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Numerical Study for Evaluation of Dynamic Stability Derivatives using Prescribed Motion

> 강제운동을 이용한 동안정 미계수 산출에 관한 수치적 연구

항공우주공학과
홍 슬 기

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# Numerical Study for Evaluation of 

## Dynamic Stability Derivatives

## using Prescribed Motion

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The numerical method for evaluating the dynamic stability coefficient using prescribed motion was investigated on the present study. The Basic Finner configuration was chosen for the subject, and CFD analysis was implemented by a density-based flow solver based on the OpenFOAM. The parametric study on the prescribed motion proposed in the precedent studies was performed, and efficient inputs on prescribed motion are proposed. The validation on the evaluation of the pitch damping coefficient and the roll damping coefficient was performed by a comparison with the reference results from the experimental and numerical analyses. The sensitivity on the location of the center of gravity was investigated.

Keyword : Computational Fluid Dynamics (CFD), Dynamic Stability Derivatives, Prescribed Motion, Dynamic Mesh, Compressible Flow, Unsteady aerodynamics, Projectile
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## Chapter 1. Introduction

Stability derivatives are the essential parameters for designing aerodynamic system. As their nomenclature implies, derivatives represent the function's instantaneous change with respect to a parameter or an input variable. In the field of flight dynamics, the instantaneous change of aerodynamic loads with respect to kinematic variables is interpreted in the sense of stability and thus called "stability derivatives". The mathematical formulation of stability derivatives was introduced by Bryan and Williams [1] in the year of the first powered, manned heavier-than-air flight. Their early work particularized the longitudinal stability of the aeroplane having a vertical plane of symmetry but further generalizations including lateral stability were cumulated in the book [2] published in 1911. Although artificial flight was successful with the help of the 3 -axis flight control system of the Wright brothers' Flyer, which mainly focused on the practice of control rather than endowed with intrinsic stability, it was later realized that stability was also key to success in flying. Bairstow's research showed that stability derivatives can be determined from the wind tunnel measurements on the Blériot monoplane model [3, 4]. On the mathematical foundations of Bryan and experimental superstructure of Bairstow, the classical stability theory was substantiated in practice by the inherently stable biplane, Royal Aircraft Factory Blériot Experimental 2c, redesigned by Busk [5, 6]. The Bryan's equation regards an aerodynamic system as a rigid body and assumes steady symmetric flight where aerodynamic forces and moments varied linearly with an increment in a linear or angular velocity component. Further
research implemented by Cowley and Glauert [7] had found discrepancy between theoretical predictions and experimentally measurements and discussed the downwash lag effect on the longitudinal stability, which introduced the acceleration derivatives term $M_{\dot{\alpha}}$ to the longitudinal dynamic stability derivative. One might be focused on the limitation of linear theory, and suggested an alternative approaches on the stability derivatives, such as an introduction of the nonlinear functional form [8]. However, this classical stability model has been widely accepted and applied in the practical application due to its mathematical simplicity.

The design a projectile or a missile is no exceptional industrial area where the prediction of stability derivatives is important. More specifically, the accurate and cost-effective acquirement of dynamic stability derivatives has been widely studied in accordance with demanding requirements to satisfying the high maneuverability of missile. Several methods have been carried out for evaluating the dynamic stability derivatives, as depicted in the Figure 1.1.


Wind tunnel test


Ballistic range test



Semi-empirical method


Figure 1.1 Methods of evaluating the dynamic derivatives

Current advances in the computing capacity and refinements on a numerical approach successfully lead to utilize the computational fluid dynamics (CFD) simulation to evaluate the dynamic stability derivatives. The present study seeks to extend the current effort on the investigation how to make an efficient numerical method to evaluating the dynamic stability derivatives.

In the chapter 1, the numerical methods used in the present study are introduced. The next chapter introduced prescribed motions which simplifies solving procedure on the governing equation. Then the parametric studies on the prescribed motion parameters and the sensitivity studies are performed and validates the numerical methods.

## Chapter 2. Numerical methods

### 2.1. Numerical schemes

An unsteady, three-dimensional compressible flow is the flow field of interest in this study. Given the number of cases required for investigating the influence of flow variables and prescribed motion parameters on evaluation of dynamic stability derivatives, an inviscid flow is assumed for taking advantages of simplicity at the expense of accuracy. Therefore, the Euler equations is used for the governing equations derived from the Reynolds-Averaged Navier-Stokes (RANS) equations with zero viscosity. In the Eulerian frame of reference, the Euler equations representing the conservation laws applied on the flow field are described as follows.

Conservation of mass:

$$
\begin{equation*}
\frac{\partial \rho}{\partial t}+\nabla \cdot[\boldsymbol{U} \rho]=0 \tag{2.1}
\end{equation*}
$$

Conservation of momentum:

$$
\begin{equation*}
\frac{\partial(\rho \boldsymbol{U})}{\partial t}+\nabla \cdot[\boldsymbol{U}(\rho \boldsymbol{U})]+\nabla p=0 \tag{2.2}
\end{equation*}
$$

Conservation of energy:

$$
\begin{equation*}
\frac{\partial(\rho E)}{\partial t}+\nabla \cdot[\boldsymbol{U}(\rho E)]+\nabla \cdot(\boldsymbol{U} p)=0 \tag{2.3}
\end{equation*}
$$

where $\rho$ is the mass density, $\boldsymbol{U}$ is the fluid velocity, and $p$ is the pressure. The total energy density $E$ is evaluated with the specific internal energy $e, E=e+|\boldsymbol{U}|^{2} / 2$.

A general numerical approach to solving the partial differential equations (2.1-(2.3) is the application of finite volume method (FVM) within the cell-centered, collocated (or non-staggered) grid system. In the cell-centered, collocated grid system, all flow variables and material properties are specified at each cell centroid. The collocated grid is usually preferred over the staggered grid because it allows a nonCartesian grid to handle complex three-dimensional geometries having curved surface, for instance. The application of the FVM begins with integrating the partial differential equation over a cell volume $V$. Convection term requires cell-interface fluxes which evaluated by interpolation in cell centered values between the owner cell and neighboring cells. Divergence term is converted into integrals over the cell surface $S$ using the Gauss's theorem. Then, the both divergence and gradient term are discretized into make a form of an algebraic equation as follows.

