ij}}{\bar{\lambda}_i} \right) - cW_i. \quad (3.3)$$

### 3.1.2 Realized Customer Average Waiting Time $W_i$

Waiting time in this problem is identical to the sojourn time as the delivery service ends when food is handed to the customer, not when couriers pick up food and start

their delivery. Moreover, we exempted food preparation time from total waiting time assuming that the utility of customers does not decrease while the food is being prepared. Therefore, the elements that comprise the total waiting time would be *service time* and *waiting time within queue*. However, the service times can be ignored and only the waiting time in the queue can be considered to suit instances like Uber and taxis, where the service time is not added to the waiting time for customers.

In this food delivery platform setting, service times change depending on the class-pool sets; therefore, it is important to incorporate such changes. However, it is difficult to reflect this dynamicity in our problem setting, thereby indicating the importance of considering the mean service times and expected waiting times differently from Bai et al. [3]. Instead, considering the ever-changing service time in the mean value would simplify the problem significantly. Referring to Nourbakhsh and Turner [17], the mean service time is obtained by dividing workload, or the total time taken for service, by the number of served demands. Here, the workload of courier pool  $j$  is given as:

$$r_j = \sum_{i \in I} \tau_{ij} x_{ij}. \quad (3.4)$$

Thus, the mean service time of courier pool  $j$  is,

$$\sigma_j = \frac{r_j}{\sum_{i \in I} x_{ij}}. \quad (3.5)$$

For expected waiting time within a queue to be served by a courier pool, there has been extensive research throughout the case of the M/M/k queue. The probability

of delay can be modeled through the Erlang-C function (Cooper [7]), given as:

$$EC(k_j, r_j) = \frac{r_j^{k_j}}{(k_j - 1)!(k_j - r_j)} \cdot \left[ \sum_{n=0}^{k_j-1} \frac{r_j^n}{n!} + \frac{r_j^{k_j}}{(k_j - 1)!(k_j - r_j)} \right]^{-1}, \quad (3.6)$$

where  $k_j$  is the realized number of couriers in pool  $j$ . Additionally, Hokstad [13] derived the expected waiting time in the queue to be served by a courier pool  $j$  using (3.6),

$$EW \left( \sum_{j \in J} x_{ij}, r_j \right) = EC(k_j, r_j) \cdot \frac{r_j}{\sum_{i \in I} x_{ij} (k_j - r_j)}. \quad (3.7)$$

Therefore, the total waiting time of a customer class for the courier pool  $j$  would be the sum of (3.5) and (3.7). Finally, the expected total waiting time for customer class  $i$   $W_i$  can be derived using the demand routing ratio of customer class  $i$  for each courier pool  $j$  and the total waiting time for the courier pool  $j$ , given as:

$$W_i = \sum_{j \in J} \left\{ \left( \frac{x_{ij}}{\sum_{j \in J} x_{ij}} \right) \cdot \left( EW \left( \sum_{j \in J} x_{ij}, r_j \right) + \sigma_j \right) \right\}. \quad (3.8)$$

### 3.1.3 Realized Courier Supply $k_j$ and Wage $w_j$

When customers choose to enter the platform, an order is placed and routed to a courier pool. For each courier pool, there is a maximum number of potential couriers  $\bar{k}_j$  for a certain period. Similarly, couriers can decide whether to enter the platform by weighing their opportunity cost  $o_j$  with earnings per unit of time. Moreover, opportunity cost  $o_j$  follows a certain cumulative distribution function  $G_{o_j}(\cdot)$ , where  $G_{o_j}(\cdot)$  strictly increases.

In the case of couriers, we assume that couriers enter the platform when the opportunity cost is smaller than earnings per unit of time. Therefore, utilization of

courier pool  $j$  for a certain period is given as follows:

$$\beta_j = \text{Prob} \{U_s(o_j) \geq 0\} = \text{Prob} \left\{ o_j \leq w_j \left( \sum_{i \in I} x_{ij} / k_j \right) \right\} = G_{o_j} \left( w_j \left( \sum_{i \in I} x_{ij} / k_j \right) \right), \quad (3.9)$$

where  $x_{ij}$  is the amount of demand of customer class  $i$  served by courier pool  $j$ .

Therefore,  $k_j$ , the realized number of participating couriers in pool  $j$ , is given as:

$$k_j = \beta_j \bar{k}_j = G_{o_j} \left( w_j \left( \frac{\sum_{i \in I} x_{ij}}{\bar{k}_j} \right) \right) \bar{k}_j. \quad (3.10)$$

Through manipulation of (3.10), wage per service for courier pool  $j$  can be expressed as follows:

$$w_j = G_{o_j}^{-1}(\beta_j) \frac{k_j}{\sum_{i \in I} x_{ij}} = G_{o_j}^{-1} \left( \frac{k_j}{\bar{k}_j} \right) \frac{k_j}{\sum_{i \in I} x_{ij}} \quad (3.11)$$

The wage per service  $w_j$  considered is an average value for each courier pool. However, if a difference in wage can be identified for serving different customer classes, each wage for serving each customer class can be acquired. Nonetheless, for analytic simplicity, we only consider changes upon average wage value.

## 3.2 Problem Formulation

The total profit of the platform is generally denoted as  $\pi(\mathbf{x}, \mathbf{p}, \mathbf{w}) = \sum_{i \in I, j \in J} x_{ij}(p_i - w_j)$ .

