Master's Thesis of Economics

Welfare implications of downward nominal wage rigidity under currency pegs

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Abstract

This paper studies the welfare implications of downward nominal wage rigidity (DNWR) to the small open economy New Keynesian DSGE model with the fixed exchange rates. During the European crisis, so-called PIGS (Portugal, Italy, Greece, and Spain) countries suffered from economic hardship more than other countries. Following the crisis, policy institutions attributed economic hardships to nominal wage rigidity in conjunction with the fixed exchange rate. However, many academic papers such as Galí and Monacelli [2016] contend that the costs of nominal wage rigidity are small or even ambiguous. In this paper, I incorporate the DNWR constraint suggested by Schmitt-Grohé and Uribe [2016] into the otherwise standard new Keynesian model to study whether DNWR with fixed exchange rates has adverse aggregate effects on the economy. In line with Galí and Monacelli [2016], I find that DNWR has a non-monotonic impact on welfare loss depending on the extent of inflation stabilization and inefficient employment fluctuation. However, the welfare loss from involuntary unemployment dominates the gain from inflation stability as the degree of DNWR gets higher, which supports the conventional wisdom regarding the effectiveness of internal devaluation.

Keywords: Downward nominal wage rigidity, Small open economy, Currency union, Occasionally binding constraint

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1 Introduction

Virtues of wage moderation in a recession are widespread in policy circles. The crisis of the Euro area and subsequent struggles to recover reignite calls for structural reforms to the labor market. The ECB Monthly Bulletin, October 2012, stated:

> "... In addition, downward wage rigidities limit the necessary flexible response of wages to labour market conditions to foster employment creation. The rise in structural unemployment underlines the urgent need for continued and further comprehensive reforms to remove rigidities in the labour markets of euro area countries"

Conventional wisdom has emphasized the roles of external and internal devaluation as an important mechanism to absorb and cushion large negative shocks. The necessity of greater wage flexibility and internal devaluation is especially called for the peripheral European countries under a fixed exchange rate regime since the exchange rate cannot be adjusted (Decressin et al. [2015]). Figure 1 shows a graphical illustration of this idea. It shows the current account relative to GDP (%), nominal exchange rate, nominal labor cost index, and log employment of PIGS countries[1].

Before the Great Recession, these European countries experienced large capital inflows, increasing wages, and employment trended upwards. Large capital inflows accompanied increased demands for tradables and lowered the unemployment rate until the crisis. However, the onset of the crisis reversed this trend and led to sudden capital outflows, shifting down the aggregate demand. Under the currency union, the exchange rate had been fixed and could not cushion out all these shocks. Despite the absence of an external adjustment mechanism, nominal hourly wage measured by the labor cost index had been increasing and peaked in 2010. As a result, the labor market collapsed, and employment trended downward until the late 2013s. In spite of the high unemployment rate and large drop in employment, nominal hourly wages were even increasing or remained high with only a modest decrease, after which employment started to rebound from its trough. This episode seemingly corroborates policy institutions' viewpoint about the importance of internal devaluation as a shock absorption mechanism when external devaluation is not available.

In line with this viewpoint, several academic research suggests consistent results with the conventional viewpoints. Schmitt-Grohé and Uribe [2016] develops a model with downward nominal wage rigidity and finds that a sizable welfare improvement of capital control policy[2]. Ahn et al. [2022] empirically validate the theoretical mechanism suggested by Schmitt-Grohé and Uribe [2016] that high unemployment can be induced by the combination of fixed exchange rates and downward nominal wage rigidity during the oil prices collapse episodes (2014-2016). Schaefer and Singleton [2022] show the existence of downward nominal wage rigidity and demonstrate that this nominal rigidity can induce considerable long-run output losses.

However, there are also many academic papers that have called the effectiveness of internal devaluation into question. De Long and Summers [1985] concludes that increased wage and price flexibility can easily be destabilizing. Ball and Romer [1990] argues that welfare costs of fluctuations arising from nominal rigidity are small for plausible parameters using the static model, while Cho et al. [1997] shows that the welfare cost of nominal wage contract is quite small in a dynamic quantitative general equilibrium model. Bils and Chang [2003] analyzes the model with an effort choice and argues that welfare costs of wage stickiness are potentially much smaller. More recently,

[1] I apply the four-quarter moving average to these data (except exchange rates) from each county and take a weighted average over these countries with their population sizes For exchange rate, I take a simple arithmetic mean over these countries

[2] This is because the combination of downward nominal wage rigidity, fixed exchange rates, and free capital mobility creates a pecuniary externality

