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TOWARDS HOLISTIC UNDERSTANDING OF MASS-ENERGY EQUIVALENCE: FOUR TYPES AND CONCEPTUAL DEVELOPMENT

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Towards Holistic Understanding of $E = mc^2$: Four Types and Conceptual Development

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Abstract

$E = mc^2$ (Mass-Energy Equivalence [MEE]), which was referred by Einstein to as the most important outcome of the special theory of relativity, has become a basis of modern physics. In spite of the current educational trends highlighting modern physics education, it has been pointed out that interpretations of MEE are still not in general agreement, and derivations of MEE gloss over some logical oversights. MEE also is often introduced only with a declarative knowledge that mc^2 represents the rest energy of a particle, making MEE more confusing.

In this study, distinguished papers on MEE were collected and examined to resolve the instructional challenges above. By specifying common features of derivations in each paper, especially specifying which physical object (particles or fields) was attributed to mass and energy, the criteria were established for categorizing its meaning, from which there were at least three types of understanding MEE: conjecture and correspondence (Type I), convertibility (Type II), and indistinguishability (Type III). By discovering the logical hierarchies between them, a new type of holistic understanding was suggested. In Type IV, not only the context by which MEE has developed, but also the context by which the two main perspectives of matter theory (fields and particles) have been closely related to be conceptually identified with each other, is explicitly revealed.

In addition, to examine if this categorization in general measures up to other examples of equivalence in physics, the semantic element of equivalence was extracted based on the three types of MEE. It was confirmed, as a result, that this categorization holds for the other examples: heat/mechanical-work equivalence (first law of thermodynamics), equivalence principle (general relativity), anti-de Sitter space/conformal field theory (AdS/CFT) correspondence (quantum theory of gravity), and matrix-mechanics/wave-mechanics equivalence (quantum mechanics). As a similar logical hierarchy could also be identified that was analogous to MEE, this categorization turned out to be, in some extent, universal.

The results of this study showed the holistic conceptual connection, intrinsic value, and historicity of content knowledge in physics by illustrating not only the conceptual relationship between mass and energy also that between fields and particles. This historicity and context of inquiry can serve as a good example of practices in physics. The result of this study, consequently, are expected to play a significant role as a conceptual framework (or theoretical framework) for the analysis of existing texts and the development of new curriculums.

Keywords: Mass-Energy Equivalence, Particle, Field, Categorization, Holistic Understanding, Conceptual Development, Quantum Field Theory

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1 Introduction

Most people have heard of mass-energy equivalence (MEE) at least once. Its mathematical expression $E = mc^2$ has become so famous that it has become a main symbol of physics. Einstein himself, who is celebrated for his development of relativity, also described MEE as "the most important upshot of the special theory of relativity" (Einstein et al. 1954, 227-232). His MEE is well known for having had a tremendous influence on philosophy and physics.

However, It has been pointed out that there have been some misleading ideas and misconceptions in learning and teaching MEE (Baierlein 1991; Lange 2001). In today's modern physics textbooks and even in high school physics curriculum, MEE has been introduced without clear specification of its meaning. Rather, it is often introduced, in a practical sense, with radioactivity (Cockcroft and Walton 1932) or only with the claim that mc^2 represents the rest energy of a particle, making MEE more ambiguous. Existing materials prompt the following questions: Are energy and mass merely the measures of convertible nature? (Baierlein 2007; Lange 2001) Why should the energy representation corresponding to the rest mass be mc^2?

In addition, unlike the equivalence principle—categorized by its conceptual distinction, in other words, its three categories (Carroll 2019) of understanding with their obvious development: the weak (Galilean) equivalence principle [WEP; $m_g = m_a$] $\rightarrow$ the Einstein equivalence principle [EEP; $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(G)$]

$\rightarrow$ the strong equivalence principle [SEP; $\mathsf{Physics_a} = \mathsf{Physics_{-g}}$]—MEE has not been given a proper categorization nor has there been general agreement on its meaning. There had been many slightly different interpretations of MEE (Torretti 1996, 283–350; Eddington 1987; Rindler 1969, 95–119; Lange 2001, 219–238; Einstein and Infeld 1966; Jammer 1997; Bondi and Spurgin 1987, 68). Even Einstein himself also made several important statements on how to understand and interpret MEE: "If a body gives off the energy L in the form of radiation, its mass diminishes by L/c^2" (Einstein 1905a), implying the two properties are convertible measure of inertia. His more credible view was well demonstrated after he completed relativity, giving a significant support on the same-property interpretation (Einstein 1948): "It followed from the special theory of relativity that mass and energy are both but different manifestations of the same thing."

The reason behind the emergence of these various different understandings may have been not only that there were a number of derivations for the MEE, but also that those derivations conclude with $E = mc^2$ without clear specification of its meaning. In addition, many interpretations of MEE have used the terms "mass" and "energy" without clearly distinguishing them from the concepts of fields or particles. To clear this ambiguity, in this study, I will present three ways to understand MEE focusing on two physical concepts that can have both mass and energy, fields and particles, by analyzing existing derivations of MEE. I will also suggest a lens of similar connotative distinction of "equivalence" in several other examples that have emerged in physics, through which it will be shown that our three ways to understand MEE directly measures up to this criteria or categorization.

Finally, just as Einstein himself developed his perspective on MEE, although each person has different points through which they feel they can sufficiently interpret "equivalence," it was necessary to count on the belief that there would be a conceptual development if the categorized types were connected. Putting the resultant categories all together in a proper context, The aim was to find a further hierarchical structure in the same way as the three types of equivalence principles (WEP, EEP, and SEP) did.

In summary, the research purpose of this study is to suggest a proper way to holistic understanding of MEE by categorizing the existing demonstrations of $E = mc^2$. In addition, the research questions in this study are as follows:

[QUESTION I] *Can understandings of MEE be categorized into a few types with some common features in deriving $E = mc^2$? And from these types, how can a holistic understanding of MEE be reached?*

[QUESTION II] *What common features do the types of understanding of MEE have, and can they be also used as criteria for categorizing other examples of equivalence in physics?*

2 Theoretical Background

2.1 Theory of Matter before Relativity: Particle and Field

"In physics we can give a cold scientific definition of reality which is free from all sentimental mystification." (Eddington 1927, 283)

Aristotle regarded matter not as matter itself, but as a combination of matter (*hyle*) and form (*eidos*). Matter (*hyle*) as "potentiality" and form (*eidos*) as "actuality" were characteristics of natural things (Agamben 2013, 46). This perspective was closer to a metaphysics of matter than physics of matter. Aristotle's metaphysical view of nature, known as "Aristotelianism," had authority for centuries and provided a dominant view of nature. Weyl (1924, 561–612) considered this epistemological idealism.

After Aristotle, the description of substance deviated somewhat from the system of metaphysics. Substantiality in metaphysics shifted from "form" to "matter," which can be regarded as a historical transition of meaning in the course of translating between "*substania*" (Latin) and "*hypo keimenon*" (Greek).

Entering the Galilean era, substantiality became a necessary condition for "Galilean physics." For Galilei, the definition of motion deviated from Aristotle's "substantial form." The concept of matter was an object being sustained in existence regardless of its motion, and motion (referred to as form in Aristotle's metaphysics) was represented as a concept independent from the change of existence. Galilei also relates the sensible representations of time and space to substantiality, and explains it as follows: Substance neither change over time, nor in the process of moving through space by motion. This space-time independence of substance not only gave birth to the concept of instantaneous velocity which he provided as the conceptual origin of "differentiation," but also provided a foothold for Aristotle's epistemological idealism to be reconstructed into a substantive theory of matter.

In the Galilean era, substances were considered to move, which led to the problem of measuring amounts of substances. This is the prototype of the concept of "mass." Contrary to contemporary physics textbooks that introduce the concept of momentum after ambiguously presenting the concept of mass, the concept of momentum—a quantity that is preserved in the process of collision—precedes the concept of mass in Galilei's kinematics.

In the Newtonian era, "mathematical empiricism," which stems from the belief that mathematical and physical laws could accurately describe phenomena through experimentation and observation was dominant. In his book *Principia*, Newton directly rebelled against the metaphysical idea of matter and epistemological idealism with the following famous quote:

> I frame no hypothesis. For whatever is not deduced from the phenomena is to be called a hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. (Newton 1687, 943)

His particle-based mechanical view of matter presupposes the hypothesis of impenetrability (two particles cannot be in the same space) and the notion of particles that are solid and immutable. Furthermore, he stated that mechanics should consist solely of the separation, combination, and motion of particles (Newton 1952). This was a detailed description of particles, and a presumption in the perspective of particle mechanics at the time.[1] One can find clues for the concept of mass from how he describes the motion of an object.

Taking over Galilei's mechanics, the emergence of the concept of mass, or the quantity of matter, resulted in matter, which only had geometric characteristics according to the hypothesis of impenetrability, being defined by inertia, a quantitative characteristic. Newton saw that both matter (Aristotle's matter) and motion (Aristotle's form) are conserved much like substances, as we learn in Newtonian mechanics today. To translate this into the language of contemporary physics, mass and momentum are conserved. In a closed system, motion and matter cannot be created, as substances cannot be created from nothingness. In other words, pure transmissibility without creation and annihilation became the standard for substantiality.

The concept of conservation also presupposes the concept of non-conservation. This is exemplified by the identification of classical physics with Newtonian mechanics: The mechanics of non-conservation is explained by Newton's second law. Newton described the change in momentum of an object as the result of a cause, which was referred to as "force," acting on it from the outside for a certain amount of time. According to Newton's second law ($\boldsymbol{a} = \boldsymbol{F}/m$), acceleration—a change in velocity with respect to force—decreases as mass increases.

In Newtonian mechanics, the concept of force is combined with the concept of particles and has two characteristics: (1) Force acts at a distance. (2) Force

[1]Paradoxically, contrary to his ambition of framing no hypothesis, he assumed the epistemological system prevalent at that time as the constituent of matter.

appears as a phenomenon when detected by particles. According to these characteristics, the concept of matter represented as particles had a serious problem: Action at a distance. For example, how can the moon and the earth interact gravitationally at a distance? The phenomenon of non-local interaction of two objects that were not in contact with each other could not be explained by the epistemological system at that time. Newton proposed a law that gravity is inversely proportional to the square of distance by observing the motion of planets, but could not provide any answer or hypothesis for the problem of action at a distance, even though he declared that he would frame no hypothesis.

Another view of matter is "field." Physicists initially assumed that aether—a medium that fills up the space—was an answer to the problem of action at a distance, taking clues from the principle of wave mechanics. Electromagnetic waves were seen as being propagated using aether as a medium. However, the hypothesis of aether was abandoned after the acceptance of the principle of relativity due to the universal nature of physics theory. It was recognized that electromagnetic waves constitute a physical entity themselves, rather than representing a hidden movement that cannot be seen. From this concept, classical physics was able to describe matter from two perspectives: particles and fields. Particles and fields clearly have different epistemological characteristics. This study will explore how the two concepts could be ontologically related from MEE.

2.2 Einstein's Relativistic Approach and its Implications

Einstein (1905) computed the dynamics of charged particles that are slowly accelerated. Assuming no radiation, he found the dynamics in the parallel direction (x-axis) of velocity were different from those in the vertical direction (y and z-axis). His computation, using today's friendly gamma notation, of the dynamics in the parallel direction was $F_{\parallel} = \gamma(\boldsymbol{v})^3 m a_{\parallel}$ and in the vertical direction was $F_{\perp} = \gamma(\boldsymbol{v})^2 m a_{\perp}$. As it turned out, he suggested two different types of mass: longitudinal mass $m_{\parallel} := F_{\parallel}/a_{\parallel}$ and transverse mass $m_{\perp} := F_{\perp}/a_{\perp}$.

The concept of transverse mass has made many other physicists criticize Einstein. However, the previous statements themselves are accurate and correct under the condition as follows:

- Defining mass as the force divided by acceleration $m = \boldsymbol{F}/\boldsymbol{a}$ (so-called force-derived mass) at that moment, it starts accelerating from rest with respect to co-moving coordinates $d^2x'^i/dt'^2$.
- Acceleration is expressed in terms of stationary coordinates d^2x^i/dt^2.

Thus, it is natural to derive the equations of motion below:

$$eE_{x'} = m_0\gamma(\boldsymbol{v})^3\frac{d^2x}{dt^2},\ \ eE_{y'} = m_0\gamma(\boldsymbol{v})^2\frac{d^2y}{dt^2},\ \text{ and }\ eE_{z'} = m_0\gamma(\boldsymbol{v})^2\frac{d^2z}{dt^2}. \tag{2.1}$$

This misunderstanding of Einstein's achievement was because there was neither general consensus on relativistic representation of momentum nor semantic agreement considering frame-dependence of kinematics at that time. In other words, Einstein could not start from $\boldsymbol{F} = d\boldsymbol{p}/dt$ properly. Instead, he started from $\boldsymbol{F} = m\boldsymbol{a}$. To correct Einstein's consequence, applying the Lorentz boost rule of force ($F_{x'} = F_x$, $F_{y'} = \gamma(\boldsymbol{v})F_y$, and $F_{z'} = \gamma(\boldsymbol{v})F_z$) leads to the correct equation (same situation as the Einstein's original paper)

$$eE_{x'} = m_0\gamma(\boldsymbol{v})^3\frac{d^2x}{dt^2} \to eE_x = m_0\gamma(\boldsymbol{v})^3\frac{d^2x}{dt^2}, \tag{2.2}$$

$$eE_{y'} = m_0\gamma(\boldsymbol{v})^2\frac{d^2y}{dt^2} \to eE_y = m_0\gamma(\boldsymbol{v})\frac{d^2y}{dt^2}, \tag{2.3}$$

$$eE_{z'} = m_0\gamma(\boldsymbol{v})^2\frac{d^2z}{dt^2} \rightarrow eE_z = m_0\gamma(\boldsymbol{v})\frac{d^2z}{dt^2}. \tag{2.4}$$

Thus one can conclude a correct force-derived transverse mass of $m_\perp^{(F)} = \gamma(\boldsymbol{v})m_0$.

Notwithstanding these surroundings, as luck would have it, in 1-D kinematics, only longitudinal mass can be defined. Thus one can write

$$\gamma(\boldsymbol{v})^3\frac{d^2\boldsymbol{x}}{dt^2} = \frac{d}{dt}\left(\gamma(\boldsymbol{v})\frac{d\boldsymbol{x}}{dt}\right), \tag{2.5}$$

which implies that Einstein's result, with no consideration of transverse mass, was only a numerical coincidence with today's well-known form of relativistic mass.

However, the result not only contains some derivational misapprehensions, but also would have indicated the serious shortcoming that an important result in mechanics was obtained from electromagnetic theory.

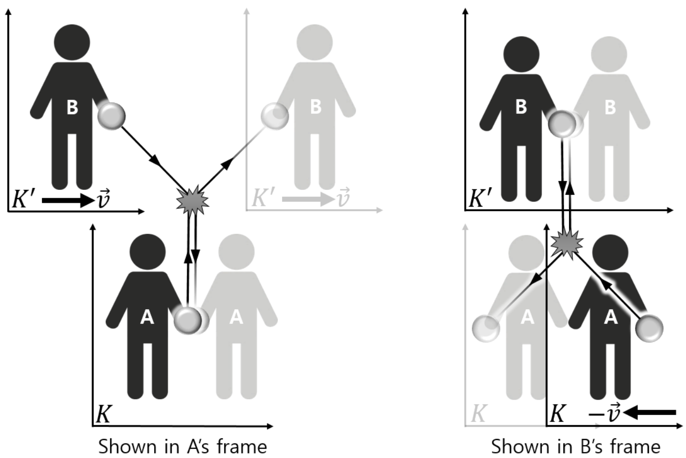


Figure 2.1 The famous situation for developing relativistic momentum suggested by Lewis and Tolman (1909): **A** is in the fixed K system, and **B** is in the moving K' system. **A** throws a ball along the $+y$ axis, and **B** throws the other ball along the $-y'$ axis. The relative speed between frames K and K' is v. The balls collide and bounce back, assuming the collision perfectly elastic.

On the other hand, the purely dynamic derivation of relativistic mass was deduced from symmetric collision (Lewis and Tolman 1909). They assumed a physical situation like Figure 2.1 and suggested a form of relativistic mass to satisfy the conservation of linear momentum:

$$m_{\mathrm{rel}}(\boldsymbol{v}) = \gamma(\boldsymbol{v})m_0, \tag{2.6}$$

which havs been accepted as today's concept of "relativistic mass."

As Einstein (1905) did in the longitudinal directions, although it turned out not to be derived correctly, relativistic extension of force can be written as $\boldsymbol{F} = \gamma(\boldsymbol{v})^3 m_0 \boldsymbol{a}$. Also in the same paper, he computed the energy of a charged particle withdrawn from an electric field, which can be expressed as equal to the relativistic extension of kinetic energy. He obtained

$$E_{\mathrm{kin}}[C, \boldsymbol{F}] = \int_C \boldsymbol{F} \cdot d\boldsymbol{x} = \left[\gamma(\boldsymbol{u})mc^2\right]_{u=0}^{u=v} = mc^2\left(\gamma(\boldsymbol{v}) - 1\right), \tag{2.7}$$

where C is the trajectory drawn by the force $\boldsymbol{F}$ and $x \in C$.

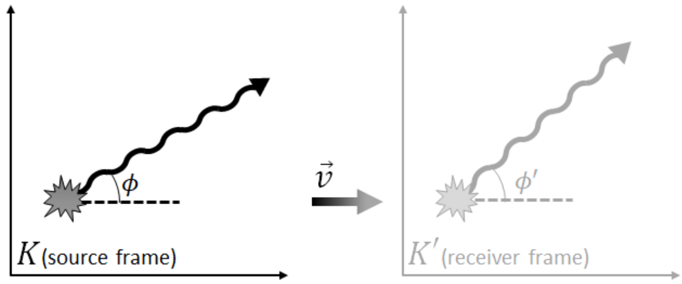


Figure 2.2 Identical light observed by two different reference frames whose relative speed is v.

In addition, he argued about the transform rule of radiation energy using his time dilation between the two different reference frames (Figure 2.2). He derived the expression of the energy (E), the azimuthal angle (ϕ)[2] and the frequency (ν) viewed from the receiver frame ($'$-added) as

$$E' = E\gamma(\boldsymbol{v})(1-\beta\cos\phi), \quad \nu' = \nu\gamma(\boldsymbol{v})(1-\beta\cos\phi), \quad \cos\phi' = \frac{\cos\phi-\beta}{1-\beta\cos\phi}. \tag{2.8}$$

Here I put $\beta = v/c$. These results, especially the energy transformation rule, were used to establish his first argument about MEE (Einstein 1905a).

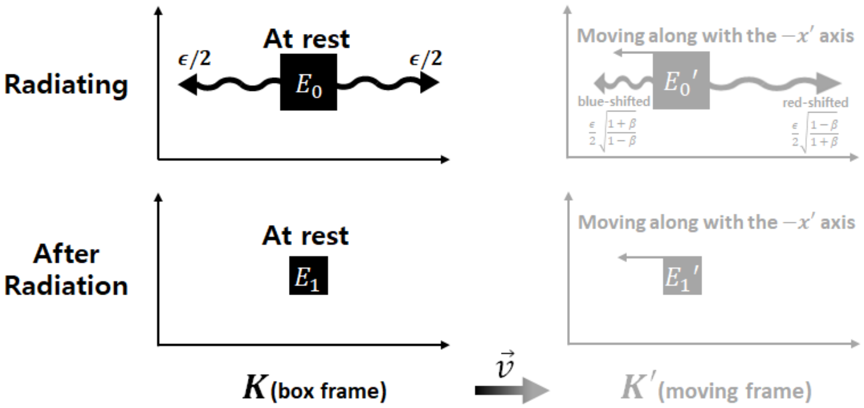


Figure 2.3 A box emitting light observed by two different reference frames whose relative speed is v. The energy of the box is denoted by E_0 (before the radiation) and E_1 (after the radiation) and the amount of energy during the radiation by ϵ. The quantities in the moving frame are written with a prime ($'$) symbol.

His reasoning was based on a comparison of the total energy of an object in two reference frames, and also on a comparison of before and after radiation. Here I reduce a spatial degree of freedom by setting $\phi = \phi' = 0$. If one set E_0 and E_0' to be the total energy of the box before the radiation, relative to K and K', respectively, and E_1 and E_1' to be the corresponding energy of the box after the radiation, respectively, and ϵ to be the energy content of the radiation

[2] Note the transverse direction means $\phi' = 0$ not $\phi = 0$ here.

viewed from K, he showed

$$(E_0' - E_0) = (E_1' - E_1) + \epsilon\left(\gamma(\boldsymbol{v}) - 1\right). \tag{2.9}$$

Here, taking a closer look at $E' - E$, which is the total energy difference between two frame, one can notice that it is just the kinetic energy of the box and that the equation (2.9) means the kinetic energy of the box decreased by $\epsilon\left(\gamma(\boldsymbol{v}) - 1\right)$. Since the relative velocity was fixed, he concluded the mass of the box decreased by its energy content. From the relativistic kinetic energy he derived in (2.7), he would have caught that the exact decrement of mass (Δm_0) is equal to the decrement of energy (ϵ/c^2). However, he used $E_{\text{kin}} = m\boldsymbol{v}^2/2$ instead of the correct relativistic expression for kinetic energy (2.7) (Einstein 1905a). It is somewhat dubious that Einstein derived the equivalence of mass and energy here, since the approximation only holds for low velocity. However, it does not matter that Einstein's intuition could not reach the answer at that time. In any case, many contemporary physicists might conclude $\Delta m_0 = \epsilon/c^2$.

2.2.1 Oversights and Shortcomings

One might conclude from this result that the equivalence of mass and energy was derived deductively. However, this result misses a critical point that most people can easily overlook. No one guaranteed the validity of the situation that Einstein hypothesized (Figure 2.3). This logically overseen problem can be reduced to the problem of inter-convertibility from mass to electromagnetic waves, which will be covered in Section 4.2.2. To determine whether this situation is physical or not, experimental or theoretical evidence to support the inter-convertibility is required, whose basic perspective requires its quantum nature, which will be covered in Appendix B.

Another significant unawareness is Newtonian reduction of relativistic kinetic expression. In Einstein's (1905) paper, he resolutely abandoned the higher order term of β, but did not mention the reduction to Newtonian kinetics at low speed. Low speed expansion was not mentioned in his June 1905 paper (Einstein 1905b). It was only after his theory of special relativity had received wide recognition that he first mentioned such an expansion in his review paper, "*Manuscript on the Special Theory of Relativity*" in 1912 (Einstein 2003):

$$E_{\text{kin}} = mc^2 + \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots - mc^2.^3 \tag{2.10}$$

Considering the low-speed contribution, the $-mc^2$ term must have drawn Einstein to endow this term with a special interpretation. It does not depend on speed, cancelled when expanded, and appears in the lower limit of the integral (2.7). He notes that "one is therefore already inclined at this point to grant a real significance to this term mc^2, to view it as the expression for the energy of the point at rest." (Einstein 1996, 49) What this history makes clear is that in his 1905 paper, he was not fully convinced of $E = mc^2$ and of how mass and energy are related intrinsically. It would have been a springboard to reach the conceptual merger of mass and energy only after giving the mc^2 term the interpretation of the energy corresponding to the rest mass. This study will give a detailed discussion about Einstein's heuristic approach to the equivalence in Section 4.2.1.

[3] I will drop the subscript $_0$ for notational convenience. The mass symbol written as m represents the rest mass rather than the relativistic mass from now on.

2.2.2 Implications and Empirical Evidence

Once the connection of mass and energy is premised as $E = mc^2$, energy and momentum can be integrated into the Lorentz covariant form as later in 1908, H. Minkowski (Rindler 1969) constructed today's four-vector structure of momentum and energy:

$$p^{\mu} = m\left(\frac{dt}{d\tau}, \frac{d\boldsymbol{x}}{d\tau}\right) = \left(\gamma(\boldsymbol{v})mc = \frac{E}{c}, \gamma(\boldsymbol{v})m\boldsymbol{v} = \boldsymbol{p}\right). \tag{2.11}$$

As the form of relativistic mass and momentum was being derived, there was no general agreement on the proper use of terms. Momentum can be defined as the product of "mass," which is short for "invariant mass" (m), and "proper velocity" ($d\boldsymbol{x}/d\tau$), but it may also be defined as the product of "mass," which is in this case short for "velocity-dependent mass" ($\gamma(\boldsymbol{v})m$), and "spatial velocity" ($d\boldsymbol{x}/dt$). In this paper, unless otherwise stated, the term "mass" will be used to refer to "invariant mass" denoted by $m_0 = m$, and "velocity" refer to "Newtonian spatial velocity." Also the term "energy" used here is generally characterized as a concept of frame-dependent energy, which varies $E \to \gamma E$ under the Lorentz boost, as is the case of Minkowski's four-vector construction.

One important significant implication of MEE is that conservation of four-momentum is expressed in each inertial frame by the separate laws of conservation of three-momentum and of relativistic mass. However, there is one important difference between the non-relativistic and relativistic situations: The presence of the factor γ shows that in contrast to invariant mass, relativistic mass is not a function of state. Let us consider the consequences of this for a collision that does not alter the inertial state of the particles involved. Since rest mass is a function of state, and indeed is an isolated invariant, the rest masses of all the particles will be unchanged by the collision. Hence both $\sum m$ and $\sum m\gamma$ will be unchanged. This has an interesting implication for the non-relativistic theory, in the limit $c \to \infty$,

$$c^2 \left(\sum m\gamma - \sum m \right) \to \sum \frac{1}{2} m \boldsymbol{v}^2. \tag{2.12}$$

It follows that kinetic energy must be conserved in such collisions, which is a correct non-relativistic result but one that had to be added as a quite separate hypothesis in the systematic development of Newtonian theory itself.

In a general collision with in Newtonian theory, kinetic energy alone is not conserved. Hence, by the reverse of the above argument, within the relativistic theory a general collision can neither preserve the rest masses of the individual particles nor even conserve their sum. This behavior of rest mass resembles that of internal energy rather than that of mass in Newtonian theory. Conservation of relativistic mass can then be expressed by the constancy of

$$\sum E_{\text{kin}} + c^2 \sum m. \tag{2.13}$$

If mc^2 is called the rest energy representation of the particle, (2.13) shows that the sum of the kinetic and rest energies of the particles involved in a collision is constant. This explicitly shows the parallel between rest mass and Newtonian internal energy, and for this reason, the law of conservation of relativistic mass is more commonly called the relativistic law of conservation of energy.

In particular, the rigorous validity of Lavoisier's law is renounced. Since another form of energy, heat, is also equivalent to mass, rest mass decreases in an exothermic reaction and it increases in an endothermic reaction. Jammer (1997) suggested a nice example: a kinetic reaction between two interacting molecules of rest masses m_1 and m_2, emerging after a reaction with masses

m_3 and m_4. For simplicity one can assume all motions to take place on the x-axis so that the conservation of momentum contains only an equation. In Newtonian picture, three conservation laws are:

- Conservation of mass (Lavoisier's law)

$$m_1 + m_2 = m_3 + m_4 \tag{2.14}$$

- Conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4 \tag{2.15}$$

- Conservation of energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_3 v_3^2 + \frac{1}{2} m_4 v_4^2 + Q \tag{2.16}$$

Here Q is the heat of reaction, which is either positive or negative. If $m_{1,2,3}$ and $v_{1,2,3}$ is given, then m_4 is determined by (2.14), v_4 by (2.15).