Convection term:

$$
\begin{equation*}
\int_{V} \nabla \cdot[\boldsymbol{U} \boldsymbol{\Psi}] d V=\int_{S} d \boldsymbol{S} \cdot[\boldsymbol{U} \boldsymbol{\Psi}] \approx \sum_{f} \boldsymbol{S}_{f} \cdot \boldsymbol{U}_{f} \boldsymbol{\Psi}_{f}=\sum_{f} \phi_{f} \boldsymbol{\Psi}_{f} \tag{2.4}
\end{equation*}
$$

Gradient term:

$$
\begin{equation*}
\int_{V} \nabla \Psi d V=\int_{S} \Psi d S \approx \sum_{f} \Psi_{f} S_{f} \tag{2.5}
\end{equation*}
$$

where $\Sigma_{\mathrm{f}}$ is a summation over cell-interfaces, $\boldsymbol{\Psi}$ is a general tensor field, $\boldsymbol{S}_{f}$ is a vector outward normal to the face surface, $\phi_{f}$ is a volumetric flux, and $\Psi$ is a general scalar field.

The characteristics of high-speed compressible flow, such as shocks, expansion fan, and contact surface, introduces discontinuities into the flow solution, which requires to capture these features, so called 'Shockcapturing method', while restraining spurious oscillations. The $2^{\text {nd }}-$ order Kurganov-Noelle-Petrova (KNP) central upwind scheme [9] is used for the convection flux scheme. In comparison with a general Godunov-type central schemes, this inviscid flux scheme does not involve characteristics decomposition at each cell-interface, which can avoid cumbersome Jacobian evaluation while performing shock-capturing reliably in compressible flows. Furthermore, an introduction of piecewise polynomial functions used for evaluating flux integrals with the process of 'reconstruction', 'evolution', and 'projection' required for the $2^{\text {nd }}$-order Godunov-type central schemes is unnecessary. Instead, interpolation procedure with total variation diminishing (TVD) schemes is used for flux evaluations. The values at cell-interfaces are split into two directions; The direction $f_{+}$is aligned with the outward face normal vector $\mathbf{S}_{f}$, while the direction $f$. is aligned with the inward face normal vector $-\mathbf{S}_{f}$. The algebraic equations of convection and gradient terms derived in Eq.(2.6)-(2.7) are interpolated with the KNP central upwind scheme as follow.

$$
\begin{align*}
\sum_{f} \phi_{f} \boldsymbol{\Psi}_{f}= & \sum_{f}\left[\alpha \phi_{f+} \boldsymbol{\Psi}_{f+}+(1-\alpha) \phi_{f-} \boldsymbol{\Psi}_{f-}+\omega_{f}\left(\boldsymbol{\Psi}_{f-}-\boldsymbol{\Psi}_{f+}\right)\right]  \tag{2.6}\\
& \sum_{f} \boldsymbol{S}_{f} \Psi_{f}=\sum_{f}\left[\alpha \boldsymbol{S}_{f} \Psi_{f+}+(1-\alpha) \boldsymbol{S}_{f} \Psi_{f-}\right] \tag{2.7}
\end{align*}
$$

where $\alpha$ is a weighting factor, and $\omega_{f}$ is a diffusive volumetric flux.

Evaluation of the both weighting factor and diffusive volumetric flux are based on one-sided local speeds of propagation, $\psi_{f_{+}}$and $\psi_{f_{-}}$. These quantities are calculated as follow.

$$
\begin{gather*}
\psi_{f+}=\max \left(c_{f+}\left|\boldsymbol{S}_{f}\right|+\phi_{f+}, c_{f-}\left|\boldsymbol{S}_{f}\right|+\phi_{f-}, 0\right) \\
\psi_{f-}=\max \left(c_{f+}\left|\boldsymbol{S}_{f}\right|-\phi_{f+}, c_{f-}\left|\boldsymbol{S}_{f}\right|-\phi_{f-}, 0\right)  \tag{2.8}\\
\alpha=\frac{\psi_{f+}}{\psi_{f+}+\psi_{f-}}  \tag{2.9}\\
\omega_{f}=\alpha(1-\alpha)\left(\psi_{f+}+\psi_{f-}\right) \tag{2.10}
\end{gather*}
$$

where the speeds of sound of the ideal gas at the face in both directions are denoted by $c_{f_{ \pm}}=\sqrt{\gamma R T_{f_{ \pm}}}$, respectively. Note that if the contribution in both direction is equal, namely, $\alpha=1 / 2$, the equation (2.6) reduced to the central scheme description. On the other hands, if the weight factor is biased, it is reduced to the upwind scheme description.

The aforementioned interpolation procedure with a TVD scheme is used for evaluating values at cell interfaces. For example, the interpolated field in outward direction, $\boldsymbol{\Psi}_{f_{+}}$is evaluated as follows.

$$
\begin{gather*}
\boldsymbol{\Psi}_{f+}=\left\{1-\beta(r)\left(1-\omega_{f}\right)\right\} \boldsymbol{\Psi}_{P}+\beta(r)\left(1-\omega_{f}\right) \boldsymbol{\Psi}_{N}  \tag{2.11}\\
r=2 \frac{\boldsymbol{d} \cdot(\nabla \boldsymbol{\Psi})_{P}}{\left(\nabla_{\boldsymbol{d}} \boldsymbol{\Psi}\right)_{f}}-1 \tag{2.12}
\end{gather*}
$$

where the subscription P denotes own cell, the subscription N denotes neighboring cell, $\beta(r)$ is a flux limiter, and $r$ is a successive gradients ratio.

The equations described in the eq. (2.1)-(2.12) are implemented in a density-based solver [10] based on the free, opensource computational fluid dynamics software OpenFOAM ${ }^{\circledR}$ [11]. This flow solver is originally used to solve the unsteady, three-dimensional, compressible ReynoldsAveraged Navier-Stokes (RANS) equation. Only the inviscid part of the solver is used for solving the Euler equations.

The Sweby diagram depicted in the Figure 2.1 represents the TVD flux limiters supported in the solver with the the $2^{\text {nd }}$ order TVD region. The Minmod function is chosen as a flux limiter for offering enough accuracy and robustness.

The time integration is performed using the explicit Gauss time integration scheme. A time step size is determined by the maximum Courant-Friedrichs-Lewy (CFL) number managed to keep below 0.8. The set of discretized algebraic equations derived from the Euler equations become linear systems of equations. These are expressed in explicit matrix form, where the exact solution is obtained with the inverse of the diagonal matrix.


Figure 2.1 Sweby diagram for available flux limiters

### 2.2. Geometry

The generic missile configuration presented in the Figure 2.2, as known as the Basic Finner, was chosen for the present numerical study. This geometry was proposed by Ballistic Research Laboratory to perform verification and validation of experimental methodology for obtaining dynamic stability derivatives from their aeroballistic range test measurements in comparison with the theoretical values calculated on the basis of linearized supersonic theory [12]. It has the advantage of simplicity from a manufacturing standpoint and ease of analysis. As a result, experimental results on obtaining dynamic stability derivatives of the Basic Finner exist in line with the purpose.