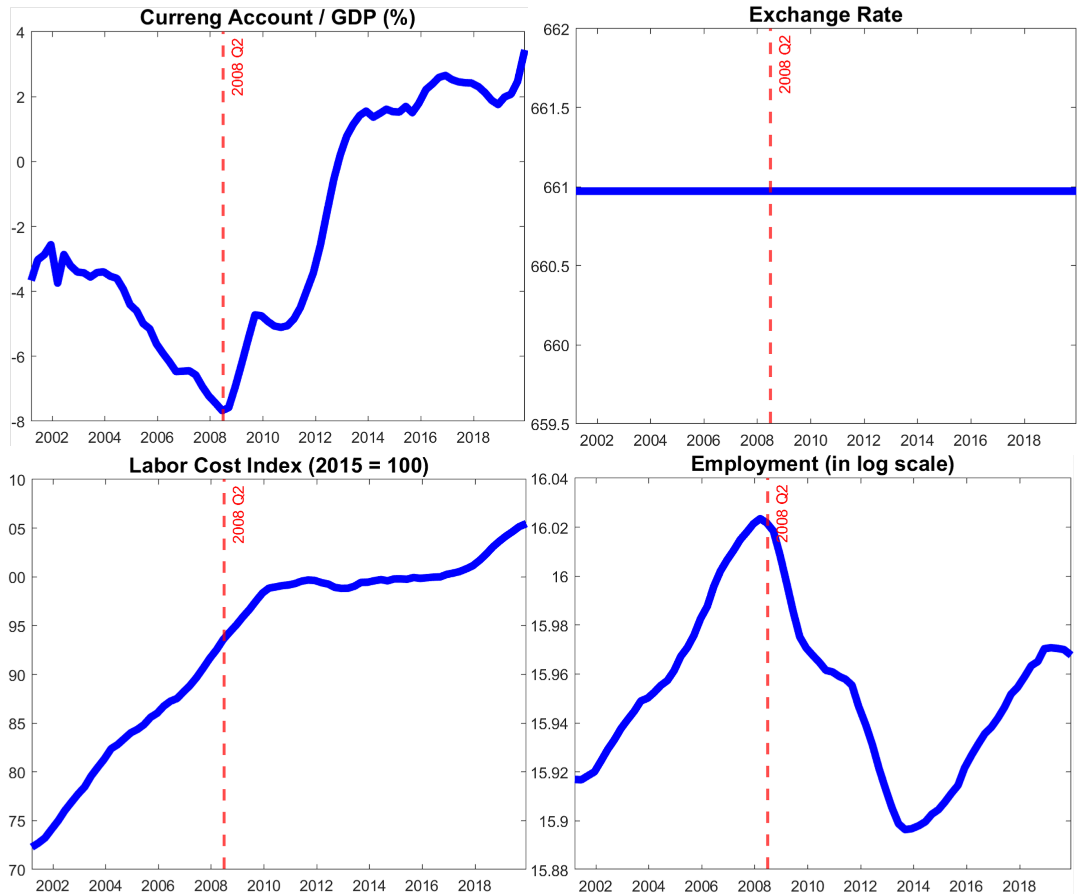

Figure 1: Boom burst cycle in Portugal, Italy, Greece, and Spain (2001-2019)

Notes. Figure 1 presents the boom burst cycle of PIGS countries. All data (2002-2019 series) comes from the Eurostat. Except for the exchange rate, the current account to GDP, nominal labor cost index, and employment are four-quarter averages. I take the weighted average of all data across the PIGS countries (Portugal, Italy, Greece, and Spain) using the population size. The vertical dotted line (red) displays the onset of the crisis. The nominal labor cost index covers the industry, services, and construction sectors. I measure employment as the aggregate hours of the economy in thousands.

Eggertsson and Krugman [2012] show that when the economy is faced with a large deleveraging shock, higher wage flexibility makes things even worse. They dub this the "paradox of flexibility." Galí and Monacelli [2016] study the standard small open economy new Keynesian model with a fixed exchange rate and show that greater wage flexibility may raise welfare losses. They demonstrate that welfare implications of wage stickiness crucially depend on policy responses, and benefits of greater wage flexibility from monetary policy responses are muted in a currency union. This is because, in the new Keynesian model, employment is determined by the demand side in contrast to the classical model, implying the welfare implications of wage rigidity are not robust to nominal price rigidity. However, these studies evaluate the welfare consequences of wage rigidity using Calvo [1983] and Erceg et al. [2000] types nominal wage stickiness[3]. On the other side, there is extensive micro evidence supporting the existence of downward nominal wage rigidity. This literature point out skewness in wage changes: wage cuts are rare, there are usually spikes at zero nominal wage changes rather than negative changes, and wages tend to be adjusted upwards (McLaughlin [1999], Lebow et al. [2003], and Kahn [1997]). Knoppik and Beissinger [2009] find

[3]Although Amano and Gnocchi [2020] study the new Keynesian model with the downward nominal wage rigidity, they focus on a closed economy with the zero lower bound constraints

empirical evidence of downward nominal wage rigidity among the twelve European countries using the European Community Household Panel (ECHP) dataset. Dickens et al. [2007], with access to individual workers' earning data in sixteen countries, find that an average of 28 percent of workers are covered by downward nominal wage rigidity across countries. Barattieri et al. [2014] finds downward nominal rigidity in the United States using the Survey of Income and Program Participation (SIPP).

This study aims to re-evaluate the effectiveness of lesser downward nominal wage rigidity during the recession and its implications on welfare losses. Specifically, I investigate how greater or lesser downward nominal wage rigidity (DNWR) affects fluctuations in output, employment, and prices while deriving welfare losses as a function of the degree of DNWR. I conduct a counterfactual experiment by recovering shock sequences and feeding them into the model without DNWR. In counterfactual analysis, without DNWR, employment and output would have been 2.56% and 1.92% higher on average respectively, and inflation would have been -8.6 basis points lower during the crisis and recovery periods (2008:2014). The average output loss of DNWR for two decades is about 1.15, which is consistent with long-run output loss (1.3 %) of Schaefer and Singleton [2022]. This result suggests output and employment gains from greater wage flexibility and supports the conventional wisdom. In the unconditional welfare evaluation, however, I find somewhat inconclusive results: DNWR has a non-monotonic impact on welfare loss. When the degree of DNWR (γ) takes a higher value than around 0.99, the welfare loss from involuntary unemployment dominates the gain from inflation stability. This result supports both Galí and Monacelli [2016] and Schmitt-Grohé and Uribe [2016]. Under the modest degree of DNWR (γ), the economy gains from higher γ and inflation stability (paradox of flexibility). However, as γ takes a higher value than around 0.99 (nominal wage can decrease up to 4% per year), the employment loss exponentially increases and outweighs the gains from inflation stability. Considering many peripheral European countries have degrees of DNWR around 0.99, this result implies that there are still potential welfare gains from lessened downward wage rigidity.

The rest of the paper is organized as follows. Section 2 outlines a simple model of a small open economy with both Calvo [1983] and Yun [1996] types of price stickiness and downward nominal wage rigidity suggested by Schmitt-Grohé and Uribe [2016]. Section 3 presents the baseline calibration and discusses the impacts of downward rigidity on output, employment, and welfare. Finally, Section 4 gives concluding remarks.

2 Model

In this section, I describe the model I study in the analysis of the welfare impacts of DNWR. The model is based on Schmitt-Grohé and Uribe [2016], incorporating staggered price setting introduced by Calvo [1983] and Yun [1996] and a domestic technology shock in the nontradables sector. As in Gali and Monacelli [2005] and other literature, the size of the home economy is negligible relative to that of the world economy. This assumption simplifies the analysis and makes it possible to treat the foreign variables as exogenous processes. In addition, it is assumed that the law of one price holds for tradable goods. The economy is driven by three exogenous shocks, a country-interest rate shock, a domestic technology shock, and a terms-of-trade shock.