However, we assumed the demand and supply were endogenous considering they were set dependent upon price and wage, respectively. This indicates that the price and wage could be in turn expressed as a function of demand and supply. Such interactions between demand and price and supply and wage were derived through (3.3), (3.8), and (3.11). Substituting price and wage with (3.3), (3.8), and (3.11), thereby



enabling the profit function  $\pi$  to be defined by  $(\mathbf{x}, \mathbf{k})$ :

$$\begin{aligned} \pi(\mathbf{x}, \mathbf{k}) = & \sum_{i \in I} x_{ij} \left[ F_{v_i}^{-1} \left( 1 - \frac{\sum_{i \in I} x_{ij}}{\bar{\lambda}_i} \right) \right. \\ & \left. - c \sum_{j \in J} \left\{ \left( \frac{x_{ij}}{\sum_{j \in J} x_{ij}} \right) \cdot \left( EW \left( \sum_{j \in J} x_{ij}, r_j \right) + \sigma_j \right) \right\} \right. \\ & \left. - G_{o_j}^{-1} \left( \frac{k_j}{\bar{k}_j} \right) \frac{k_j}{\sum_{i \in I} x_{ij}} \right]. \end{aligned} \quad (3.12)$$

Hence, the profit maximization model would be formulated as below:

$$\begin{aligned} & \text{maximize} \quad \pi(\mathbf{x}, \mathbf{k}) \\ & \text{subject to} \quad \sum_{i \in I} \tau_{ij} x_{ij} \leq k_j \quad \forall j \in J \end{aligned} \quad (3.13)$$

$$\sum_{j \in J} x_{ij} \leq \bar{\lambda}_i \quad \forall i \in I \quad (3.14)$$

$$k_j \leq \bar{k}_j \quad \forall j \in J \quad (3.15)$$

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in J \quad (3.16)$$

$$k_j \geq 0 \quad \forall j \in J. \quad (3.17)$$

Constraint (3.13) ensures that the workload of each courier pool does not exceed the number of realized couriers, which guarantees that the system is stable and queues do not explode. Constraint (3.14) assures that the total realized demand of each customer class  $i$  does not exceed the maximum potential demand of that class, considering demands are endogenously determined through observation of the service price. Constraint (3.15) states that the number of participating couriers for each transportation mode  $j$  should not surpass the maximum potential number of couriers of that mode. Finally, Constraints (3.16) and (3.17) ensure all decision

variables are non-negative.

## Chapter 4

### Model : Two-class Two-pool system

To solve this problem, a compact case is dealt with in advance. This case comprises two types of customer distance (i.e. close and distant) and two types of courier transportation modes (i.e. on-foot and motorcycle). As shown in Fig. 4.1, couriers on foot cannot travel long distances whereas those on motorcycles can reach any distance. Therefore, demand from close customers may be routed to either couriers on foot or motorcycles. Contrastingly, demand from distant customers is only handled by couriers on motorcycles.

#### 4.1 Mean service time and Expected waiting time

In this compact case, two types of service times occur as courier pool 2, i.e., couriers on motorcycles handle two types of customer classes. Therefore, instead of using two types of service time  $\tau_{11}$  and  $\tau_{12}$  individually upon each customer, mean service time is used. By adopting (3.5), the mean service time of courier pool 2 would be  $\frac{\tau_{12}x_{12}+\tau_{22}x_{22}}{x_{12}+x_{22}}$  whereas mean service time of courier pool 1, i.e., couriers on-foot would be  $\tau_{11}$ .

In the case of close customers, the customers face two different service times from two different courier types. When they are served by couriers on-foot, the customers experience a service time of  $\tau_{11}$ . Conversely, customers go through a service time of

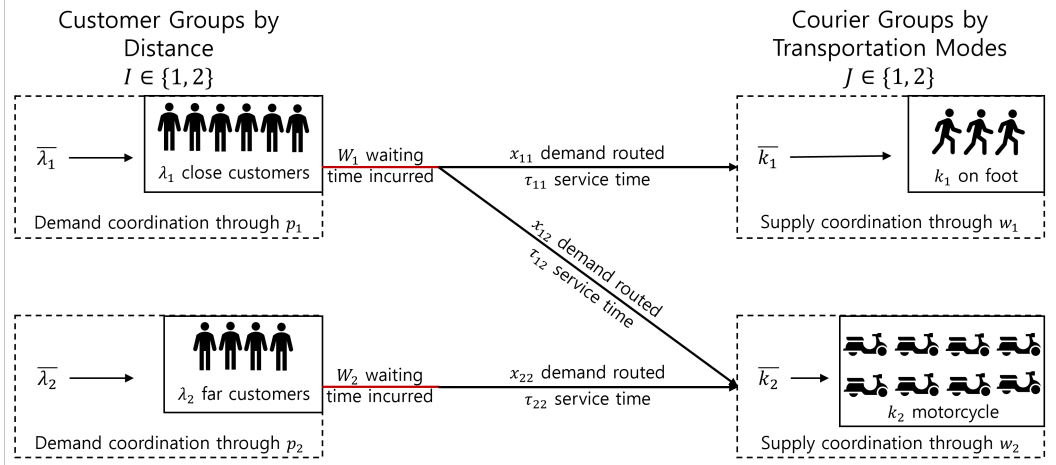


Figure 4.1: Schematic representation of two-class two-pool system

$\tau_{12}$  when served by couriers on the motorcycle. Therefore, the expected waiting time that customer class 1, i.e., close customers encounter can be modeled by applying (3.5) and (3.7) as:

$$W_1 = \left( \frac{x_{11}}{x_{11} + x_{12}} \right) \cdot [EW(x_{11}, r_1) + \tau_{11}] + \left( \frac{x_{12}}{x_{11} + x_{12}} \right) \cdot \left[ EW(x_{12} + x_{22}, r_2) + \frac{\tau_{12}x_{12} + \tau_{22}x_{22}}{x_{12} + x_{22}} \right]. \quad (4.1)$$

Likewise, the waiting time for customer class 2, i.e., distant customers can be modeled as:

$$W_2 = EW(x_{12} + x_{22}, r_2) + \frac{\tau_{12}x_{12} + \tau_{22}x_{22}}{x_{12} + x_{22}}. \quad (4.2)$$

## 4.2 Model Formulation

For simple analysis, assumptions were made on the distributions of customer's value  $v_i$  and courier's opportunity cost  $o_j$ . We assume that all customer values and courier opportunity costs follow uniform distribution for each customer class and courier

pool. The range of the specified distributions are as follows:  $v_1 \sim U[\alpha, \beta]$ ,  $v_2 \sim U[\gamma, \sigma]$ ,  $o_1 \sim U[\epsilon, \zeta]$ ,  $o_2 \sim U[\eta, \theta]$ . The parameters of uniform distribution may vary considering the analytical and numerical results in this study can be extended to more general instances; therefore, any value may be fitted in the parameters of the distribution.