The missile design is composed of conical nose, cylindrical fuselage, and four fins in cruciform arrangement. The dimension of the Basic Finner is calibrated in the fuselage diameter, D. The total body length is 10 D . The spherically blunt conical nose has the cone half-angle of $10^{\circ}$ with bluntness radius of 0.004 D . Each fin has a square planform and a wedge-shaped cross section with the chord length of 1 D , the leading edge bluntness radius of 0.004 D , and the trailing edge thickness of 0.08 D .


Figure 2.2 The geometry of the Basic Finner

### 2.3. Computation domain

The computational domain of the Basic Finner consisted of 2.47 M structured hexahedral cells, constructed using the commercial meshing generation software Cadence Fidelity Pointwise ${ }^{\circledR}$ [13]. The CH-type grid topology is used around the Basic Finner and the x-z section of the computational grid is shown in the Figure 2.3. The upstream grid having the radius of 45 D is hemispherically extended from the nose tip and followed by the downstream grid cylindrically extruded to the length of 90 D . The sufficient size of the computational domain size is chosen for avoiding the adverse influence of boundary conditions on numerical accuracy.


Figure 2.3 Computational domain of the Basic Finner
Surface grid of the Basic Finner is shown in Figure 2.4. Small size cells are clustered around the nose tip, cone-cylinder junction, leading edge of fins and fuselage base, where the steep gradient of flow variables was expected to be observed or the high geometric fidelity is required to capture flow characteristics around the blunt edge.


Figure 2.4 Surface grid of the Basic Finner
A characteristic wave propagated in a high-speed compressible flow should smoothly transmitted as it arrives at the computational domain boundary. Any reflection wave at the boundary violates the physics and contaminates a flow solution. Non-reflecting boundary condition offered in the OpenFOAM is called 'waveTransmissive'. This boundary condition treats a material derivative of a physical quantity, $\phi$ as zero as if the transmitted wave transports an increased amount of physical quantity in a control volume through the control surface. The advection speed of the transmitted wave, $U_{n}$ is approximately expressed with the sum of surface-normal velocity component, $\mathrm{u}_{n}$ and the sound speed, $c$.

$$
\begin{equation*}
\frac{D \phi}{D t} \approx \frac{\partial \phi}{\partial t}+U_{n} \cdot \frac{\partial \phi}{\partial n}=0 \tag{2.13}
\end{equation*}
$$

The cell-interface value at the computational boundary is evaluated by the finite discretization of the Eq. (2.13) as follows.

$$
\begin{align*}
& \frac{\phi_{\text {face }}^{n+1}-\phi_{\text {face }}^{n}}{\Delta t}+U_{n} \frac{\phi_{\text {face }}^{n+1}-\phi_{\text {center }}^{n+1}}{\Delta x}=0  \tag{2.14}\\
& \phi_{\text {face }}^{n+1}=\frac{1}{1+U_{n} \frac{\Delta t}{\Delta x}} \phi_{\text {face }}^{n}+\frac{U_{n} \frac{\Delta t}{\Delta x}}{1+U_{n} \frac{\Delta t}{\Delta x}} \phi_{\text {center }}^{n+1} \tag{2.15}
\end{align*}
$$

Types of boundary condition applied in pressure, temperature, and velocity are summarized in the Table 2.1. Inlet and outlet boundary region are depicted in the Figure 2.3. The wall region is designated on the surface of the Basic Finner. The freestream boundary condition with a uniform flow is applied on the inlet and the non-reflecting boundary condition is applied on the outlet. On the surface of the Basic Finner, an adiabatic wall and flow-tangency condition is used. The magnitude of each physical quantity in boundary conditions was assigned individually in line with each reference. Those values are suggested along with the results.

Table 2.1 Types of boundary conditions applied on each boundary region

|  | Inlet | Outlet | Wall |
| :---: | :---: | :---: | :---: |
| P | freestreamPressure | waveTransmissive | zeroGradient |
| T | inletOutlet | inletOutlet | zeroGradient |
| U | freestreamVelocity | freestreamVelocity | movingWallSlip |

### 2.4. Dynamic mesh

The dynamic mesh is a technique how meshes accommodate the motion of a subject with movable or deformable meshes for every time step. Mesh morphing, overset mesh [14], sliding mesh [15] are the representative examples of the dynamic mesh technique. Numerical simulation of the prescribed motion involves the dynamic mesh, inevitably. Therefore, handling the dynamic mesh in a robust and reliable manner is of primary importance.

Mesh morphing changes the position of interior mesh points to accommodate moving boundaries of computational domain with maintaining mesh topology; Addition, removal or modification of connectivity for points, faces, or cells is not considered. Several mesh morphing techniques were used to determine the new position of mesh points. Algebraic interpolation method [16] formulates the boundaryfitted rectangular coordinate systems on distinct moving boundaries of physical domain with algebraic function and generates the interior grid points with a unidirectional interpolation from the boundaries. This method provides an explicit control of physical grid distribution with a uniform algebraic function but has a limitation on the application in an unstructured mesh. Transfinite interpolation (TFI) based on blending function [17] is a major interpolation algorithm used in the algebraic interpolation method. Spring analogy method is another mesh morphing technique, which regards each mesh point connection as a virtual elastic spring and solves equilibrium equations to determine the grid spacing in response to boundary loading. Unlike the algebraic interpolation method, the spring analogy method can serve an unstructured mesh. Early work [18] had introduced a linear spring to constitute the mesh connectivity
but later realized potential problems in cell topology; edge collapse and face or cell inversion [19]. Replacement of the linear spring to a torsional spring having the limit of torsional stiffness goes to infinity as the angle between the adjacent edges approaches $0^{\circ}$ or $180^{\circ}$ with additional corrections on unloading alleviates the issues [20,21] but expensive iterative calculations are involved to solve the non-linear equations of equilibrium. The Arbitrary Lagrangian Eulerian (ALE) method with a local mesh smoothing algorithm applying the laplace operator [22] or the biharmonic operator [23] on the mesh motion is the other option for the mesh morphing.

The overset mesh constitutes a computational domain with hierarchical subdomains. Distinct subdomains are overlapped and communicates each other with interpolation of flow variables at the interfaces. The hierarchical structure determines inclusion and exclusion of the subdomains to naturally treat embedded grids. This method takes an advantage of the improved local mesh resolution and quality and is usually used to accommodate the mesh around a complex geometry or moving object [24].