Now in the relativistic revision of this situation, two conservation laws are acquired:

- Conservation of four-momentum (spatial)

$$\gamma(v_1) m_1 v_1 + \gamma(v_2) m_2 v_2 = \gamma(v_3) m_3 v_3 + \gamma(v_4) m_4 v_4 \tag{2.17}$$

- Conservation of four-momentum (temporal)

$$\gamma(v_1) m_1 c^2 + \gamma(v_2) m_2 c^2 = \gamma(v_3) m_3 c^2 + \gamma(v_4) m_4 c^2 \tag{2.18}$$

Here it should be noted that in (2.18), no additional heat term Q can be posited, since the equation states the conservation of the total energy. Same as the previous, let $m_{1,2,3}$ and $v_{1,2,3}$ be given. If one can assume for a moment that Lavoisier's law holds also in this situation like (2.14), where m_4 is determined by (2.14) and v_4 by (2.17). But (2.18) would then in general be inconsistent with the preceding equations. Thus in relativistic dynamics, the factor γ arising disproves Lavoisier's law. This breakdown can also be checked by measuring the mass decrement $\Delta E/c^2$ of the two systems after combining chemically and giving off the energy ΔE. Actually, earlier than the advent of special relativity, Lavoisier's law was regarded with a certain amount of skepticism. In 1891, Kreichgauer (Verband Deutscher Physikalischer Gesellschaften 1891, 13–16) expressed doubts concerning the correctness of Lavoisier's law, with certain experiments on reactions involving mercury, bromine, and iodine. An experiment can also be designed that raises the temperature corresponding to the heat transferred by Q and measure the weight increased by the amount of Q/c^2. The problem with these experiments, however, is that the quantity to be measured is too small to be detected experimentally.

After Einstein's demonstration of the equivalence of mass and energy, a number of experiments to study atomic transformation were carried out that were of great significance, since the energy given off is on a far larger scale than that given off chemically.

These physical processes—relativistic breakdown of Lavoisier's law and atomic transformation—are not hypothetical situations. They were physically and experimentally verified. In particular it should be noted that the breakdown of Lavoisier's law involved the unprecedented case that mass is converted into another form of energy: thermal energy. Although, to specify thermal energy in modern physics, on a quantum time scale, it may have been converted into a form of light energy or possibly gauge bosons, and then converted into a different reactant. The energy given off is then transformed into heat as a result of interaction with electrons and atoms outside. Nonetheless, to justify MEE sufficiently, an example of conversion to types of energy other than light must be shown. For educational purposes, the example of relativistic breakdown of Lavoisier's law could play its role.

Some years after the first atomic transformations were investigated, new particles, called positrons, were discovered by Blackett and Occhialini (Chadwick et al. 1934). It was found that when an electron meets a positron, the two particles can annihilate each other to leave two photons giving off the total combined energy of the electron and positron. Symbolically, this pair annihilation process is written as follows:

$$e^+e^- \to 2\gamma. \tag{2.19}$$

Pair annihilation is known as the first case that showed all of the rest energy being transformed. Commonly known as pair production, the inverse process can also physically occur and has been verified by many experiments. Pair production was the first example showing not only that matter with mass is convertible into energy, but also that it was possible in the opposite direction. In addition, MEE was once again shown through a physically possible process rather than through the hypothetical situation suggested by Einstein.

2.3 Field Theoretical Approach to Find the Origin of Mass

Thomson (1881), when studying the dynamics of a uniformly charged and conductive sphere moving in a dielectric medium, found that, based on the energy and momentum expression for electromagnetic fields, it was harder to set the charged sphere in motion than an uncharged one. He assumed no frictional dissipation of energy, which is analogous to the case of bodies moving through a perfect fluid, and no distortion on the moving sphere. The latter has come to be considered fallacious since relativity. Due to this self-induction effect, electrostatic energy increases the inertia of an object, making it seem as if it has apparent momentum and energy. Thomson showed in his article that the apparent excessive induced mass (m_{ind}) where a sphere has a radius of a is ($q^2 = e^2/4\pi\epsilon_0$, where e is electric charge in SI unit)

$$m_{\text{ind}} = \frac{4}{15}\frac{q^2}{ac^2}. \tag{2.20}$$

To estimate the scale of this induced mass, he applied this result to the Earth orbiting the Sun and showed that the scale was remarkably negligible. At this stage, all inertial mass could not be interpreted as induced mass.

Thomson's result was improved up to the higher order of β by Heaviside's (1889) investigation of point charge. He derived a magnetic field due to a charge moving with velocity $\boldsymbol{v} = v\mathbf{e}_x$ and immediately assumed his result to also be valid for the case of a uniformly charged conductive sphere. He derived the excessive energy E_{exc} due to the electromagnetic field outside the moving sphere as

$$E_{\text{exc}} = \frac{q^2}{3ac^2}v^2. \tag{2.21}$$

Comparing this result with $\frac{1}{2}mv^2$, he obtained the increase of inertia by the form of excessive mass m_{exc}

$$m_{\text{exc}} = \frac{2}{3}\frac{q^2}{ac^2}. \tag{2.22}$$

Since the total electromagnetic field energy $E_{\rm EM}$ outside a stationary sphere with the overall surface charge q and radius a can be calculated by

$$E_{\rm EM} = \frac{1}{8\pi}\int \mathbf{E}^2 d\tau = \frac{1}{8\pi}\int_a^\infty 4\pi r^2 \mathbf{E}^2 dr = \frac{q^2}{2a}, \tag{2.23}$$

the increment of mass corresponds to 4/3 of its stationary field energy $E_{\rm EM}$ divided by c^2:

$$m_{\rm exc} = \frac{4}{3}\frac{E_{\rm EM}}{c^2}. \tag{2.24}$$

This problem indeed arises equivalently with simple calculation of momentum density of the electromagnetic field. When $v \ll c$, the momentum-derived mass is computed to be

$$m_{\rm EM}^{(p)} = \frac{\mathbf{E}\times\mathbf{H}}{\boldsymbol{v}} = 2\pi\int_a^\infty dr\, r^2 \int d\theta\, \frac{\epsilon_0}{c^2}\mathbf{E}^2 \sin^3\theta = \frac{2}{3}\frac{q^2}{ac^2}. \tag{2.25}$$

In contrast to Thomson, Heaviside spoke explicitly of the "electric origin of inertia." Jammer (1997) commented on Heaviside's viewpoint: This excessive mass is a physically significant phenomenon analogous not only to mechanical inertia but an inertial effect itself. Wien (1901) generalized Heaviside's result and was convinced that mechanical inertia can be derived from electromagnetic theory (Miller 1986). His derivation of inertial mass was based on Searle's (1896) result concerning the field energy $E(\boldsymbol{v})$ corresponding to ellipsoidal body moving with velocity $\boldsymbol{v}$ and energy at rest $E(0)$:

$$E(\boldsymbol{v}) = E(0)\frac{1+\beta^2/3}{\sqrt{1-\beta^2}\arcsin\beta}. \tag{2.26}$$

Wien expanded this result and obtained

$$\frac{E(\boldsymbol{v})}{E(0)} = 1 + \frac{2}{3}\beta^2 + \mathcal{O}(\beta^4). \tag{2.27}$$

Comparing the increment in energy, due to the motion of charged body, up to leading order, with $\frac{1}{2}mv^2$ he obtained the corresponding inertial mass

$$m = \frac{4}{3}\frac{E(0)}{c^2}. \tag{2.28}$$

Abraham (1903) was concerned with the energy transfer in the electromagnetic field theory. The concept of momentum is directly interpreted as the field-theoretical point of view. He conceptually linked Maxwell's theory and Lorentz formula by showing the force density $\mathbf{f}$ can be expressed by

$$f_i = \rho\left(E_i + \frac{1}{c}\varepsilon_{ijk}v_j B_k\right) = \frac{\partial}{\partial x^j}T_{ij} - \frac{dg_i^{(f)}}{dt} \tag{2.29}$$

where $\mathbf{T}$ is the Maxwell stress tensor and $\mathbf{g}^{(f)}$ is the electromagnetic momentum density of field. Abraham and Lorentz (1892) applied this to the electron using Abraham-Lorentz force to get the mass-energy relation

$$m_{\mathrm{EM}} = \frac{4}{3}\frac{E_{\mathrm{EM}}}{c^2}. \tag{2.30}$$

Right after this result, Wien (1901) and Abraham (1903) tried to demonstrate the electrical origin of matter and that electromagnetic mass should be identical to inertial mass of bodies. However, two problems remained unsolved. One can imagine the point particle model of the electron: The total energy (2.23) with a point source ($a \to 0$) tells us that the energy in the electromagnetic field is infinite since the field near the source has divergent contribution. In addition, the 4/3 factor in (2.30) was still unacceptable for contemporary physics.

After the quantum theory of electrons was developed, the first divergent problem was solved by showing that due to quantum fluctuation, the charge of an electron is smeared out in its position. This problem might also be solved by the uncertainty principle. The ideal point particle model of the electron certainly breaks down for the scale of $a < \hbar/mc$, called the Compton wavelength. At the momentum scale of $p > \hbar/r$, the velocity p/m approaches order c, thus the electron cannot be regarded as a static point. If one can set a cut-off at the Compton wavelength, the electrostatic contribution to the mass of an electron is of the order of the fine structure constant $\alpha = q^2/\hbar c \simeq 1/137$, that is $\delta m/m \sim \alpha$.

2.3.1 Resolution of Discrepancy between Electromagnetic Mass Theory and Special Relativity

The 4/3 factor has been attributed to neglecting the mechanical momentum flow inside the charge distribution. For a surface-charged sphere moving with velocity $\boldsymbol{v} = v\mathbf{e}_x$, the energy-momentum tensor has the form ($c = 1$ for notational convenience)

$$T^{00} = \rho, \quad T^{0i} = g_i, \quad \text{and} \quad T^{ij} = t_{ij}, \tag{2.31}$$

where ρ is energy density, g_i's are the momentum flow, and t_{ij} is the mechanical stress. For a fixed sphere, $T^{\mu\nu}$ would be of the form $T^{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$. A Lorentz transformation of the tensor gives the tension transformation rule in a co-moving frame as

$$g_x = \gamma^2(\rho v + t_{xx} v). \tag{2.32}$$

What has been overlooked is the mechanical stress component in the second term, making the exact total mechanical momentum

$$p_x = \int dx\, v \int dy\, dz\, t_{xx} = -\frac{e^2 v}{32\pi\epsilon_0 a^4}\int_{-a}^{a}(a^2 - x^2)\, dx = -\frac{1}{6}\frac{q^2 v}{ac^2}. \tag{2.33}$$

Thus, the ponderomotive stress acting on the surface charge elements leads to the overall pressure of $-\pi(a^2 - x^2)\epsilon_0 \mathbf{E}^2/2$. This additional contribution is negative, making the momentum-derived mass $m_{\mathrm{EM}}^{(p)} = \boldsymbol{p}/\boldsymbol{v}$ consistent with the energy-derived mass $m_{\mathrm{EM}}^{(p)} = E_{\mathrm{EM}}/c^2$.

2.4 Interpretations of Mass-Energy Equivalence

Equals, a symbol for numerical agreement, are represented in various connotations in physics. This section introduces the inconsistent views of prominent physicists or philosophers of the time who understand MEE.

Bunge (1967, 199–202) objected to the conceptual equivalence between mass and energy. He argued $E = mc^2$ does not mean conceptual equivalence as in a mathematical identity, but it can hold only in some cases that are assigned a mass to begin with, emphasizing its use in technical aspects according to physical phenomena in nature. For instance, mass is to energy in $E = mc^2$ what force is to displacement in Hooke's law, $F = -kx$.[4] He also believed that the apparent mass of light did not exist and stuck to the concept of mass as an invariant quantity.[5] For an example of quantum-electrodynamic interactions such as pair annihilation, he disproved their conceptual equivalence, saying that while energy is conserved, mass dissipates through the process; light was not considered a mechanical object.

Torretti (1996, 283–350) focused on the arbitrariness of unit systems to argue the meaning of equivalence. The units that are used to measure mass and energy seem to be distinguished from each other as different physical properties. However, such a distinction, according to his book, is the result of "convenient

[4]This is not a proper analogy because there exist many different meanings for how two physical quantities are connected by an equal sign.

[5]It may be helpful to consider an important semantic issue about mass. The current concept of mass is generally agreed not to be a Lorentz invariant like energy, with the concept of rest mass being utilized for the purpose of distinguishing it from energy.

but deceitful act of the mind" (Torretti 1996, 298). Speaking against Bunge's argument, he subsequently claimed that the conceptual distinction between mass and energy depends on the unit system used, and that the two concepts could be on an equal footing by setting $c = 1$. Therefore, he explained mass and energy as the same properties.

Eddington (1987) likewise recognized that mass and energy differ only in units (with only artificial differences) and that mass and energy have no measurement difference using a unit system with $c = 1$.

Both Eddington and Torretti did not explicitly state whether or not properties are on the same ontological ground, for instance, that one property can be substituted for the other. Even though they acknowledged that the connotations of these terms are obviously different, they recognized that what is denoted by the two terms is the same. That is to say, two properties are the same.

Like Bunge's interpretation, Bondi and Spurgin (1987, 68) argued that $E = mc^2$ should not be interpreted as the two properties being the same, just as $m = \rho V =$ (density) $\times$ (volume) is not interpreted as mass and volume being the same property. In other words, they did not recognize the particularity of c (speed of light) as a global constant specifying the special theory of relativity and the arbitrariness of unit system, therefore disagreeing with Torretti's and Eddington's argument. They claimed that mass and energy, as well as the significant conceptual (or connotative) distinction between time and space, are also conceptually (connotatively) distinguished, and that $E = mc^2$ merely tells the linear transition between the two properties.

Contrary to the same-property interpretation, Rindler's (1969, 95–119) comments are essentially different. He presumed the explicit convertibility between the two attributes. Since $E = mc^2$ can be regarded as a significant corollary of special relativity and special relativity itself tells us neither about the ontological identification of mass and energy nor about whether two properties are same, he put the interpretation on hold while leaving a challenge for future physicists. Contemporary (in the early 20th century) physics was basically able to determine little about the composition of matter. With a careful stance about this controversial topic, his only assertive argument was that it is reasonable to interpret that two properties are inter-convertible to each other.

Lange (2001, 219–238) pointed out that the ontological implications of $E = mc^2$ have been misunderstood so widely and in so many ways. He directly opposed the convertibility or inclusion interpretation (mass is only a form of energy). For Lange, Lorentz invariance is the sufficient condition of reality since in all reference frames the Lorentz scalar always gives the same value. According to this condition, (rest) mass could be regarded as the only "real property," whereas energy is not. However, according to the definition of mass (extent to inertia), inertiality itself is not Lorentz invariant, thus mass cannot also be Lorentz invariant. Since so-called relativistic mass or the transformation rule of mass is $m \to \gamma(\boldsymbol{v})m$ under the Lorentz boost, his reason to confine the semantic definition of mass to rest mass is in some sense reasonable (Lorentz invariance), but is not in the sense of the definition of mass. This can also be somewhat misleading.

2.5 Pragmatic Distinction of Mass and Energy

It is now clear that in relativistic picture, energy and mass are somewhat equivalent physical attributes. This suggests that any energy-carrying field such as the electromagnetic field has as such right to be considered as matter as does the ordinary substantial matter that is usually designated as such. From here, I will discuss why the pre-relativistic distinction between energy and mass is still valid. Before we can see why mass and energy aspects were separated in the Newtonian limit, we must consider what are the properties of substantial matter are so that it can be clearly distinguished in everyday experience from fields carrying energy. At a phenomenological level, there are primarily two such properties: the locality of substantial matter and the dominant contribution of mass to energy being inert. The former is responsible for point particles being approximately realizable, and for the conception of a particle as identifiable. In the continuous medium theory, this leads to a particular material being recognizable enough that it can be followed during its motion, so the velocity of the material is well defined everywhere. Let us now turn our attention to the latter property.

A more specific description of this property is that for a given particle, the variation in mass over the range of states that are being considered is small compared to its total mass in any of these states. Let one such state be chosen as a reference state, and let m_0 be its rest mass. If the relativistic internal energy U of a state of mass m relative to the reference state is defined by

$$U = (m - m_0)c^2, \tag{2.34}$$

then in any collision, the law of conservation of energy can be expressed as the conservation of kinetic plus internal energy. In this way, the contribution of $\sum m_0 c^2$ to (2.13) is removed from consideration. This is useful as this contribution, which is constant, is normally far larger than either of the variable contributions $\sum E_{\text{kin}}$ or $\sum U$. This is why m_0 is called the inert mass of the particle.

Such a distinction of inert mass and internal energy can be made in a pragmatic sense. Let us consider a few examples. In atomic physics, the natural reference state for a nucleus is that in which all the nucleons are entirely separated. The internal energy is then known as the binding energy of the nucleus. It is a useful concept even though it may be of the order of one percent of the rest energy. The nucleus is treated as inert and attention is centered on the electronic structure of the atoms. The natural reference state now is that in which the nucleus is in its ground state and the electrons are widely separated from it and from one another. Relative to this, the binding energies of atomic states are of far smaller order than those in atomic physics. This illustrates the dependence of the internal energy concept on the situations being considered, as atomic and nuclear physics both treat the same physical system but they concern different ranges of states.

Also, one can consider that there exists an inert mass that is conserved in all collisions and is constant as long as a particle preserves its identity. This fact will be referred to as the "conservation of inert mass." It enables the dominant inert mass contribution to be excluded from the conservation of energy in all collisions, not only in the identity-preserving ones. Rather than the conservation of relativistic mass, it is the true analogue of the Newtonian conservation of mass. It is less fundamental than conservation of four-momentum in that at any conceptual level, it is a phenomenological explanation of details of the internal structure of the particles that are outside the scope of the theory at that level. Inert mass thus transcends dynamics in the same way as energy does in Newtonian theory.

When considering more particles than one in a fixed inertial frame, another qualitative distinction is possible. Taking a close look at pair creation, due to the concept of quanta, the incoming energy of the photon above a threshold is needed, at least the total rest mass energy of the two particles created. However, in the case of pair annihilation, the kinetic energy of the reactant particles can also contribute to the energy of the photon created, so in this case the energy has a threshold but is continuous above the threshold. On the scope of subatomic scale, the discontinuity of particles (matters) is, in other words, the discontinuity of mass. But energy, as long as it is unbound so that it is not discrete, has a continuous spectrum. As long as the threshold is always kept in mind when considering the transformation from energy to mass or its inverse, energy is at least partially continuous, but total rest mass is always discrete. Of course, mass can be considered quasi-continuous, like a conductor's energy band, but the total rest mass of the particles is clearly discrete as long as the constituents of substantial matter are literal particles.

3 Research Procedure and Methods

3.1 Requirements for Literature Search and Collection

In order to extract the features of the existing methods of understanding MEE, an online search was performed to collect existing papers. All data were primarily collected using Google Scholar, and a snow ball search method—a method of searching for additional major authors and referenced papers—was used to sufficiently collect relevant literature. The searched terms and conditions were as follows:

(a) keyword: French, German, Italian, and Spanish translations were included considering the historical background at the time of the emergence of the theory and the diversity of academic languages.

- [Formula] `E=mc2`
- [English] `mass-energy equivalence (equivalence between mass and energy), mass-energy relation, energy of inertia (energy and inertia)`
- [French] `l'équivalence masse-énergie (d'équivalence entre la masse et l'énergie), la relation entre masse et énergie, l'énergie de l'inertie`

- [German] `Masse-Energie-Äquivalenz (Äquivalenz von Masse und Energie), Masse-Energie-Beziehung, Trägheit und Energie`
- [Italian] `equivalenza massa-energia (equivalenza tra massa ed energia), la relazione tra l'energia e la massa, energia di inerzia`
- [Spanish] `Equivalencia masa-energía (Equivalencia entre masa y energía), Relación entre masa y energía`

(b) To ensure the authority of the study, papers published in SCI(E)-indexed journals before December 2022 and papers written by scientists who were given a Nobel Prize in recognition of their outstanding achievements in physics were included.

3.2 Literature Selection and Criteria of Analysis

Seven hundred and twenty-seven papers were collected using an online search method focusing on keywords and titles. Among them, papers that meet the following selection criteria were included to be analyzed in this study.

(a) Papers that include a derivation that states that mass and energy are equivalent and those that originally did not aim to derive MEE but did so as a by-product were included. The semantic category of MEE was limited to the clear linear relationship between mass and energy. (This included papers that did not explicitly identify the coefficient as c^2.)

(b) Papers with logical oversights were included because they could be used as a common feature. (This included papers that used knowledge of quantum theory or includeed circular reasoning by using principles that can be derived through MEE.)

(c) Papers that obtained an assymptotic relation through derivation in a limit situation, for example $\lim_{v\to\infty} E(m) \propto m$, were excluded since they could not show the direct linear relation.

(d) Where there were equivalent approaches, only the paper published earlier was included.

(e) Papers with demonstrations that did not differ from those presented in modern physics textbooks were excluded.

Supplementary material was selected according to the needs of the research process at the discretion of the researcher, even if all of the above criteria were not met. According to the criteria above, 14 different deviations and 47 supplementary materials were collected.

3.3 Standards for Analysis of Literature

The selected materials were analyzed based on the following four standards:

(a) Physical situations presented for derivation: What physical situations were presented and analyzed as the starting point for deriving MEE? For example, Einstein's 1905 derivation assumed a radiating object and analyzed the physical system.

(b) Assumptions needed for development of logic and logical oversights: When limiting to the contemporary physics knowledge (excluding the quantum mechanical background), what were the additionally required assumptions for the derivation process (or the physical situation provided in [a]) to be valid, or what were the logical oversights in the development of the derivation? For example, Einstein's 1905 derivation process assumes a situation in which the mass of matter reduces and converts into radiation—in other words, a quantum interaction between light and matter—and energy was thought to be conserved in the process.

(c) The meaning of equality in the derived results: Connecting the two quantities with equalities can be a representation of various meanings. In Einstein's 1905 derivation, this means that the physical quantities on the left and right sides can be converted to each other.

(d) How the field and particle are semantically connected to energy and mass in the derived results: In Einstein's 1905 derivation, the energy of the electromagnetic field is connected to the mass of the particle: a metaphysical entity constituting the massive body. In other words, (mass of particle) $\times$ c^2 $=$ (energy of field). The manner of connection was specified to allow extraction of the implications of the qualitative distinction between fields and particles—the two representative perspectives of classical matter theory—suggested by the apparent form of connection.

3.4 Classification and Use of Supplementary Materials

Based on the standards above, similarities that could be used to classify the existing derivations were explored. Fourteen derivations were classified for each of the above four standards. Thereafter, a common pattern among the classification of each standard was found for generality of classification. The common pattern was defined as a method of understanding MEE. This resulted in three methods of understanding being discovered. However, according to standard 3.3(d), the classified understandings of MME had a missing link. For example, in one of the classified types, when classifying the connection

$m_{\text{ptcl}}c^2 = E_{\text{ptcl}}$ used in relation to particles as a method of understanding, studies that drew $m_{\text{field}}c^2 = E_{\text{field}}$ as a conclusion could not be found. The missing link was supplemented using 24 of the 47 supplementary materials. The characteristics of the three methods of understanding were thus discovered.

3.5 Verification of Logical Hierarchy

Based on the understanding of mass and energy prior to the emergence of relativity, whether the three methods above could be considered independent was verified. Considering that the principles of equivalence were classified into three types in general relativity and that there was a logical hierarchy among them, the three methods of understanding MEE were examined in order to identify if there was a context of inquiry. For example, in terms of the power of equivalence, a direction from weak significance to strong significance for equivalence may be one local hierarchy.

3.6 Extracting Features and Semantic Elements of Equivalence

In this study, 3.3(a)-(d) were used as standards for understanding MEE. To verify that these standards were universally applicable to other equivalences, standard 3.3(d) and the features found in 3.4 were used as standards and were verified in terms of whether they could be applied to other examples of equivalences in physics. Five examples of equivalences in physics (including MEE) were analyzed in this study:

(a) mass-energy equivalence (MEE)

(b) equivalence principle (in general relativity): (b1) WEP and (b2) SEP

(c) the mechanical equivalent of heat (first law of thermodynamics)

(d) Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence (in string theory)

(e) theoretical equivalence of matrix mechanics and wave mechanics (in quantum mechanics).

The above can generally classified into

- equivalence of two quantities: (b1) and (c)
- equivalence of two theories: (b2), (d), and (e).

How the features found in 3.3.4 were alternately applied according to the classification above was also examined.

4 A Road to Understanding Mass-Energy Equivalence

Let us look over the concepts of mass and energy *before relativity* a priori. For Einstein, there were two physical realities: "matter" and "fields" (Einstein and Infeld 1966; Einstein and Born 2005, 170). Mass was often used to describe the quantity of particles, which was on the conceptual level of the discrete ingredient of matter; Matter was considered substance in the metaphysical sense, and on the other hand the this was technically defined model of existing things insofar as we borrow the classical particle concept as an idealization of the common-sense notion of matter. In classical physics, particles are described as ideal massive points. Massiveness of a particle can be considered a quantitative description of Locke's (1690) primary qualities.[1] In other words, the concept of mass served as a quantitative measure of particles.

Energy was in its early stage, a stage of matter. Classically, it consisted of two parts: kinetic and potential. There was no energy concept corresponding to the rest particle's mass itself. The rest energy E_0 in Galilei-invariant theory is unrelated to the rest mass m. Their relation $E_0 = mc^2$ was first formalized by Einstein.

[1]Properties of matter that are independent of any observer; solidity, extension, motion, number and, figure.

Though the concept of a field had been mathematically formulated in Newtonian mechanics as an answer to the problem of action at a distance, the concept of *fields* was still an ambiguous compared to particles. Before Maxwell's theory of electromagnetism, a field was just considered invisible entity placed in space that has wave properties. However, with the failure of aether theories and with the role of space in which light itself can be propagated without a medium, the concept of field gained its ontological status: a continuous ingredient of matter.

As the classical field theory having the significance to explain the basis of material things, fields also became established as a physical entity and were also considered substances. After the study of electromagnetism, the conceptual construction of energy density u_{EM} (Jackson 2021, 259) and momentum density $\mathbf{S}_{\mathrm{EM}}$ (Poynting 1884; Jackson 2021, 258–292) of electromagnetic fields was made:

$$u_{\mathrm{EM}} = \frac{1}{2}\left(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}\right), \quad \mathbf{S}_{\mathrm{EM}} = \mathbf{E}\times\mathbf{H}. \tag{4.1}$$

These are related to the conservation of energy momentum (continuity equation) as

$$\frac{d}{dt}\int d^3x\ \mathbf{J}\cdot\mathbf{E} + \frac{d}{dt}\int d^3x\ u_{\mathrm{EM}} + \int d^3x\ \nabla\cdot\mathbf{S} = 0. \tag{4.2}$$

Thus, energy no longer inheres only to the dynamics of particles, in the field-theoretic point of view; rather, it has also become an inherent concept of fields. However, before relativity emerged, the concept of inertia was still confined to a physical attribute of particles since the mass concept of fields was far beyond human recognition: fields do not exist the same way a solid or liquid does. Since fields also have no definite shape, they can fill in any space.[2]

[2] These properties of fields directly violate Locke's primary qualities. See Chapter 4.3.1.