Using the above assumptions and the derivation of expected waiting times, the functions of price, wage, and profit of (3.3), (3.11), (4.7) can be defined as follows:

$$p_1 = \left\{ \alpha + (\beta - \alpha) \cdot \left( 1 - \frac{x_{11} + x_{12}}{\bar{\lambda}_1} \right) \right\} - c \left[ \left( \frac{x_{11}}{x_{11} + x_{12}} \right) \cdot (EW(x_{11}, r_1) + \tau_{11}) + \left( \frac{x_{12}}{x_{11} + x_{12}} \right) \cdot \left( EW(x_{12} + x_{22}, r_2) + \frac{\tau_{12}x_{12} + \tau_{22}x_{22}}{x_{12} + x_{22}} \right) \right], \quad (4.3)$$

$$p_2 = \left\{ \gamma + (\sigma - \gamma) \cdot \left( 1 - \frac{x_{22}}{\bar{\lambda}_2} \right) \right\} - c \left( EW(x_{12} + x_{22}, r_2) + \frac{\tau_{12}x_{12} + \tau_{22}x_{22}}{x_{12} + x_{22}} \right), \quad (4.4)$$

$$w_1 = \left\{ \epsilon + (\zeta - \epsilon) \cdot \frac{k_1}{\bar{k}_1} \right\} \cdot \frac{k_1}{x_{11}} \quad (4.5)$$

$$w_2 = \left\{ \eta + (\theta - \eta) \cdot \frac{k_2}{\bar{k}_2} \right\} \cdot \frac{k_2}{x_{12} + x_{22}} \quad (4.6)$$

$$\begin{aligned}
\pi(\mathbf{x}, \mathbf{k}) = & (x_{11} + x_{12}) \left\{ \alpha + (\beta - \alpha) \cdot \left( 1 - \frac{x_{11} + x_{12}}{\lambda_1} \right) \right\} \\
& + x_{22} \left\{ \gamma + (\sigma - \gamma) \cdot \left( 1 - \frac{x_{22}}{\lambda_2} \right) \right\} \\
& - k_1 \left\{ \epsilon + (\zeta - \epsilon) \cdot \frac{k_1}{k_1} \right\} - k_2 \left\{ \eta + (\theta - \eta) \cdot \frac{k_2}{k_2} \right\} \\
& - cx_{11} \left\{ \frac{r_1^{k_1}}{(k_1-1)!(k_1-r_1)} \cdot \left[ \sum_{n=0}^{k_1-1} \frac{r_1^n}{n!} + \frac{r_1^{k_1}}{(k_1-1)!(k_1-r_1)} \right]^{-1} \right. \\
& \quad \left. \times \frac{r_1}{x_{11}(k_1-r_1)} + \tau_{11} \right\} \\
& - c(x_{12} + x_{22}) \left\{ \frac{r_2^{k_2}}{(k_2-1)!(k_2-r_2)} \cdot \left[ \sum_{n=0}^{k_2-1} \frac{r_2^n}{n!} + \frac{r_2^{k_2}}{(k_2-1)!(k_2-r_2)} \right]^{-1} \right. \\
& \quad \left. \times \frac{r_2}{(x_{12}+x_{22})(k_2-r_2)} + \frac{\tau_{12}x_{12} + \tau_{22}x_{22}}{x_{12}+x_{22}} \right\} \quad (4.7)
\end{aligned}$$

Hence, optimization problem below must be solved:

$$(P1) \quad \text{maximize} \quad (4.7)$$

subject to Constraints (3.13), (3.14), (3.15), (3.16), (3.17)

### 4.3 Approximation Scheme

Before using various concave optimization techniques to determine the optimal set of  $(\mathbf{x}, \mathbf{k})$ , the concavity of the model must be considered. (4.7) is the element-wise concave regarding  $x_{ij}$  according to Grassmann [10] under fixed  $\tau_{ij}$  and  $k_j$ . However, showing the concavity of  $k_j$  is difficult considering the expected waiting time function incorporates factorials and summations of  $k_j$ , making it impossible to show its hessian. Therefore, we use an approximation of the expected waiting time function proposed by Sakasegawa [18], which provides a solid estimate of (3.7) when  $k \geq 1$ :

$$E\hat{W}(x_{12} + x_{22}, r_j) = \frac{\rho_j^{\sqrt{2(k_j+1)}}}{(x_{12} + x_{22})(1 - \rho_j)}, \quad (4.8)$$

where  $\rho_j = \frac{r_j}{k_j}$  represents system utilization. Additionally, the system utilization  $\rho_j$  cannot exceed 1 owing to the constraint (3.13), which ensures that the system does not overflow.

Using the above, equation (4.7) can be simplified:

$$\begin{aligned} \pi(\mathbf{x}, \mathbf{k}) = & (x_{11} + x_{12}) \left\{ \alpha + (\beta - \alpha) \cdot \left( 1 - \frac{x_{11} + x_{12}}{\bar{\lambda}_1} \right) \right\} \\ & + x_{22} \left\{ \gamma + (\sigma - \gamma) \cdot \left( 1 - \frac{x_{22}}{\bar{\lambda}_2} \right) \right\} \\ & - k_1 \left\{ \epsilon + (\zeta - \epsilon) \cdot \frac{k_1}{\bar{k}_1} \right\} - k_2 \left\{ \eta + (\theta - \eta) \cdot \frac{k_2}{\bar{k}_2} \right\} \\ & - c \left( \frac{\rho_1^{\sqrt{2(k_1+1)}}}{1 - \rho_1} + r_1 \right) - c \left( \frac{\rho_2^{\sqrt{2(k_2+1)}}}{1 - \rho_2} + r_2 \right). \quad (4.9) \end{aligned}$$