The sliding mesh decomposes a computational domain into independent cell zones, wherein each region independently generates an internal mesh. The grid points on the interface of two adjacent cell zones may be either coincident (conformal mesh) or discrepant (non-conformal mesh). In line with overset mesh, information of flow variables is transferred by interpolation between nonoverlapped neighboring cell zones and both the complex-topology and the selective-grid-refinement problems are mitigated.

Given that the subject is assumed as a rigid body and the relatively large displacements along a prescribed motion, mesh morphing or topological mesh changes involving a local remeshing with reconstructing mesh connectivity may have difficulties in maintaining the mesh quality and numerical accuracy. Therefore, moving mesh in a whole computational domain without any topological change is only considered in the present study.

Additional reviews on the governing equations discretized by the FVM formulated in a fixed mesh is required due to the relative velocity of moving mesh interface. Calibration is performed in the conservative equations with respect to moving mesh by using the relative velocity of moving mesh interface in the convection flux term over the control volume. Then, the governing equations in eq. (2.1-(2.3) are redescribed in conservative integral form as follows [25, 26].

$$
\begin{gather*}
\frac{\partial}{\partial t} \int_{V} \rho \partial V+\oint_{S} \rho\left(\boldsymbol{U}-\boldsymbol{U}_{s}\right) \cdot \partial \boldsymbol{S}=0  \tag{2.16}\\
\frac{\partial}{\partial t} \int_{V} \rho \boldsymbol{U} \partial V+\oint_{S}\left[\rho\left(\boldsymbol{U}-\boldsymbol{U}_{s}\right) \boldsymbol{U} \cdot \partial \boldsymbol{S}+\oint_{S} p \partial \boldsymbol{S}=0\right.  \tag{2.17}\\
\left.\frac{\partial}{\partial t} \int_{V} \rho E \partial V+\oint_{S}\left[\rho\left(\boldsymbol{U}-\boldsymbol{U}_{s}\right) E+p \boldsymbol{U}\right)\right] \cdot \partial \boldsymbol{S}=0 \tag{2.18}
\end{gather*}
$$

where $\boldsymbol{U}_{s}$ is a mesh velocity.

The velocity of the dynamic mesh is determined by satisfying the both the conservation laws of fluid and the geometric conservation law [27, 28, 29] in that the time rate of volumetric change in a dynamic mesh control volume is equal to the volume swept by the velocity of the mesh interface in a unit time.

$$
\begin{equation*}
\frac{\partial}{\partial t} \int_{V} \partial V=\oint_{S} \boldsymbol{U}_{s} \cdot \partial \boldsymbol{S} \tag{2.19}
\end{equation*}
$$

The interior points of dynamic mesh are updated in every time step. The coordinate transformation of mesh points consists of the displacement component and the rotational component. Quaternions form of the rotation matrix $R(\vec{Q})$ is used to avoid the gimbal lock problem that may arise out of using the Euler angle expression.

$$
\begin{gather*}
\vec{Q}=\left[\begin{array}{l}
q_{0} \\
q_{1} \\
q_{2} \\
q_{3}
\end{array}\right]=\left[\begin{array}{c}
\cos (\theta / 2) \\
v_{x} \sin (\theta / 2) \\
v_{y} \sin (\theta / 2) \\
v_{z} \sin (\theta / 2)
\end{array}\right]  \tag{2.20}\\
R=\left[\begin{array}{ccc}
q_{0}^{2}+q_{1}^{2}-q_{2}^{2}-q_{3}^{2} & 2\left(q_{1} q_{2}+q_{0} q_{3}\right) & 2\left(q_{1} q_{3}+q_{0} q_{2}\right) \\
2\left(q_{1} q_{2}-q_{0} q_{3}\right) & q_{0}^{2}-q_{1}+q_{2}^{2}-q_{3}^{2} & 2\left(q_{2} q_{3}-q_{0} q_{1}\right) \\
2\left(q_{1} q_{3}-q_{0} q_{2}\right) & 2\left(q_{2} q_{3}+q_{0} q_{1}\right) & q_{0}^{2}-q_{1}-q_{2}^{2}+q_{3}^{2}
\end{array}\right] \tag{2.21}
\end{gather*}
$$

where $\vec{Q}$ is a quaternion vector, $\vec{v}$ is a unit vector aligned with the rotation axis and $\theta$ is a rotation angle.

## Chapter 3. Evaluation methods

The dynamic stability derivatives of the flying subject are evaluated by an analysis on the aerodynamical response to an arbitrary motion. A motion of the flying subject and the aerodynamic response are coupled inherently. The simultaneous calculation of the both coupling equations are cumbersome and costly. An alternative approach is using a prescribed motion. If the motion of the flying subject is given in priori for every time step, solving the subsequent motion induced by aerodynamic response is unnecessary.

The overview of evaluating dynamic stability derivatives using prescribed motion is described in Figure 3.1. A prescribed motion with a constant rate of rotation or a simple harmonic function (called forced harmonic motion) are introduced in the next sections. These prescribed motions are implemented in CFD simulation with an unsteady, threedimensional compressible flow solver. Then, aerodynamic responses to the prescribed motion are obtained and analyzed in a suitable postprocessing process.


Figure 3.1 The overview of evaluation methods

### 3.1. Force harmonic motion

A forced harmonic motion is a simple harmonic oscillation with a uniform amplitude and angular frequency about the center of gravity. A forced harmonic motion in the pitch direction is performed to evaluate the pitch damping coefficient $C_{m_{q}}+C_{m_{\tilde{\alpha}}}$. The pitch angle $\theta$ of an object is a state variable under control. If the direction of a uniform flow is parallel with the ground surface, the angle of attack $\alpha$ is equal to the pitch angle. Then, the pitch angle of an object in a forced harmonic motion is expressed in the angle of attack as follows.

$$
\begin{gather*}
\theta(t)=\alpha\left(t ; M a_{\infty}, \alpha_{A}, k\right)=\alpha_{0}+\alpha_{A} \sin \left(t \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}}\right)  \tag{3.1}\\
k=\frac{\omega D}{2 M a_{\infty}} \sqrt{\frac{\rho_{\infty}}{\gamma p_{\infty}}} \tag{3.2}
\end{gather*}
$$

where $M a_{\infty}$ is the freesteam Mach number, $p_{\infty}$ is the freesteam pressure, $\rho_{\infty}$ is the freesteam density, $\alpha_{0}$ is the angle of attack of interest, $\alpha_{A}$ is the amplitude of the forced harmonic motion, $\omega$ is the angular frequency of the forced harmonic motion, $\gamma$ is the heat capacity ratio of an ideal gas, and $k$ is the reduced frequency. The angle of attack of an object in a forced harmonic motion is described by the freesteam Mach number, the amplitude of the forced harmonic motion, and the reduced frequency of the forced harmonic motion. The period of oscillation is in inverse proportion to the Mach number or the reduced frequency.