2.1 Households

A continuum of identical households of unit measure populates the economy. Household preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{1}$$

where c_t is a consumption index and h_t is an amount of labor input. Parameter $\beta \in (0,1)$ is the subjective discount factor. The symbol $\mathbb{E}_t$ denotes the conditional expectations operator using the information available in period t. Period utility u takes the form:

$$u(c_t, h_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{(\bar{h} - h_t)^{1-\chi}}{1-\chi} \tag{2}$$

The consumption good (c_t) is a composite of tradable consumption, c_t^T and nontradable consumption, c_t^N. Specifically, the aggregation technology has the form:

$$c_t = [\nu (c_t^T)^{\frac{\xi-1}{\xi}} + (1-\nu)(c_t^N)^{\frac{\xi-1}{\xi}}]^{\frac{\xi}{\xi-1}} \tag{3}$$

where $\xi > 1$ is the elasticity of substitution between tradable and nontradable consumption goods. Note that when both goods are complementary, any factors increasing the demand for one of them also increase the demand for the other. Here, c_t^N is an index of nontradable consumption goods given by $c_t^N = (\int_0^1 c_t^N(i)^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}}$, where $i \in [0,1]$ denotes the good variety and $\epsilon > 1$ is the elasticity of substitution among differentiated nontradable consumption goods. The sequential budget constraint is given by:

$$P_t^T c_t^T + \int_0^1 P_t^N(i) c_t^N(i) di + E_t d_{t-1} \leq P_t^T y_t^T + W_t h_t + \Phi_t + \frac{E_t d_t}{R_t} \tag{4}$$

for $t = 0, 1, \cdots$, where P_t^T denotes the nominal price of tradables, $P_t^N(i)$ the nominal price of nontradables variety i, E_t the nominal exchange rate. y_t^T is the exogenous endowment of tradables, W_t is the nominal wage rate, h_t^s is hours worked, and Φ_t is the nominal profits from the ownership of firms. Households may not be able to work as much as they want because of DNWR constraint ($h_t \leq h_t^s$).

It is assumed that the law of one price holds for tradables, so that $P_t^T = E_t P_t^{T*}$ at each period t where P_t^{T*} is the foreign currency price of tradable goods. I further assume P_t^{T*} to be constant and normalized to unity for all t[4]. Unlike Gali and Monacelli [2005], Households have access to one-period state-noncontingent bonds (d_t) denominated in tradables which are traded internationally. Here, R_t is the gross interest rate between periods t and $t+1$. To induce stationarity in the small open economy, I assume the external debt-elastic interest rate (EDEIR) following Schmitt-Grohé and Uribe [2003]:

$$R_t = R_t^* + \psi[\exp(d_t - \bar{d}) - 1] \tag{5}$$

where $\bar{d}$ is the steady state debt level and R_t^* is the exogenously given world interest rate in conjunction with the country risk premium. Letting $p_t(i) = \frac{P_t^N(i)}{P_t^T}$ denotes the relative price of nontradable goods in terms of tradable goods and using the law of one price, one can rewrite budget constraints as:

$$c_t^T + \int_0^1 p_t(i) c_t^N(i) di + d_{t-1} \leq y_t^T + w_t h_t + \phi_t + \frac{d_t}{R_t} \tag{6}$$

[4]Hence, the nominal price of tradable goods in domestic currency is equal to the nominal exchange rate, $P_t^T = E_t$

where $w_t = \frac{W_t}{P_t^T}$ and $\phi_t = \frac{\Phi_t}{P_t^T}$. Cost-minimizing optimal allocation of any given expenditure on nontradable goods gives the expression for demand functions:

$$c_t^N(i) = (\frac{P_t^N(i)}{P_t^N})^{-\epsilon} c_t^N \tag{7}$$

for all $i \in [0,1]$ and $\int_0^1 P_t^N(i)c_t^N(i)di = P_t^N c_t^N$ where $P_t^N = (\int_0^1 P_t^N(i)^{1-\epsilon}di)^{\frac{1}{1-\epsilon}}$ is the nontradable price index. The optimality conditions associated with households maximization problem are:

$$(\frac{c_t^N}{c_t^T})^{-\frac{1}{\xi}} = p_t \tag{8}$$

$$\frac{\lambda_t}{R_t} = \beta E_t[\lambda_{t+1}] \tag{9}$$

$$c_t^T + p_t c_t^N + d_{t-1} = y_t^T + w_t h_t + \phi_t + \frac{d_t}{R_t} \tag{10}$$

along with aggregation technology (3) and optimal allocation between nontradable varieties (7). Here, the Lagrangian multiplier (λ_t) is given by $\lambda_t = c_t^{-\sigma}\nu(\frac{c_t}{c_t^T})^{\frac{1}{\xi}}$ and relative aggregate price index for nontradable goods (p_t) is defined by $p_t = \frac{P_t^N}{P_t^T}$. Households optimization condition (8) describes the intratemporal optimization condition between tradable and nontradable goods. The marginal rate of substitution (MRS) is equal to the relative price of nontradable goods in terms of tradable goods. Equation (9) describes a standard intertemporal optimization condition (Euler equation) given gross interest rate (R_t), and Equation (10) is the budget constraint denominated in tradable goods. Combining (8)-(10) yields the demand function for tradables and nontradables. It follows that increase in endowment y_t^T or decrease in interest rate R_t increase the demand for tradables (c_t^T).

2.2 Firms

A continuum of firms indexed by $i \in [0,1]$, are assumed to produce differentiated nontradables in the monopolistic competitive market, using the technology:

$$y_t^N(i) = A_t h_t(i)^\alpha, \quad \alpha \in (0,1] \tag{11}$$

where $y_t^N(i)$ is nontradable output, h_t is the amount of labor input used by firm i, and A_t is a stochastic technology shifter common across all firms. I assume that A_t follows AR(1) process in logs:

$$\log(\frac{A_t}{A_s}) = \rho_A \log(\frac{A_{t-1}}{A_s}) + \epsilon_{A,t} \tag{12}$$

Here, ρ_A is the AR(1) coefficient governing the persistence of TFP shock, and A_s is the steady state level of TFP in the nontradables sector. Each firm of a variety i is subject to employment subsidy τ, which offset steady-state distortions caused by imperfect competition so that the effective cost of hiring labor service is $(1-\tau)W_t$. Profits of firms are $\Phi_t(i) = P_t^N(i)y_t^N(i) - (1-\tau)W_t h_t(i)$ for each variety i and total amount of profits transferred to each household is $\Phi_t = \int_0^1 \Phi_t(i)di$.