Therefore, the final optimization problem that should be solved is:

$$(P2) \quad \text{maximize} \quad (4.9)$$

$$\text{subject to} \quad \text{Constraints (3.13), (3.14), (3.15), (3.16), (3.17)}$$

The joint concavity of  $\mathbf{x}$  under fixed  $\mathbf{k}$  can be shown using the property of convexity mentioned in Theorem 1. Therefore, defining  $g(t) = \pi([\mathbf{x} + t\mathbf{y}])$  and obtaining

its second derivative regarding  $t$ :

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial t^2} = & -c \left( \frac{\left\{ 2(k_1 + 1) - \sqrt{2(k_1 + 1)} \right\} y_1^2 \rho_1^{\sqrt{2(k_1+1)}}}{(x_{11} + ty_1)^2 (1 - \rho_1)} \right. \\
& + \frac{2\sqrt{2(k_1 + 1)} \tau_{11} y_1^2 \rho_1^{\sqrt{2(k_1+1)}}}{k_1 (x_{11} + ty_1) (1 - \rho_1)^2} + \frac{2(\tau_{11} y_1)^2 \rho_1^{\sqrt{2(k_1+1)}}}{k_1^2 (1 - \rho_1)^3} \\
& + \frac{\left\{ 2(k_2 + 1) - \sqrt{2(k_2 + 1)} \right\} (y_2 + y_3)^2 \rho_2^{\sqrt{2(k_2+1)}}}{(x_{12} + ty_2 + x_{22} + ty_3)^2 (1 - \rho_2)} \\
& + \frac{2\sqrt{2(k_2 + 1)} (\tau_{12} y_2 + \tau_{22} y_3)^2 \rho_2^{\sqrt{2(k_2+1)}}}{k_2 (x_{12} + ty_2 + x_{22} + ty_3) (1 - \rho_2)^2} \\
& + \frac{2(\tau_{12} y_2 + \tau_{22} y_3)^2 \rho_2^{\sqrt{2(k_2+1)}}}{k_2^2 (1 - \rho_2)^3} \\
& \left. - \frac{2(\beta - \alpha)(y_1 + y_2)^2}{\lambda_1} - \frac{2(\delta - \gamma)y_3^2}{\lambda_2} \right), \quad (4.10)
\end{aligned}$$

its negativity is evident from the above.

The joint concavity of  $\mathbf{k}$  under fixed  $\mathbf{x}$  can be shown through the hessian matrix of (4.9). Second derivatives of  $k_j$ :

$$\begin{aligned}
\frac{\partial^2 \pi}{\partial k_j^2} = & -c \left\{ \frac{\rho_j^{\sqrt{2(k_j+1)}} \left( \frac{\ln \rho_j}{\sqrt{2(k_j+1)}} - \frac{\sqrt{2(k_j+1)}}{k_j} \right)^2}{1 - \rho_j} \right. \\
& - \frac{2r_j \rho_j^{\sqrt{2(k_j+1)}} \left( \frac{\ln \rho_j}{\sqrt{2(k_j+1)}} - \frac{\sqrt{2(k_j+1)}}{k_j} \right)}{(1 - \rho_j)^2 k_j^2} \\
& + \frac{\rho_j^{\sqrt{2(k_j+1)}} \left( \frac{\sqrt{2(k_j+1)}}{k_j^2} - \frac{\sqrt{2}}{k_j \sqrt{k_j+1}} - \frac{\ln \rho_j}{\{2(k_j+1)\}^{\frac{3}{2}}} \right)}{1 - \rho_j} \\
& \left. + \frac{2r_j \rho_j^{\sqrt{2(k_j+1)}}}{(1 - \rho_j)^2 k_j^3} + \frac{2r_j^2 \rho_j^{\sqrt{2(k_j+1)}}}{(1 - \rho_j)^3 k_j^4} - \frac{2}{k_j} \right), \quad (4.11)
\end{aligned}$$



is always non-positive. This can be easily seen based on the fact that  $\rho_j < 1$ , which is ensured by constraint (3.13). Moreover, (4.11) does not contain joint functions of  $k_j$ , which guarantees the hessian matrix of (4.9) to be a diagonal matrix that is negative semi-definite. Therefore, the objective function of the proposed model is concave to  $\mathbf{x}$  and  $\mathbf{k}$ .

#### 4.4 Approximation Algorithm

The maximization problem ( $P2$ ) that we face is nonlinear programming, in which both objective function and constraints are nonlinear. As computational complexity significantly increases with only one nonlinear constraint, the computational burden of ( $P2$ ) is significant for solving commercial convex optimization programs. Furthermore, variables in this problem tend to increase at a high rate as scale increases and nonlinear constraint is added whenever there is an extra server pool. Therefore, it is important to find an algorithm that could handle this problem even when there are many classes and pools.

To address the aforementioned problem, we propose an approximation algorithm that uses Newton's method to achieve the optimal value. Each  $x_{ij}$  is updated via Newton's method, and optimal  $\mathbf{k}^*$  under given  $\mathbf{x}$  is obtained at every update. For any  $x_{ij}$  that turns negative after the update, it is bounded to the smallest positive value possible. Other  $x_{ij}$  keep updating till the Euclidean distance between  $\mathbf{x}$  of the present and the previous is smaller than  $\epsilon = e^{-5}$ . After obtaining optimal  $\mathbf{k}^*$ , the integer set that derives the largest objective value is determined by exploring every possible integer  $\hat{\mathbf{k}}$  around optimal  $\mathbf{k}^*$ .

The pseudo-code of the approximation algorithm is as follows.