The concepts of mass and energy as physical attributes were apparently limited to particles and fields, respectively. The concept of mass was only in possession of particles, while energy was in possession of fields or was defined dynamically. Obviously, there was neither a rest energy interpretation nor the intrinsic energy of a particle before Einstein (See Figure 4.1).

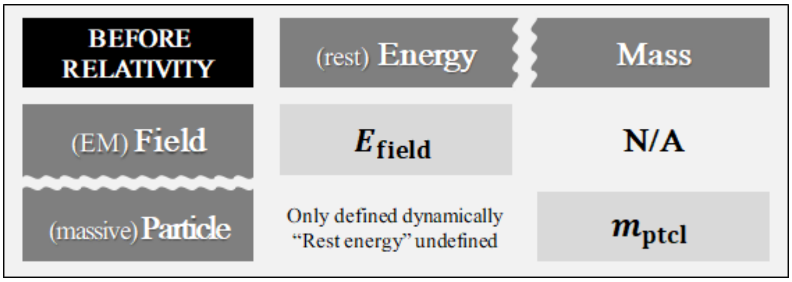


Figure 4.1 Understanding of mass and energy before relativity

It is exactly the same way as mass and energy are conceptually separated in Newtonian limit, we must consider what are the properties of substantial matter that so clearly distinguish it in everyday experience from fields carrying energy. At a phenomenological level, there are primarily two such properties. One is the locality of substantial matter, and the other is that the dominant contribution to the mass is inert. The former is responsible for point particles being approximately realizable, and for the conception of a particle as identifiable. In the theory of continuous medium, it leads to a particular material being sufficiently recognizable to be followed during its motion, so that the velocity of the material is well defined everywhere. The latter shows that such a distinction of inert mass and internal energy can also be made in a pragmatic sense.

4.1 Mass-Energy Equivalence Categorized

To ensure the quality of the collected data, I collected literature published in SCI(E)-indexed science (education) journals that included the derivation of MEE that could be found in Google Scholar searches for the topic of "mass-energy equivalence." I narrowed the scope of the target literature by limiting the meaning of "derivation" to papers involving derivations that resulted in a linear relationship between mass and energy.

A review of various derivations of MEE, indicated that most assumed a certain situation as the starting point of the derivation. In general sense, all situations can be classified into two groups: non-dynamic situations (Einstein 1906; Born 1962; Perez and Ribisi 2022; Laue 1911; Einstein 1905a; Rohrlich 1990; Steck and Rioux 1983; Leary and Ingham 2007; D'Abramo 2020), including non-classical concept like quanta, and purely dynamic situations (Einstein 1935; Dai and Dai 2018; Adkins 2008; Feigenbaum and Mermin 1988). In non-dynamic situations, further categorical presence of presuppositions can also be suggested: one major assumption has been that fields can be identified with a bundle of particles (or matter) to have a property of inertial mass (Einstein 1906; Born 1962; Perez and Ribisi 2022; Laue 1911), while the other is a straightforward assumption that particles (or matter) can be radiated by being transformed into a form of (electromagnetic) field or vice versa (Einstein 1905a; Rohrlich 1990; Steck and Rioux 1983; Leary and Ingham 2007; D'Abramo 2020). The former involves deducing with a kind of conjecture that there might be a mass corresponding to the energy of fields whose amount should be expressed as $m_{\text{field}} = E_{\text{field}}/c^2$. The latter involves drawing a conclusion that mass (particle) can be converted into energy (field) satisfying $E_{\text{field}} = m_{\text{ptcl}}c^2$.

On the other hand, purely dynamic derivations conclude the MEE of a particle as $E_{\text{ptcl}} = m_{\text{ptcl}}c^2$, implying in general no need to conceptually distinguish between mass and energy. (See Table 4.1)

Table 4.1 Characterization of MEEs from reviewing the literature

Situation posited	**Basic assumption**	**Concluded by**	**Type**
non-dynamic (including non-classical concept like quanta)	Fields can be identified with a bundle of particles	$m_{\text{field}} = E_{\text{field}}/c^2$	I
	Particles can be radiated into the form of field	$m_{\text{field}} = m_{\text{ptcl}}c^2$	II
purely dynamic	nothing in common	$E_{\text{ptcl}} = m_{\text{ptcl}}c^2$	III

I have found three types of proper connections between four quantities (m_{ptcl}, E_{ptcl}, m_{field} and E_{field}), making individual connections become its own category. All three types have distinct meanings in their way of equating both sides. The equality in Type I contains the meaning of conjecture. Rather than seeing it as a logical proof, this type emphasizes the correspondence between mass and energy through an assumption or a claim. The equality in Type II represents the convertibility between mass and energy. The last, Type III, sees that no more distinction between the mass and energy is possible, at least for the particle.

However, so far, there have still been empty connections to formulate each type: No connection exists for particles in Type I, $E_{\text{ptcl}} = m_{\text{ptcl}}c^2$, and no for fields in Type III, $E_{\text{field}} = m_{\text{field}}c^2$. All the empty connections with some supplementary materials make it obvious that there are at least three credible types of understanding: Conjecture and correspondence (Type I), convertibility (Type II), and indistinguishability (Type III). I will introduce Einstein's works representing each type and, at the same time, introduce a context in which they have been historically justified or conceptually developed. In addition, I will show that the resultant context does a good job of illustrating simplicity and universality as the nature of physics, working towards a holistic understanding.

4.2 Three Types of Mass-Energy Equivalence

4.2.1 Type I: Conjecture and Correspondence

In Type I understanding, conjecture and correspondence, MEE is defined as a conjecture that there might be a mass corresponding to the energy of a field, (for example, an electromagnetic field), $E_{\text{field}} = m_{\text{field}}c^2$, or as the same correspondence between energy and mass, $E_{\text{ptcl}} = m_{\text{ptcl}}c^2$ for particles (see Figure 4.2). This type contains no clear verification for $E = mc^2$, but a powerful assumption under which one can identify fields with particle or can imagine the existence of the non-kinetic portion of the particle's energy. For this reason, with no verification, neither constructing an integrated conservation law of mass and energy nor the conceptual integration of mass and energy also are possible.[3]

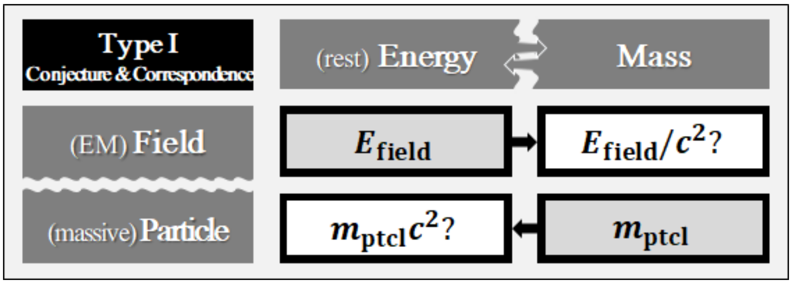


Figure 4.2 MEE (Type I): Conjecture and correspondence

First, many studies derived MEE with the assumption that a mass m_{field} exists by considering the field to be a particle and concluding that $E_{\text{field}} =$

[3]The integrated conservation law can be postulated, but it would not be possible to suggest a reasonable proof at this stage.

$m_{\text{field}}c^2$. This kind of understanding can be categorized as a sort of conjecture and correspondence. Studies that concluded $E_{\text{EM}} = m_{\text{EM}}c^2$ (Einstein 1906; Born 1962; Perez and Ribisi 2022; Laue 1911) by viewing an electromagnetic field as a single particle with a mass m_{EM} correspond to this understanding.[4]

One famous example describing MEE as conjecture and correspondence is Einstein's (1906) thought experiment involving abduction. Imagining a tube-

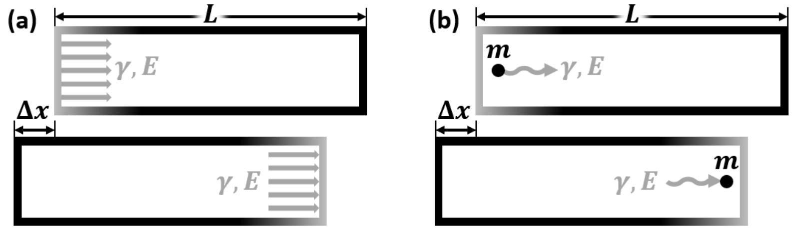


Figure 4.3 Einstein's thought experiment: radiation confined in a box

like hollow box of mass M and length L with an electromagnetic source on the left side, the radiation inside was absorbed on the wall of opposite side (see Figure 4.3a). Since a propagating electromagnetic wave has its momentum of E_{EM}/c, of which amount the box experiences recoil that makes momentum conservation hold, the total recoil is expressed by $\Delta x = E_{\text{EM}}L/Mc^2$. During this entire process, the center of mass is not preserved even though no external force was exerted. Einstein suggested two possible options to solve this dynamic contradiction: The concept of a massless carrier body that can violate momentum conservation can solve this mechanical contradiction, or it can also be solved with existence of inertia corresponding to the radiation. In other words, it was necessary to abrogate the conservation law between particle

[4]Since it presupposes the existence of mass that was originally an attribute of particles, what I emphasize here is that MEE (Type I) is not a deductive result but a correspondence mapping $m_{\text{EM}} \stackrel{?}{=} E_{\text{field}}/c^2$ (Figure 4.2). A deductive connection between mass and energy would be given in MEE (Type III).

and field or to give up the special right of inertiality that is vested only in the concept of particles.[5] Born (1962) also suggested mass m_{EM} being transported to resolve this contradiction: $M\Delta x - m_{\mathrm{EM}}L = 0$, resulting in $E_{\mathrm{EM}} = m_{\mathrm{EM}}c^2$.

Since the mass property was still attributed only to particles, their solution to this contradiction is the same as making a bold assumption that one can identify fields with particles. Some other studies (Perez and Ribisi 2022; Laue 1911) have also argued for $E_{\mathrm{field}} = m_{\mathrm{field}}c^2$. In case of a closed electromagnetic system, in particular, one would get a transformation rule of electromagnetic field energy of $E_{\mathrm{EM}}(\boldsymbol{v}) = \gamma(\boldsymbol{v})E_{\mathrm{EM}}(0)$, and by comparison with the second-order term $\mathcal{O}(\boldsymbol{v}^2)$ of its velocity-expansion to the non-relativistic kinetic contribution $m_{\mathrm{EM}}\boldsymbol{v}^2/2$, one would obtain $E_{\mathrm{EM}} = m_{\mathrm{EM}}c^2$, which is the same as Einstein's prescription. To show a closed system of fields behaving (when in uniform motion) as a particle, it is necessary to confirm whether it functions as a measure of inertia according to the definition of mass.

Physicists, including Einstein, studied the generalization of this result for a few decades after this with a focus on universality as a nature of physics. At the Salzburg conference in September 1909 (Stachel and Penrose 1998, 118; Isaacson 2007, 675), Einstein was strongly convinced that electromagnetic radiation should have its mass contribution. Claiming the wave theory of light was no longer complete, that is to say, light can be regarded as a bundle of particles (or packets of energy), he was inclined to generalize that inertiality is a property of a general form of energy, therefore electromagnetic radiation also has inertia.

[5] Einstein's prescription in this paper is an example of abduction as a scientific methodology, which is an educationally significant methodology (Adúriz-Bravo and Pinillos 2022).

This idea that a field, regardless of its massiveness, can be considered a particle transcends the classical concepts of mass and energy. This correspondence relies on the ideas of a simplified non-quantum mechanical version of the color confinement in quantum chromodyanmics. Concurrently, assuming the existence of an apparent mass contribution to an electromagnetic field eventually implies a simplified field theoretical model for particles: Particles are an intense accumulation of field energy. A further generalization was carried out as a by-product of investigating a time-independent energy-momentum tensor in general relativity (Klein 1918).

However, treating light as a bundle of massless (since a photon has no rest mass) particles can also bring up the issue that the stress-energy tensor of a massless particle is incompatible with that of the electromagnetic field. Lo (2006) pointed out that this intrinsic difference had no effect on the coupling of gravity.

Likewise, the conjecture and correspondence that energy congruous with the mass of a rest particle might exist may also be included in this category. Einstein (1907) formulated the energy content of a massive particle at rest as $E_0 = mc^2$ for the energy of a moving mass to be simplest form $E = \gamma(\boldsymbol{v})mc^2$. There was also no justification for this expression. However, he would have been convinced of the expression for rest energy. Hecht (2012) proposed that Einstein's assertion was based on the expression for kinetic energy of a particle, $E_{\text{kin}} = [\gamma(v) - 1]mc^2$, being expressed as a difference form of the integration $\left[\gamma(v')mc^2\right]_{v'=0}^{v'=v}$. Influenced by Einstein's heuristics, not only many modern physics textbooks have borrowed this approach as a way to introduce $E = mc^2$ (Beiser 2003, 26–27; Serway et al. 2004, 45–46; Tipler and Llewellyn 2012, 71–72; Harris 2007, 36; Krane 2012, 50–51; Thornton and Rex 2012, 62–63), also some papers (Duarte and Lima 2021) have so, with the purpose of giving instructional guidance. However, this argument is nothing but a claim and does not prove anything since it arbitrarily distinguishes the velocity-independent part from the velocity-dependent part.

Einstein was obviously inclined towards the conceptual integration of two concepts through his heuristics, but at that time, no clear derivation that supported Einstein's indulgence was suggested. The equivalence he wanted to get his hands on was Type III MEE, which perfectly holds for any energy and mass in an equal reference frame, regardless of the concepts of particle and field.

4.2.2 Type II: Convertibility

In Type II understanding, convertibility, MEE is defined as a linear convertibility between field energy and material (or constituent particle) mass: $E_{\text{field}} = m_{\text{ptcl}}c^2$ (see Figure 4.4). Once the hypothetical situation that particle can be converted into a radiation is verified, one can construct an integrated conservation law of mass and energy, even though particle-field identity could not be preserved. However, in this type, no conceptual replacement possible of one with the other. MEE (Type II) can also be further categorized in two macroscopic senses: One is actual convertibility, $E_{\text{field}} = \Delta m_{\text{body}}c^2$ (Einstein 1905a; Leary and Ingham 2007; Rohrlich 1990), and the other is potential convertibility, $E_{\text{field}} = m_{\text{body}}c^2$ (Steck and Rioux 1983). The former reserved generalization that all mass could be converted into energy, and the latter throws away the meticulousness that all mass may not be converted into energy.

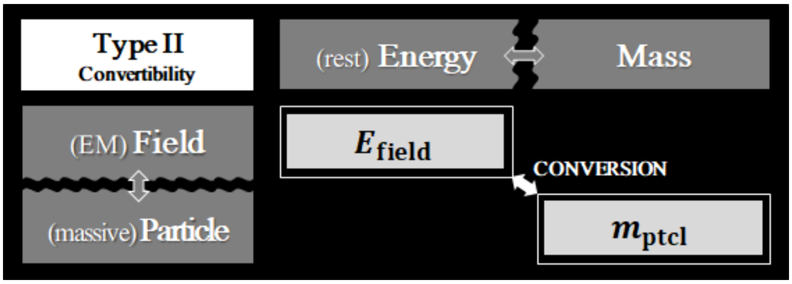


Figure 4.4 MEE (Type II): Convertibility

Derivations of MEE (Type II) have some common features: they all start with the assumption that matter can be converted into light (or other gauge bosons), with the exact linear connection of mass and energy coming from the belief that the conservation law of energy and momentum also holds for field-particle conversion.

Einstein (1905) is a well-known study showing MEE. It supposes a box emitting light observed by two different reference frames whose relative velocity is $\boldsymbol{v}$. The energy of the box is denoted by E_0(before the radiation) and E_1(after the radiation), and the amount of energy during the radiation is denoted by ϵ. The quantities in the moving frame are written with a prime ($'$) symbol (see Figure 2.3). Einstein's reasoning was based on the comparison of both the total energy of an object in two reference frames and before/after radiation. He designated E_0 and E'_0 as the total energy of the box before the radiation relative to K and K', respectively; E_1 and E'_1 as the energy of the box after the radiation; and ϵ as the energy content of the radiation viewed from K. Using his previous paper on the relativistic Doppler effect (Einstein 1905b), he showed

$$(E'_0 - E_0) = (E'_1 - E_1) + \epsilon\left(\gamma(\boldsymbol{v}) - 1\right). \qquad (4.3)$$

$E' - E$ is total energy difference between two frames, meaning the kinetic energy of the box. Since the relative velocity was fixed, Einstein concluded that the mass of the box Δm decreases by its energy content ϵ. The meaning of $\epsilon = \Delta mc^2$ is nothing but the partial conversion, leaving room for the extension to $E = mc^2$, which means latent ability to work is proportional to rest mass. His contemporary derivation of MEE required the energy conservation law to equate the left-hand side energy $\epsilon = \epsilon_{\mathrm{rad}}$ of electromagnetic propagation and the right-hand side mass decrement $\Delta m = \Delta m_{\mathrm{box}}$ of the box.

This paper was widely cited as the being the first "proof" of the "inertia of energy as such." However it was criticized due to its elusive assumption that the total energy of a body can be described by the additive composition of its kinetic and rest energy (Planck 1908), and later, due to its assuming the conclusion *petitio principii* (Ives 1952; Jammer 1997). The posited situation in which the box is represented by the bundle of particles being reduced to become radiated is, according to Figure 4.4, from the beginning, totally equivalent to assuming the conversion between mass and energy. The use of the conservation law of energy has the effect of directly assuming a linear relationship between them. Additionally, people were fully accustomed to the hypothetical process in which matter becomes light. From the viewpoint of interacting quantum field theory, the problem of whether or not this process is physically valid comes down to the problem of writing down a possible interaction.[6] With no consideration of internal degree of freedom, such as spin, relativistic field theory itself is required to satisfy the Klein-Gordon equation, which is nothing more than an operator form of the mass-energy relation, $E^2 = p^2c^2 + m^2c^4$. The

[6] (Scalar) quantum electrodynamics might be a descriptive model of this situation.

same went for the other derivations of MEE (Type II; Rohrlich 1990; Steck and Rioux 1983; Leary and Ingham 2007; D'Abramo 2020). It is a clear logical oversight.

Later, many experiments were performed and showed that many elementary particles, not only electrons (Klemperer 1934), interact with their anti-particles to produce gauge bosons. Matter is convertible into light. On this point, one can say that MEE would not be considered a result of special relativity alone or rather a result of special relativity and the quantum nature of physical interaction. The process of particle-particle interaction can be considered an exchanging process of mass and energy (whose exchanging rate is c^2) in which the sum of mass and energy is preserved. Light has only the attribute of energy, not the attribute of rest mass in the view of contemporary physics. Perhaps this was the reason why we say mass and energy are equivalent today. After Einstein's desired equivalence of Type III MEE, introduction of light quanta gives a possibility of non-trivial finite apparent mass $\gamma(v)m_{\mathrm{rad}}$ of an electromagnetic field: as $v \to c$ and $m_{\mathrm{rad}} \to 0$, $\gamma(v)m_{\mathrm{rad}} = \infty \times 0$ can converge to a non-trivial finite apparent mass whose exact value can be obtained by quantum physics.

4.2.3 Type III: Indistinguishability

The Type III understanding of MEE, indistinguishability, indicates the ontological merger of mass and energy. Mass and energy are conceptually integrated to make conceptual replacement with each other possible and to make an integrated conservation law (see Figure 4.5).

An important example is Einstein's (1935) well-known derivation that was given in a lecture in Pittsburgh in 1934 (Topper and Vincent 2007, 2016).

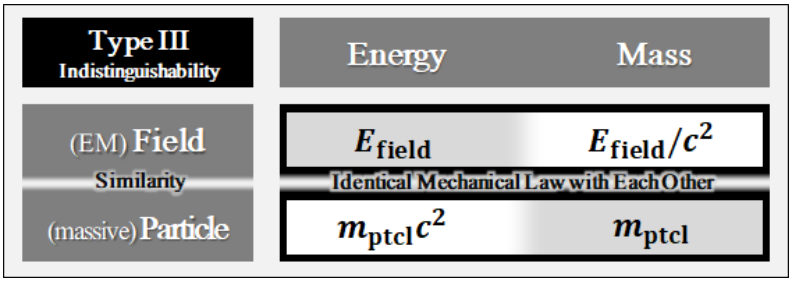


Figure 4.5 MEE (Type III): Indistinguishability

Einstein deduced that $m\left[\gamma(\boldsymbol{v})c, \gamma(\boldsymbol{v})\boldsymbol{v}\right]$ is covariant under a Lorentz transformation using covariant formulation of the space-time differential $(dt, \boldsymbol{dx})$. He then argued that if the conservation law of momentum and energy holds for arbitrary frames that are connected with Lorentz transformation, it is nothing but a possible form of energy-momentum covariant formulation for a massive particle

$$m\left[\gamma(\boldsymbol{v})c, \gamma(\boldsymbol{v})\boldsymbol{v}\right] = \left[(E_{\text{rest}} + E_{\text{kin}})/c, \boldsymbol{p}_{\text{rel}}\right]. \tag{4.4}$$

where Einstein used m with no subscript (instead of m_0) for invariant mass.

His argument is as follows: He first assumed $p^i_{\text{rel}} = mv^i f(v)$ and $E = E_{\text{rest}} + mg(v)$, with f and g being an arbitrary even function of $v = |\boldsymbol{v}|$. With a situation of arbitrary elastic head-on collisions of two equal masses with opposite velocities of equal magnitude $\boldsymbol{v}_+ + \boldsymbol{v}_- = 0$, due to the conservation laws, the situation after the collision should be symmetric, as it was before the collision. One can have

$$\gamma(\boldsymbol{v}_+) + \gamma(\boldsymbol{v}_-) = \gamma(\bar{\boldsymbol{v}}_+) + \gamma(\bar{\boldsymbol{v}}_-) \tag{4.5}$$

$$\gamma(\boldsymbol{v}_+)v^i_+ + \gamma(\boldsymbol{v}_-)v^i_- = \gamma(\bar{\boldsymbol{v}}_+)\bar{v}^i_+ + \gamma(\bar{\boldsymbol{v}}_-)\bar{v}^i_- \tag{4.6}$$

where the barred quantities refer to those after the collision. No other set of symmetric or anti-symmetric functions $f(v)$ and $g(v)$ is possible except

$f(\boldsymbol{v}) = \gamma(\boldsymbol{v})$ and $g(\boldsymbol{v}) = \gamma(\boldsymbol{v}) - 1$, which can be regarded as the correct form of relativistic momentum and kinetic energy. Next, Einstein considered inelastic collision to make the final product at rest, with an assumption that internal changes of each mass make a gain of constituent particles, leading to an increase in total mass. If conservation of energy holds for this process, after some lengthy algebra, he argued

$$\bar{E}_{\text{rest}} - E_{\text{rest}} = (\bar{m} - m)c^2. \tag{4.7}$$

With the stipulation that $E_{\text{rest}}(m = 0) = 0$, one can conclude that $E_{\text{rest}} = mc^2$. Combining this with the relativistic representation of kinetic and velocity-dependent mass, it could be easily shown that $E(\boldsymbol{v}) = m(\boldsymbol{v})c^2$ always holds in all frames.

In his very first derivation in 1905, Einstein may have recognized the clear limitations of the derivation using light—its masslessness—and again derived MEE using only the dynamics of a particle system. Inspired by the Einstein's radiating object situation, it is also possible to derive MEE (Type III) through the dynamic ejection of massive particles (Feigenbaum and Mermin 1988), rather than through radiation, to overcome this limitation. This ejecting particle situation is no different from the time reversal of inelastic collisions. Derivations with inelastic collision (or their reverse) had mainly been based on the belief that the energy conservation law holds not only for the elastic collision but also for arbitrary inelastic processes. This was not a big problem because it had been well established in classical collision. In order for energy to be conserved during an inelastic dynamic process, the total mass should increase to make a gain in the corresponding form of energy. However, no one could explain through which kind of physical interaction the mass increased (or even decreased) inside the reactants, violating the preservation of reactants' identities. Other demonstrations without the ad hoc situation were suggested, with another assumption: linearity of energy-momentum transformation (Dai and Dai 2018) and a direct linear relation between mass and energy (Adkins 2008).

Einstein's approach using an inelastic collision was adopted a proof of MEE that was fairly satisfactory compared to his previous arguments. Unlike his other demonstrations, he emphasized the logical validity of the process of showing MEE by adopting the word "derivation" from the title with great confidence. In MEE (Type I), only the implications of indistinguishability were given, without proper justification, and in MEE (Type II), the two concepts were still distinguishable. In MEE (Type III), mass and energy are no longer ontologically distanced.

However, this is clearly valid only for massive particles. The assumption that particles and fields are identified in MEE (Type I) have also been seen, and the possibility of transition between particles and fields in MEE (Type II). It is time to discuss MEE in relation to the other physical object: fields. For the mass of a field to be well defined, relativistic particle dynamics should be able to apply at least for the electromagnetic field. Like particle mass was well defined by $m = \boldsymbol{F}/\boldsymbol{a} = \boldsymbol{p}/\boldsymbol{v} = E/c^2$, there are also three ways to define the apparent mass of the electromagnetic field: momentum-derived mass, $m_{\mathrm{EM}}^{(p)} \overset{\mathrm{def}}{=} \boldsymbol{p}_{\mathrm{EM}}/\boldsymbol{v}_{\mathrm{EM}}$; energy-derived mass, $m_{\mathrm{EM}}^{(E)} \overset{\mathrm{def}}{=} E_{\mathrm{EM}}/c^2$; and force-derived mass, $m_{\mathrm{EM}}^{(F)} \overset{\mathrm{def}}{=} \boldsymbol{F}_{\mathrm{EM}}/\boldsymbol{a}_{\mathrm{EM}}$. In order to confirm MEE (Type III) for an electromagnetic field, the following two conditions should be considered:

- Existence of inertia: An electromagnetic field has inertia if and only if exertion of a force is necessary to accelerate it. It intuitively suggests that a group of fields can be treated as a particle.
- Mechanical consistency: same as for particles, $m_{\mathrm{EM}}^{(p)} = m_{\mathrm{EM}}^{(E)} = m_{\mathrm{EM}}^{(F)}$, i.e., $\boldsymbol{p}_{\mathrm{EM}}/\boldsymbol{v}_{\mathrm{EM}} = E_{\mathrm{EM}}/c^2 = \boldsymbol{F}_{\mathrm{EM}}/\boldsymbol{a}_{\mathrm{EM}}$.

In studying the dynamics of a uniformly charged and conductive sphere moving in an inductive medium, based on the energy and momentum expression for the electromagnetic fields in (4.1), Thomson (1881) found that it was harder to set in motion than an uncharged one as if the electromagnetic field itself underwent inertia (existence of inertia holds). This idea was developed further in many other studies (Searle 1897; Heaviside 1889; Abraham 1903).