The aforementioned pitch damping coefficient is the derivative of the pitching moment coefficient with respect to the angle of attack. Linear approximation of the pitching moment coefficient is described in the eq. (3.3). The substitution of the eq. (3.1) in the eq. (3.3) results in the pitching moment coefficient of an object with the aerodynamic response to the forced harmonic motion in pitch direction modeled with the sum of transcendental functions.

$$
\begin{align*}
& C_{m}(t) \approx C_{m_{0}}+ C_{m_{\alpha}} \alpha+\left(C_{m_{\dot{\alpha}}}+C_{m_{q}}\right) \frac{D}{2 M a_{\infty}} \sqrt{\frac{\rho_{\infty}}{\gamma p_{\infty}}} \dot{\alpha}  \tag{3.3}\\
& \begin{aligned}
C_{m}\left(t ; M a_{\infty}, \alpha_{A}, k\right) & =C_{m_{0}}+C_{m_{\alpha}} \alpha_{A} \sin \left(t \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}}\right) \\
& +\left(C_{m_{\dot{\alpha}}}+C_{m_{q}}\right) k \alpha_{A} \cos \left(t \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}}\right)
\end{aligned}
\end{align*}
$$

The pitch damping coefficient in the above equation is estimated by the least square method or the Fourier coefficient method.

Least square method:

$$
\begin{align*}
C_{m}\left(t_{i}\right) & =C_{m_{A}} \sin \left(\frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t_{i}+\delta\right)+\epsilon_{i} \\
& =A_{0}+A_{1} \sin \left(\frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t_{i}\right)+A_{2} \cos \left(\frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t_{i}\right)+\epsilon_{i} \tag{3.5}
\end{align*}
$$

where $A_{0}=C_{m_{0}}, A_{1}=C_{m_{A}} \cos \delta, A_{2}=-C_{m_{A}} \sin \delta$

$$
\begin{gather*}
\left\{C_{m}\right\}=[X]\{A\}+\{\epsilon\}  \tag{3.6}\\
\{A\}=\left([X]^{T}[X]\right)^{-1}[X]^{T}\left\{C_{m}\right\}  \tag{3.7}\\
\therefore C_{m_{\dot{\alpha}}}+C_{m_{q}}=\frac{A_{2}}{k \alpha_{A}}=\frac{-C_{m_{A}} \sin \delta}{k \alpha_{A}} \tag{3.8}
\end{gather*}
$$

where $\epsilon$ is the least square error and $\delta$ is the phase shift.

Fourier coefficient method:

$$
\begin{align*}
C_{m_{\dot{\alpha}}}+C_{m_{q}}= & \frac{2}{k \alpha_{A} T} \int_{0}^{T} C_{m}(t) \cos \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t d t \\
\simeq & \frac{1}{k \alpha_{A} T} \sum_{n=1}^{N-1}\left(C_{m}\left(t_{n+1}\right) \cos \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t_{n+1}+\right.  \tag{3.9}\\
& \left.\quad C_{m}\left(t_{n}\right) \cos \frac{2 k M a_{\infty}}{D} \sqrt{\frac{\gamma p_{\infty}}{\rho_{\infty}}} t_{n}\right)\left(t_{n+1}-t_{n}\right)
\end{align*}
$$

### 3.2. Constant rate of rotation motion

A constant rate of rotation motion rotates about the center of gravity with a constant rate of rotation. Constant rate of rotation motion has been used for evaluating roll damping coefficient, but it was recently found that it is possible to use for evaluating the pitch damping coefficient when an object rotates about its center of gravity [30, 31]. The two distinct pitch rates $q_{1}, q_{2}$ are applied for evaluating the pitch damping coefficient. The same pitch angular displacement is reached with a different time for each pitch rate. As previously noted, if the flow direction is parallel with the ground surface, the angle of attack can replace the pitch angle.

$$
\begin{equation*}
\Delta \theta=\Delta \alpha=\dot{\alpha_{1}} \Delta t_{1}=q_{1} \Delta t_{1}=\dot{\alpha_{2}} \Delta t_{2}=q_{2} \Delta t_{2} \tag{3.10}
\end{equation*}
$$

The pitching moment coefficient of an object with the aerodynamic response to the two constant pitch rate motions is modeled as follows.

$$
\begin{gather*}
C_{m}\left(t_{1}\right)=C_{m_{0}}+C_{m_{\alpha}} \alpha+\left(C_{m_{\dot{\alpha}}}+C_{m_{q}}\right) q_{1}^{*}{ }^{C_{m}\left(t_{2}\right)=C_{m_{0}}+C_{m_{\alpha}} \alpha+\left(C_{m_{\dot{\alpha}}}+C_{m_{q}}\right) q_{2}^{*}} \\
q^{*}=\frac{q D}{2 M a_{\infty}} \sqrt{\frac{\rho_{\infty}}{\gamma p_{\infty}}} \tag{3.11}
\end{gather*}
$$

where $q^{*}$ is a nondimensionalized pitch rate.

The pitch damping coefficient is evaluated by difference in the distinct pitching moment coefficients.

$$
\begin{equation*}
C_{m_{\dot{\alpha}}}+C_{m_{q}}=\frac{C_{m_{1}}-C_{m_{2}}}{q_{1}^{*}-q_{2}^{*}} \tag{3.13}
\end{equation*}
$$

The roll damping coefficient of an object with the aerodynamic response to the aforementioned constant roll rate motion is evaluated as follows.

$$
\begin{align*}
& C_{l}=C_{l_{0}}+C_{l_{p}} p^{*}  \tag{3.14}\\
& p^{*}=\frac{p D}{2 M a} \sqrt{\frac{\rho_{\infty}}{\gamma p_{\infty}}} \tag{3.15}
\end{align*}
$$

where $p$ is a constant roll rate, $C_{l_{0}}$ is the rolling moment coefficient in a static state, $C_{l_{\mathrm{p}}}$ is the roll damping coefficient, and $p^{*}$ is a nondimensionalized roll rate. The roll damping coefficient is evaluated by difference in the distinct rolling moment coefficients.

$$
\begin{equation*}
C_{l_{p}}=\frac{\bar{C}_{l_{2}}-\bar{C}_{l_{1}}}{p_{2}^{*}-p_{1}^{*}} \tag{3.16}
\end{equation*}
$$

The rotational symmetry of the Basic Finner in the roll direction suggest that $C_{l_{0}}$ equals to zero. It simplifies the eq. (3.16) if $p_{1}=0$. Then, the equation is reduced as follows.

$$
\begin{equation*}
C_{l_{p}}=\frac{\bar{C}_{l}}{p^{*}} \tag{3.17}
\end{equation*}
$$

## Chapter 4. Results

### 4.1. Steady-state flow analysis

The objective of the steady-state flow analysis is the validation of three-dimensional compressible flow solver and obtaining the converged initial flow field solution used for unsteady simulation. The shockcapturing feature was investigated by observing the Mach contour distribution and the numerical schlieren image of the solution in the transonic and supersonic flow condition. The aerodynamic coefficients were obtained and compared with the result of references.