---

**Algorithm 1** Approximation Algorithm ( $P2, \mathbf{x}, \mathbf{k}$ )

---

$t \leftarrow 0$ ,  
 $\mathbf{x}^0 \leftarrow \mathbf{1}$ ;  
**while**  $\|\mathbf{x}^{t-1} - \mathbf{x}^t\|_2 \geq \epsilon$  and  $|\pi^{t-1} - \pi^t| \geq \epsilon$  and  $t \leq T$  **do**  
    solve ( $P2$ ) using  $\mathbf{x}^{t-1}$  and obtain solution  $\mathbf{k}^{t-1}$ ;  
    **if**  $x_{ij}^{t-1} > 0$  **then**  
        Newton Update on  $x_{ij}^{t-1}$ ;  
        **foreach**  $i \in I, j \in J$  **do**  
            **if**  $x_{ij}^t < 0$  **then**  
                bound  $x_{ij}^t$  to smallest positive value  
            **end if**  
        **end for**  
    **end if**  
    solve ( $P2$ ) using  $\mathbf{x}^t$  and obtain solution  $\mathbf{k}^t$ ;  
     $t \leftarrow t + 1$ ;  
**end while**  
**return** feasible solution for  $\mathcal{P}$ ;

---

## 4.5 Scalability

Thus far, we have only analyzed the two-class two-pool case. However, the algorithm proposed can be easily adopted in the general case of  $n$  class  $m$  pool. Generalizing the profit maximization model, the objective function can be expressed as:

$$\begin{aligned} \pi(\mathbf{x}, \mathbf{k}) = \sum_{i \in I} \left\{ \left( \sum_{j \in J} x_{ij} \right) \cdot F_{v_i}^{-1} \left( 1 - \frac{\sum_{j \in J} x_{ij}}{\bar{\lambda}_i} \right) \right\} - \sum_{j \in J} k_j G_{o_j}^{-1} \left( \frac{k_j}{\bar{k}_j} \right) \\ - c \sum_{j \in J} \left\{ \frac{\rho_j^{\sqrt{2(k_j+1)}}}{(1 - \rho_j)} + \sum_{i \in I} \tau_{ij} x_{ij} \right\} \quad (4.12) \end{aligned}$$

Under the assumption of uniform distribution, it is clear that the objective function is a linear combination of concave functions. This is supported by the concavity proofs derived above. Therefore, an increase in the scale of the model would only be shown as an addition of concave functions. In other words, the proposed approximation

algorithm can be applied regardless of the scale of the problem.

# Chapter 5

## Numerical Experiments

### 5.1 Two-class Two-pool

For simpler analysis, numerical experiments were initially done on two-class two-pool case. Our objective was to check the movements of important indicators as parameters changed and subsequently derive managerial insights from them. The parameters that changed were the waiting cost  $c$ , maximum customer class size  $\bar{\lambda}_i$ , and maximum courier pool size  $\bar{k}_j$ ; the indicators we focused on were the profit value  $\pi$ , demand routing probability  $\frac{x_{11}}{x_{11}+x_{12}}$ , realized customer demand  $\lambda_i$ , realized courier supply  $k_j$ , optimal price  $p_i^*$ , and the optimal wage per service  $w_j^*$ . For this set of numerical experiment, we set  $\tau_{11} = \frac{1}{3}, \tau_{12} = \frac{1}{5}, \tau_{22} = \frac{1}{4}, v_1 = U[0.5, 0.7], v_2 = U[0.6, 0.8], o_1 = U[0.7, 1.0], o_2 = U[1.0, 1.5], \sum_{i \in I} \bar{\lambda}_i = 400, \sum_{j \in J} \bar{k}_j = 100$ . Here, customer class 1 and 2 were referred to as *close* and *distant* customers, respectively. Additionally, courier pool 1 was indicated as *on-foot* couriers whereas the courier pool was *motor* couriers. Based on the numerical experiments, important managerial insights were acquired.

When  $c$  ranged from 0.02 to 0.4 and the other parameters were set as  $\bar{\lambda}_1 = 160, \bar{\lambda}_2 = 240, \bar{k}_1 = 30, \text{ and } \bar{k}_2 = 70$ , some counter-intuitive results were driven. To begin with, as shown in Fig 5.1 (e), the number of realized couriers decreased as the

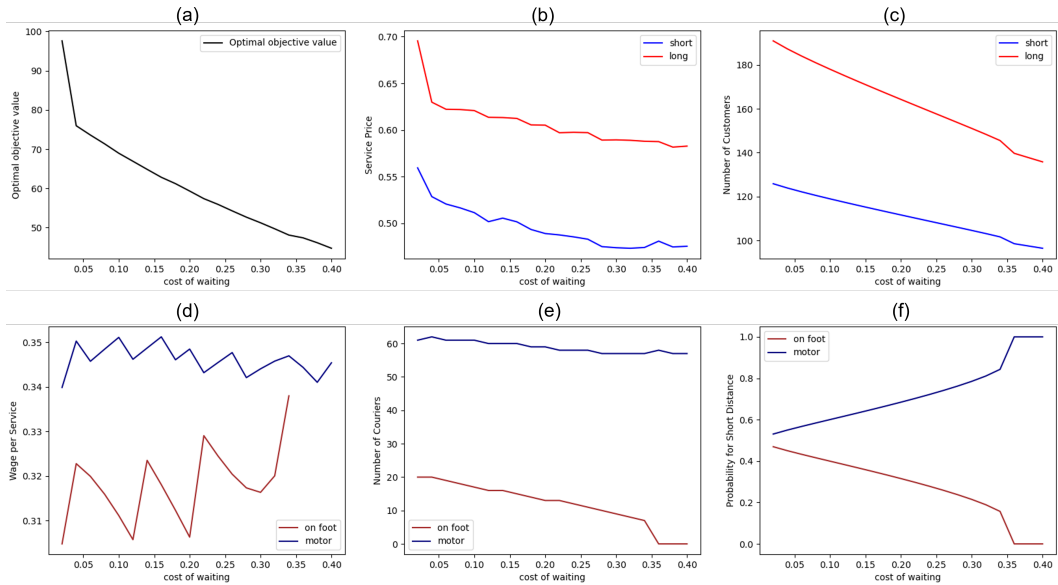


Figure 5.1: Movements of indicators under changes in the cost of waiting

cost of waiting increased. Furthermore, reducing waiting time by employing more couriers was an appropriate measure for increasing the profit of a platform when the cost of waiting increased. However, the service price was significantly lower than the minimum wage of couriers in delivery platforms owing to the nature of the industry. Therefore, it is more economical for the platform to reduce their extra employment cost than to reduce waiting time for customers through more employment. In other words, platforms should offer a lower wage per service  $w_j$  to employ fewer couriers as  $c$  increases.