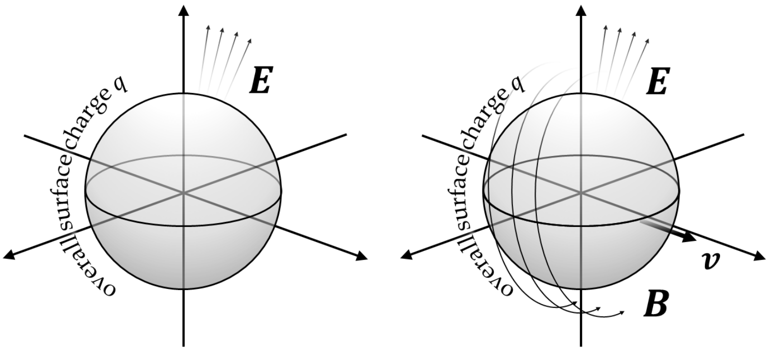


Figure 4.6 A surface-charged sphere (L: stationary, R: moving with constant velocity $\boldsymbol{v}$)

Consistency with particle analogy can now be demonstrated, showing $m_{\mathrm{EM}}^{(p)} = m_{\mathrm{EM}}^{(E)}$ first. Considering a simple uniformly surface-charged massless sphere with radius a and total charge q (see Figure 4.6), energy and momentum of an electromagnetic field can be acquired from (4.1):

$$E_{\mathrm{EM}} = \frac{1}{2}\int \mathbf{E}\cdot\mathbf{D}\, d^3x = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0}\frac{1}{a}, \tag{4.8}$$

$$\boldsymbol{p}_{\mathrm{EM}} = \int \mathbf{E}\times\mathbf{H}\, d^3x = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0}\frac{\boldsymbol{v}}{ac^2}. \tag{4.9}$$

The Lorentz boost transformation of E_{EM} makes no change under the low-speed approximation ($v \ll c$). The discrepancy between momentum-derived mass and energy-derived mass appears:

$$m_{\mathrm{EM}}^{(p)} = \frac{\boldsymbol{p}_{\mathrm{EM}}}{\boldsymbol{v}_{\mathrm{EM}}} = \frac{2}{3}\frac{q^2}{4\pi\epsilon_0 ac^2}, \quad m_{\mathrm{EM}}^{(E)} = \frac{E_{\mathrm{EM}}}{c^2} = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0 ac^2} \tag{4.10}$$

which was well-known "4/3-paradox," since their relation is $m_{\mathrm{EM}}^{(p)} = \frac{4}{3}m_{\mathrm{EM}}^{(E)}$. It seemed to disprove the MEE for the electromagnetic field. This paradox was solved in many papers (Fermi 1922; Dirac 1938; Schwinger 1983; Rohrlich 1960; Medina 2006) in that the factor of 4/3 was compensated for by a non-electromagnetic contribution to make consistent with $E_{\mathrm{EM}} = m_{\mathrm{EM}}c^2$. The Lorentz boost of rest-frame energy-momentum tensor $T_{\mu\nu}$ alongside the x-axis (velocity direction) gives additional mechanical momentum inside the sphere:

$$p_x = \gamma(v)^2 \int_{\mathrm{out}} v\frac{u_{\mathrm{EM}}}{c^2}\, d^3x + \gamma(v)^2 \int_{\mathrm{in}} v\frac{T_{xx}}{c^2}\, d^3x \tag{4.11}$$

$$\simeq \frac{2}{3}\frac{q^2}{4\pi\epsilon_0}\frac{\boldsymbol{v}}{ac^2} - \frac{1}{6}\frac{q^2}{4\pi\epsilon_0}\frac{\boldsymbol{v}}{ac^2} = \frac{1}{2}\frac{q^2}{4\pi\epsilon_0}\frac{\boldsymbol{v}}{ac^2}, \tag{4.12}$$

resulting in $m_{\mathrm{EM}}^{(p)} = m_{\mathrm{EM}}^{(E)}$.

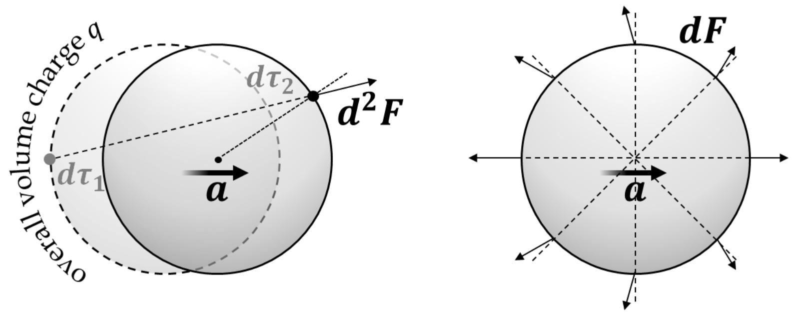


Figure 4.7 An accelerated charged sphere (Feynman et al. 1965, 28–5) (L: influenced by retarded time, R: angular distribution of dF)

Next let us show $m_{\mathrm{EM}}^{(E)} = m_{\mathrm{EM}}^{(F)}$. I now bring up an acceleration. When a volume-charged sphere is being accelerated, differential forces acting on each charge element becomes angle dependent (Figure 4.7R) due to their influence by retarded time (Figure 4.7L). All these differential forces being integrated over the sphere creates a kind of self-force. Let the state of being infinitely spread out and not interacting with each fragmented charge be chosen as a reference state. Then the self-energy to collect all these fragments is

$$E_{\text{self}} = \frac{1}{8\pi\epsilon_0} \int\int \frac{\sigma^2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} \, d\tau_1 \, d\tau_2, \tag{4.13}$$

where σ represents the uniform charge density. The self-energy to accelerate this sphere is calculated ($\boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$) after some lengthy algebra as

$$\begin{aligned} \boldsymbol{F}_{\text{self}} &= \frac{1}{4\pi\epsilon_0} \int\int \left(\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2\,|\boldsymbol{x}|^3}\boldsymbol{x} - \frac{\boldsymbol{a}}{c^2\,|\boldsymbol{x}|} \right) \left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{2c^2} \right) \sigma^2 \, d\tau_1 \, d\tau_2 \\ &\simeq -\frac{\boldsymbol{a}}{8\pi\epsilon_0 c^2} \int\int \frac{\sigma^2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} \, d\tau_1 \, d\tau_2, \end{aligned} \tag{4.14}$$

resulting in $m_{\text{EM}}^{(E)} = m_{\text{EM}}^{(F)}$ when $v \ll c$. For a detailed calculation, see Appendix D. This self-force example is a good illustration of electromagnetic fields having mass since, by definition, force has to be exerted in order to accelerate it. One can also consider the situation where the radius of the sphere approaches zero: This became the first non-quantum (classical) model of the electron (Rohrlich 1960; Abraham 1932; Heaviside 1905, 1906; Fermi 1921, 1923; Mandel 1926; Wilson 1936; Pryce 1938; Kwal 1949) Further, in this model, inertia was considered an inherent phenomena of electron's self-electromagnetic interaction. With the advent of quantum physics, this was naturally aborted, leaving some partially unsolved problems. Despite the incompleteness of the classical electromagnetic model of the electron, its result still shows exact evidence of the existence of electromagnetic inertia in accordance with the particle's MEE, $E_{\text{EM}} = m_{\text{EM}}c^2$.

I have shown that the electromagnetic field itself feels inertia, and that the laws of mechanics for electromagnetic fields are all the same as those of particles: $\boldsymbol{F} = m\boldsymbol{a}$, $\boldsymbol{p} = m\boldsymbol{v}$ and $E = mc^2$. Therefore, what is meant by these relations is that, at least for electromagnetic fields, the corresponding apparent mass exists to be considered as an ontologically same property as energy, since $E_{\text{field}} = m_{\text{field}}c^2$ always holds.

In quasi-stationary motion, these results above intuitively suggest that a group of fields can be treated as a particle, leaving serious concerns about whether to choose particles or fields as a proper model of matter. In other words, what is conjectured by this mechanical similarity is that the particle concept may not be a reliable model to describe matter, or rather, that fields may have all the particle-like properties as a unique physical reality. Great questions preceded all great theories. Physicists have always made nature true, not in a phenomenological sense, but rather in what is presumably a logical sense: not having a sense of "why" but a sense of "how." The phenomenal fact that the gravitational mass and inertial mass were numerically equivalent later eliminated the conceptual distinction between gravity and acceleration. Likewise, these arguments in the laws of mechanics later might have eliminated the conceptual distinction between fields and particles as a model of matter.

I have so far discussed three ways to understand MEE, summarized in Table 4.2. Next I will find a logical hierarchy and discover a new type of understanding MEE: Ontological merger of particles and fields (Type IV).

Table 4.2 Summary: three possible categories of understanding MEE

MEE Type		Graphical Rep. (rest) Energy	Graphical Rep. Mass	Understood by	
I (Conjecture & Correspondenc)	(EM) Field	E_{field} →	E_{field}/c^2?	$m_{\text{field}} \overset{?}{=} E_{\text{field}}/c^2$	• Conjecture that ∃ m_{field} corresponding to E_{field}
	(massive) Particle	$m_{\text{ptcl}}c^2$?	← m_{ptcl}	$E_{\text{ptcl}}^{(\text{rest})} \overset{?}{=} m_{\text{ptcl}}c^2$	• Conjecture that ∃ $E_{\text{ptcl}}^{(\text{rest})}$ corresponding to m_{ptcl}
II (Convertibility)	(EM) Field	E_{field}	CONVERSION	$E_{\text{field}} \overset{\leftrightarrow}{=} m_{\text{ptcl}}c^2$	• Inter-convertibility of mass and energy
	(massive) Particle		m_{ptcl}		• Presupposed by field-particle conversion
III (Indistinguishability)	(EM) Field	E_{field}	E_{field}/c^2	$m_{\text{field}} = E_{\text{field}}/c^2$	• $m_{\text{field}} = \boldsymbol{p}/\boldsymbol{v} = \boldsymbol{F}/\boldsymbol{a}$ is indistinguishable to E_{field}
	(massive) Particle	$m_{\text{ptcl}}c^2$	m_{ptcl}	$E_{\text{ptcl}} = m_{\text{ptcl}}c^2$	• Mass and energy are indistinguishable to each other

4.3 Logical Hierarchy and a New Type of MEE

I will begin our discussion by arranging the three types of understanding the MEE. It would be natural to put them in the order of before relativity, MEE (Type I), and MEE (Type III) according to the extent to which mass and energy are said to be equivalent:

- Before relativity: No relevance to each other
- MEE (Type I): Correspondence to each other
- MEE (Type III): Indistinguishability to each other

They could also be in the local hierarchy of their conceptual development:

- Before relativity: No concept of equivalence
- MEE (Type I): Conjecture regarding equivalence
- MEE (Type III): Assured and proven equivalence with minimal assumption

The conceptual relation between particle and field could be considered as well:

- Before relativity: No relevance to each other
- MEE (Type I): Conjecture that fields can also be treated like a particle and have the attribute of mass
- MEE (Type III): Mathematical similarity in describing physics

In addition, MEE (Type II) presents a new understanding of MEE by connecting energy with mass conceptually, with the assumption of convertibility between fields and particles. It would be also natural consequence to order them as before relativit then MEE (Type II) in the local conceptual development:

- Before relativity: No conceptual connection between mass and energy
- MEE (Type II): Connection with convertibility between energy and mass

However, if one takes a close look at MEE (Type II) and MEE (Type III), it seems that the conceptually separated “particles” and “fields” are connected by some means. MEE (Type II) assumes a transition between them and suggests an intrinsic relationship between particles and fields, with phenomenological evidence presented later. Historically, theory precedes experimentation. Likewise, MEE (Type III) seems to suggest a fundamental question of the distinction between fields and particles, showing that the laws of mechanics can be applied equally in both fields and particles. MEE (Type II) and MEE (Type III) together seem to imply so-called field-particle equivalence, as if speculation about equivalence between energy and mass originated from the compliance with the conservation laws or direct assumption of particle-field equivalence itself in MEE (Type I). Similarly, as a fourth type of MEE, tentatively, a Type implying particle-field equivalence can be suggested.

In fact, we know that historically two perspectives or theoretical model of matter or things, fields and particles, have been at odds with each other and reconciled to be integrated conceptually. Prior to the emergence of relativity, particles (in the classical sense) were adopted as a fundamental model that explained existing things, in the form of molecules and atoms. However, the emergence of MEE became a conjecture, suggesting a connection between particle and field in quantum theory. In the general theory of relativity, the emergence of Einstein’s field equation first showed that field theory can be an actual candidate for matter. Quantum field theory describes particles as local quantum excitation of the field and serves as a model to directly propose the conceptual equivalence of particles and fields for the first time. Even within the integrated perspective of field-particle, the conceptual position of mass was incomplete when considering symmetry. Later, as the Higgs mechanism was experimentally verified, mass, the old measure of inertia, had its conceptual position taken away in the perspective of quantum field theory (see 4.3.1).

A schematic representation of all these processes can play a role of a conceptual development map (see Figure 4.8).

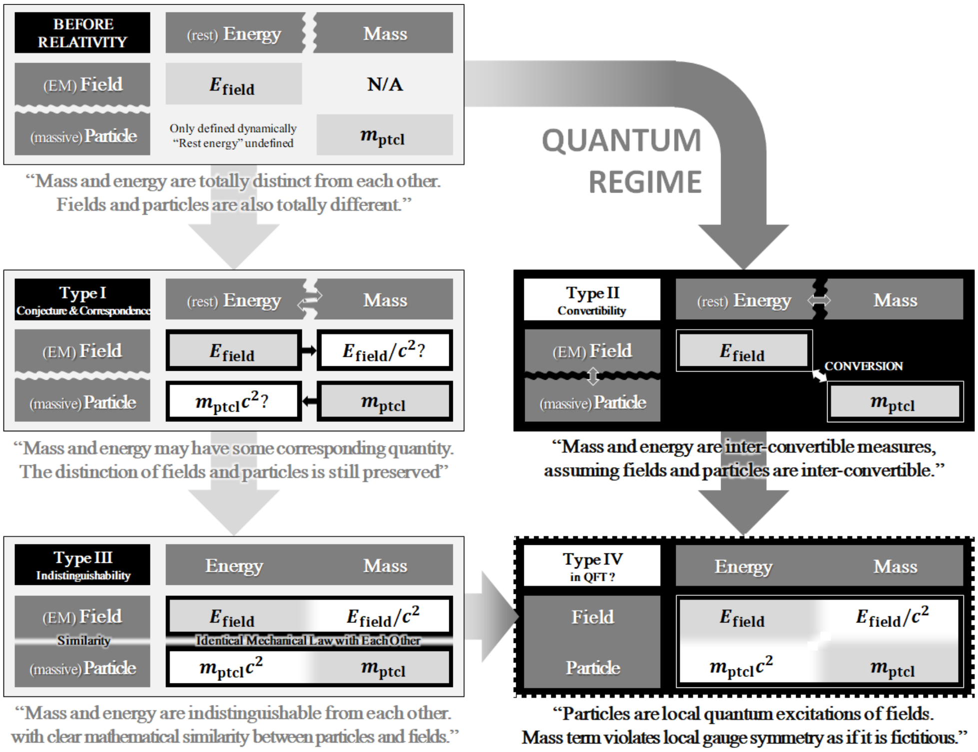


Figure 4.8 Conceptual development map: towards holistic understanding

4.3.1 Type IV: Beyond MEE, Towards Conceptual Merger of Particle and Field

Another important implication from the categorized understanding and the visualized connection of four distinct physical attributes is the conceptual merger of particle and field. Before the Einsteinian era, both particles and fields maintained sound distinction and upheld their conceptual authorities respectively.

In the Theory of Gravitation

The first connection between particles and fields was suggested during the birth of Einstein's field equation. Tidal force in two different description brought about a clue. The tidal correspondence (Wald 2010, 66–74) was given as

$$R_{\mathbf{abcd}}u^{\mathbf{a}}u^{\mathbf{c}} \longleftrightarrow \partial_{\mathbf{b}}\partial_{\mathbf{d}}\phi, \tag{4.15}$$

where ϕ represented by a potential form of gravitational field and $u^{\mathbf{a}}$ is an abstract-indexed 4-velocity of a particle. In general relativity, the action of space-time continuum M should be in the form

$$-\mathcal{S}\left[g_{\mathbf{ab}}\right] = \int_M \frac{1}{2}\left(\frac{R}{8\pi G}\right)\epsilon - \mathcal{S}_{\text{matter}}. \tag{4.16}$$

Continuous matter distributions and fields are described by a stress-energy tensor analogous to a perfect fluid ($c = G = 1$):

$$2\frac{\delta S_{\text{matter}}}{\delta g^{\mathbf{ab}}} = T_{\mathbf{ab}} = (\rho + p)\, u_{\mathbf{a}}u_{\mathbf{b}} + pg_{\mathbf{ab}}, \tag{4.17}$$

which directly implies $T_{\mathbf{ab}}u^{\mathbf{a}}u^{\mathbf{b}} = \rho$. From the Poisson equation of Newtonian gravitation $\nabla^2\phi = 4\pi\rho$, the prototype of a field theoretical view of matter emerges:

$$R_{\mathbf{ab}}u^{\mathbf{a}}u^{\mathbf{b}} \longleftrightarrow \nabla^2\phi = 4\pi\rho \longleftrightarrow 4\pi T_{\mathbf{ab}}u^{\mathbf{a}}u^{\mathbf{b}}. \tag{4.18}$$

This was actually what Einstein first postulated. The continuous ideal fluid representation of matter has become a stepping stone to two ways of representing matter: particle description and field description. Einstein's field equation was the very first formalized viewpoint of the field description of matter. This was also described well in his famous quote (Einstein et al. 1954, 371, 348):

> Physical objects are not in space, but these objects are spatially extended (as fields). . . . The field thus becomes an irreducible element

> of physical description, irreducible in the same sense as the concept of matter (particles) in the theory of Newton. ... The physical reality of space is represented by a field whose components are continuous functions of four independent variables—the coordinates of space and time. Since the theory of general relativity implies the representation of physical reality by a continuous field, the concept of particles or material points cannot play a fundamental part, nor can the concept of motion.

The same analogy between classical point-particle mechanics and dynamics of perfect fluid was also discussed (Spiegel 1982). Their direct resemblance was first pointed out with the formal similarity between the Hamilton-Jacobi equation and the Bernoulli integral (Truesdell 1954).

In Quantum Field Theory

The ontological merger of the field and particle concepts was formally achieved in quantum field theory. Although the conceptual integration between the field and particle has not been explicitly indicated in existing materials, in some literature demonstrating MEE, the implications of MEE, as a protoType Idea of ontological merger, have been suggested. Born (1962, 234) showed MEE in his book and put: "Matter itself loses its primary character as an indestructible substance and is nothing more than points of concentrated energy." For contemporary physicists, at least for Born, a convincing model of matter was particle that has an indestructible property. He continued: "Whenever electric and magnetic fields or other effects lead to intense accumulations of energy the phenomenon of inertial mass presents itself. Electrons and atoms are examples of such places at which there are enormous concentrations of energy." Born vaguely identified particles with intense accumulations of energy and inertiality with the energetic phenomenon. Born was not the only one who implied the quantum origin of matter from MEE. Einstein (1954, 348) also said: "*The* *particle can only appear as a limited region in space in which the field strength or the energy density are particularly high.*"

However, it was probably not satisfactory for physicists that energy, which was defined with the abstract definition of "capacity for doing work" was the source of matter theory. Energy was not physical reality in itself, but an attribute or property of a system. Furthermore, the phenomenon of inertia (or the property of mass) would only have been substituted as an attribute of energy and would not have given any ontological or real information about matter itself.

Without an exact quantum field theoretical definition of particle, derivations in MEE (Type I) also implicitly showed it might have a consequent mass contribution to the electromagnetic field. One might introduce the concept of quanta to give an image like the one shown in Figure 4.3b. It was a very naive model of the particle. According to Born (1962), this correspondence also might have implied the birth of the field theoretical view of the particle.

The dualism of classical physics, particle and field, was finally replaced by monism with the advent of quantum field theory. What had been considered a particle was identified with the quantum excitation of the corresponding field, and ontological integration as a matter theory was achieved. Mathematically the idea was formalized by the Fock space (F)[7]: the direct sum of Hilbert spaces (H), each of which describes a different number of particles (n) in the language of field theory. It can be denoted by

$$F_{\pm}(H) = \overline{\bigoplus_{n=0}^{n=\infty} S_{\pm} H^{\otimes n}} \tag{4.19}$$

[7]Since for free theories, not for general interacting theories, a suitable Fock representation of the canonical commutation relations satisfies the Wightman axioms, it is possible for Hilbert (Fock) space to be the proper space of quantum field theory.

where S denotes operator that (anti)symmetrizes a tensor, overbar the Hilbert space completion and $\bigotimes n$ in the exponent the tensor powers. Creation and annihilation operators, which played a significant role to raise the energy level of a simple harmonic oscillator in quantum mechanics, can also act on a Fock state to create or annihilate a particle in the ascribed state. The state $|\Psi\rangle$ with n_i particles of state p_i $(i = 1, 2, 3, \cdots)$ is described as

$$|\Psi\rangle = \prod_i \frac{\left(a^\dagger(p_i)\right)^{n_i}}{\sqrt{n_i!}} |\text{state with no particle}\rangle \, . \tag{4.20}$$

For more detailed mathematical descriptions, see Appendix B. Wave-particle duality (in quantum mechanics) and a brief mathematical description of quantum field theory are attached.

An epistemological change of matter was brought by quantum field theory. For instance, in terms of the primary qualities (solidity, extension, motion, number, and figure), which were suggested by Locke (1690) as measurable aspects of physical reality, each quality is reinterpreted in an advanced sense. Let us take a look at the classical dualism first.

For the classical concept of particle, these qualities were considered quite suitable. A particle was described as a point-like figure with its physical attribute of mass. Its massiveness and indivisibility apply well to solidity. Due to its figure, it takes up nearly zero space or at least a finite region. Thus the concept of particle naturally contains locality. Also, since particles are considered discrete, countability sufficiently holds. Its motion can also be fully determined by the equations of motion.

Whereas Locke's primary qualities break down for the classical concept of field, fields obviously established their own ground ontologically. A field appears to be neither solid nor massive, can fill in any space (thus non-localizable), and is uncountable and continuous. Due to its space-filling property, fields have no inherent figure. A field does not set in a motion; rather, it propagates through the space. The classical epistemology of reality seems to make fields, which obviously recognized by Einstein as reality, a questionable conclusions. Nevertheless, regardless of the foundation of field theory, the concept of fields and Maxwell's electromagnetic field theory were crucial steps in the direction of making constructive speculation based on what we can experience with our senses (Einstein 1936).

In quantum field theory, Locke's primary qualities can be replaced or reinterpreted in a somewhat modern sense. The solidity can be replaced by the permanence with the conservation law of physical attributes, for example, charges and mass-energy. Furthermore, motion, suited for particles but rejected for fields, is replaced by local motion (dynamics) in space-time. As inherited from classical fields, quantum fields also have no figure. The concept of figure had already been rejected in classical field theory. For a particle-like figure, instead, localizability should be required when it comes to particle-like consideration. Similar to the classical particle, in quantum field theory, occupation number representation ensures that excitations of a certain mode of field can be regarded as particles (Ryder 1996, 126–153). Considering the relativity of the four fundamental interactions, the concept of extension becomes somewhat subtle. Rules of quantum interactions reduce the generality. For example, neutral particles do not interact electromagnetically, but always couple with gravity. The extension has now been developed into a relative concept rather than a universal concept (see Table 4.3).

However, ontology of quantum field theory still has many issues. Even though countability directly supports particle-like property, Fock space rep-

Table 4.3 Epistemological changes of matter brought by quantum field theory

Primary Qualities	Classical Dualism		Monism
	Classical Particle	Classical Field	in Quantum Field Theory
solidity	massive, indivisible	apparently not solid and not massive	replaced by permanence with conservation law of physical attributes
extension	taking up a finite region	possible to fill in any space	dependent to interaction (relative)
motion	trajectory determined by equation of motion	no setting in a motion, only propagating	replaced by local dynamics of field with quantum indeterminacy of its particle-like trajectory
number	countable and discrete (finite dynamical degrees of freedom)	uncountable and continuous (infinite dynamical degrees of freedom)	countable and continuous (occupation number representation ensures countability)
figure	point-like (localized)	no inherent figure, space-filling, determined by boundary and source (not localized)	no inherent figure, space-filling, rejected in classical field (localizability required)

resentation is only valid for free theory (Haag 1955; Fraser 2008). However, concerning the asymptotically free state in scattering (interacting) theory, an alternative perspective was also suggested (Bain 2000). The relativistic vacuum state has been pointed out as having an intrinsic defect in its own interpretation. The expectation value of physical quantities does not vanish even though vacuum is considered to have no particles according to the occupation number representation of quantum field theory. That is to say, physics does exist while particles do not. Wald attributed this inconsistency to the geometric properties of Minkowski space-time.[8] One another crucial problem is non-localizability (Malament 1996, 1–10). Quantum field theory was developed from these scattering experiments. However, it turns out to have significant issues in terms of particle trajectories. Determinacy of trajectories was already prohibited by Heisenberg's uncertainty relations for position and momentum coordinates. Thus, non-localizability seems to require withdrawal of some epistemological properties of particles that had been classically accepted.

[8]Research on QFT in curved space-time suggests that the concept of a particle number operator may only apply in flat Minkowski space-time, because using Poincaré symmetry to select a specific representation of canonical commutation relations is equivalent to choosing a specific vacuum state. See (Wald 2010, 399–416; Wald 1994)

Another significant issue is the problem of measurement, which arises from the fact that the theory cannot explain how measurements of quantum systems are performed in a way that is consistent with the principles of quantum mechanics. There are many approaches to this problem, for example, the decoherent histories approach (Griffiths 1984; Isham et al. 1998) and holographic principle ('t Hooft 1993, 2001; Zaanen et al. 2015 or see Chapter 5.3), which try to resolve the problem from different perspectives. Various other ontological issues with quantum field theory have been widely suggested (Unruh 1976; Unruh and Wald 1984; Halvorson and Clifton 2002), and there have been many interpretations posed as an alternative ontology to those issues (Teller 1997; Kuhlmann et al. 2002, 127–133; Wayne 2008, 1–15; Lupher 2010). As of the current theoretical basis of modern physics, quantum field theory is the best and only existing candidate of the theory of matter.

Mass-Energy Equivalence Revisited

In quantum field theory, fields and particles have been conceptually integrated to form a monistic theory of matter. In the language of quantum field theory, physicists basically adopt another expression of MEE $(p^2 =) - E^2 + \boldsymbol{p}^2 = -m^2$ $(c = \hbar = 1)$, which is called the "relativistic dispersion relation," considering both the quantum and relativistic nature of fields. Mass is now used a form of frame-independent "rest mass" and energy remains a frame-dependent concept, resulting in a distinction in terms of the usefulness between them. However, despite this conceptual individuality, the classical concept of mass has lost its theoretical status, not through the concept of energy, but through the language of field theory, called the Higgs mechanism (Higgs 1964). That is to say "mass without mass" (Hobson 2005). It demonstrates, given a gauge symmetry, how the mass of the corresponding gauge boson comes out.