Each period, $(1-\theta)$ fraction of firms drawn randomly from the unit measure of population reoptimize the price of their goods subject to isoelastic demand given by equation (7). Since all firms are symmetric, they choose the same reoptimized price regardless of their varieties $i \in [0,1]$. Prices of the remaining fraction $\theta \in (0,1)$ are kept constant in domestic currency, and all firms are assumed to just meet the demand for their goods at the posted prices[5]. When reoptimizing the posted price $\tilde{P}_t^N$, each firm i seeks to optimize the discounted sum of profits at each period t

$$E_t[\sum_{k=0}^{\infty}\theta^k Q_{t,t+l}\{\tilde{P}_t^N y_{t+k|t}^N - C(y_{t+k|t}^N)\}] \tag{13}$$

subject to

$$y_{t+k|t}^N \leq (\frac{P_t^N(i)}{P_t^N})^{-\epsilon} c_t^N \tag{14}$$

and the cost function from production technology (11)

$$C(y_{t+k|t}^N) = (\frac{y_{t+k|t}^N}{A_{t+k}})^{\frac{1}{1-\alpha}}(1-\tau)W_{t+k} \tag{15}$$

for $k = 0, 1, 2, \cdots$. Here, $Q_{t,t+k} = \beta^k(\frac{\lambda_{t+k}}{\lambda_t})(\frac{P_t^T}{P_{t+k}^T})$ is the state-contingent nominal (stochastic) discount factor. The first order condition gives the resulting optimality condition with respect to $\tilde{P_t^N}$:

$$\sum_{k=0}^{\infty}\theta^k E_t[Q_{t,t+l} y_{t+k|t}^N\{\tilde{P}_t^N - \mathcal{M}\Psi_{t+k|t}\}] = 0 \tag{16}$$

where $\Phi_{t+k|t} = \frac{(1-\tau)W_{t+k}}{(1-\alpha)A_{t+k}h_{t+k|t}^{-\alpha}}$ is the marginal cost, in period $t+k$, of a firm with the posted price kept constant at the level set in period t and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the optimal price mark-up without price rigidity. Equation (16) implies that the price set in period t equates the discounted sum of marginal revenues and marginal costs with flexible price markup weighted by the output level and probability of keeping the posted price constant.

2.3 Downward nominal wage rigidity

Following Schmitt-Grohé and Uribe [2016], I impose the following constraint

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0 \tag{17}$$

where the parameter γ determines the degree of downward nominal wage rigidity. As it takes a higher value, the nominal wage becomes more downwardly rigid. For example, nominal wage becomes perfectly flexible if $\gamma = 0$ whereas $\gamma \geq 1$ implies absolute downward rigidity, and nominal wage never declines. The presence of downward rigidity in nominal wage causes disequilibrium in the labor market because nominal wage cannot be downwardly adjusted to equate the supply and demand of labor. Specifically, actual employment is lower than labor supply in general at all times, as follows:

$$h_t \leq h_t^s \tag{18}$$

where h_t^s is labor supply of households equating marginal substitution of leisure and consumption and its relative price

$$(\frac{W_t}{P_t^T})\lambda_t = \varphi(\bar{h} - h_t^s)^{-\chi} \tag{19}$$

[5] I follow the conventional assumption that the probability of posted prices falling below marginal costs is negligible and zero

where $\bar{h}$ is total endowment of time per each period. The slackness condition regarding nominal wage and employment is

$$(W_t - \gamma W_{t-1})(\bar{h} - h_t^s) = 0 \tag{20}$$

so that periods of unemployment ($h_t^s < \bar{h}$) is accompanied by binding wage constraint ($W_t = \gamma W_{t-1}$). The degree of DNWR (γ) will be calibrated to match the nominal wage growth when the unemployment rate steadily increases. I further assume that both workers and firms are price takers in the labor market[6].

2.4 Monetary regime

The home country is assumed to be a member of the world currency union, so the exchange rate is pegged to the world currency indefinitely and credibly. Hence, the net devaluation rate ($\log(\frac{E_t}{E_{t-1}})$ satisfies

$$\log(\frac{E_t}{E_{t-1}}) = 0 \tag{21}$$

for all period t. There are two sources of nominal wage rigidity, one is structural and is given by downward nominal wage rigidity in the labor market, and the other one is policy-induced because of the monetary authority. Combined together, these nominal frictions result in a sluggish downward adjustment in real wages. In specific, the real wage can fall at a rate no larger than $1-\gamma$. This affects both employment and inflation. On the one hand, wage rigidity leads to involuntary unemployment when the current market-clearing real wage is sufficiently lower than the past real wage. On the other hand, it stabilizes the inflation rate, especially during economic downturns, moderating deflationary pressure.

2.5 Equilibrium

In equilibrium, the market for nontradables clears at all times. In other words,

$$c_t^N = y_t^N \tag{22}$$

holds for $\forall t$. Combining the market clearing condition for nontradables with household budget constraint and definition of aggregate profit yields the market clearing condition for tradables:

$$c_t^T + d_{t-1} = y_t^T + \frac{d_t}{R_t} \tag{23}$$

After log-linearizing around the zero (net) inflation steady state, the equilibrium can be characterized by the system of difference equations (with hat variables denoting the natural logarithms of the deviation from the steady state).