Nonetheless, the wage per service of on-foot couriers  $w_2$  tends to increase as  $c$  increases. As a result, motor couriers cross-service close customers to reduce the waiting time for customers. Owing to this, the amount of work that on-foot couriers service decreases. Therefore, the wage per service for on-foot couriers must be increased to match the high minimum wage when there is less work. This movement

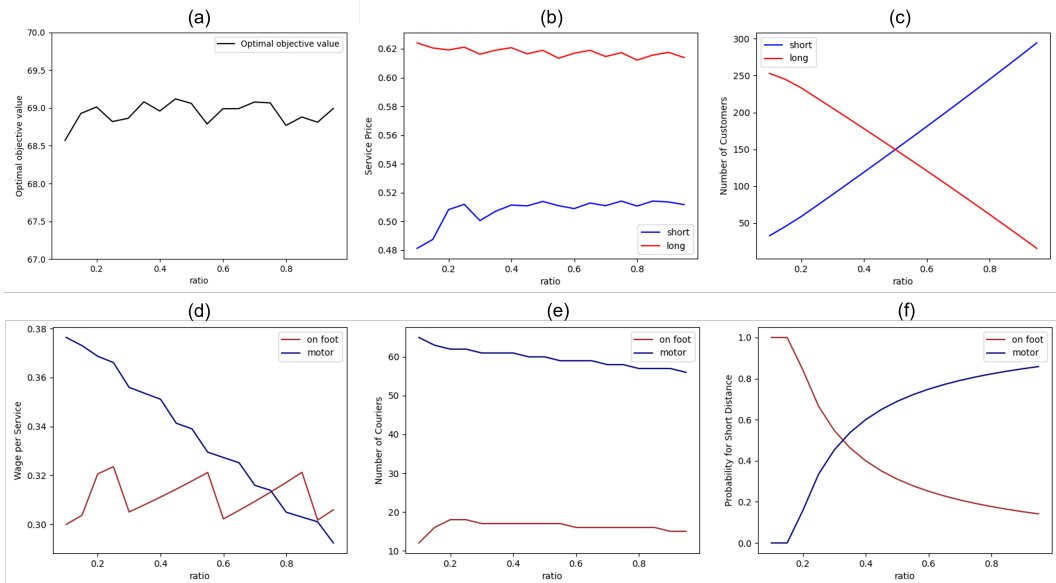


Figure 5.2: Movements of indicators under changes in maximum potential demand ratio.  $\bar{k}_1 = 30$ ,  $\bar{k}_2 = 70$ .

can be clearly seen in Figs 5.1 (d) and (f), where the probability of demand being served by on-foot couriers decreases whereas wage per service for on-foot couriers increases. Therefore, the platform should lower the overall expected earnings per unit time for the couriers as  $c$  increases, while there may be a slight increase in wage per service owing to the routing of demands.

In contrast, other indicators showed intuitive results where profit, price, and realized customers decrease as the cost of waiting  $c$  increases. The decrease in profit and realized customers is obvious as fewer customers enter the platform owing to higher waiting penalties. Simultaneously, the platform will lower the price to bring in customers as much as possible.

In a different experiment setting, the maximum potential demand ratio  $\frac{\bar{\lambda}_1}{\lambda_1 + \lambda_2}$  ranged from 0.1 to 0.95 and the waiting cost was fixed at  $c = 0.1$ . For courier pool

parameters, we divided it into two cases where case 1 had on-foot couriers as the majority while case 2 had motor couriers as the majority. For case 1, we set  $\bar{k}_1 = 30$ ,  $\bar{k}_2 = 70$ . First, the platform should have a sufficient size of maximum potential motor couriers to be guaranteed a certain level of profit. As shown in Fig 5.2 (a), the profit of the platform remained independent of changes in the demand ratio when there were more motor couriers than on-foot couriers.

Second, the platform reduced the wage per service for motor couriers as the demand for close customers increased. A steep decrease in wage per service for motor couriers can be seen in Fig 5.2 (d) owing to an increase in the workload for motor couriers as cross-service of close customers increases, as shown in Fig 5.2 (f). Although the workload increased, a steep decrease in wage per service decreased the overall earnings per unit time. Therefore, the number of realized motor couriers decreased (Fig 5.2 (e)). However, this is reasonable as the platform does not have to maintain a high employment rate of expensive motor couriers just to serve relatively cheap close customer demands when the demands of distant customers decrease.

However, movements in the service price for distant customers and the wage of on-foot couriers are not monotonic to demand ratio changes but they appear to oscillate. The oscillation of service price of the distant customers in Fig 5.2 (b) is due to an increase in the cross-service of close customers, which eliminates the need for the platform to draw more distant customers. Therefore, our numerical results indicate that the platform should maintain its high price for distant customers as the proportion of distant customers decreases when there are more motor couriers than on-foot couriers.

Additionally, the oscillation of the wage of on-foot couriers in Fig 5.2 (d) is a