The steady-state transonic flow condition is summarized in the Table 4.1. The size of the Basic Finner diameter is identical to that of the reference experimental model [32]. The Mach numbers was chosen within the transonic flow range with the increment of 0.1 but the fine increment was used around the Mach number 1 to avoid the singular point.

The steady-state supersonic flow condition is summarized in the Table 4.2. The conditions are identical to that of the reference experiment [33].

Table 4.1 The conditions of steady-state transonic flow analysis

| Diameter | 30 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
| Mach number | $0.8,0.9,0.95,1.05,1.1,1.2,1.3,1.4$ |

Table 4.2 The conditions of steady-state supersonic flow analysis

| Diameter | 45.72 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}$ |
| Mach number | 2.49 |

Mach contour distributions around the Baic Finner in transonic flow conditions are depicted in the Fig. 4.1. The range of contour is customized based on the freesteam Mach number. The both magnitude of the limits are increased or decreased by 0.4, respectively. Numerical Schlieren images around the Basic Finner in transonic flow conditions are depicted in the Fig. 4.2.


Figure 4.1 Mach contour distribution in transonic flow


Figure 4.2 Numerical schlieren image in transonic flow

The axial force coefficient of the Basic Finner in the transonic flow condition is presented in the Figure 4.3. The reference area for evaluating the aerodynamic coefficient is the base area of the missile fuselage. The comparison with the result of the wind tunnel, aeroballistic range [26] and the CFD analyses $[34,35,36]$ was performed.


Figure 4.3 Axial force coefficient in the transonic flow condition
The locally supersonic flow region is formed in Mach number 0.8 condition and observed at around of the cone-cylinder junction and the trailing-edge of fins. in the Figure 4.1 and Figure 4.2. The strength of normal shock located at the aft of the locally supersonic region is gradually increased as the Mach number is increased until the 1. In that transonic region, the axial force coefficient of the Basic Finner is abruptly increased and the peak is observed around the Mach number 1. The characteristic waves are gradually inclined to the downstream as the further increase in the Mach number. The axial force coefficient of the Basic Finner is also gradually decreased as the further increase in the

Mach number. The results of present study followed the trend of compressible flow characteristics and match well with the results of the references.


Figure 4.4 Aerodynamic coefficients in the supersonic flow condition
The aerodynamic coefficients of the Basic Finner in the supersonic flow condition is presented in the Figure 4.4. The evaluation of the axial force coefficient, the normal force coefficient, and the pitching moment coefficient was performed. The comparison with the result of the wind
tunnel [35] and the CFD analyses [36, 37] was also performed. The general trends of changes with respect to the angle of attack is well matched with the references but the discrepancy between the present result and the experiment was increased for higher angles. This discrepancy at the high angle of attack may have arisen of viscosity effect. However, the present results in the steady-state flow condition followed the general trend of change in the flow characteristics and matched well with the reference results, we conclude that the numerical methods used in the present study is appropriate to solve the compressible flow.

### 4.2. Parametric study on the prescribed motion

The objective of parametric study is investigation on the sensitivity of prescribed motion parameters and determination of valid range of those parameters. Two types of prescribed motion were used; A forced harmonic motion and a constant rate of rotation motion introduced in the chapter 3. The pitch and roll direction of the Basic Finner were considered but the yaw direction was excluded due to the rotational symmetry of the configuration.

The parametric study of the reduced frequency and the amplitude of a forced harmonic motion was performed. The flow conditions and the motion parameter are summarized in the Table 4.3 and the Table 4.4. The upper diagram of the Figure 4.5 described the angle of attack of the forced harmonic motion. It is a sine function and maintains its oscillation up to three periods $\tau$. This input condition is invariant unless specifically described.

The effects of the reduced frequency on the pitching moment are observed in the Figure 4.5 and Figure 4.6. In the Figure 4.6, only the
pitching moment of the third period is depicted to improve the visibility, which all loops are overlapped each other. The peak magnitude of the pitching moment is increased as the reduced frequency is increased, but it is shown that the relation is not linear. Note that the $C_{m_{A}}$ in the eq. (3.4). The model shows the relation, $C_{m_{A}} \propto$ $\sqrt{a k^{2}+b}$ where $\mathrm{a}, \mathrm{b}$ is a constant. The hysteresis loops are observed and the vertical width of the loops is also increased as the reduced frequency is increased. Only remaining is the last term in the eq. (3.4) when the angle of attack is at zero, and it implies that the vertical width of the hysteresis loops is proportional to the reduced frequency.

The effect of the reduced frequency on the pitch damping coefficient is shown in the Figure 4.7. Overall, the results are insensitive to the reduced frequency except the outlier point of $\mathrm{k}=0.001$. It was hard to resolve the magnitude of the coefficient of the last term in the eq. (3.4) when the magnitude of the reduced frequency is too small. The hysteresis loop of the outlier is close to a line and hard to distinguish from the noise. Therefore, it is concluded that the appropriate range of the reduced frequency is from 0.005 to 0.02 and the choice of the arbitrary reduced frequency within the range is insignificant on the evaluation of the pitch damping coefficient.

The effect of the amplitude on the pitching moment is observed in the Figure 4.8 and that on the pitch damping coefficient is observed in the Figure 4.9. The width of the hysteresis loops is also broadened as the amplitude is increased. It is observed that the amplitude of the forced harmonic motion smaller than $3^{\circ}$ is also insignificant on the result of the pitch damping coefficient.