Definition 1. *The equilibrium is a set of processes* $\{\hat{y}_t^N, \hat{h}_t, \hat{c}_t^T, \hat{\pi}_t^N, \hat{R}_t, \hat{w}_t, \hat{d}_t, \hat{\varepsilon}_t, \hat{p}_t^N, \hat{p}_t^T\}_{t=0}^{\infty}$*, for any exogenously given* $\{\hat{A}_t, \hat{y}_t^T, \hat{R}_t^*, \hat{p}_t^{*T}\}_{t=0}^{\infty}$ *satisfying*

Aggregate demand block:

[6] As pointed out in Schmitt-Grohé and Uribe [2016] using the EU Klems data, it is less likely that strategic wage compression of households played a significant role in peripheral Europe during the crisis. Nominal hourly wages increased over 60 percent during 2000-2008 despite low inflation and weak TFP growth. Spain, for example, experienced a 4 percent fall in value-added TFP during 2000-2007, and TFP in Ireland fell by 1 percent

$$\underbrace{(\hat{p}_t^N - \hat{p}_t^T)}_{=\hat{p}_t} = -\frac{1}{\zeta}(\hat{y}_t^N - \hat{c}_t^T) \tag{24}$$

$$\hat{c}_t^T + (\frac{\sigma_N}{\sigma_T})\hat{y}_t^N = E_t[\hat{c}_{t+1}^T] + (\frac{\sigma_N}{\sigma_T})E_t[\hat{y}_{t+1}^N] - (\frac{1}{\sigma_T})\hat{R}_t \tag{25}$$

$$\hat{c}_t^T + [\frac{\bar{d}}{1-(1-\beta)\bar{d}}]\hat{d}_{t-1} = [\frac{1}{1-(1-\beta)\bar{d}}]\hat{y}_t^T + [\frac{\bar{d}/R}{1-(1-\beta)\bar{d}}](\hat{d}_t^T - \hat{R}_t) \tag{26}$$

Aggregate supply block:

$$\hat{\pi}_t^N = \beta E_t[\hat{\pi}_{t+1}^N] + \kappa(\frac{1-\alpha}{\alpha})\hat{y}_t^N + \kappa(\hat{w}_t - (\hat{p}_t^N - \hat{p}_t^T) - \hat{A}_t) \tag{27}$$

$$\hat{\pi}_t^N = \hat{p}_t^N - \hat{p}_{t-1}^N \tag{28}$$

$$\hat{p}_t^T = \hat{\varepsilon}_t + \hat{p}_t^{*T} \tag{29}$$

with the downward nominal wage rigidity constraint,

$$\hat{w}_t = \begin{cases} \chi(\frac{h_s}{h-h_s})\hat{h}_t - \sigma_T(\hat{c}_t^T + \frac{\sigma_N}{\sigma_T}\hat{y}_t^N) & \textbf{not binding} \\ \\ \log(\gamma) + \hat{w}_{t-1} - \hat{\pi}_t^T & \textbf{binding} \end{cases} \tag{30}$$

where $\hat{\pi}_t^T = \hat{p}_t^T - \hat{p}_{t-1}^T$ and $\hat{h}_t = (\frac{\hat{y}_t^N - \hat{A}_t}{\alpha})$[7]. Here, $\sigma_N = (\frac{1}{\xi} - \sigma)(1-\nu)(\frac{c^N}{c})^{\frac{\xi-1}{\xi}}$ and $\sigma_T = (\frac{1}{\xi} - \sigma)(1-\nu)(\frac{c^T}{c})^{\frac{\xi-1}{\xi} - \frac{1}{\xi}}$. Monetary regime and exogenous processes are characterized by the following:

Monetary regime

I assume that the economy pegs the exchange rate to the world economy indefinitely and credibly:

$$\hat{\varepsilon}_t = 0 \tag{31}$$

and (external) debt-elastic interest rate (EDEIR) takes the form of:

$$\hat{R}_t (= \log(\frac{R_t}{R})) = \hat{R}_t^* + \psi\frac{\bar{d}}{R}\hat{d}_t \tag{32}$$

Shock process

Driving forces of exogenous shocks are:

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{A,t}, \quad \varepsilon_{A,t} \sim (0, \sigma_A^2) \tag{33}$$

for nontradables productivity, and

$$\begin{bmatrix} \hat{y}_t^T \\ \\ \hat{R}_t^* \end{bmatrix} = \begin{bmatrix} \rho_{y,y} & \rho_{y,r} \\ \\ \rho_{r,y} & \rho_{r,r} \end{bmatrix} \begin{bmatrix} \hat{y}_t^T \\ \\ \hat{R}_{t-1}^* \end{bmatrix} + \Gamma\varepsilon_t \tag{34}$$

[7] log-linearized price dispersion around the zero net inflation rate is 0

where $\Sigma_\epsilon = I_2$ and $\Gamma\Gamma' = \begin{bmatrix} \sigma^2_{y,y} & \sigma_{y,r} \\ \\ \sigma_{r,y} & \sigma^2_{r,r} \end{bmatrix}$

where y_t^T is the exogenous tradables output (endowment or terms-of-trade shock) and $R_t{}^*$ is the country risk-premium (world interest rate shock).

Foreign price level is assumed to be constant normalized to unity ($P_t^{*T} = 1$)

$$\hat{p}_t^{*T} = 0 \tag{35}$$

The aggregate demand block consists of the intratemporal optimization condition for tradables and nontradables (24), intertemporal optimization condition (25), and market clearing condition for tradables[8]. These conditions imply that increases in interest rate lead to decreases in tradables and nontradables, while tradables (and subsequently nontradables) increase if the amount of tradables endowments get higher.

Figure 2 depicts the demand schedule for nontradables and illustrates the aggregate demand block. It is downward sloping since the demand for nontradables decreases as its relative price increases. Also, an increase in the absorption of tradables, an increase in y_t^T for example, shifts the demand function to the right. This is because both goods are complementary. For example, suppose the world interest rate R_t^* in equation (32) decreases by Δ amount. Then the demand schedule for nontradables shifts to the right (from the dashed line to the solid line). Similarly, if tradables endowments increase by Δ, the demand schedule moves to the right. Thus, any shocks involving a decrease in the interest rate or terms-of-trade improvement increase the demand for nontradables.

The aggregate supply block describes the evolution of nontradables price inflation, the law of one price for tradables, and downward nominal wage rigidity constraint denominated in tradables. Equation (27) is the standard New Keynesian Phillips curve (NKPC) for nontradables showing a positive association between inflation and output. Increases in productivity or devaluation of the currency shift the NKPC to the right, while a decrease in wage shifts the curve to the right. Thus, either external or internal devaluation is necessary to increase the supply if there is no improvement in productivity.