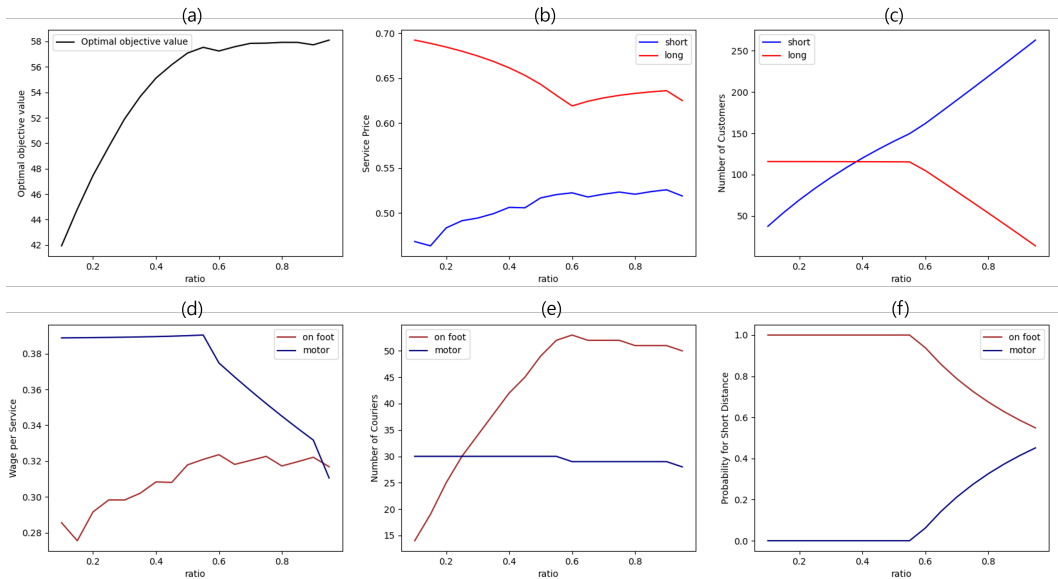


Figure 5.3: Movements of indicators under changes in maximum potential demand ratio.  $\bar{k}_1 = 70$ ,  $\bar{k}_2 = 30$ .

result of two simultaneous actions. The workload of on-foot couriers increases owing to an increase in maximum potential close customer size. However, simultaneously, motor couriers mitigate the burden by cross-services as the proportion of close customers increases. Therefore, the low variance of workload per on-foot courier results in the wage being fairly steady, which also maintains consistency for the number of realized on-foot couriers. In other words, the platform must maintain its wage per service level for on-foot couriers as the number of distant customers decreases when there are more motor couriers than on-foot couriers.

Apart from this factor, there are only intuitive results that showed an increase in the service price for close customers owing to an increase in the proportion of maximum potential close customers and the number of realized customers proportional to the maximum potential customer sizes.



For case 2 where the parameters are set as  $\bar{k}_1 = 70$ ,  $\bar{k}_2 = 30$ , portrayed profit increased as close customer ratio increased, unlike case 1 (Fig 5.3 (a)). This is clear considering on-foot couriers cannot serve distant customers, unlike motor couriers. However, the platform should change its action at the point where cross-service occurs as the demand ratio changes when there are more on-foot couriers than motor couriers.

The non-monotonic movement was seen in the service price for distant customers in Fig 5.3 (b). Initially, the service price for distant customers decreased considering the price necessary to control the level of demand decreases as the maximum potential distant customer size decreases. However, at the point where motor couriers cross-service close customers, distant service price increases to adjust the distant customer level so that motor couriers can handle both close and distant customers. Therefore, in Fig 5.3 (c), we observed an increase in realized close customers and decrease in realized distant customers at the point of cross-service.

Coupled with price changes, the wage per service for motor couriers and the number of realized on-foot couriers show non-monotonic movements. At first, as the maximum potential distant customer size is large compared to the maximum potential motor courier size, the constraint is on the supply side. Therefore, at first, the wage for motor couriers maintains its level as no changes are observed in the realized motor courier numbers and distant customers. Then, at the point where cross-service is engaged, the wage of motor couriers decreases with an increase in the workload (Fig 5.3 (d)).

The non-monotonicity of the number of realized on-foot couriers shown in Fig 5.3 (e) is due to cross-service. Before cross-service, the number of on-foot couriers

continuously increases owing to an increase in the ratio of close customers. However, cross-service alleviates their workload, eventually causing the number of on-foot couriers to decrease. Furthermore, the point where non-monotonic movements start is identical to the point where demand routing probability changes (Fig 5.3 (f)). Therefore, the platform needs to review its actions when cross-service takes place.

Furthermore, only intuitive results that showed an increase in the service price for close customers and the wage for on-foot couriers as the proportion of the maximum potential close customers increases, are driven.

## 5.2 Real Data Implementation

Actual data from a food delivery platform in China were collected from Mao et al. [14] to emulate real-life situations and obtain realistic optimal values upon price and wage. This data contained the delivery data from Hangzhou, China from July 1 to August 31, 2015, which included information about order placements, order deliveries, restaurants, drivers, weather and traffic conditions, and more. However, because we were not familiar with China's food delivery platform, we adopted many features from a Korean food delivery platform, Baemin. Considering Hangzhou and Seoul have many urban features like dynamicity and high population density in common, we only used the population and delivery data from Hangzhou, and the platform framework was adopted from Baemin for numerical analysis. We assumed there are three types of couriers (on-foot, bicycle, and motorcycle), each having a different delivery range. For instance, on-foot couriers can only deliver orders that have a delivery distance of under 1 km, whereas a bicycle delivery range is within 2 km. Finally, a motorcycle has no boundaries. Similarly, we divided customer class

into three according to their delivery distances. Therefore, close, middle, and distant customer class refers to distance between 0-1 km, 1-2 km, and over 2 km. Under this framework, we compared peak and non-peak hours upon the optimal value of price and wage using the proposed approximate algorithm.

During the experiment, we updated each customer class' maximum potential size and service time of couriers every hour. While the update on customer class was fairly reasonable, the update on service time was to convey the different traffic throughout the day. Conversely, the parameters of the distribution of service value, opportunity cost, and the maximum potential size of each courier pool were fixed regardless of the time. Parameters of distribution of service value for each customer class were set depending on the average service price and distance surcharge rate. Parameters of distribution of opportunity cost for each courier pool were set according to the minimum wage, average fuel cost per hour, and insurance cost per hour. Finally, we set the value of waiting cost between 0-5000 won. According to Gomez-Ibanez et al. [9], a working class passenger in San Francisco quantifies one's waiting cost to be 195% of their after-tax wages. We provided a range based on this study and income inequality prevalent in Korea. Additionally, we recalibrated the range by combining the fact that the average delivery price is approximately 12% of the delivery food price.

Using the parameter setting above, we obtained the values of optimal price and wage under the range of the waiting cost. As a result, peak hours had higher values of optimal price and wage, as shown in Figs 5.4 and 5.5, considering the peak hours have a greater size in maximum potential customer class and longer service time owing to traffic in peak hours.