Table 4.3 Parametric study on the reduced frequency

| Mach number | 1.89 |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
| Amplitude | $0.2^{\circ}$ |
| Reduced frequency | $0.001,0.005,0.01,0.05,0.1,0.2$ |

Table 4.4 Parametric study on the amplitude

| Mach number | 1.89 |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
| Amplitude | $0.2^{\circ}, 0.25^{\circ}, 0.5^{\circ}, 1^{\circ}, 2^{\circ}, 3^{\circ}$ |
| Reduced frequency | 0.01 |




Figure 4.5 Forced harmonic motion in pitch and $C_{m}$ with respect to $k$


Figure 4.6 Phase diagram with respect to $k$


Figure 4.7 Pitch damping coefficient with respect to $k$
soll wional ungran


Figure 4.8 Phase diagram with respect to $\alpha_{A}$


Figure 4.9 Pitch damping coefficient with respect to $\alpha_{A}$

The parametric study of the nondimensionalized pitch rate of a constant pitch rate motion (CPRM) and the nondimensionalized roll rate of a constant roll rate motion (CRRM) was performed. The flow and the motion parameter are summarized in the Table 4.5 and the Table 4.6, respectively. The first value of the angle of attack or the roll angle denotes the initial posture angle and the last value denotes the final posture angle.

The pitching moment coefficient with respect to the nondimensionalized pitch rate are observed in the Figure 4.10. The initial transient response of the CPRM is gradually prominent and the magnitude of the pitching moment coefficient is increased as the nondimensionalized pitch rate is increased. The dashed lines in the Figure 4.10 are tangent lines at the angle of attack of interest. The linearity is observed in comparison with an auxiliary line when the initial transient response is attenuated. The smaller nondimensionalized pitch rate, the smaller the range and the magnitude of the deviation is observed. The last values of the pitching moment coefficient are depicted in the Figure 4.11. Those points are in line with linear regression, which the pitch damping coefficient evaluated by using the eq. (3.13) would be nearly identical regardless of choice.

The rolling moment coefficient with respect to the nondimensionalized roll rate are observed in the Figure 4.12. The initial transient response of the CRRM is gradually prominent and the magnitude of the rolling moment coefficient is increased as the nondimensionalized roll rate is increased. The last values of the rolling moment coefficient are depicted in the Figure 4.13. Those points are in line with linear regression, which the roll damping coefficient evaluated by using the eq. (3.17) would be nearly identical regardless of choice.

Table 4.5 Parametric study on the nondimensionalized pitch rate

| Mach number | 1.89 |
| :--- | :--- |
| Angle of attack | $-4^{\circ} \rightarrow 0^{\circ}$ |
| Nondimensional pitch rate | $0.001,0.002,0.005,0.01,0.02$ |



Figure 4.10 Pitching moment coefficient with respect to $q^{*}$


Figure 4.11 Last values of $C_{m}$ and its linear regression line

Table 4.6 Parametric study on the nondimensionalized roll rate

| Mach number | 1.89 |
| :--- | :--- |
| Roll angle | $0^{\circ} \rightarrow 45^{\circ}$ |
| Nondimensional roll rate | $0.001,0.005,0.01,0.02$ |



Figure 4.12 Rolling moment coefficient with respect to $\mathrm{p}^{*}$


Figure 4.13 Last values of $C_{1}$ and its linear regression line

The influence of the prescribe motion parameter on the aerodynamic coefficients and the dynamic stability derivatives has been observed. The choice of the reduced frequency and the amplitude of the forced harmonic motion is insignificant for evaluating dynamic stability derivatives in the standpoint of precision. Similarly, the choice of the nondimensionalized pitch rate and the nondimensionalized roll rate is insignificant for evaluating dynamic stability derivatives.

On the other hands, those prescribed motion parameter determines the computation time as shown in the Table 4.7. The period of the forced harmonic motion is determined by the reduced frequency. Therefore, high reduced frequency is preferred for calculating three periods of the harmonic oscillation. The computational time of the CPRM is determined by the nondimensionalized pitch rate and the angular displacement. Given that the angle of attack of interest is fixed, the high pitch rate is preferred, likewise. However, there is a limitation on the pitch rate due to the initial transient response. As a result, the nondimensionalized pitch rate has a trade-off relation between the accuracy and the economy of the computation.

The reduced frequency of 0.1 and the amplitude of $1^{\circ}$ of the forced harmonic motion were chosen for the further study. The pitch angle displacement of $2^{\circ}$ of the CPRM was chosen for comparing with the forced harmonic motion. The nondimensionalized pitch rate or the roll rate was carefully chosen to satisfying the both criteria.

Table 4.7 Prescribed motion parameter effect on the computation time

| Prescribed motion | Parameter | Computation time |
| :--- | :---: | :---: |
| Forced harmonic motion | $k, a_{A}$ | $T=\pi D \sqrt{\rho_{\infty}} / M a_{\infty} k \sqrt{\gamma p_{\infty}}$ |
| Constant rate of rotation | $q^{*}, \Delta \alpha$ | $\Delta t=\Delta \alpha D \sqrt{\rho_{\infty}} / 2 M a_{\infty} q^{*} \sqrt{\gamma p_{\infty}}$ |

### 4.3. Evaluation of dynamic stability derivatives

In this sub-section, dynamic stability derivatives were evaluated using the prescribed motion while the Mach number, the angle of attack, and the center of gravity are changed, respectively. Those flow conditions and geometrical feature are important in the aerodynamic design. The scope of flow condition is limited on the transonic and supersonic region and low angle of attack. The location of the center of gravity is shifted to the downstream direction from the center of the fuselage.

The pitch damping coefficient is affected by the longitudinal location of the center of gravity. Two cases from the references were investigated. The flow conditions and the prescribed motion parameters of the center of gravity of 5.5 D case [29] are described in the Table 4.8. That of the 6.0D case [41, 42] are described in the Table 4.9. The results of the pitch damping coefficient are depicted and compared with the references in the Figure 4.14 and the Figure 4.15. The peak value of the pitch damping coefficient is observed around the Mach number of 0.9 in the both cases. A drastic decrease in magnitude of the pitch damping coefficient is observed around the Mach number of 1.2. That change is affected by the conversion of the normal shock into the oblique shock [56]. Overall, the pitch damping coefficient evaluated by using the both prescribed motions are match well with the result of the references.

The roll damping coefficient is evaluated using the CRRM. The flow conditions and the CRRM parameters are described in the Table 4.10. The result of the roll damping coefficient is depicted and compared with the references in the Figure 4.16. The drastic increase of the roll damping coefficient is observed around the Mach number of 1.2. The peak value of the roll damping coefficient is observed around the Mach number of 1.4.

Overall, the roll damping coefficient evaluated by using the CRRM are match well with the result of the references.