Figure 3.3 illustrates the supply curve of nontradables. It is obtained by combining equations (27) and (28) and arranging the terms to drive the expression for price level instead of inflation. The supply schedule is upward-slopping, and it shifts to the right if there is any decrease in cost factors denominated in tradables in the efficiency unit, including nominal wage, exchange rate, and productivity. For example, consider a decrease by Δ in real wage denominated in tradables (internal devaluation). It implies a decrease in the marginal cost in equation (16). As a result, as the optimal reset price setting condition shows, it becomes profitable to set lower prices and produces more.

Similarly, currency devaluation (increase in exchange rate by Δ amount) decreases the real wage denominated in tradables, and subsequently decreases the real marginal cost. Both changes shift the supply schedule to the right (from the dashed line to the solid line).

Figure 4 plots the demand and supply schedules and illustrates the role of downward nominal wage rigidity during the boom and bust cycles. Suppose that the initial output level of nontradables is y_0 and suddenly, the world interest rate (R^*) decreases by Δ amount. In response to the lower interest rate, tradables absorption (c_t^T) increases while shifting up the demand for complementary nontradables to the right. In panel A, price and output increase to p_{boom} and y_{boom} respectively in

[8] Aggregate demand block already impose nontradables market clearing condition by substituting c_t^N with y_t^N

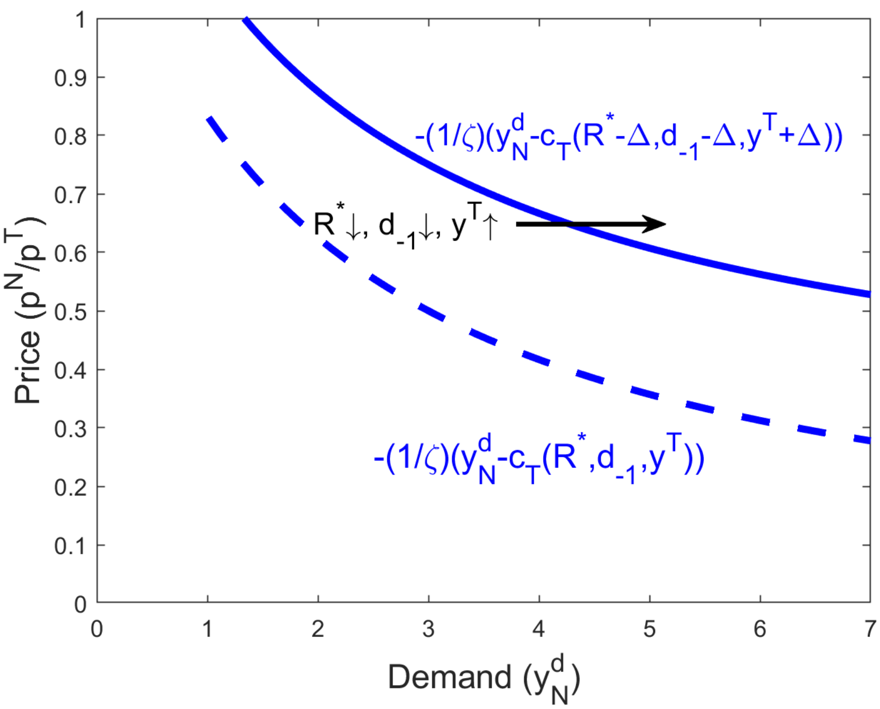

Figure 2: Demand curve for nontradables

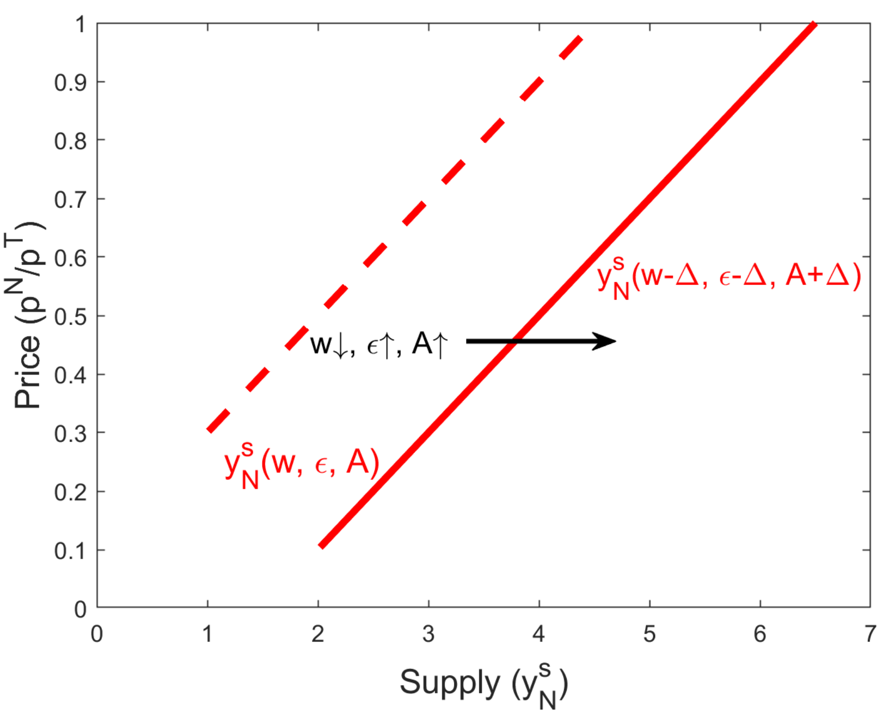

Figure 3: Supply curve for nontradables

the new equilibrium (Boom). Also, the nominal wage increases from w_0 to $w_0 + \Delta$ because of no upward wage rigidity, causing the supply schedule to shift to the left.

Suppose now that the expansionary shock of the decline in interest rate disappeared and the world interest rate increases by Δ amount. It shifts down the demand schedule to the left in panel B. If the wage were fully flexible, the economy could have returned back to the initial equilibrium point or achieved a higher output than y_{burst} at the worst. However, the nominal wage does not decline from its boom level, $w_0 + \Delta$, and the exchange rate is fixed at ε_0. As a result, the supply curve does not shift down to the right, and the economy achieved the lower output level y_{burst} and higher price level p_{burst}. This is because the nominal wage is downwardly rigid, and the exchange rate is pegged at a fixed level. The new equilibrium point (Burst) is even lower than the initial level.