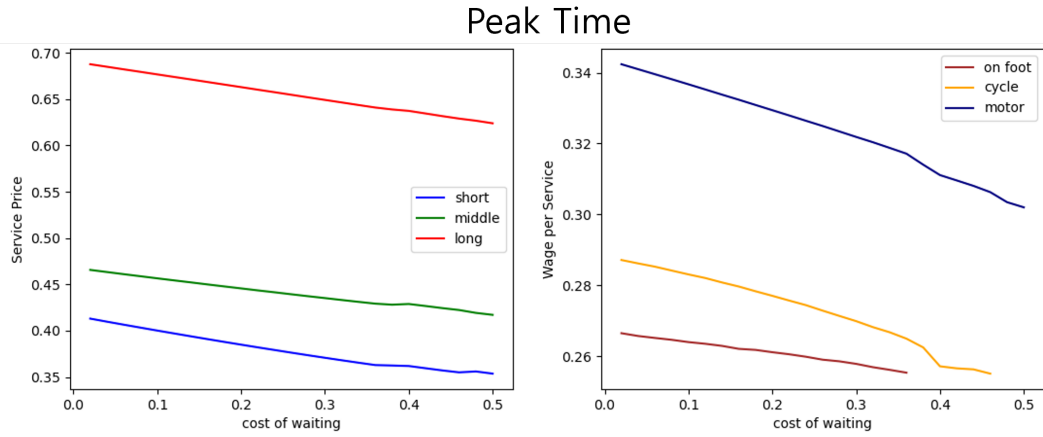


Figure 5.4: Values of optimal price and wage under changes in the cost of waiting during peak hours

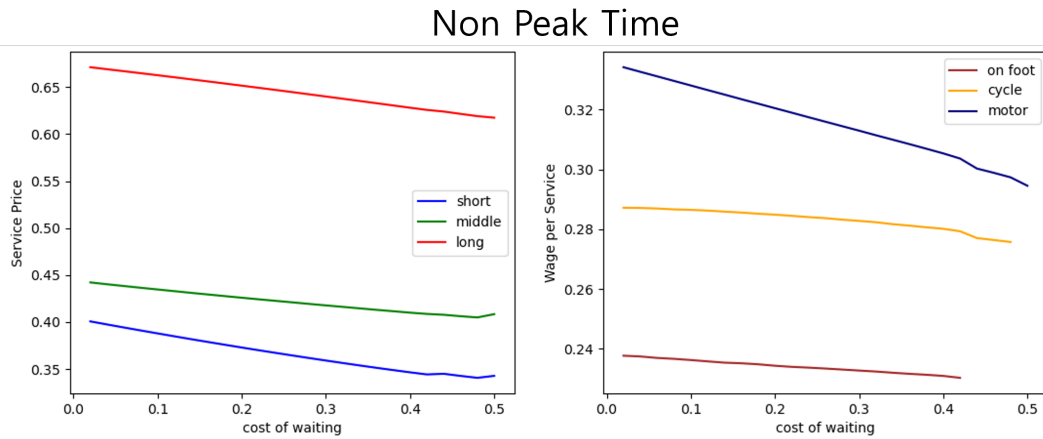


Figure 5.5: Values of optimal price and wage under changes in the cost of waiting during peak hours

The numerical results indicate that higher prices and wages should be charged during rush hours or peak times when customer demands are high and service time is low owing to traffic. These results go hand in hand with what is being practiced in real delivery platforms, which usually run a promotion on wage per delivery to engage more couriers. These results not only validate our results but also prove that our model can be used as a guideline for crowdsourced delivery platforms in increasing their profitability.

# Chapter 6

## Conclusion

### 6.1 Summary

In this study, we modeled a food delivery platform having multiple types of customers and couriers. To incorporate the unique characteristics that customers and couriers have in a food delivery platform, we used multi-class multi-pool system. Our framework harnessed the complexity of the network and combined the time- and price-sensitive nature of customers and wage sensitive nature of couriers, thereby providing an analytic framework that showed platforms how to set their prices, wages, and optimal demand routing probabilities. Moreover, we proposed a solution approach for a profit maximization problem, which is nonlinear programming having high computational complexity. This was further followed by extensive numerical experiments that provided practical managerial insights, and the validity of our model was shown through real data implementation.

### 6.2 Future Direction

Possible directions for further extension of this problem are three-fold. First, our results were based on assumption that the service value distribution of the customer and the opportunity cost distribution of the courier are uniform. However, other

more realistic distributions such as exponential and gamma distributions are to be addressed in later works. Second, we can incorporate dynamic pricing strategies and dynamic demand routing strategies so that the platform can offer dynamic prices and wages based on real-time status. Finally, we plan to extend the problem situation where competition is present to better depict how the platform acts and sets its price and wage in the presence of competitors.

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## 국문초록

본 논문에서는 여러 유형의 고객과 배송업체가 있는 음식 배달 플랫폼을 고려한다. 플랫폼의 목표는 이익 극대화를 달성하기 위해 최적 서비스 가격과 서비스 당 임금을 도출하는 것이다. 그러나 플랫폼은 다양한 배송 거리와 서로 다른 서비스의 가치 평가를 가진 가격과 시간에 민감한 고객과 다양한 운송수단과 서로 다른 기회 비용을 가진 수익에 민감한 배달원에 직면해 있다. 본 논문에서는 multi-class multi-pool 시스템을 사용하여 모델링을 진행하여 고객-배달원 조합에 따라 달라지는 서비스 시간을 모델에 적용하였다. 이 복잡한 문제를 해결하기 위해 최적해를 도출하기 위한 근사 알고리즘을 제안한다. 광범위한 수치 실험이 실시되었고 이를 통해 실질적인 경영적 통찰이 도출되었다.

**주요어:** 음식 배달 서비스, 내생적 수요와 공급, multi-class multi-pool 시스템, 대기행렬 모델, 볼록 최적화

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