The Higgs boson was actually discovered in 2012 (Aad et al. 2012), and for a decade after the first discovery, more precise experiments have been conducted (CMS Collaboration 2022; ATLAS Collaboration 2022). The linear relation they experimentally showed is

$$m(\text{mass}) = \langle H \rangle \, (\text{Higgs V.E.V.}) \times \kappa(\text{coupling intensity}). \tag{4.21}$$

This relation is precisely predicted in the standard model, which is the most accurate theory we currently know about the origin of the mass of the elementary particles, at least up to ten thousandths the scale of nucleon size. For more theoretical detail, see Appendix E.

5 Other Examples of Equivalence in Physics

Confining the semantic definition of mass to rest mass or invariant mass, Type I MEE means a conjecture that energy might be thought of as mass by the amount of the corresponding quantity, whereas, in Type II MEE, energy and mass are considered inter-convertible. However, in these types of MEE, it is still difficult to choose only the concept of mass with no concept of energy or vice versa. In Type I and II MEE it is still impossible to abandon one of the two concepts. However, in Type III MEE it suffices to abolish one of the concepts to establish an ontological interpretation of mass and energy. In Type III MEE, there is also no general requirement to confine the semantic definition of mass to invariant mass m_0. No matter what inertial frame is taken, $E = mc^2$, of which Lorentz boost transformation gives the covariant form, always holds. In order to argue the non-arbitrariness of our categorization, I have to clarify the validity of these conditions for other equivalences in physics.

I would like to take a close look at the term "equivalence," which is used or meant by many cases in physics, in more detail:

- mass-energy equivalence (MEE)
- equivalence principle (in general relativity)
- the mechanical equivalent of heat (first law of thermodynamics)
- anti-de Sitter space/conformal field theory (AdS/CFT) correspondence (in string theory)

- theoretical equivalence of matrix mechanics and wave mechanics (in quantum mechanics).

In MEE, weak equivalence principle (WEP), and heat-mechanical energy equivalence, it is two physical quantities that are related to each other conceptually (equivalence between two quantities). On the other hand, strong equivalence (SEP), AdS/CFT correspondence, and the equivalence of matrix mechanics and wave mechanics are all the relation between two theories (equivalence between two theories). All these examples are briefly introduced first in the sense of conceptual development to compare and find some common features.

5.1 The Equivalence Principle: From Weak to Strong, the History of Extensions and Abandonment of Concepts

In the principle of equivalence, it is well known that regardless of a particle's constitution, all particles fall with the same acceleration when acted upon by a gravitational field, and thus there can be no free particle with which to test a reference frame for inertiality. This formed the theoretical foundation of the theory of gravitation, which goes further to be general theory of relativity (GR). Let us consider the weak one first.

There are a few methods to express the weak equivalence principle (WEP), sometimes referred to as the universality of free fall or the Galilean equivalence principle. Astronomical bodies with gravitational self-binding energy are included in the strong equivalence principle (SEP), which is a generalization of the weak equivalence principle. Instead, the WEP postulates that the non-gravitational forces self-bind falling bodies. Treating the motion of particles based on a space-time continuum, it is convenient to think of the world line in four-dimensional space-time. The weaker interpretation is that the world-line trajectory of a moving particle in a gravitational field depends only on its initial position and velocity, and is independent of the material of which the particle is constructed. However this says nothing about the behavior of particles projected simultaneously from the same point but with different initial velocities. To be in accordance with Newtonian gravity, such particles also have the same initial acceleration when measured in an inertial frame. This common acceleration given to all particles is called the "gravitational acceleration" regardless of their motion. It is the stronger interpretation that is needed in the foundation of general relativity, but in its present form, it relies on Newtonian concepts. It must be revised in such a way that it defines its own frames of validity, as was done in the principle of inertia. In the foundation of special relativity, it was found to be necessary to extend the range of phenomena considered in the Newtonian principle of inertia to extend the principle. This was because the principle was given a larger task to perform, in that the inertial frames had to be selected from a more general class of reference frames than was the case in Newtonian gravity. Similarly, the scope of this gravitational principle needs to be generalized when it is given the additional task of selecting its own frames of validity.

First considering uniform gravitational field, the gravitational acceleration will then be independent of space-time from the Newtonian viewpoint. Supposing a reference frame that now has this same acceleration is used, all particles that are acted on only by gravitation will appear to be either at rest or moving uniformly in a straight line. Their behavior is thus same as that of free particles in an inertial frame. According to Newtonian theory, a dynamic experiment cannot distinguish between a freely falling frame in a static uniform gravitational field and an inertial frame in the absence of gravitation. The revised principle of inertia thus holds under "free from external forces" condition and then can be revised as "free from all external forces except gravitation." Since the extended principle of inertia is free from Newtonian preconceptions, it is necessary to modify our basic gravitational principle for uniform fields.

Before non-uniform fields are considered, it must be noted that there is an alternative form for the case of a uniform field that is less precise but historically significant. In the above discussion, two situations were compared. One used a freely falling frame in a uniform field, and the other used an inertial frame under no consideration of gravitation. Suppose now that the first of these situations are referred to a Newtonian inertial frame. If the mathematical transformation that this involves is also applied to the second situation, the frame that results is uniformly accelerated. The result of all dynamic experiments will necessarily still be the same when performed in either of the new frames. Thus, for dynamic experiments, placing a reference frame in a uniform gravitational field is equivalent to giving it a uniform acceleration. It is from this version that the principle that we are being led towards takes its name: the principle of dynamic equivalence. Its lack of precision in comparison with the previous version arises from its retention of the Newtonian concept of an inertial frame.

There is another crucial step in the historical development of relativity. The development of special relativity taught the dangers of believing that situations that are equivalent for dynamics could be distinguished by far-ranging physical experiments. Einstein thus put forward the more general hypothesis that the above equivalence holds for all physical phenomena, which is called the principle of equivalence. However, it must be treated carefully because of its essentially approximate nature in dealing only with uniform fields. Einstein's (1911) first application of it was to predict the deflection of light by a gravitational field, but although the effect has been verified, its magnitude disagrees by a factor of two with that given by a naive application of the principle.

The following study of space-time in a gravitational field is based on dynamic equivalence. But this does not remove the need for care. The first step must be to make a precise statement of the equivalence principle that is meaningful even in an in-homogeneous field. An in-homogeneous field can be considered uniform to better and better approximation as the size of the region of space-time scale of interest is reduced. The freely falling frames of the principle can thus be defined more and more accurately as their domain of definition is reduced. This suggests that the only possible precise definition is a "freely falling frame at a point in space-time." Such a frame will extend throughout a region that is exactly satisfied at one point. This is the first indication that gravitation theory needs the full generality of the concept of reference frame. A reference frame must be extended over a finite region of space-time, and yet in a realistic gravitational field, a frame can only satisfy the preferred free fall condition at one point. Elsewhere, the frame will be arbitrary. Any theory built on this basis will thus have no preferred class of observers unless their observations are restricted to their immediate neighborhood. This goes a long way towards Einstein's pursuit of a universality where all observers should be on an equal footing; in other words, no general preference can exist in our nature.

5.2 The Mechanical Equivalent of Heat: Entropy as a Constraint on Convertibility

In the early 19th century, scientists began to experiment with the conversion of heat into mechanical work. Carnot ([1824], 1986) proposed the concept of a perfect engine that could convert heat into mechanical work with perfect efficiency. This idea laid the foundation for the study of thermodynamics, which would become a promising field of contemporary physics. Clausius formulated the first law of thermodynamics (Truesdell 1980), which states that (today's concept of) energy "cannot be created or destroyed," but "can be converted" from one form to another. This law formalized the idea that heat and mechanical work are equivalent forms of energy $dU = \delta Q + \delta W$ (known as the first law of thermodynamics).

He also introduced the concept of entropy, which measures the amount of disorder or randomness in a system. Heat is supplied to the engine from a higher-temperature reservoir, and the work is done by expanding a gas or fluid against a piston or turbine. The waste heat is then rejected to a lower-temperature reservoir. The efficiency of a heat engine is limited by the difference in temperature between the hot and cold reservoirs $T_h - T_c$. On the other hand, mechanical work can be converted into heat energy by processes such as friction, where the work done to overcome the friction force is converted into heat energy in the form of thermal energy.

Entropy is a constraint on the convertibility of heat into mechanical work. According to the second law of thermodynamics, which is also known as the law of entropy, the total entropy of a closed system cannot decrease over time.

This means that the entropy of the system will either remain the same or increase.

In the context of heat engines, this means that a heat engine cannot convert all of the heat energy supplied to it into mechanical work. Some of the heat energy must be rejected as waste heat, which increases the entropy of the system. The efficiency of a heat engine is defined as the ratio of the work done by the engine to the heat supplied to it. The maximum efficiency of a heat engine is given by the Carnot efficiency, which is determined by the temperatures of the hot and cold reservoirs, and is given by $1 - T_c/T_h$. This means that the efficiency of a heat engine is limited by the second law of thermodynamics and that the second law of thermodynamics is a consequence of the fact that entropy is generated during adiabatic processes and that the Carnot cycle is the most efficient heat engine cycle.

Maxwell (1860; 1860; 1867) and Boltzmann (1877) developed the kinetic theory of gases, which proposed that heat is a form of energy associated with the random motion of particles (molecules) in a substance. They showed that the temperature of a gas is proportional to the average kinetic energy of its molecules and that the heat capacity of a gas depends on the number of degrees of freedom of its molecules. This theory provided a particle theory for the macroscopic statistical laws of thermodynamics.

5.3 AdS/CFT Correspondence: Conjectured Isomorphism without General Proof

Now let us examine the equivalence between two theories, instead of the equivalence between two physical quantities.[1] The equivalence between theories, rather than the equivalence between the two physical quantities, could be conjectured first through some examples, unless each theory's bijective relation to the other is clearly identified and unless proof from one theory to another is provided. One good example is AdS/CFT correspondence, which is stemmed from the study of black hole thermodynamics.

Black holes, as their name implies, are known as always absorbing and not radiating. However, according to Hawking (1975), considering the quantum mechanical effect, black holes are not completely black and produces weak thermal radiation. Known as Hawking radiation, this was unacceptable based on contemporary physics knowledge. Through this mechanism, the temperature and entropy of non-rotating black holes could also be calculated (Hawking 1975; Bekenstein 1972, 1973):

$$T(M) = \frac{\hbar c^3}{8\pi k_B G M}, \quad S(A) = \frac{k_B c^3 A}{4\hbar G} \tag{5.1}$$

Those equations above have special properties. They contain all fundamental theories in physics: quantum mechanics ($\hbar$), special relativity (c), theory of gravity or general relativity (G), and statistical mechanics (k_B). This property shows strong significance for the existence of a well-behaved theory of quantum gravity, implying that general relativity and quantum mechanics are applied

[1]The equivalence between two theories is often expressed in the sense of duality.

simultaneously. However, if one tries to fuse general relativity with quantum field theory, renormalization becomes impossible. Therefore, quantum field theory seems to be unable to carry its weight in quantum gravity. In other words, it seems that quantum field theory only has value as a low-energy effective theory and is incomplete as a fundamental theory since in high-energy phenomena such as black holes another approach is required.

According to the second equation in (5.1), the temperature of a black hole only depends on its mass (M), and the microstate for the entropy of a black hole comes only from the surface area of black hole. However, this is a very mysterious point, because the number of possible microstates in a given space is naturally proportional to the exponential of its volume. For example, the number of microstates $\Omega(E, \delta E, N, V)$ of a system of N identical particles in 3D volume V with total energy between E and $E + \delta E$ can be represented as

$$\Omega(E, \delta E, N, V) = \frac{V^N E^{(3N-2)/2}}{N!\Gamma(3N/2)} \left(\frac{2m\pi}{\hbar^2}\right)^{3N/2} \delta E. \tag{5.2}$$

Thus in this case, entropy has a logarithmic power of volume V. Inspired from the area dependence of black hole's entropy, 't Hooft (1993) and Susskind (1995) suggested the holographic principle, which states that the quantum theory of gravity behaves like a hologram, thus all the information about the volume of space can be determined by the surface surrounding it. This can be understood from the super-string theory, which is a strong candidate for a theory of quantum gravity.

AdS/CFT correspondence or gauge/gravity duality is a conjectured relationship between two theories from quantum gravity theory. On one side of the correspondence is anti-de Sitter space (AdS), formulated by the super-string theory of M-theory. On the other side of the correspondence is conformal field theory (CFT), which is a quantum field theory, including a theory similar to the Yang-Mills theory that describes the elementary particles. According to this duality, the following two theories are considered quantum mechanically equivalent (Maldacena 1999; Witten 1998):

- quantum gravity in $(d+1)$-dimensional AdS space-time.
- d-dimensional conformal quantum field theory without gravity.

Due to the feature of the boundary of AdS, on which it locally looks like (non-gravitational) Minkowski space, space-time for a CFT can be treated as given by the boundary of AdS. The claim is that all the calculation in one theory has a corresponding counterpart in the other theory. It is generally very difficult to verify AdS/CFT correspondence since if the calculation is easy (weak-coupling) on one side, the calculation is very difficult on the other side (strong-coupling or many gauge fields). Therefore, this correspondence is of course not obvious, but in the case of a super-symmetric system, it is possible to verify. String theorists have discovered many examples of this correspondence (Maldacena 1999; Gubser et al. 1998; Witten 1998). All the examples describe the correspondence between conformal field theories and compactifications of string theory or M-theory.

5.4 Dirac–von Neumann Axiomatic Formulation: Theoretical Bijection between Matrix Mechanics and Wave Mechanics

Hilbert space is used to describe a quantum mechanical system. A single 1-D particle's pure states, for instance, can be described by components of the Hilbert space $\mathfrak{L}^2(\mathbb{R})$, which is a concept covered in introductory quantum mechanics courses. It is the Hilbert space that plays a role of theoretical equivalence between Schrödinger's wave mechanics and Heisenberg's matrix mechanics.

Schrödinger (1926) established his wave mechanics. The key idea of wave mechanics was that a wave function had to be specified in order to fully characterize quantum phenomena. Schrödinger's equation was the wave function analogue of the classical equation of motion with the operator replacement of $p_j \to -i\hbar\partial_j$. Thus, Schrödinger equation directly leads to a partial differential equation. In the stationary case, the eigenvalues of time-independent Schrödinger's equation were the energy level, according to Schrödinger.

Given that only specific energy states were observed in spectroscopy, the wave-mechanical treatment of the atom as a charge cloud by Schrödinger, instead of an electron as a particle orbiting around the nucleus (Bohr's early model), did not initially accurately account for radiation of the atom while Bohr's model did. The cloud's electric density varied from location to location but stayed constant. Schrödinger proposed that the charge cloud vibrates in two or more distinct modes with various frequencies in order to explain the radiation in corresponding atomic energy levels. As a result, the radiation only emits wave-packets with specific energies that conform to Bohr's condition of frequency.

From Heisenberg's perspective, Bohr's quantization rule proved to be a useful, although it was an imperfect first approximation after he constructed matrix mechanics with Born and Jordan. Once Heisenberg teamed up with Born and Jordan, matrix mechanics became an entirely separate approach (Born et al. 1926). The direct guideline for matrix mechanics' initial construction would have been Bohr's correspondence principle. Matrix Mechanics could be thought of as an enhanced version of Bohr's approach.

Heisenberg sought to create a quantum mechanical formalism that matched classical mechanics as similarly as possible. He then thought about the equation of motion substituted by its quantum analogue, in other words, the position q is substituted by the matrix form of position $\hat{Q}$ and the momentum p by $\hat{P}$, such that these matrices satisfy the commutation relation of $[\hat{Q}, \hat{P}] = i\hbar\hat{I}$ (Heisenberg 1925). These quantum correspondences, similar to classical mechanics, make Hamiltonian equations of motion also hold. In order to make the eigenvalues of Hamiltonian matrix give energy levels, proper diagonalization was the main problem since the time evolution can be fully determined with the matrix extension of the classical Hamiltonian equation of motion.

In Schrödinger's theory, continuous functions and integration play an important role. The total configuration space integration must be equal to the identity:

$$\int \left|\phi(q_1, \cdots, q_k)\right|^2 dq_1 \cdots dq_k = 1 \tag{5.3}$$

in order that ϕ can be given a proper physical interpretation. On the other hand, the vector $x_1, x_2, \cdots$ plays a significant role in the matrix mechanics. The conditions from the eigenvalue problems of Hilbert space with no trivial solution suggest also a similar normalization as

$$\sum_{\nu} |x_\nu|^2 = 1. \tag{5.4}$$

Neumann showed that the Heisenberg's matrix mechanics and Schrödinger's wave mechanics are algebraically isomorphic to each other, realized in the same Hilbert space (Neumann 2018). He formalized that the wave functions are defined in the space

$$\mathfrak{L}^2(\mathbb{R}) = \Bigg\{ f : \mathbb{R} \to \mathbb{C} \mid f \text{ Lebesgue measurable and}$$
$$||f||_2 = \left(\int \left| f(x) \right|^2 dx \right)^{1/2} < \infty \Bigg\} \quad (5.5)$$

and that matrices in matrix mechanics are generated in the space

$$\mathfrak{l}^2 = \left\{ (z_n) \mid ||z_n||_2 = \left(\sum_{n=1}^{\infty} |z_n|^2 \right)^{1/2} \right\}, \quad (5.6)$$

which was first postulated by Dirac (1947). He also argued from the Riesz-Fischer theorem,[2] given a complete and orthonormal system $\{\phi_k\}$,

$$\Phi_{\{\phi_k\}} : \mathfrak{L}^2(R) \to \mathfrak{l}^2,\ \psi \mapsto \left(\langle \psi, \phi_k \rangle \right)_{k=0}^{+\infty} \quad (5.7)$$

is a linear and isometric isomorphism.[3] He commented:

> F_Z (state-space of matrix mechanics) and F_Ω (state-space of wave mechanics) are isomorphic, i.e., identical in their intrinsic structure ... and since they are the real analytical substrata of the matrix and wave theories (mechanics), this isomorphism means that the two theories must always yield the same numerical results.

That is, this occurs whenever the isomorphism lets the matrix

$$\hat{H} = H\left(\hat{Q}_1, \cdots, \hat{Q}_k, \hat{P}_1, \cdots, \hat{P}_k \right)$$

[2]The set $\left(L_p\,[a,b]\,,\ ||\cdot||_p\right)$ with $1 \leq p < \infty$ is a Banach space (Rudin et al. 1976).

[3]Here, linearity means, if $x_1, x_2, \cdots$ corresponds to $\phi(q_1, \cdots, q_k)$ and $y_1, y_2, \cdots$ to $\psi(q_1, \cdots, q_k)$, then $ax_1+by_1, ax_2+by_2, \cdots$ corresponds to $a\phi(q_1, \cdots, q_k)+b\psi(q_1, \cdots, q_k)$. And isometry means, if $x_1, x_2, \cdots$ and $\phi(q_1, \cdots, q_k)$ correspond to one another then $\sum_\nu |x_\nu|^2 = \int \left|\phi(q_1, \cdots, q_k)\right|^2 dq_1 \cdots dq_k$. In addition, if $x_1, x_2, \cdots$ and $y_1, y_2, \cdots$ correspond respectively to $\phi(q_1, \cdots, q_k)$ and $\psi(q_1, \cdots, q_k)$, then $\sum_\nu x_\nu y_\nu^* = \int \phi(q_1, \cdots, q_k)\psi^*(q_1, \cdots, q_k) dq_1 \cdots dq_k$, and both sides are absolutely convergent.

and the operator

$$\mathbf{H} = H\left(q_1, \cdots, q_k, -i\hbar\partial_1, \cdots, -i\hbar\partial_k\right)$$

correspond to one another. He finally concluded two theories are "mathematically equivalent."

5.5 Some Common Features of Equivalence

The essence of the principle of equivalence is that no one can suggests clear distinction between inertia and gravitation. Keeping this idea in mind, a particle that is acted upon alone by gravitation should be considered freely moving. Likewise, a frame that is falling freely and without rotation on a gravitational field should be considered inertial. What the principle of equivalence tells us is that either the concept of gravity or that of acceleration can be discarded though a practical distinction might still be needed. This study suggests the definition of ontological equivalence as a negative answer to whether two properties are possible to distinguish conceptually. In other words, two different physical attributes are considered to be ontologically equivalent if and only if one of the two concepts can be completely abandoned. The indistinguishability of the concept of gravity and the concept of acceleration, which is formulated as the strong equivalence principle (SEP), which is $\mathsf{Physics}_{\mathbf{a}} = \mathsf{Physics}_{-\mathbf{g}}$, is another good example of an ontological merger. Here one can notice that ontological equivalence is far beyond the stage of numerical conjecture or coincidence, for instance, $m_a = m_g$, which is known as the Galilean or weak equivalence principle (WEP) and which will be discussed soon. However, it should be noted again that WEP and SEP also have a logical hierarchy, as Type I MEE and Type III MEE did.

Next, the first law of thermodynamics tells us that the internal energy of a system has two channels to convert into—heat or mechanical work[4]—through a thermodynamic process. Following natural questions of why the internal energy has double channels and whether an entire transition between mechanical work and heat is possible, the concept of entropy[5] had emerged to become a constraint on the convertibility. This was formulated by the second law of thermodynamics: regulating the convertibility. However, still its equivalence connotes the convertible nature in an obvious way. Moreover, the first law of thermodynamics directly guarantees the integrated conservation: $dU = \delta Q + \delta W$. Thus, it is concluded that their equivalence refers to the inter-convertibility through an isothermal process, indisputably beyond the numerical coincidence that the ratio of calorie per joule is 4.184 without any reason why. However, it is preposterous that heat and work (energy) are regarded as conceptually equivalent. The concept of heat is not defined in the scale of a particle, but is defined statistically through the scale of the number of particles, whereas the concept of energy (work) is well defined in any scale. The relation between heat and mechanical energy is in an inter-convertibility, but heat is only a form of (general) energy. Thus it can be seen that both concepts stand their own ontological grounds. That is to say, they are remarkably distinguished,

[4]One can also consider a magnetic system analogous to mechanical system. The work W_{M} performed by the external e.m.f. $\mathbf{E}$ in the short time interval δt is

$$\delta W_{\mathrm{M}} = \delta t \int \mathbf{J} \cdot \mathbf{E} d\tau \simeq \int \mathbf{H} \cdot \delta \mathbf{B} d\tau$$

where one supposes switching on the magnetizing current $\mathbf{J}$ so slowly that eddy-current dissipation term becomes neglected.

[5]Clausius theorem states that a cyclic thermodynamic system exchanging heat with an external reservoir should satisfy $\oint \delta Q/T \geq 0$, deducing that in order for a thermodynamic cycle to become reversible, $\delta Q/T$ has a great significance whose value integrated over to be zero.

thus remaining in the equivalence as convertibility, like Type II MEE. The equivalence as convertibility is defined such that the two different attributes are under the ontological distinction, and their quantity can be preserved to become able to construct an integrated law of conservation.

Likewise, one can consider the conjecture and correspondence in terms of a primitive equivalence. This type of equivalence can be also be considered numerically equivalent or empirically equivalent. With no general proof suggested, only numerical or empirical coincidences are suggested with some experiments or examples. MEE (Type I) was derived with a thought experiment (Einstein 1906) or ad hoc prescription (Einstein 1906; Born 1962; Perez and Ribisi 2022) with no logical proof. WEP also measures up to those criteria since gravitational mass and inertial mass are numerically and empirically equal (Ciufolini and Wheeler 1995, 117–119; Touboul et al. 2017) with no proof. AdS/CFT correspondence can also be an example since it conjectures a bijective relation between $(d + 1)$-dimensional AdS space and d-dimensional CFT (Maldacena 1999; Gubser et al. 1998; Witten 1998). Same as the previous examples, many calculations in both theories give numerically equal results: $\mathrm{AdS}_7/\mathrm{CFT}_6$ (Section 3.1 of Maldacena 1999; Section 6.1.1 of Aharony et al. 2000; Sezgin and Sundell 2002), $\mathrm{AdS}_4/\mathrm{CFT}_3$ (Section 3.2 of Maldacena 1999; Section 6.1.2 of Aharony et al. 2000), $\mathrm{AdS}_3/\mathrm{CFT}_2$ (Section 4 of Maldacena 1999; Section 5 of Aharony et al. 2000; Dibitetto and Petri 2018; Dibitetto and Petri 2019), $\mathrm{AdS}_2/\mathrm{CFT}_1$ (Maldacena et al. 2016; Maldacena and Stanford 2016; Sachdev 2010; Almheiri and Polchinski 2015; Cotler et al. 2017), and others. The AdS/CFT correspondence continues to be an active area of research, with many ongoing efforts to understand and extend the duality to a wider range of theories and phenomena. However, they provide neither the mathematical isomorphism between two theories nor the proper justification of it.

It is noteworthy that this conjecture and correspondence soon resulted in a different type of equivalence with the emergence of a proper theory of how nature works. Like Type I MEE's implication that there might be no clear conceptual distinction between mass and energy would soon be formalized by Type III, the WEP gave a crucial hint towards eliminating the distinction between acceleration and gravity.Areas that had been considered separate turned out to be closely connected. AdS/CFT correspondence suggests a very interesting point that quantum field theory includes theoretical aspects of both quantum gravity and particle physics, and the phenomena in condensed matter physics can be described and understood through it. If the AdS/CFT correspondence becomes indistinguishable like MEE in the future, quantum field theory will embrace quantum gravity and further solidify its universal and fundamental grounds.

Similarly, in the history of quantum mechanics, before the Dirac-von Neumann axioms were introduced and accepted, Heisenberg's matrix approach to quantum mechanics and Schrödinger's wave theory of quantum mechanics had provided independent but equal solutions for the quantum simple harmonic oscillator ($V \propto r^2$) and hydrogen atom ($V \propto 1/r$), respectively. Schrödinger conjectured mathematical equivalence between wave theory and matrix theory, but failed to demonstrate it. There were only a few empirical coincidences between them. At that time, since there was no satisfactory proof, their correspondence might have been conjectured, and could be called MM/WM correspondence or wave-matrix duality, similar to today's AdS/CFT correspondence or gauge-gravity duality. It was von Neumann who axiomatically formulated mathematical foundations of quantum mechanics (Neumann 2018) by connecting Heisenberg's matrix theory and Schrödinger's wave theory. The idea of MM/WM correspondence became clearer as they became indistinguishable to each other in terms of operators on a Hilbert space, as both approaches have been used to solve problems in quantum mechanics textbooks.

Table 5.1 shows the summarized conditions that the three types of equivalencies should satisfy. Our categorization might seem artificial and original. However, this section has shown that it is nothing new. Many other examples of equivalence in physics can also be categorized this way.