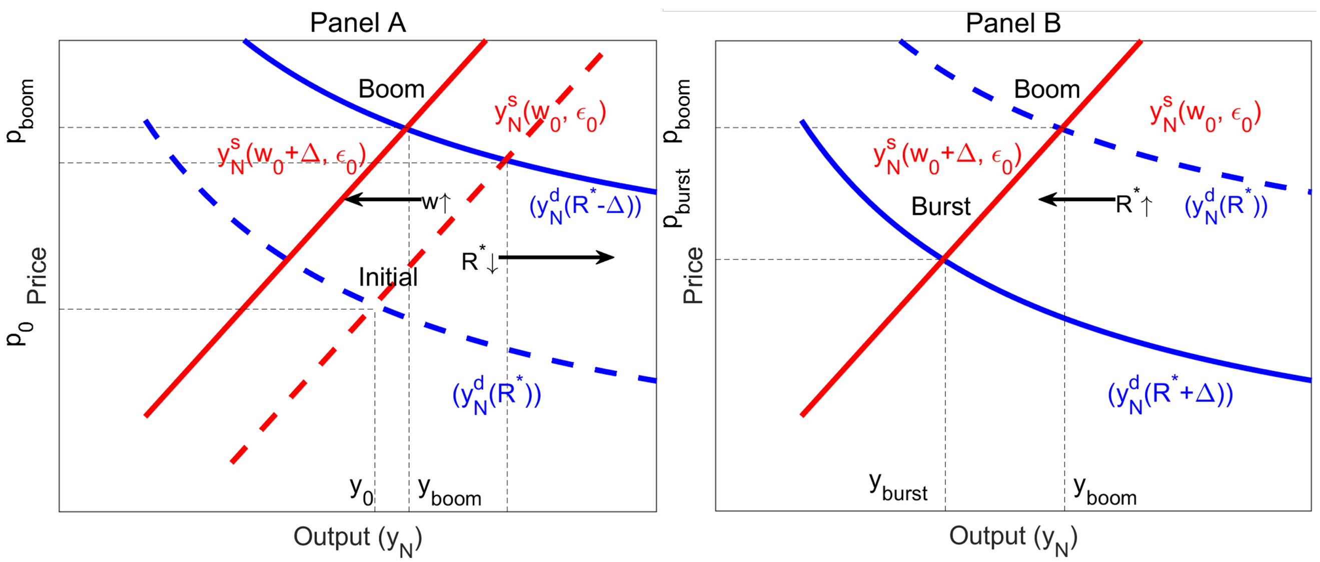

Figure 4: Boom burst in the model

This result is consistent with what peripheral European countries experienced during the 2000-2011 boom-burst cycle. In the early 2000s, these countries experienced capital inflows, low unemployment rates, and nominal wage increases. The large capital inflows and subsequent current account deficit can be modeled as a decrease in the world interest rate (R^*), So the equilibrium changed from the initial point to the boom point. However, the global financial crisis in 2008 led to a sudden stop, and the contractionary shocks on domestic absorption caused a collapse in aggregate demand. Meanwhile, the nominal hourly wage did not sufficiently decrease or even increase, and the unemployment rate has trended upward since the onset of the crisis.

Seemingly, the downward nominal wage rigidity, in conjunction with the fixed exchange rate, monotonically deteriorates the overall welfare level of the economy. It is true that in the flexible price economy, the severity of nominal wage rigidity has a monotonic detrimental effect on welfare. However, the downward rigidity of nominal wage can stabilize inflation and prevent deflation during the downturn. Figure 5 compares the economy with a low degree of rigidity with the one with severe downward rigidity of nominal wage. Starting from the initial output level y_0, both economy achieve the same output level y_{boom} during the boom period.

In contrast, equilibrium outputs in the recession (Burst) are different. In the left panel (Low DNWR (γ)), the economy almost returns to its initial output level (Low Δy_N) in response to negative shock because of a sufficient decline in the nominal wage. However, this stabilization

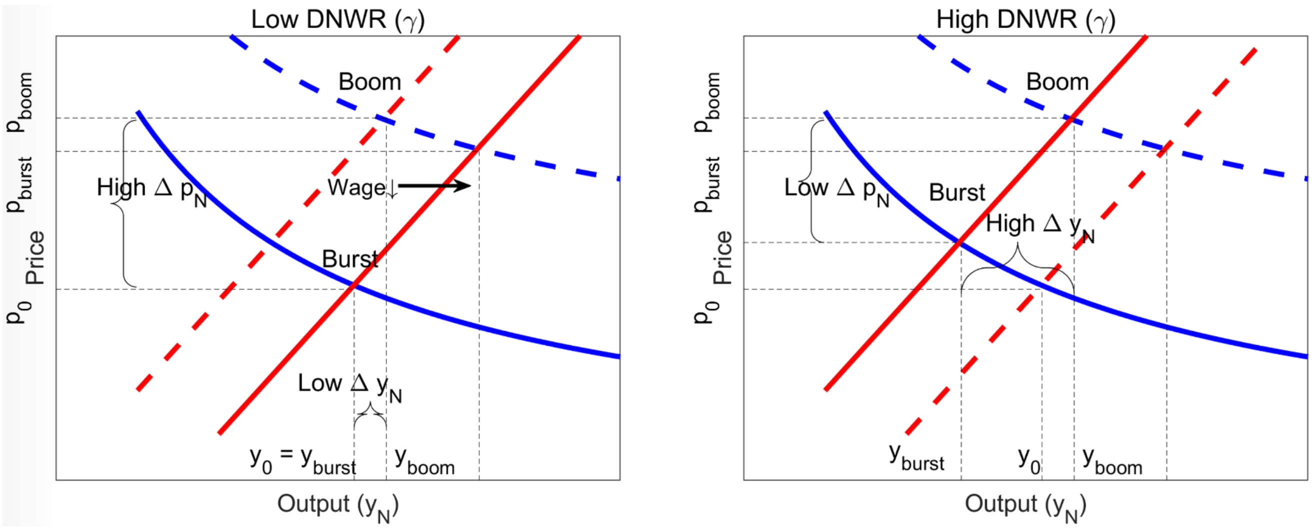

Figure 5: Low DNWR vs High DNWR

benefit comes at the cost of volatile adjustment in the price level (High Δp_N). In the right panel (High DNWR (γ)), downward rigidity causes a large adjustment in the output (High Δy_N) during the recession in contrast to less decline in the price level (Low Δp_N). In other words, there is a trade-off between inflation and output stability regarding the degree of downward nominal wage rigidity. Hence, the overall unconditional welfare impacts are determined by these two conflicting factors.