Table 4.8 Pitch damping coefficient with respect to Ma (C.G. 5.5D)

| Diameter | 30 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
|  | $0.8,0.9,0.95,1.05,1.1,1.2,1.3$ |
| Mach number | $, 1.4,1.5,1.58,1.76,1.89,2.16$, |
|  | $2.48,2.88,3.24$ |
| Nondimensionalized pitch rate | $-0.002,-0.003$ |
| Angular displacement | $2^{\circ}$ |
| Reduced frequency | 0.1 |
| Amplitude | $1^{\circ}$ |



Figure 4.14 Pitch damping coefficient with respect to Ma (C.G. 5.5D)

Table 4.9 Pitch damping coefficient with respect to $M a$ (C.G. 6.0D)

| Diameter | 38.1 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
|  | $0.8,0.9,0.95,1.05,1.1,1.2,1.3$ |
| Mach number | $, 1.4,1.5,1.58,1.76,1.89,2.16$, |
|  | $2.48,2.88,3.24$ |
| Nondimensionalized pitch rate | $-0.0015,-0.002,-0.0025$ |
| Angular displacement | $2^{\circ}$ |
| Reduced frequency | 0.1 |
| Amplitude | $1^{\circ}$ |



Figure 4.15 Pitch damping coefficient with respect to $M a$ (C.G. 6.0D)

Table 4.10 Roll damping coefficient with respect to $M a$

| Diameter | 38.1 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
|  | $0.8,0.9,0.95,1.05,1.1,1.2,1.3$ |
| Mach number | $, 1.4,1.5,1.58,1.76,1.89,2.16$, |
|  | $2.48,2.88,3.24$ |
| Nondimensionalized roll rate | $0.005,0.01$ |
| Angular displacement | $30^{\circ}$ |



Figure 4.16 Roll damping coefficient with respect to $M a$

The pitch damping coefficient with respect to $\alpha$ is evaluated using the both prescribed motion. The flow conditions and the both prescribed motion parameters are described in the Table 4.11. The result of the pitch damping coefficient is depicted and compared with the references in the Figure 4.17. Overall, the pitch damping coefficient evaluated by using the both prescribed motions are match well with the result of the references except the angle of attack of $0^{\circ}$. This discrepancy is caused by the contamination of experimental results due to the interference effects of the strut on the pressure distribution of the Basic Finner model [46].

The roll damping coefficient with respect to $\alpha$ is evaluated using the CRRM. The flow conditions and the both prescribed motion parameters are described in the Table 4.12. The result of the roll damping coefficient is depicted and compared with the references in the Figure 4.18. Overall, the roll damping coefficient evaluated by using the CRRM is match well with the result of the references except the high angle of attack. This discrepancy is caused by viscosity effect, which caused by the assumption in the governing equation.

The longitudinal location of the Basic Finner calibrated by its diameter is shown in Figure 4.19. The center of the gravity is shifted from the center of the fuselage to 8.5 D aft from the nose apex. The detailed conditions are described in the Table 4.13. The pitch damping coefficient with respect to the C.G. location is depicted in Figure 4.20. The power of the pitch damping arisen from the fins are the stronger than that of fuselage when the C.G. is located adjacent to the center of the fuselage. However, the contribution of fins is getting weaker as the moment arm is getting shorter. On the other hands, the contribution of the fuselage is insensitive to the shift of the center of gravity.

Table 4.11 Pitch damping coefficient with respect to $\alpha$ (C.G. 6.1D)

| Diameter | 31.75 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}$ |
| Mach number | 1.96 |
| Nondimensionalized pitch rate | $0.002,0.003$ |
| Angular displacement | $2^{\circ}$ |
| Reduced frequency | 0.1 |
| Amplitude | $1^{\circ}$ |



Figure 4.17 Pitch damping coefficient with respect to $\alpha$ (C.G. 6.1D)

Table 4.12 Roll damping coefficient with respect to $\alpha$

| Diameter | 45.72 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}, 20^{\circ}$ |
| Mach number | 2.49 |
| Nondimensionalized roll rate | $0.01,0.02$ |
| Angular displacement | $360^{\circ}$ |



Figure 4.18 Roll damping coefficient with respect to $\alpha$


Figure 4.19 The longitudinal location calibrated by the diameter

Table 4.13 Pitch damping coefficient with respect to the C.G. location

| Diameter | 30 mm |
| :--- | :--- |
| Angle of attack | $0^{\circ}$ |
| Mach number | 2.49 |
| Center of gravity | $5 \mathrm{D}, 5.5 \mathrm{D}, 6 \mathrm{D}, 6.5 \mathrm{D}$, |
|  | $0.01,0.02$ |
| Angular displacement | $360^{\circ}$ |



Figure 4.20 Pitch damping coefficient with respect to the C.G. location

## Chapter 5. Conclusion

The present study had investigated the numerical methods of evaluating the dynamic stability derivatives of the generic missile configuration using a prescribed motion. The forced harmonic motion and the constant pitch rate motion were used to evaluate the pitch damping coefficient and the constant roll rate motion was used to evaluate the roll damping coefficient. The constant pitch rate motion was recently utilized in the evaluation method.

Parametric studies on the prescribed motion parameters were performed. The reduced frequency and the amplitude of the forced harmonic motion and the nondimensionalized pitch rate and the nondimensionalized roll rate was investigated. It was shown that the choice of those parameter within the scope of the present parametric study revealed that dynamic stability derivatives are insensitive to those parameters. Given that the computational time is affected by those parameters, the high values of the reduced frequency is preferred. On the other hands, there is a tradeoff between the accuracy and the economy in the choice of the nondimensionalized rate of rotation.

Sensitivity studies on the dynamic stability derivatives to Mach number, angle of attack, and the longitudinal location of the center of gravity were performed. The evaluation method of dynamic stability derivatives using the both prescribed motion is validated by comparison with the result of references. The contribution of components to the pitch damping coefficient is also analyzed. If the center of gravity were shifted further aft, the pitch damping provided by the fins would be weaken.

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## 초 록

본 연구에서는 강제운동을 이용한 동안정 미계수 산출에 관한 수치적 방법에 대한 연구를 수행하였다. 연구 대상은 Basic Finner 형상을 선정하였으며, OpenFOAM 밀도 기반 해석자를 활용하여 CFD 해석을 수행하였다. 기존 연구에서 제시된 강제운동 방법의 입력값에 대한 민감도 분석을 수행하고, 효율적인 강제운동 입력값을 제시하였다. 피치 및 롤 방향 동안정 미계수를 산출하여 실험 및 해석결과와 비교 검증하였으며, 무게중심 변화에 의한 민감도 분석도 수행하였다.

주요어 : 전산유체역학, 동안정 미계수, 강제운동, 동적격자, 압축성 유동, 비정상 공기역학, 발사체
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