Table 5.1 Qualitative distinction of equivalences

Equivalence as…	**Convertibility**	**Conjecture & Correspondence**	**Indistinguishability**
numerically or empirically equivalent?	**O**	**O** (without general proof)	**O**
inter-convertible? (Can an integrated conservation law be constructed without any assumption?)	**O**	**X**	**O** (Equivalence of Two quantities) **X** (Equivalence of Two theories)
ontologically equivalent? (Is one of the concepts able to be inter-replaced?)	**X**	**X**	**O**
(Examples) **Equivalence of which…** — **two quantities** (conceptual relation of two quantities)	**MEE** (Type II) $\delta Q - \delta W_{\text{Mech}}$ **Equiv.** Thermodynamic 1st Law	**MEE** (Type I) **MEE** (Type IV) Particle-field correspondence **WEP** ($m_g = m_a$)	**MEE** (Type III) Particle-Field Equivalence?
Equivalence of which… — **two theories** (bijective relation of two theories)		**AdS/CFT** Correspondence **Mat. Mech. /Wave. Mech.** Correspondence	**SEP** ($\text{Physics}_a = \text{Physics}_{-g}$) **AdS/CFT equivalence?** ($\text{Physics}_{\text{AdS}} = \text{Physics}_{\text{CFT}}$) Dirac-von Neumann Axioms

6 Summary and Discussion

$E = mc^2$ (Mass-Energy Equivalence, [MEE]) has significant educational standing in terms of its enormous and profound implications as the basis of modern physics and the current educational trends in which modern physics education has been highlighted. In spite of its educational importance, demonstrations of MEE and its understandings are still insufficient and partialized. This study was made to resolve these problems.

In this study, these problems were attributed ignoring the relevance between field and particle, two physical objects having intrinsic energy and mass, respectively, which has given rise to disagreement on understanding or interpreting MEE. Accordingly, in this study, literature mainly focusing on the derivation of MEE was collected and reviewed. (The collected papers are listed in Appendix A.) The result showed that at least three distinct types of understanding exist. Each type shared a common feature in terms of connection between particle and field or mass and energy. Moreover, assumptions and concluding equations ($E_{\text{of which}} = m_{\text{of which}} c^2$) are also clearly distinguished (see Table 4.1).

[Conclusion I]

In the first section of results (Section 4), as an answer to [Question I], the characteristics of the three types of MEE were identified, introducing Einstein's derivations corresponding to each type briefly. The types were expressed as conjecture and correspondence (Type I), convertibility (Type II), and indistinguishability (Type III). Closely examining the supplementary papers to fill the blank connections, each type can be interpreted as follows:

- Type I: Mass and energy might have quantities corresponding to each other. The distinction between fields and particles is still preserved.
- Type II: Mass and energy are inter-convertible measures, assuming fields and particles are inter-convertible.
- Type III: Mass and energy are indistinguishable from each other, with clear mathematical similarity between particles and fields.

In addition, some logical hierarchies between these types (Figures 4.2, 4.4, and 4.5)and before relativity (Figure 4.1) were found and a conceptual development map was suggested (Figure 4.8). MEE was implied to serve as a stepping stone to suggest particle-field equivalence (or duality). Considering that fields and particles are closely related to particle theory and wave theory, respectively, in the history of quantum physics, the resultant map naturally provides the following holistic understanding of MEE as Type IV.

- Type IV: Just as mass and energy are conceptually integrated on the same ontological ground, fields and particles are indistinguishable from each other as if they have theoretical duality.

Quantum field theory tried to formalize this viewpoint, with the concept of a particle described as a local quantum excitation of the corresponding field. This can also be construed as the field supremacy of a candidate for matter theory. The two physical objects, which seemed to have distinct epistemological properties from each other, have achieved an ontological integration. In addition, the Standard Model, which is believed to be the most theoretically self-consistent and successful theory of modern physics, is written with the language of quantum field theory. As MEE implies, the concept of mass (inertiality) in the Standard Model was actually substituted by the Higgs mechanism. Therefore, the first result can be summarized by saying that the perspective change from classical dualism of fields and particles to monism in quantum physics was implied as an example of the desired holistic understanding of MEE.

[Conclusion II]

Starting with [Question II], whether our method of categorizing MEE holds for the other examples of equivalence in physics, I extracted the semantic elements of equivalence based on the three resultant types of MEE and examined whether they are also included in other examples of equivalence in physics. The results (Section 5) showed that not only were the semantic elements extracted earlier, but also features in the three types of MEE hold for some other examples (see Table 5.1). Further, analogous to MEE, a similar logical hierarchy could be identified between examples in conjecture and Correspondence and examples in indistinguishability. This suggests that empirical evidence or unproven correspondence have been hinted at, and the context of inquiry leading to the ontological merger between the two theories (or two physical quantities) has generally existed in physics.

[Discussion and Outlook]

Although the equation $E = mc^2$ might seem simple at a glance, there are many implications behind its conceptual development. Nevertheless, the existing arguments for MEE in most modern physics textbooks are declarative and not logically persuasive. To compensate for the insufficient understanding, I have found some papers that derived MEE. However, each of the conclusions implies different meanings for $E = mc^2$, making it more confusing.

Contrary to the existing arguments, this study suggests a context of inquiry in which two perspectives have been merged ontologically. To be concrete, what I have studied well illustrates not only the conceptual relation of mass and energy, but also that of fields and particles. Perspective change in physics naturally represents the intrinsic value, historicity, and holistic connection of content knowledge. Consequently, the results of this study, the historical process and context of inquiry in physics, can be thought of as a good example of "practices"[1] in physics, which describe the essence of physics.

By highlighting the differences between the study results and existing instructional materials, this study is expected to play a significant role as a conceptual framework (or theoretical framework) for the analysis of existing textbooks and the development of new curriculums.

[1]According to MacIntyre (1981), practices are important because it is only within the context of a practice that human beings can practice the virtues. Practice-based education follows the concept of practices proposed by MacIntyre to criticize the 18th century Enlightenment project. Lee (2022) also pointed out that the practices of science (from his "essential-holistic" perspective) provides students an important context in which they can learn many elements of science collectively, such as unpartialized knowledge, intrinsic value, and historicity.

A Review of Selected Literature

Table A.1 lists the selected literature. It can be seen that there was no perfectly deductive derivation and there were some degrees of logical oversight. Perhaps MEE shows that human belief played a big role in its theoretical foundation rather than that it was perfectly axiomatic. Starting with the existence of rest energy, the natural belief in linearity between mass and energy might be expected as an aspect of the nature of physics pursuing simplicity. This requires further discussion.

Table A.1 Selected literature and supplementary materials (denoted by 'sup')

literature (author & year)	situation posited	assumption or logical overshadowing	derived MEE	type
A. Einstein, 1906	electromagnetic field confined in a finite volume	energy itself has the corresponding mass (field can be considered as a particle)	$E_{\mathrm{EM}} \overset{?}{=} m_{\mathrm{EM}}c^2$	I
M. Born, 1962	electromagnetic field confined in a finite volume	energy itself has the corresponding mass (field can be considered as a particle)	$E_{\mathrm{EM}} \overset{?}{=} m_{\mathrm{EM}}c^2$	I
M. Laue, 1911	electromagnetic field confined in a finite volume	field can be considered as a particle (derived $E_{\mathrm{EM}}(\boldsymbol{v}) = \gamma(\boldsymbol{v})E_{\mathrm{EM}}(0)$ and compare its 2nd order with the particle kinetic)	$E_{\mathrm{EM}} \overset{?}{=} m_{\mathrm{EM}}c^2$	I
A. Perez and S. Ribisi, 2022	electromagnetic field confined in a finite volume	field can be considered as a particle (derived $E_{\mathrm{EM}}(\boldsymbol{v}) = \gamma(\boldsymbol{v})E_{\mathrm{EM}}(0)$ and compare its 2nd order with the particle kinetic)	$E_{\mathrm{EM}} \overset{?}{=} m_{\mathrm{EM}}c^2$	I
(sup) F. Klein, 1918	generalization of Born's result with a time-independent energy-momentum tensor in GR			I
(sup) C. Y. Lo, 2006	intrinsic difference between massless particles and electromagnetic field had no effect on inertiality			I
A. Einstein, 1907	-	particle itself has an intrinsic form of energy	$E_{\mathrm{ptcl}} \overset{?}{=} m_{\mathrm{ptcl}}c^2$	I
S. Duarte and N. Lima, 2021	inelastic head-on collision	particle itself has an intrinsic form of energy only claim exists without proof	$E_{\mathrm{ptcl}} \overset{?}{=} m_{\mathrm{ptcl}}c^2$	I
(sup) Modern Physics textbook	presence of claim $E_0 = mc^2$ without any proof might have been affected by Einstein's paper in 1907			I
(sup) E. Hecht, 2012	relativistic kinetic expression has the form of a difference; $E_{\mathrm{kin}}(v) = \left[\gamma(v')mc^2\right]_{v'=0}^{v'=v}$ and its velocity-independent term was given an interpretation of rest-energy by Einstein			I
A. Einstein, 1905b	radiating object	light-matter interaction was presupposed photon model ($E = \hbar k$) was not used	$E_{\mathrm{EM}} \overset{\longleftrightarrow}{=} m_{\mathrm{ptcl}}c^2$	II
D. J. Steck and F. Rioux, 1983	radiating object	light-matter interaction was presupposed photon model ($E = \hbar k$) was not used	$E_{\mathrm{EM}} \overset{\longleftrightarrow}{=} m_{\mathrm{ptcl}}c^2$	II
F. Rohrlich, 1990	radiating object	light-matter interaction was presupposed photon model ($E = \hbar k$) was not used	$E_{\mathrm{EM}} \overset{\longleftrightarrow}{=} m_{\mathrm{ptcl}}c^2$	II
J. J. Leary, 2007	radiating atom	light-matter interaction was presupposed photon model ($E = \hbar k$) was not used	$E_{\mathrm{EM}} \overset{\longleftrightarrow}{=} m_{\mathrm{atom}}c^2$	II
(sup) M. Planck, 1908	criticism about assuming the additive decomposition of kinetic and rest energy			II
(sup) O. Klemperer, 1934	first directly supporting evidence or matter-light conversion $e^+e^- \leftrightarrow \gamma\gamma$ was suggested			II
(sup) H. E. Ives, 1952 and M. Jammer, 1997	criticism about *petitio principii*			II
A. Einstein, 1935	head-on collision	reactant's internal interaction to make a resultant gain of constituent	$E_{\mathrm{ptcl}} = m_{\mathrm{ptcl}}c^2$	III
M. J. Feigenbaum and N. D. Mermin, 1988	parallel particle-ejection	reactant's internal interaction to lead particle-ejection	$E_{\mathrm{ptcl}} = m_{\mathrm{ptcl}}c^2$	III
Y. Dai and L. Dai, 2018	-	linearity of energy-momentum transformation	$E_{\mathrm{ptcl}} = m_{\mathrm{ptcl}}c^2$	III
G. S. Adkins, 2008	symmetric Lewis & Tolman collision	Linear relation between mass and energy	$E_{\mathrm{ptcl}} = m_{\mathrm{ptcl}}c^2$	III
(sup) J. J. Thomson, 1881 and others	surface-charged conductor	supplementary for existence of field inertia		III
(sup) E. Fermi, 1922 and others	surface-charged conductor	supplementary for $m_{\mathrm{EM}}^{(p)} = m_{\mathrm{EM}}^{(E)}$	$E_{\mathrm{field}} = m_{\mathrm{field}}c^2$	III
(sup) B. Podolsky et al., 2009 and others	charged sphere	supplementary for $m_{\mathrm{EM}}^{(F)} = m_{\mathrm{EM}}^{(E)}$		III

B Brief Introduction to Quantum Mechanics and Quantum Field Theory

In classical physics, all physical quantities were considered to satisfy the commutation relation. For example, in classical mechanics, two quantities $A(x, p)$ and $B(x, p)$ have the commutative property of multiplication. But in the quantum mechanics, the commutation of A and B is not zero in general.

$$\big[A(x, p), B(x, p)\big] \neq 0 \tag{B.1}$$

The history of quantum physics is the history of abandoning commutative properties. The first abandonment occurred when the Fourier transform of space-time $x^\mu = (ct, \boldsymbol{x})$ was given a physical meaning: energy-momentum $p^\mu = (E/c, \boldsymbol{p})$.

First, considering an arbitrary field $\phi(x) = \phi(t, \boldsymbol{x})$, one can choose an arbitrary orthogonal basis that satisfies the Sturm-Liouville condition. A physical quantity whose domain is space-time can be expanded by linear summation of the basis. Taking the trigonometric basis to see the wave-like behavior, Fourier transformation in Minkowski space is:

$$\phi(x) = \frac{1}{(2\pi)^4} \int d^4p \; e^{ip\cdot x/\hbar} \tilde{\phi}(p), \tag{B.2}$$

$$\tilde{\phi}(p) = \int d^4x \; e^{-ip\cdot x/\hbar} \phi(x). \tag{B.3}$$

Here, p is written with a four-vector notation, but it does not mean energy-momentum here. After claiming $p^\mu = (E/c, \boldsymbol{p})$, the wave nature of quantum physics inheres in these formulae:

$$\phi(t, \boldsymbol{x}) = \int \frac{dE\, d^3p}{(2\pi)^4 c}\, e^{(-iEt + i\boldsymbol{p}\cdot\boldsymbol{x})/\hbar} \tilde{\phi}(E, \boldsymbol{p}), \tag{B.4}$$

$$\tilde{\phi}(E, \boldsymbol{p}) = \int c\, dt\, d^3x\, e^{(iEt - i\boldsymbol{p}\cdot\boldsymbol{x})/\hbar} \phi(t, \boldsymbol{x}). \tag{B.5}$$

After quantization, particle nature in the space-time x^μ will perfectly correspond with wave nature in the energy-momentum p^μ. In hindsight, no special meaning to ϕ was given. It has only been chosen arbitrarily to see the wave-like property of a field. Fourier transform and quantization then play a role of the mediator to connect particle nature with wave nature. From these relations, operator relations in quantum physics can be obtained naturally for the appropriate quantity to be extracted:

$$p_i \to p_i = \frac{\hbar}{i} \frac{\partial}{\partial x^i}, \qquad E \to H = i\hbar \frac{\partial}{\partial t} \tag{B.6}$$

which satisfies $[x_i, p_j] = i\hbar\, \delta_{ij}$ and agrees well with (B.1). When this operator being acted on a wave-function ψ to satisfy energy-momentum relation $E = E(p_i, V)$, it is natural to get the Schrödinger equation as follows:

$$i\hbar \frac{\partial}{\partial t} \psi = H\left(\frac{\hbar}{i}\frac{\partial}{\partial x^i}, V\right)\psi. \tag{B.7}$$

When the Hamiltonian is given by $H(\boldsymbol{x}, \boldsymbol{p}) = \boldsymbol{p}^2/2m + V(\boldsymbol{x})$, assuming our wave-function is exponentiated with some action functional $S\left[\boldsymbol{x}(t)\right]$ or considering a scattered system by the action $S(t, \boldsymbol{x})$, the wave-function using the WKB approximation can be write down as:

$$\psi(t, \boldsymbol{x}) \sim \exp\left(\frac{i}{\hbar} S(t, \boldsymbol{x})\right). \tag{B.8}$$

Then it can be deduced that

$$-\partial_t S = \frac{1}{2m}(\nabla S)^2 - \frac{i\hbar}{2m}\nabla^2 S + V. \tag{B.9}$$

If the term that multiplies $\hbar$ is neglected, one can have the Hamilton-Jacobi equation, where the Hamiltonian is given by:

$$H(\boldsymbol{x}, \nabla S) = \frac{1}{2m}(\nabla S)^2 + V(\boldsymbol{x}), \tag{B.10}$$

illustrating well the reduction to classical mechanics.

The core idea of quantum mechanics is to promote the two physical quantities in a canonical conjugate relation to a higher mathematical dimension so that the commutative property no longer holds true. Since the physical quantities were promoted to the operators, the uncertainty relation between the coordinates and conjugate momenta $\Delta x_i \Delta p_j \geq \hbar\delta_{ij}/2$ is naturally acquired from the Fourier transformation relation. But time in quantum mechanics plays a role as a parameter that labels the specific moment. Time is not on an equal footing to space, since relativity has not yet been imposed, although there is an uncertainty relation between energy and time $\Delta E \Delta t \geq \hbar/2$. However, there is no operator corresponding to time. Because there is a relation between energy and momenta (e.g., $E = \boldsymbol{p}^2/2m$), this relation becomes a mathematical constraint to reduce the degree of freedom by one. Equivalently, the boundedness of the Hamiltonian precludes the existence of conjugate time operator.[1] Since there is no observable time, the uncertainty relation is written in a slightly different manner. Just as the generator of time translation was Hamiltonian in classical mechanics, quantum mechanics describes the time translation for

[1]Stone–von Neumann theorem assures that there is no well-behaved time operator T that is conjugate pair of physically realistic Hamiltonian H.

the same manner. That means one can describe the energy-time uncertainty relation as

$$\Delta A \Delta H \geq \frac{1}{2}\left|\frac{d\langle A\rangle}{dt}\right|. \tag{B.11}$$

Therefore, energy-time uncertainty is one of the anomalies where one cannot deal with time and energy exactly for the same manner as did with position and momentum.

As I have seen before, imposing Lorentz invariance on the energy-momentum relation gives $-E^2/c^2 + \boldsymbol{p}^2 = -m^2c^2$ or $p^2 = -m^2$ using four-vector notation and natural unit$(c = \hbar = 1)$, which is the simplest isolated free particle system.[2] From this relation and (B.2), one can describe how to evolve a wave-function relativistically in the space-time domain:

$$\left(-\partial^2 + m^2\right)\phi(x) = 0. \tag{B.12}$$

This equation is called Klein-Gordon equation named after Oskar Klein (1926) and Walter Gordon (1926). It was originally proposed to describe the dual nature of electrons, but it could not consider the internal spin degree of freedom. $\left(-\partial^2 + m^2\right)$ is the operator form of relativistic energy-momentum relation $p^2 + m^2 = 0$. It is a mathematical constraint that the Klein-Gordon operator is acted on an arbitrary field ϕ to become zero, thus equivalently it can be said that the energy-momentum relation should be satisfied in the integrand of (B.2). Then the Fourier transformation rule requires the constraint as

$$\phi(t, \boldsymbol{x}) = \int \frac{dE\, d^3p}{(2\pi)^4}\delta\left[E^2 - \left(\boldsymbol{p}^2 + m^2\right)\right] e^{-iEt + i\boldsymbol{p}\cdot\boldsymbol{x}}\tilde{\phi}(E, \boldsymbol{p}). \tag{B.13}$$

[2]One can also write $E = \pm\sqrt{\boldsymbol{p}^2c^2 + m^2c^4}$. However, if it is translated into the operator form using (B.6), expanding the square root to all power of ∇^2 leads to a violation of the locality of the theory.

Two solutions of inside the delta-function are

$$E = \pm\sqrt{\boldsymbol{p}^2 + m^2}. \tag{B.14}$$

Two solutions give rise to the introduction of one another basis, which corresponds to the negatively signed energy expression. Putting $\tilde{\phi}_-$ for the negative energy basis and $\tilde{\phi}_+$ for the positive energy basis, (B.2) with delta function is solved to give

$$\phi(t, \boldsymbol{x}) = \frac{1}{(2\pi)^3} \int \frac{d^3p}{2E} \left[e^{-iEt + i\boldsymbol{p}\cdot\boldsymbol{x}} \tilde{\phi}_+(\boldsymbol{p}) + e^{iEt - i\boldsymbol{p}\cdot\boldsymbol{x}} \tilde{\phi}_-(\boldsymbol{p}) \right]_{E=\sqrt{\boldsymbol{p}^2+m^2}} \tag{B.15}$$

where in the negative energy part of the integrand $p \to -p$ was reversed and amplitude $\tilde{\phi}_-(-\boldsymbol{p}) \to \tilde{\phi}_-(\boldsymbol{p})$ was renamed. If the energy-momentum relation was given in the implicit form of $f(E, \boldsymbol{p}) = 0$ and its solution was given as $E(\boldsymbol{p}) = E_1(\boldsymbol{p}),\ E_2(\boldsymbol{p}),\ \cdots, E_m(\boldsymbol{p})$ ($m = 1$ or 2 in general[3]), solving the delta functions gives

$$\phi(t, \boldsymbol{x}) = \frac{1}{(2\pi)^3} \sum_{i=1}^{m} \int \frac{d^3p}{(\partial f/\partial E)_{E=E_i}} e^{-iE_it + i\boldsymbol{p}\cdot\boldsymbol{x}} \tilde{\phi}_i(\boldsymbol{p}) \tag{B.16}$$

where $\tilde{\phi}_i(E_i, \boldsymbol{p}) \to \tilde{\phi}_i(\boldsymbol{p})$ was also renamed. It also gives a general solution for $f\left(i\partial/\partial t, -i\partial/\partial x^i\right)\phi = 0$.

Here note that the wave-function ϕ is a Lorentz scalar, that is Lorentz transformation $(t, \boldsymbol{x}) \to (t', \boldsymbol{x}')$ implies $\phi \to \phi' = \phi$. Hence this scalar field is only valid for particles with no orientation, that is, particles with no spin. Since $|\phi|^2$ is also a Lorentz scalar, there is a problem with the probability density interpretation. To match up the definition of density, it should transform as the

[3]A degree of freedom more than two is not allowed since a higher derivative requires more parameters than those needed to specify the theory.

temporal component of a four-vector in terms of the contraction of a volume element. Another problem arises in the negative energy solution, which was physically not allowed in classical field theory. But in quantum field theory (QFT), the negative energy part in (B.15) can be interpreted physically by introducing the concept of an anti-particle. This is another reason for making probability density interpretation impossible.

In 1934, Pauli and Weisskopf (1988) solved this problem by reinterpreting from quantum field theoretical view. The density is no longer positive definite but it can also be negative. Possible four-current form $j^\mu = (\rho, \boldsymbol{j})$ for ϕ which satisfies the continuity equation $\partial_\mu j^\mu = 0$ and Klein-Gordon equation $(-\partial^2 + m^2)\phi = 0$ is

$$j^\mu = \frac{1}{2mi}\phi^* \overleftrightarrow{\partial^\mu} \phi = \frac{1}{2mi}\left(\phi^* \overrightarrow{\partial^\mu} \phi - \phi^* \overleftarrow{\partial^\mu} \phi\right). \tag{B.17}$$

The spatial component coincides with the case of Schrödinger equation, but the temporal component does not:

$$\rho_{\text{K-G}} = \frac{i}{2m}\left(\phi^* \partial_t \phi - \phi \partial_t \phi^*\right) \neq \rho_{\text{Schrödinger}} = \phi^* \phi. \tag{B.18}$$

Mathematically, this is because the Klein-Gordon equation has one more higher order of time derivative, thus it needs another initial condition corresponding to $\partial_t \phi$ as well as the initial condition corresponding to ϕ. But in the non-relativistic limit, $i\partial_t \phi = E\psi$ gives $\rho_{\text{K-G}} = (E/m)\phi^*\phi$. In quantum field theory, this current gives no interpretation of probability density current anymore. Instead, multiplying by charge e, this current also promotes to be an operator that gives charge density and current:

$$j^\mu = \frac{e}{2mi}\phi^* \overleftrightarrow{\partial^\mu} \phi. \tag{B.19}$$

The field equation is then reinterpreted not by the wave-function ϕ but the operator field ϕ, whose excitation makes the occupation number of particles. Promoting the amplitudes $\tilde{\phi}_-$ and $\tilde{\phi}_+$ in (B.15) to become quantum harmonic oscillators, one can formulate quantum field theory, which is currently used to describe physics at a very short distance scale and a very high energy scale. The second abandonment of commutativity arises in this step,

$$[\phi_1(x), \phi_2(x)] \neq 0. \tag{B.20}$$

I am now going to make our field ϕ real. Imposing $\phi^* = \phi$ reads

$$\phi(x) = \int \frac{d^3p}{(2\pi)^3 2p^0} \left[e^{ip\cdot x} \tilde{\phi}_+(\boldsymbol{p}) + e^{-ip\cdot x} \tilde{\phi}^*_+(\boldsymbol{p}) \right]. \tag{B.21}$$

Let us take up the quantum theory. Promoting $\tilde{\phi}_+(\boldsymbol{p})$ to $a(\boldsymbol{p})$, and its complex conjugate to $a^\dagger(\boldsymbol{p})$, which I will soon show them to be ladder operators. From (B.21), it can be easily shown that

$$a(\boldsymbol{p}) = i \int d^3\, x e^{-ip\cdot x} \overleftrightarrow{\partial_t} \phi(x). \tag{B.22}$$

Then the Heisenberg equation of motion $[H, \phi(x)] = -i\partial_t \phi(x)$ leads us to desired relation for ladder operators.

$$[H, a(\boldsymbol{p})] = i \int d^3x\; e^{-ip\cdot x} \left(ip^0 + \partial_t \right) (-i\partial_t)\, \phi(x) \tag{B.23}$$

$$= ip^0 \int d^3x\; e^{-ip\cdot x} \left(ip^0 + \partial_t \right) \phi(x) \tag{B.24}$$

where I integrated by parts in the second line. Now I have shown $a(\boldsymbol{p})$ and $a^\dagger(\boldsymbol{p})$ really behave like ladder operators:

$$[H, a(\boldsymbol{p})] = -p^0 a(\boldsymbol{p}), \qquad [H, a^\dagger(\boldsymbol{p})] = -p^0 a^\dagger(\boldsymbol{p}). \tag{B.25}$$

One can also consider the appropriate generalization of canonical conjugate of ϕ as $\pi = \partial\mathcal{L}/\partial(\partial_t\phi) = \partial_t\phi$ (for Klein-Gordon Lagrangian $\mathcal{L}$) thus can get a non-trivial equal-time commutation relation as

$$\left[\phi(x), \phi(x')\right]\Big|_{x^0=x'^0} = 0, \tag{B.26}$$

$$\left[\pi(x), \pi(x')\right]\Big|_{x^0=x'^0} = 0, \tag{B.27}$$

$$\left[\phi(x), \pi(x')\right]\Big|_{x^0=x'^0} = i\delta^3(\boldsymbol{x} - \boldsymbol{x}'). \tag{B.28}$$

One can check between two canonically conjugate field operators; the field version of non-commutativity (B.20) automatically holds. If one considers a fermionic version of these relations, anti-commutators instead of commutators in (B.26-B.28) should be used, but not in (B.25).

One remarkable feature is that compared to quantum mechanics, ladder operators in quantum field theory are interpreted as creating and annihilating particles rather than as raising and lowering the energy level of one particle system. Like a harmonic oscillator in quantum mechanics, non-commutativity

$$\left[a(\boldsymbol{p}), a^\dagger(\boldsymbol{p}')\right] = 2p^0(2\pi)^3\delta^3\left(\boldsymbol{p} - \boldsymbol{p}'\right) \tag{B.29}$$

leads to non-zero ground state energy, which apparently represents quantum nature. The Hamiltonian of Klein-Gordon system is given by the manifestly time-independent expression

$$H = \int \frac{d^3p}{(2\pi)^3 2p^0} \frac{p^0}{2} \left[a^\dagger(\boldsymbol{p})a(\boldsymbol{p}) + a(\boldsymbol{p})a^\dagger(\boldsymbol{p})\right]. \tag{B.30}$$

And for the total momentum $P^i = \int d^3x\, \Theta^{0i}$, also one can get the expression

$$P^i = \int \frac{d^3p}{(2\pi)^3 2p^0} \frac{p^i}{2} \left[a^\dagger(\boldsymbol{p})a(\boldsymbol{p}) + a(\boldsymbol{p})a^\dagger(\boldsymbol{p})\right]. \tag{B.31}$$

What we have in (B.30) and (B.31) are the Hamiltonian and momentum appropriate to an infinite set of oscillators; thus, these expressions both diverge since each oscillator has a non-zero ground state energy. However this is an avoidable infinity since in reality we always measure only the difference in energy. The infinity arises from the momentum integral of the commutator of the integrand, defining the true Hamiltonian after erasing zero-point energy or equivalently ordering normally. Then we have a positive-definite and finite expression for the Hamiltonian and a finite expression for the total momentum as

$$: H := \int \frac{d^3p}{(2\pi)^3 2p^0} p^0 a^\dagger(\boldsymbol{p}) a(\boldsymbol{p}), \quad : P^i := \int \frac{d^3p}{(2\pi)^3 2p^0} p^i a^\dagger(\boldsymbol{p}) a(\boldsymbol{p}). \tag{B.32}$$

With this Hamiltonian, the ground state of the theory $|0\rangle$ is annihilated by all $a(\boldsymbol{p})$'s. Also, one may represent the eigenstates of $: H :$ and $: P^i :$ in terms of the occupation-number representation $\left|n_{\boldsymbol{p}}\text{'s}\right\rangle$, constructed as

$$\left|n_{\boldsymbol{p}}\text{'s}\right\rangle = \left|n_{1,(\boldsymbol{p}_1)}\right\rangle \left|n_{2,(\boldsymbol{p}_2)}\right\rangle \cdots \left|n_{m,(\boldsymbol{p}_m)}\right\rangle = \prod_i \frac{\left(a^\dagger(\boldsymbol{p}_i)\right)^{n_i}}{\sqrt{n_i!}} |0\rangle . \tag{B.33}$$

The state vectors should also satisfy the eigenvalue conditions

$$: H : \left|n_{\boldsymbol{p}}\text{'s}\right\rangle = \left(\sum_i p_i^0 n_i\right) \left|n_{\boldsymbol{p}}\text{'s}\right\rangle , \quad : \boldsymbol{P} : \left|n_{\boldsymbol{p}}\text{'s}\right\rangle = \left(\sum_i \boldsymbol{p}_i n_i\right) \left|n_{\boldsymbol{p}}\text{'s}\right\rangle . \tag{B.34}$$

Here I also note that for a restricted Lorentz transformation $\Lambda \in \mathrm{SO}^+(1,3)$, one can have a unitary operation $U(\Lambda)$ such that

$$U(\Lambda)^{-1} \phi(x) U(\Lambda) = \phi(\Lambda^{-1} x). \tag{B.35}$$

Thus the Lorentz transformation of $a(\boldsymbol{p})$ and $a^\dagger(\boldsymbol{p})$ is

$$U(\Lambda)^{-1} a(\boldsymbol{p}) U(\Lambda) = a(\Lambda^{-1}\boldsymbol{p}), \quad U(\Lambda)^{-1} a^\dagger(\boldsymbol{p}) U(\Lambda) = a^\dagger(\Lambda^{-1}\boldsymbol{p}). \tag{B.36}$$

This directly implies

$$U(\Lambda)\,|\boldsymbol{p}_1\cdots\boldsymbol{p}_m\rangle = |\Lambda\boldsymbol{p}_1\cdots\Lambda\boldsymbol{p}_m\rangle\,. \tag{B.37}$$

It can be referred that the occupation-number representation in quantum field theory illustrates well the relativistic nature of particles.

C Completing Einstein's First Derivation

Unfortunately, the Einstein's 1905 derivation has been still considered "proof" of MEE, despite suffering from begging the question. His derivation is repeated today as being correct and physically informational without any mention of the above problem of it. Herewith, a quantum treatment of this problem is suggested. The hypothesized situation now can be applied to the neutral pion decay, $\pi^0 \to \gamma\gamma$,[1] or scalar fermion decay. In this chapter, how the matter-light interaction is theorized and how the light quanta could manifest particle-like properties will be rectified.

Some mathematical descriptions of quantum field theory will be used without any detailed introduction. I will examine the validity of this situation and the scattering amplitude when matter is converted into energy using only scalar quantum electrodynamics (QED) under no consideration of the spin of the constituent particles. Here I shall actually play with an example to show how theoretically this hypothetical situation can actually be possible from the interaction term obtained from the result of imposing gauge symmetry. First consider an action for the theory introducing the covariant gauge-fixing term

[1] More knowledge preceded explanation of the interaction: Since a neutral pion is uncharged, it needs to undergo an electromagnetic interaction to produce photons, or it can be easily modeled by scalar quantum electrodynamics (QED).

$\mathcal{S}_{\text{gf}} = -(1/2\xi)\int d^4x \left(\partial_\mu A^\mu\right)^2$.

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\mathcal{L}_\gamma} \underbrace{- \left(D_\mu\Phi\right)^\dagger \left(D^\mu\Phi\right) - m_\Phi^2\Phi^\dagger\Phi - \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2}_{\mathcal{L}_\Phi+\mathcal{L}_{\text{int}}} \underbrace{-\frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2}_{\mathcal{L}_{\text{gf}}} \tag{C.1}$$

where $D_\mu = \partial_\mu - ieA_\mu$. The action can be separated into two free field parts $\mathcal{S}_\gamma$ and $\mathcal{S}_\Phi$ with an interaction term of

$$\mathcal{S}_{\text{int}} = \int d^4x \left\{-ie\Phi^\dagger A^\mu\partial_\mu\Phi + ieA^\mu(\partial_\mu\Phi^\dagger)\Phi + e^2A_\mu A^\mu\Phi^\dagger\Phi - \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2\right\}. \tag{C.2}$$

First I evaluate the vacuum functional for A_μ. Considering $\mathcal{S}_\gamma+\mathcal{S}_{\text{gf}}+\int d^4x J_\mu A^\mu$, varying with respect to A_μ gives us the equation of motion

$$\partial^2 A_\mu - \left(1-\frac{1}{\xi}\right)\partial_\mu\partial_\nu A^\nu + J_\mu = 0. \tag{C.3}$$

Hence the photon propagator $D_{\mu\nu}(x-y)$ should satisfy

$$\left[\eta_{\mu\nu}\partial^2 - \left(1-\frac{1}{\xi}\right)\partial_\mu\partial_\nu\right] D^\nu{}_\lambda(x-y) = -\eta_{\mu\lambda}\delta^4(x-y). \tag{C.4}$$

Or equivalently in the energy-momentum space

$$\left[\eta_{\mu\nu}k^2 - \left(1-\frac{1}{\xi}\right)k_\mu k_\nu\right] D^\nu{}_\lambda(q) = \eta_{\mu\lambda} \tag{C.5}$$

which is invertible. Thus the proper energy-momentum expression of the propagator is acquired:

$$D_{\mu\nu}(k) = \frac{1}{k^2 - i\epsilon}\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]. \tag{C.6}$$

Now I can obviously get the vacuum functional for A_μ.

$$Z_0[J^\mu] = \exp\left[\frac{i}{2}\int d^4x\, d^4y\; J^\mu(x)D_{\mu\nu}(x-y)J^\nu(y)\right]. \tag{C.7}$$

Next consider the vacuum functional for the complex scalar Φ:

$$\mathcal{S}^{J}_{\Phi,\text{free}} = \int d^4x \left\{ -\partial^\mu \Phi^\dagger \partial_\mu \Phi - m^2 \Phi^\dagger \Phi + \eta \Phi^\dagger + \eta^* \Phi \right\}. \tag{C.8}$$

The equation of motion now reads the Klein-Gordon equation with a source as $\left(-\partial^2 + m^2\right)\Phi = \eta$. Hence its propagator is nothing but the Feynman propagator Δ_F. And obviously I get the vacuum functional expression for Φ as

$$Z_0[\eta, \eta^*] = \exp\left[i \int d^4x\, d^4y\, \eta^*(x) \Delta_F(x-y) \eta(y) \right]. \tag{C.9}$$

Thus the full generating functional for the free theory is given by

$$\begin{aligned}
Z_0[J^\mu, \eta, \eta^*] = &\int [\mathcal{D}A] \int [\mathcal{D}\Phi][\mathcal{D}\Phi^\dagger] \\
&\qquad \exp\left[i\mathcal{S}_\gamma + i\mathcal{S}_\Phi + i \int d^4x \left(A_\mu J^\mu + \eta^* \Phi + \eta \Phi^\dagger \right) \right] \\
&= Z_0[0,0,0] \exp\left[\int d^4x\, d^4y\, \left\{ \frac{i}{2} J^\mu(x) D_{\mu\nu}(x-y) J^\nu(y) \right\} \right] \\
&\qquad \exp\left[\int d^4x\, d^4y\, \left\{ i\eta^*(x) \Delta_F(x-y) \eta(y) \right\} \right]
\end{aligned} \tag{C.10}$$

where I do not need to consider the $(\Phi^\dagger\Phi)^2$ term since the interaction trying to be evaluated is just matter-to-photon process. Next let us establish the Feynman rules. Starting with the generating functional for our theory:

$$Z[J^\mu, \eta, \eta^*] = N \exp\left[i \int d^4z \mathcal{L}_{\text{int}} \left(\frac{1}{i}\frac{\delta}{\delta J^\mu}, \frac{1}{i}\frac{\delta}{\delta \eta}, \frac{1}{i}\frac{\delta}{\delta \eta^*} \right) \right] Z_0[J^\mu] Z_0[\eta, \eta^*] \tag{C.11}$$

$$N^{-1} = \exp\left[i \int d^4z \mathcal{L}_{\text{int}} \left(\frac{1}{i}\frac{\delta}{\delta J^\mu}, \frac{1}{i}\frac{\delta}{\delta \eta}, \frac{1}{i}\frac{\delta}{\delta \eta^*} \right) \right] Z_0[J^\mu] Z_0[\eta, \eta^*] \Bigg|_{(J^\mu, \eta, \eta*)=(0,0,0)}. \tag{C.12}$$

I can ignore the other Φ's self-interaction terms since there are only three terms associated with QED:

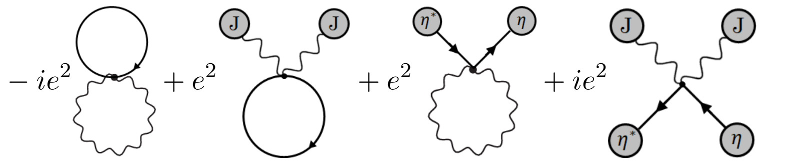
$$\begin{aligned}
\mathcal{S}_{\text{int}}^{(1)} &= e^2 \int d^4z\ A_\mu A^\mu \Phi^\dagger \Phi \\
&\to\quad e^2 \int d^4z \left(\frac{1}{i}\frac{\delta}{\delta J^\mu(z)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J_\mu(z)}\right)\left(\frac{1}{i}\frac{\delta}{\delta \eta(z)}\right)\left(\frac{1}{i}\frac{\delta}{\delta \eta^*(z)}\right), \\
\mathcal{S}_{\text{int}}^{(2)} &= -ie^2 \int d^4z\ \Phi^\dagger A^\mu \partial_\mu \Phi \\
&\to\quad -ie^2 \int d^4z \left(\frac{1}{i}\frac{\delta}{\delta \eta(z)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J_\mu(z)}\right)\partial_\mu\left(\frac{1}{i}\frac{\delta}{\delta \eta^*(z)}\right), \\
\mathcal{S}_{\text{int}}^{(3)} &= ie \int d^4z\ A^\mu \left(\partial_\mu \Phi^\dagger\right)\Phi \\
&\to\quad ie \int d^4z \left(\frac{1}{i}\frac{\delta}{\delta J_\mu(z)}\right)\partial_\mu\left(\frac{1}{i}\frac{\delta}{\delta \eta(z)}\right)\left(\frac{1}{i}\frac{\delta}{\delta \eta^*(z)}\right).
\end{aligned} \tag{C.13}$$

First computing $\mathcal{S}_{\text{int}}^{(1)}$ as

$$\begin{aligned}
&\left(i\mathcal{S}_{\text{int}}^{(1)} Z_0[J^\mu] Z_0[\eta,\eta^*]\right) / \left(Z_0[J^\mu] Z_0[\eta,\eta^*]\right) \\
&= ie^2 \int d^4x \left[-i\Delta_F(0) + \int d^4z_1\ d^4z_2\ \Delta_F(x-z_1)\,\eta(z_1)\,\Delta_F(x-z_2)\,\eta(z_2)\right] \\
&\quad \left[-iD_\mu^\mu(0) + \int d^4z_1\ d^4z_2\ D^{\mu\nu}(x-z_1)\,J_\nu(z_1)\,D_{\mu\lambda}(x-z_2)\,J^\lambda(z_2)\right]
\end{aligned} \tag{C.14}$$

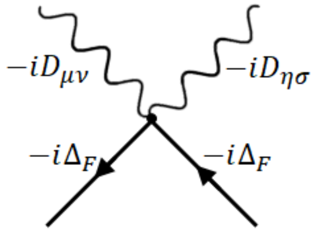
leads the diagram naturally by connecting the space-time points with the sources:

$-ie^2$ … $+e^2$ … $+e^2$ … $+ie^2$ … (C.15)



I want to take only the scattering process into account in which matter (Φ, $\Phi^\dagger$) transforms to light (2γ), in other words, in this theory, pair annihilation process. Thus the only contribution to the scattering is the last diagram of

(C.15). Functional differentiation then reads

$2ie^2$ 

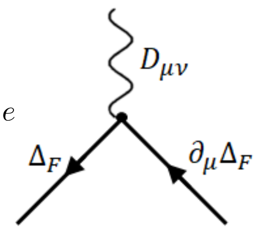
(C.16)

Since $-iD_{\mu\nu}$ is assigned to each photon line, each vertex has the value of $-2ie^2\eta_{\mu\nu}$ in order for our diagram to have the coefficient of $2ie^2$.

Next, in the same procedure, the contribution of the term $\mathcal{S}^{(2)}_{\text{int}}$ can be obtained by the followings:

$$\begin{aligned}\left(i\mathcal{S}^{(2)}_{\text{int}} Z_0[J^\mu] Z_0[\eta,\eta^*]\right) &\left(Z_0[J^\mu] Z_0[\eta,\eta^*]\right) \\ &= -ie\int d^4x\, d^4z_1\, \left(\partial_\mu \Delta_F(0)\right) D^{\mu\nu}(x-z_1)\, J_\nu(z_1) \\ &\quad + e\int d^4x\, d^4z_1\, d^4z_2\, d^4z_3\, \left(\partial_\mu \Delta_F(x-z_1)\right) \\ &\qquad \eta(z_1)\,\Delta_F(x-z_2)\,\eta^*(z_2)\, D^{\mu\nu}(x-z_3)\, J_\nu(z_3)\,. \end{aligned} \tag{C.17}$$

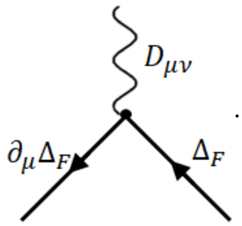
Also reading the result gives the connected diagram with sources η, η^* and J^μ. Ignoring all the tadpole-like diagrams, the only contribution to the scattering is

e  .

(C.18)

Similarly, the only effective diagram for $\mathcal{S}_{\text{int}}^{(3)}$ is

$- e$



. (C.19)

Some of the interactions that connect A_μ to Φ and $\Phi^\dagger$ have derivatives in them, which will be shown to extract momentum factors in the Feynman rules. Looking back at the quantized fields of Φ and $\Phi^\dagger$, one can notice which momentum factors I get.

$$\Phi(x) = \int \frac{d^4p}{(2\pi)^3 2p^0} e^{ip\cdot x} a(\boldsymbol{p}) + e^{-ip\cdot x} b^\dagger(\boldsymbol{p}), \tag{C.20}$$

$$\Phi^\dagger(x) = \int \frac{d^4p}{(2\pi)^3 2p^0} e^{-ip\cdot x} a^\dagger(\boldsymbol{p}) + e^{ip\cdot x} b(\boldsymbol{p}). \tag{C.21}$$

Φ in the $\mathcal{L}_{\text{int}}$ creates an anti-particle or annihilates a particle at space-time x. $\Phi^\dagger$ then implies a particle creation or an anti-particle annihilation. When a space-time derivative ∂_μ acts on the above fields, a factor of $\pm ip_\mu$ is pulled down to give a Feynman rule of a vertex. To speculate the exact meaning of $\partial_\mu \Delta_F$, consider the LSZ formula below:

$$\begin{aligned} {}_{\text{out}}\left\langle p' | p \right\rangle_{\text{in}} = \left(\frac{i}{\sqrt{Z}}\right)^2 \int d^4x'\, d^4x\, e^{i(p\cdot x - p'\cdot x')} \\ \left(-\partial'^2 + m^2\right)\left(-\partial^2 + m^2\right) \langle 0 | T\left(\Phi(x')\Phi^\dagger(x)\right) | 0 \rangle . \end{aligned} \tag{C.22}$$

In the lowest order of e, setting $Z \sim 1$, use the relations below:

$$\langle 0 | T\left(\Phi(x')\Phi^\dagger(x)\right) | 0 \rangle = \frac{\delta^2 Z[\eta, \eta^*]}{\delta\eta^*(x')\delta\eta(x)} \tag{C.23}$$

$$Z[\eta, \eta^*] = \int d^4z\, d^4z_1\, d^4z_2 \left[\partial_\mu^{(z)} \Delta_F(z - z_1)\right] \eta(z_1) \Delta_F(z - z_2)\, \eta^*(z_2). \tag{C.24}$$

Then the result illustrates well putting the momentum $-ip_\mu$ out in the Feynman rule desired. Using $\left(\partial_\mu^{(z)} + \partial_\mu^{(x)}\right) \Delta_F(z-x) = 0$ and integrating by parts leads to

$$
\begin{aligned}
{}_{\text{out}}\langle p'|p\rangle_{\text{in}} &= -\int d^4x'\, d^4x\, d^4z \left[\partial_\mu^{(z)} \Delta_F(z-x)\right] \Delta_F\left(z-x'\right) && \text{(C.25)}\\
&= -ip_\mu \int d^4x'\, d^4x\, d^4z\, e^{i(p\cdot x - p'\cdot x')} \Delta_F(z-x)\, \Delta_F\left(z-x'\right). && \text{(C.26)}
\end{aligned}
$$

Thus $\partial_\mu \Delta_F$ of (C.18) gives $-ip_\mu$, and that of (C.19) gives ip'_μ. Rearranging all these factors, now I can read off the final rules in the energy-momentum space (other rules are the exact same as the spinor QED):

- for each internal particle line,

$$\frac{1}{i}\Delta_F(p) = \frac{1}{i}\frac{1}{p^2+m^2-i\epsilon}.$$

- for each external particle line,

$$1.$$

- for each internal photon line,

$$\frac{1}{i}D_{\mu\nu}(k) = \frac{1}{i}\frac{1}{k^2-i\epsilon}\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right].$$

- for each external photon line,

$$e_\mu(k,\lambda).$$

- for each 3-vertex (2 particles - 1 photon),

$$-ie(p+p')_\mu.$$

- for each 4-vertex (2 particles - 2 photons),

$$-2ie^2\eta_{\mu\nu}.$$

Now I can read off the diagrams to get the scattering amplitude $\mathcal{T}$ of $\Phi(p_1)\Phi^\dagger(p_2) \to \gamma(k_1, \epsilon_1)\gamma(k_2, \epsilon_2)$. Using the derived rules above, the amplitude is given as

$$\begin{aligned}\mathcal{T} = e^2 \Bigg[& \frac{4\left\{p_1 \cdot \epsilon_1(k_1, \lambda_1)\right\}\left\{p_2 \cdot \epsilon_2(k_2, \lambda_2)\right\}}{(p_1 - k_1)^2 + m^2} \\ & + \frac{4\left\{p_1 \cdot \epsilon_1(k_2, \lambda_2)\right\}\left\{p_2 \cdot \epsilon_2(k_1, \lambda_1)\right\}}{(p_1 - k_2)^2 + m^2} + 2\epsilon_1(k_1, \lambda_1) \cdot \epsilon_2(k_2, \lambda_2)\Bigg]. \quad \text{(C.27)}\end{aligned}$$

In case of pair creation $2\gamma \to \Phi\Phi^\dagger$, the amplitude can be obtained by reading off the diagram in the opposite direction of time.

D Calculating Self-Force and Consistency with Energy-Derived Mass

Consider the radius vector of a sphere $\boldsymbol{r}$. When the vector $\boldsymbol{r}$ is affected by the anisotropic gravitation $\boldsymbol{g}$, the average proper velocity of light also affected to become

$$c_{\text{avg}} = c\left(1 + \frac{\boldsymbol{g}\cdot\boldsymbol{r}}{2c^2}\right). \tag{D.1}$$

For small gravity $\boldsymbol{g}\cdot\boldsymbol{r} \ll c^2$, since $r = ct$, the proper radius vector to reach a point around the sphere can be expressed as

$$r_{\text{avg}}^{-1} \simeq r^{-1}\left(1 - \frac{\boldsymbol{g}\cdot\boldsymbol{r}}{2c^2}\right). \tag{D.2}$$

From the straight forward application to situation of Figure 4.7, with the equivalence principle $\boldsymbol{a} = -\boldsymbol{g}$, the anisotropic volume element can be derived:

$$d\tau^{(a)} = d\tau_{\text{avg}}\Big|_{\boldsymbol{a}=-\boldsymbol{g}} = d\tau\left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{r}}{2c^2}\right). \tag{D.3}$$

Also I get the expression for the infinitesimal scalar potential considering $\boldsymbol{a} = -\boldsymbol{g}$

$$d\phi^{(a)} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, d\tau_{\text{avg}}}{r_{\text{avg}}}\Bigg|_{\boldsymbol{a}=-\boldsymbol{g}} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, d\tau}{r}\left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{r}}{2c^2}\right)^2, \tag{D.4}$$

and upto the leading order,

$$d\phi^{(a)} = \frac{1}{4\pi\epsilon_0}\frac{\sigma\, d\tau}{r}\left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{r}}{c^2}\right). \tag{D.5}$$

Thus the self-force that the electromagnetic field exerts upon the other volume charge element is

$$\begin{aligned}\boldsymbol{F}_{\text{self}} &= -\int \sigma \, d\tau_2^{(a)} \nabla \int d\phi_1^{(a)} \\ &= \frac{1}{4\pi\epsilon_0} \int\int \left(\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2\,|\boldsymbol{x}|^3}\boldsymbol{x} - \frac{\boldsymbol{a}}{c^2\,|\boldsymbol{x}|} \right) \left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{2c^2} \right) \sigma^2 \, d\tau_1 \, d\tau_2,\end{aligned}$$

where $\boldsymbol{x} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. Here I have derived (4.14). Similarly, one also can derive (4.13) as

$$E_{\text{self}} = \frac{1}{8\pi\epsilon_0} \int\int \frac{\sigma^2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} \, d\tau_1 \, d\tau_2.$$

Now let us shift our attention to the problem of $m_{\text{EM}}^{(F)} = m_{\text{EM}}^{(E)}$. The expression (4.14) can be written within the leading order of c^-2 as

$$\boldsymbol{F}_{\text{self}} = \frac{1}{8\pi\epsilon_0} \int\int \left(\frac{\boldsymbol{x}}{|\boldsymbol{x}|^3} + \frac{3}{2}\frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2\,|\boldsymbol{x}|^3}\boldsymbol{x} - \frac{\boldsymbol{a}}{c^2\,|\boldsymbol{x}|} \right) \sigma^2 \, d\tau_1 \, d\tau_2. \tag{D.6}$$

This system has the interchanging symmetry $\boldsymbol{x}_1 \leftrightarrow \boldsymbol{x}_2$, yielding $\boldsymbol{x} \to -\boldsymbol{x}$ and invariant $\boldsymbol{F}_{\text{self}}$. Exchange-odd term thus has no contribution when integrated over the sphere as

$$\boldsymbol{F}_{\text{self}} = \frac{1}{8\pi\epsilon_0} \int\int \left(\frac{3}{2}\frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2\,|\boldsymbol{x}|^3}\boldsymbol{x} - \frac{\boldsymbol{a}}{c^2\,|\boldsymbol{x}|} \right) \sigma^2 \, d\tau_1 \, d\tau_2. \tag{D.7}$$

Using the decomposition $\boldsymbol{x} = \boldsymbol{x}_\parallel + \boldsymbol{x}_\perp$ such that $\boldsymbol{x}_\perp \perp \boldsymbol{a}$, then, in the second term, $(\boldsymbol{a}\cdot\boldsymbol{x})\,\boldsymbol{x} = \left(\boldsymbol{a}\cdot\boldsymbol{x}_\parallel\right)\boldsymbol{x}_\parallel + \left(\boldsymbol{a}\cdot\boldsymbol{x}_\parallel\right)\boldsymbol{x}_\perp$. Integrating over the sphere, the rotated contribution that the vector $\boldsymbol{x}$ is rotated 180° about $\boldsymbol{a}$ perfectly cancels out to leave only the parallel term as

$$\boldsymbol{F}_{\text{self}} = \frac{1}{8\pi\epsilon_0} \int\int \left(\frac{3}{2}\frac{\boldsymbol{a}\cdot\boldsymbol{x}_\parallel}{c^2\,|\boldsymbol{x}|^3}\boldsymbol{x}_\parallel - \frac{\boldsymbol{a}}{c^2\,|\boldsymbol{x}|} \right) \sigma^2 \, d\tau_1 \, d\tau_2. \tag{D.8}$$

Since the magnitude of $\left(\boldsymbol{a}\cdot\boldsymbol{x}_\parallel\right)\boldsymbol{x}_\parallel$ is $\left|\left(\boldsymbol{a}\cdot\boldsymbol{x}_\parallel\right)\boldsymbol{x}_\parallel\right| = ax^2\cos^2\theta$ where θ is the intersection angle between $\boldsymbol{a}$ and $\boldsymbol{x}$, one can write $\left(\boldsymbol{a}\cdot\boldsymbol{x}_\parallel\right)\boldsymbol{x}_\parallel = \boldsymbol{a}\,(\hat{\boldsymbol{e}}_{\boldsymbol{a}}\cdot\boldsymbol{x})^2$.

Then the first term can be rewritten as ($\hat{\boldsymbol{e}}_{\boldsymbol{x}} = \boldsymbol{x}/|\boldsymbol{x}|$ and $\hat{\boldsymbol{e}}_{\boldsymbol{a}} = \boldsymbol{a}/|\boldsymbol{a}|$)

$$\int\!\!\int \frac{\boldsymbol{a} \cdot \boldsymbol{x}_\|}{|\boldsymbol{x}|^3} \boldsymbol{x}_\| \, d\tau_1 \, d\tau_2 = \int\!\!\int \frac{(\hat{\boldsymbol{e}}_{\boldsymbol{x}} \cdot \hat{\boldsymbol{e}}_{\boldsymbol{a}})^2}{x} \boldsymbol{a} \, d\tau_1 \, d\tau_2. \tag{D.9}$$

For the spherical charge distribution, all directions are indistinguishable. Therefore one can extract the average directional integral out from (D.9) as

$$\int\!\!\int \frac{(\hat{\boldsymbol{e}}_{\boldsymbol{x}} \cdot \hat{\boldsymbol{e}}_{\boldsymbol{a}})^2}{x} \boldsymbol{a} \, d\tau_1 \, d\tau_2 = \frac{1}{4\pi} \int\!\!\int \frac{\boldsymbol{a} d\tau_1 \, d\tau_2}{x} \int (\hat{\boldsymbol{e}}_{\boldsymbol{x}} \cdot \hat{\boldsymbol{e}}_{\boldsymbol{a}})^2 \, d\Omega \tag{D.10}$$

$$= \frac{1}{4\pi} \int\!\!\int \frac{\boldsymbol{a} d\tau_1 \, d\tau_2}{x} \int \cos^2 \theta \, d\Omega \tag{D.11}$$

$$= \frac{1}{3} \int\!\!\int \frac{\boldsymbol{a} d\tau_1 \, d\tau_2}{x}. \tag{D.12}$$

Thus I get the consistent result:

$$\boldsymbol{F}_{\text{self}} = -\frac{\boldsymbol{a}}{8\pi\epsilon_0 c^2} \int\!\!\int \frac{\sigma^2}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|} \, d\tau_1 \, d\tau_2 = -\frac{E_{\text{self}}}{c^2} \boldsymbol{a}. \tag{D.13}$$

E Field-Theoretical Origin of Mass

The operator form of the relativistic dispersion relation leads to the dynamics of field, yielding the form of the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu \tag{E.1}$$

for a massive (abelian) gauge boson A_μ, and

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \tag{E.2}$$

for a massive fermion ψ. For more generality, let us consider the non-abelian case:

$$F^\alpha_{\mu\nu} = \frac{1}{2}\partial_{[\mu}A^\alpha_{\nu]} - gf^{\alpha\beta\gamma}A^\beta_\mu A^\gamma_\nu. \tag{E.3}$$

A vector boson A^α_μ requires the form of the mass term to be $-m_A^2 \mathrm{tr} A_\mu A^\mu$ to satisfy the Klein-Gordon equation. However, if one attempts to impose the gauge invariance to the mass term with respect to the gauge transformation of

$$A_\mu \to GA_\mu G^{-1} - \frac{i}{g}\left(\partial_\mu G\right)G^{-1}. \tag{E.4}$$

(where $G \in \mathcal{G}$ is an element of gauge group($\mathcal{G}$) and g is the coupling constant), the mass term then is not invariant. This violation requires a gauge boson to be massless. In other words, the concept of mass confronted an essential skepticism about its ontological status. The basic perspective that inertiality appears as field interactions was formulated by Higgs and explained by the idea called the Higgs mechanism (Higgs 1964). This was actually verified when the Higgs boson was later discovered in 2012 (Aad et al. 2012). That is to say, the concept of mass was fabricated by the Higgs-field interaction for the sake of the symmetry of the nature. It is obvious that imposing gauge symmetry has been an code of today's human intelligence, since laws of physics have evolved following the trend of satisfying gauge in(co)variance, which contains the universal nature of physics.

In the same manner, the Standard Model, which still has remarkable power to explain our nature, describes mass as an interaction with the Higgs field:

$$\begin{aligned}
\mathcal{L}_{\mathrm{SM}} = \; & \underbrace{-\frac{1}{4}W^{\alpha}_{\mu\nu}W^{\alpha\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{\substack{\text{kinetic terms and self-interactions of} \\ U(1)\otimes SU(2) \text{ gauge bosons}}} \\
& + \underbrace{G_{\mathrm{Yuk}}\bar{\psi}_L\phi\psi_R + G'_{\mathrm{Yuk}}\bar{\psi}_L\tilde{\phi}\psi_R + h.c.}_{\text{Yukawa couplings with fermions and Higgs}} \\
& + \underbrace{i\bar{\psi}_L\gamma^\mu\left(\partial_\mu + i\frac{g}{2}\sigma\cdot W_\mu + i\frac{g'}{2}Y_L B_\mu\right)\psi_L}_{\text{kinetic terms and electroweak ...}} \\
& \underbrace{+i\bar{\psi}_R\gamma^\mu\left(\partial_\mu + i\frac{g'}{2}Y_R B_\mu\right)\psi_R}_{\text{... interactions of quarks and leptons}} \\
& + \underbrace{\left|\left(i\partial_\mu - \frac{g}{2}\sigma\cdot W_\mu - \frac{g'}{2}YB_\mu\right)\phi\right|^2 - V(\phi)\,.}_{\substack{\text{gauge bosons } W^\pm,\ Z,\ \gamma \text{ and Higgs couplings} \\ \text{with the potential of } V(\phi)=-\mu^2\phi^\dagger\phi+\lambda\left(\phi^\dagger\phi\right)^2}}
\end{aligned} \tag{E.5}$$

The exact meaning of each term will be discussed later. The purpose of this section is to examine how the concept of mass as an extent to the inertia appears in the interaction field theoretical view.

Consider the simplest example, an abelian gauge boson:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left|(\partial_\mu - igA_\mu)\phi\right|^2 + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2, \tag{E.6}$$

where $g(=e)$ is a coupling constant. It is nothing but the scalar QED I have examined. Under the gauge transformation of

$$A_\mu \to A_\mu - \partial_\mu \Lambda, \quad \phi \to e^{ig\Lambda}\phi \tag{E.7}$$

our Lagrangian is invariant. But one can easily show that if $\mu^2 > 0$, the theory has non-zero vacuum, generating the expectation value of $\langle\phi\rangle = \mu/\sqrt{2\lambda}$. Spontaneously $U(1)$ symmetry will be broken to require the vacuum refinement. Since the original ϕ was a complex scalar, having two degrees of freedom, one can replace it with two real scalars: the direction of argument χ and the radial direction near the true vacuum ϕ_H

$$\phi = \left[\langle\phi\rangle + \frac{\phi_H}{\sqrt{2}}\right] e^{\frac{i\chi}{\sqrt{2}\langle\phi\rangle}}. \tag{E.8}$$

Changing variables gives our refined Lagrangian,

$$\mathcal{L} = \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g^2\mu^2}{4}A_\mu A^\mu}_{\text{kinetic term of massive bosons}} + \underbrace{\frac{1}{2}\partial_\mu\chi\partial^\mu\chi}_{\text{massless Goldstone}} \tag{E.9}$$

$$+ \underbrace{\frac{1}{2}\left(\partial_\mu\phi_H\partial^\mu\phi_H - 2\mu^2\phi_H^2\right)}_{\text{Higgs with mass } \sqrt{2}\mu} + \text{(interactions)}. \tag{E.10}$$

Thus the gauge boson A_μ has acquired the mass $g\mu/\sqrt{2}$. After the symmetry has been broken, the gauge boson A_μ has gained a degree of freedom due to its massiveness and the scalar field ϕ has lost a degree of freedom to grant a mass to the field A_μ, preserving the total four degrees of freedom.

Now, consider a gauge theory with $U(1) \otimes SU(2)$ electro-weak interactions, the second and third lines in (E.5). Here, the left and right chiral projections of fermions are

$$\psi_L = \frac{1-\gamma^5}{2}\psi, \quad \psi_R = \frac{1+\gamma^5}{2}\psi. \tag{E.11}$$

The left-handed quarks and leptons ψ_L are chiral doublets, and the right-handed quarks and leptons ψ_R are chiral singlets. Each doublets and singlets transforms as

$$\psi_L \to \psi'_L = e^{iY_L\theta} e^{i\sigma^i\eta^i/2}\psi_L, \quad \psi_R \to \psi'_R = e^{iY_R\theta}\psi_R \tag{E.12}$$

where $\sigma^i/2$ denote the generators of $SU(2)_L$ Lie algebra whose structure constants are levi-civita $f^{\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}$ in (E.3). The transformation of W_μ and B_μ are determined by the above gauge transformation.

$$\sigma\cdot W_\mu \to \sigma\cdot W'_\mu = e^{i\sigma\cdot\eta/2}\sigma\cdot W_\mu e^{-i\sigma\cdot\eta/2} + \frac{1}{g}\left(\partial_\mu e^{i\sigma\cdot\eta(x)/2}\right)e^{-i\sigma\cdot\eta/2} \tag{E.13}$$

$$B_\mu \to B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu\theta(x). \tag{E.14}$$

The four gauge parameters η^i and θ contain the $SU(2)$ gauge bosons W^i coupling with weak-isospin $T = \sigma/2$ and a $U(1)$ gauge boson B coupling with hypercharge Y. A given fermion's electric charge Q is determined by adding its weak-isospin T^3 and hypercharge Y, which have fixed values of $Q = T^3 + Y/2$. Since for ψ_L the eigenvalue of T^3 is $\pm1/2$ and for ψ_R is 0, the lepton's eigenvalues are fully determined as $Y_L^{\text{leptons}} = -1$, $Y_R^{\text{leptons}} = -2$, $Y_L^{\text{quarks}} = 1/3$, $Y_R^{(\text{u})} = 4/3$, $Y_R^{(\text{d})} = -2/3$.

The gauge kinetic term is the two terms in the first line in (E.5), where

$$W^\alpha_{\mu\nu} = \frac{1}{2}\partial_{[\mu}W^\alpha_{\nu]} - g\epsilon^{\alpha\beta\gamma}W^\beta_\mu W^\gamma_\nu, \;\; B_{\mu\nu} = \frac{1}{2}\partial_{[\mu}B_{\nu]}. \tag{E.15}$$

The last two terms in (E.5) are the Higgs terms, where ϕ is a complex scalar field. In $SU(2)_L$ spinor representation, $\phi = (\phi^+, \phi^0)^T$ has the $U(1)$ weak hypercharge $Y_\phi = 1$. The $U(1)_Y$ symmetry is henceforth required to be a system with massless gauge boson. In order for the theory to be renormalizable and $SU(2)_L \otimes U(1)_Y$ invariant, the Higgs potential has to be the same potential form as (E.6). Like the abelian example above, if $\mu^2 > 0$, our theory has non-zero vacuum, leading to a spontaneous symmetry breaking. Since there are an infinitely degenerate vacuum with $\phi^\dagger\phi = \langle\phi\rangle^2/2 = -\lambda^2/2\mu$, one can take $(\phi^+, \phi^0) = \left(0, \left(\langle\phi\rangle + \phi_H\right)/\sqrt{2}\right)$ by specifying the direction. Conservation of charge leads ϕ^0 to be interpreted as an electrically neutral and the total charge to become neutral, meaning the $U(1)_Q$ symmetry stays unbroken. In the unitary gauge, the Goldstone boson does not exist. Thus the only physical one is Higgs after symmetry breaking. Calculating kinetic terms of Higgs in (E.5) gives (after lengthy algebra)

$$
\begin{aligned}
&\phi^\dagger \left(\frac{g}{2}\sigma\cdot W_\mu + \frac{g'}{2}YB_\mu\right)^\dagger \left(\frac{g}{2}\sigma\cdot W_\mu + \frac{g'}{2}YB_\mu\right)\phi \\
=&\frac{1}{8}\left|\begin{pmatrix} gW^3_\mu + g'B_\mu & g\left(W^1 - iW^2\right)_\mu \\ g\left(W^1 + iW^2\right)_\mu & gW^3_\mu + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ \langle\phi\rangle \end{pmatrix}\right|^2 \\
=&\underbrace{\frac{g^2\langle\phi\rangle^2}{8}\left\{\left(W^1_\mu\right)^2 + \left(W^2_\mu\right)^2\right\}}_{:=\left(g\langle\phi\rangle/2\right)^2 W^+_\mu W^-_\mu/2} + \underbrace{\frac{\langle\phi\rangle^2}{8}\left(gW_\mu - g'B_\mu\right)^2}_{:=\left(\langle\phi\rangle\sqrt{g^2+g'^2}/2\right)^2 Z_\mu Z^\mu/2}
\end{aligned}
\tag{E.16}
$$

where I have only considered the contribution for gauge boson masses. For the charged boson $W^\pm_\mu$ and neutral boson Z_μ and A_μ, they are defined as

$$
W^\pm_\mu := \frac{1}{\sqrt{2}}\left(W^1 \mp W^2\right)_\mu \quad \text{with mass} \quad m_W = \frac{g\langle\phi\rangle}{2} \tag{E.17}
$$

$$
Z_\mu := \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2 + g'^2}} \quad \text{with mass} \quad m_Z = \frac{\langle\phi\rangle}{2}\sqrt{g^2 + g'^2} \tag{E.18}
$$

$$
A_\mu := \frac{g'W^3_\mu + gB_\mu}{\sqrt{g^2 + g'^2}} \quad \text{with mass} \quad m_A = 0. \tag{E.19}
$$

Counting the degrees of freedom gives $4(\phi) + 2(B_\mu) + 6(W^\alpha_\mu) = 12$ before spontaneous symmetry breaking, and $1(\phi_H) + 3(Z_\mu) + 6(W^\pm_\mu) + 2(A_\mu) = 12$.

Fermion masses comes from Yukawa coupling terms in the second line of (E.5). For each quark and lepton, writing down all the Yukawa terms including all spinor indices reads

$$\mathcal{L}_{\text{Yuk}} = G^{(\text{q,d})}_{\alpha\beta}\bar{\psi}^{(\text{q})}_{\alpha,L}\phi\psi^{(\text{d})}_{\beta,R} + G^{(\text{q,u})}_{\alpha\beta}\bar{\psi}^{(\text{q})}_{\alpha,L}\tilde{\phi}\psi^{(\text{u})}_{\beta,R} + G^{(\text{l,e})}_{\alpha\beta}\bar{\psi}^{(\text{l})}_{\alpha,L}\phi\psi^{(\text{e})}_{\beta,R} + G^{(\text{l},\nu)}_{\alpha\beta}\bar{\psi}^{(\text{l})}_{\alpha,L}\tilde{\phi}\psi^{(\nu)}_{\beta,R} + h.c. \tag{E.20}$$

where $\tilde{\phi}_\alpha = \epsilon_{\alpha\beta}\phi^*_\beta$. Definitely, Yukawa terms have no gauge symmetry violator since they consist of $SU(2)_L$ singlet. Mass terms are required to be a trivial hypercharge. The up quarks $\psi^{(\text{u})}$ and electrons $\psi^{(\text{e})}$ can have mass with proper mix up of ϕ and $\tilde{\phi}$ due to the eigenvalue $Y_\phi = 1/2$ and $Y_{\tilde{\phi}} = -1/2$. But, in case of neutrino $\psi^{(\nu)}$, there is no right-handed partner in the Standard Model. Neutrinos, thus, were expected to be massless[1] (Cottingham and Greenwood 2007). Under $U \in SU(2)$, each representation transforms[2] as

$$\phi \to \phi' = U\phi, \quad \tilde{\phi} \to \tilde{\phi}' = U\tilde{\phi}. \tag{E.21}$$

The same procedure, putting $\phi = \left(0, \langle\phi\rangle + \phi_H\right)^T/\sqrt{2}$, gives

$$\begin{aligned}\mathcal{L}_{\text{Yuk}} &= G^{(\text{q,d})}\bar{\psi}^{(\text{q})}_L\phi\psi^{(\text{d})}_R + G^{(\text{q,u})}\bar{\psi}^{(\text{q})}_L\tilde{\phi}\psi^{(\text{u})}_R + G^{(\text{l,e})}\bar{\psi}^{(\text{l})}_L\phi\psi^{(\text{e})}_R + h.c.\\ &= \frac{G^{(\text{d})}\langle\phi\rangle}{\sqrt{2}}\left(\bar{\psi}^{(\text{d})}_L\psi^{(\text{d})}_R + \bar{\psi}^{(\text{d})}_R\psi^{(\text{d})}_L\right) + \frac{G^{(\text{u})}\langle\phi\rangle}{\sqrt{2}}\left(\bar{\psi}^{(\text{u})}_L\psi^{(\text{u})}_R + \bar{\psi}^{(\text{u})}_R\psi^{(\text{u})}_L\right)\\ &\quad + \frac{G^{(\text{e})}\langle\phi\rangle}{\sqrt{2}}\left(\bar{\psi}^{(\text{e})}_L\psi^{(\text{e})}_R + \bar{\psi}^{(\text{e})}_R\psi^{(\text{e})}_L\right) + h.c.\end{aligned} \tag{E.22}$$

Comparing with the fermionic mass term in (E.2), masses for the fermions are determined as

$$m_{\text{d}} = \frac{G^{(\text{d})}\langle\phi\rangle}{\sqrt{2}}, \quad m_{\text{u}} = \frac{G^{(\text{u})}\langle\phi\rangle}{\sqrt{2}}, \quad \text{and} \quad m_{\text{e}} = \frac{G^{(\text{e})}\langle\phi\rangle}{\sqrt{2}}. \tag{E.23}$$

[1] However, neutrinos should have nonzero masses in order to exhibit the empirically proven phenomena of neutrino oscillation, which combines neutrino flavour states with neutrino mass states. For more information, see Schechter and Valle (1980) and Grossman and Lipkin (1997).

[2] $\tilde{\phi}'_\alpha = \epsilon_{\alpha\beta}\phi'^*_j = \epsilon_{\alpha\beta}U^*_{\beta\gamma}\phi^*_\gamma = (U^\dagger)_{\gamma\beta}\epsilon_{\alpha\beta}\phi^*_\gamma = U_{\alpha\beta}\epsilon_{\beta\gamma}\phi^*_\gamma$ since $\epsilon_{\alpha\beta}(U^\dagger)_{\gamma\alpha}(U^\dagger)_{\delta\beta} = \epsilon_{\gamma\delta}\det(U^\dagger)$

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초 록

질량-에너지 등가성은 현대물리학의 기저로서 현대 물리 교육이 강화되고 있는 현재의 추세를 고려했을 때 중대한 교육적 가치를 지닌다. 이러한 교육적 가치에도 불구하고, 질량-에너지 등가를 해석하는 데에 있어서 여전히 비일관된 진술들이 만연하며, 여러 연구에서 등장한 유도과정은 여전히 일부는 논리적 비약을 포함한 채 그 논리적 연결고리를 호도하고 있다. 현대물리학 교재에서도 마찬가지로 mc^2 항을 정지 에너지라 정의하며 선언적 지식(declarative knowledge)으로 제시하고 있으며 예시를 통해 활용하는 수준에서 소개한다.

본 연구에서는 질량-에너지 등가에 관한 기존 물리 교재의 설명과 여러 연구의 유도과정이 가지는 문제점들을 해결하기 위하여, 질량-에너지 등가에 관한 물리학 및 물리교육학 분야의 주요 논문들을 수집하여 분석하였다. 또한 질량과 에너지가 장과 입자 중 어떤 물리적 대상에 귀속된 것인지를 명시함으로써 질량-에너지 등가성을 분류하는 기준을 마련하였고, 그 결과 해당 기준에 따라 질량-에너지 등가성을 이해하는 세 가지의 유형이 존재함을 보였다: 상응성에 대한 추측, 전환 가능성, 상호구분 불가능성. 또한 분류된 유형들의 논리적 위계성을 발굴함으로써 질량-에너지 등가성을 총체적으로 이해하는 새로운 유형의 이해 가능성을 제안하였다. 네 번째 유형은 에너지와 질량 두 개념 사이의 등가성이 발전하게 된 과정을 함축할 뿐만 아니라 물질 이론으로서 장과 입자의 두 관점이 서로 밀접하게 연관되며 개념적으로 동일시되어 온 과정을 명시적으로 드러내었다.

또한 위 분류 방식이 물리학에서 등장하는 다른 등가성에서도 보편적으로 적용되는지를 확인하기 위해 먼저 질량-에너지 등가성의 각 유형의 의미 요소를 추출하여 다음의 등가 개념들에 적용해 보았다: 열-역학적 일 등가 (열역학 제 1 법칙), 등가 원리 (일반상대론), AdS/CFT 대응성 (양자 중력이론), 행렬역학과 파동역학의 동등성 (양자역학). 그 결과 질량-에너지 등가성과 마찬가지로 다른 물리학에서의 등가성에도 어느 정도 유사한 분류가 가능했다. 또한 개념 발전상의 유사성도 함께 발견됨에 따라 본 연구에서 제시한 분류 기준들이 임의적이지 않으며 어느 정도 보편성을 지님을 확인할 수 있었다.

본 연구의 결과는 질량과 에너지 사이의 개념적 관계를 넘어 장과 입자 사이의 개념적 관계까지 조망함으로써, 자연스럽게 물리학 내용 지식의 총체적 연결성과 내적 가치, 역사성 등을 함께 보여주었다. 이러한 역사적 과정과 탐구 맥락은 물리학의 본질을 잘 보여주는 물리학의 실천전통의 좋은 예시가 될 수 있을 것이다. 향후 본 연구의 결과는 기존 물리교재의 분석과 새로운 교육과정 및 교재개발을 위한 개념틀이나 이론틀로서 중요한 역학을 수행할 수 있을 것으로 기대된다.

주요어: 질량-에너지 등가성, 입자, 장, 분류,
총체적 이해, 개념 발전, 양자장론

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감사의 글

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예컨대, 저는 이 석사논문을 작성하면서도 많은 난점을 마주하였습니다. 본 연구의 결론 중 하나는 실체성을 지닌 두 가지 상반되는 관점이 융화되는 과정으로서 질량-에너지 등가성이 그 시발점이 되었다는 것입니다. 장과 입자의 인식론적 이질성(heterogeneity)은 마치 그들이 존재론적 통합이 이루는 것이 불가능해 보이게 합니다. 석사 첫 학기에 양자장론을 수강하며, 혹은 그 이전에 이론물리학을 공부하면서도 늘 불만족스러웠던 부분은 이 두 가지 개념의 통합에 대한

정당성입니다. 다시 말해 조금 난해한 용어로 포장하자면 “양자장론의 존재론적 문제” 정도가 되겠습니다.

2022년 하반기 무렵, 우리 연구실 세미나 시간에 김홍빈 박사님께서 양자역학의 본질을 꿰뚫는 질문을 던지신 것이 기억납니다: “조화진동자의 양자수 n이 도대체 무엇을 의미하는가?”, “과연 이산성(discreteness)이 양자역학을 특정하는 특징에 해당할 것인가?” 등 많은 해석적 문제에 대한 의문을 제기해 주셨다고 생각합니다. 양자장론의 존재론적 근원을 입자로 보는 해석에 따르면 모든 세상이 조화진동자의 연속체(continuum)이며, 양자수 n은 입자의 수로 해석이 됩니다. 그 근원을 입자로 보기에는 다양한 문제를 안고 있으며, 현재로서는 물질을 구성하는 일원론적 실체가 장이 아닐까 하는 입장에서 현재 연구가 이루어지고 있다는 것을 알게 되었습니다. 이산성은 익히 알고 있던 고전역학의 특성이기도 했으며, 양자역학을 특정하는 본질은 어쩌면 비-영점 진공(non-zero vacuum)에 있는 것이 아닐까 하는 합리적 대안도 논의되었습니다. 이러한 양자역학적 유산(legacy)은 양자장론에 와서는 수많은 무한 물리량으로 나타났으며, 재규격화(renormalization)의 필요성에 따라 오늘날 우리는 유효장론(effective field theory)의 시대에 살고 있습니다. 이러한 논의들은 결코 견고하며 완성된 형태로서 만족을 주지 않습니다. 결론은 “아무도 모른다”였습니다. 지적 호기심을 좇아 짧게는 몇 달이고 길게는 몇 년 동안 열심히 연구 결과를 추적해온 결과가 인류 지성의 무지라니, 배우는 입장에서 이런 비극이 어디 있겠습니까?

배우는 과정도 한 편으로 즐거움도 있었지만, 마냥 즐거웠던 것은 아니었던 것 같습니다. 대부분의 사람들은 배움을 인간다운 삶을 영위하기 위한 필요조건 정도로 여깁니다. 오늘날의 상실된 학습의 내적 가치는 어쩌면 도구적 이성의 비극일지도 모르겠습니다. 그러나, 한 편으로 진리를 추구한다는 것은 매우 배고픈 일일지도 모른다는 생각이 듭니다. 하고 싶은 일을 업으로 삼는다는 것은 정신적으로 만족적이지만 육신의 불만족을 불러일으킵니다. 육신의 불만족은,

그러니까 생활고에 시달리고, 일과 학업을 병행하며 힘든 시간을 보내면서, 결국 정신적 만족또한 소실시키는 것 같습니다. 지성의 패배가 낳은 반지성주의와 도구적 이성이 만연하는 오늘날 인류의 비극적 모습을 고려한다면 아주 당연한 모습이 아닐까요? 배우는 과정이 만약 영혼을 살찌우는 일이라면, 살찐 영혼은 어찌 염세와 인간 불신을 낳게 되는지 모르겠습니다.

그럼에도 저에게는 물리학과 물리교육을 공부해온 과정이 생각하는 힘을 길러주었다고 생각합니다. 생각하는 힘을 운동에 비유하자면, 뇌에 근육이 붙는 것과 같습니다. 아마 머리로는 3대 500을 치지 않을까 싶습니다. 특히나 비판적 사고에 대해서는 거의 불신에 가까울 정도로 검증을 거치는 습관을 기르게 되었습니다. 지금 여기서도 소크라테스의 격언을 비판적으로 검토하는 것만 보아도 그러하지요. 마찬가지로 아인슈타인도 스스로 수 많은 오류에 빠지고, 본인의 진술을 번복하기도 하며 생각하는 힘을 길러왔다고 생각합니다. 이러한 비판적 사고의 방향이 스스로에게 향할 때, 비로소 객관적으로 자아를 성찰하며 발전할 수 있는 계기가 되는 것 같습니다.

생각하는 힘은 곧 세상을 바라보는 심미적 안목이었습니다. 우리가 아름다운 것을 아름답다고 인식하는 그 자체를 위하여 생각하는 힘을 기르는 것이라 보아도 무방할 것입니다. 그것은 때때로 매우 중독적이어서 배움의 과정이 주는 육신의 불만족을 어느 정도 무디게 해주는 것 같습니다. 이것이 소크라테스가 말한 즐거움일까요.

어찌되었든 학습은 그저 추상적인 표현으로서의 앎에 그치지 않는다는 것과, 그것의 내재화가 가져다주는 안목의 형성까지임을, 그로써 한 인간의 인격적 도야까지 이루어낼 수 있다는 교수님의 가르침에 이제서야 어느 정도 공감할 수 있게 된 것 같습니다. 그러한 가르침을 이어 받아, 저는 박사과정 진학과 함께 세상을 바라보는 시각이 길러지지 못한 사람들에게 일종의 시력을 부여하는 일을 하게 됩니다.

그간 종잡을 수 없는 제 혈기를 감당해주시고, 자신감을 잃고 마음이 흔들릴 때마다 많은 조언과 격려를 해 주신 교수님과 박사님, 그리고 연구실 선생님들, 많은 편의를 봐주셨던 과사 선생님들께도 이 지면을 빌려 깊은 감사의 인사를 드립니다. 박사과정에서도 또 한번 잘 부탁드리겠습니다. 이토록 오랜 기간 공부를 하게 될 줄은 몰랐는데, 여의치 않은 상황임에도 긴 세월동안 물심양면으로 지원해주신 부모님께도 감사 인사를 올립니다. 어찌되었든 이걸로 한 인간으로서의 몫을 다할 수 있게 되지 않았나 생각합니다. 또 연구의 힘든 전 과정을 항상 마음 한 켠에서 지지해주고 지켜봐준 SKK에게도 마음 깊이 감사의 마음을 전합니다.