3 Downward rigidity and unconditional welfare

3.1 Calibration

Table 2 shows the baseline calibration for the model parameters. I calibrate the model at quarterly frequency using the Eurostat data from 2000:Q1 to 2011:Q3. Following Schmitt-Grohé and Uribe [2016], I estimate a bivariate AR(1) process for the exogenous driving forces regarding foreign shocks (y_t^T, R_t^*) by ordinary least square procedure. Here, y_t^T denotes Hodrick-Prescott (HP) filtered log tradables output. It is the cyclical component of each country's agriculture, forestry, mining, and manufacturing at 2015 prices. I estimate the world interest rate in terms of tradables using

$$R_t = (1 + i_t)\mathbb{E}_t[\frac{\varepsilon_t P_t^T}{\varepsilon_{t+1} P_{t+1}^T}] \tag{36}$$

where R_t is the gross real interest rate in terms of the tradables unit, i_t is the nominal net interest rate in terms of the national currency, which is measured as 10-year government treasury bonds of each country. ε_t denotes the nominal exchange rate of domestic currency per unit of EURO. P_t^T denotes the German consumer price index, which is a proxy for the foreign-currency price of tradables. I measure $\mathbb{E}_t[\frac{\varepsilon_t P_t^T}{\varepsilon_{t+1} P_{t+1}^T}]$ by the one step ahead forecasting value of $\frac{\varepsilon_t P_t^T}{\varepsilon_{t+1} P_{t+1}^T}$ from an commensurate estimated AR(2) process. All data is from the Eurostat. Both output and interest rate are highly volatile, and the unconditional correlation between the two variables is negative. These results imply that real interest rate is counter-cyclical and borrowing terms tend to deteriorate during the recession. The implied value for steady-state real interest rate is 2.42 percent per year, implying that the quarterly subjective discount factor (β) is 0.994. The estimated process is

$$\begin{bmatrix} \hat{y}_t^T \\ \\ \hat{R}_t^* \end{bmatrix} = \begin{bmatrix} 0.8620 & -0.2857 \\ \\ 0.0671 & 0.9154 \end{bmatrix} \begin{bmatrix} \hat{y}_t^T \\ \\ \hat{R_{t-1}^*} \end{bmatrix} + \Gamma\varepsilon_t \tag{37}$$

where $\Gamma = \begin{bmatrix} 0.0132 & 0 \\ \\ 0.0037 & 0.0067 \end{bmatrix}$

To measure the cyclical productivity of nontradables, I approximate the log productivity as $\log(A_t) = \log(y_t^N) - \alpha\log(h_t)$ first where y_t^N is per capita real GDP except tradables and h_t is employment. Using the HP-filtered variables, I fit an AR(1) process for log productivity:

$$\log(A_t) = 0.5963\log(A_{t-1}) + \varepsilon_t^A, \quad \varepsilon_t^A \sim (0, 0.0059^2) \tag{38}$$

The corresponding values for ρ_A and σ_A are reported in the table.

Parameter ε, the elasticity of substitution across different varieties of nontradables, is calibrated as 3.8, which is implied from the steady-state markup of 35 percent[9]. The parameter α is set to 0.75 following Galí and Monacelli [2016] and Schmitt-Grohé and Uribe [2016][10]. The parameter regarding the utility from leisure, φ, is set to match steady-state hours worked with 1 out of total endowment of time $\bar{h} = 3$. I adopt a Frisch wage elasticity of labor supply of 2 so that the implied value of χ is $\chi = 1$. Baseline Calvo parameter (θ) is set to 0.83, indicating that prices do not change for 5.75 quarters on average. The parameter ν, approximately the share of tradables in total output, is set to match the average share of tradables in GDP over the sample period. I set the elasticity of substitution between tradables and nontradables (ξ) to 0.44 following Mendoza [1991] and Corsetti et al. [2008]. The steady-state debt level ($\bar{d}$) is set to 1.1537, implied from $\bar{d} = 1.1537$ after rearranging terms in market clearing conditions for tradables. The constant relative risk aversion (CRRA) parameter (σ) is set to $\sigma = 1$ to one.

Last but not least, the degree of downward nominal wage rigidity (γ) is set to 0.99, implying at most a 4 percent annual decrease in nominal wage. From 2007:Q4 to 2013:Q4, there were large increases in the unemployment rate, whereas nominal wages even increased or showed modest declines.

Figure 6 depicts the nominal hourly wage and unemployment during the crisis. It has trended upward until 2013:Q4 since it reached its lowest value in 2007:Q4. However, the nominal hourly wage even increased until 2011 and slightly decreased thereafter.

Table 1 shows unemployment, nominal wages, and implied values of γ. W is a nominal (average) hourly labor cost index in manufacturing, construction, and services (including the public sector). The DNWR slackness condition $(W_t - \gamma W_{t-1})(h_t^s - h_t) = 0$ implies that if unemployment rate is increasing, $(h_t^s - h_t) > 0$, the nominal wage should change at γ rate, so that $W_t = \gamma W_{t-1}$.

During 2007:Q4-2013:Q4, the implied value of $\gamma = (\frac{W_t}{W_{t-1}})^{\frac{1}{24}}$. After considering foreign inflation and long-run growth, which are not explicitly incorporated into the model, $\gamma \in [0.9906, 1.0024]$ for the PIGS countries. I use a conservative value of $\gamma = 0.99$ for quarterly frequency.

[9]This is consistent with the European Central Bank's New Area Wide Model of Christoffel et al. [2008]

[10]This value is close to 0.25 implied by share of labor in nontradables sector estimated by Schmitt-Grohé and Uribe [2007] and observed 0.55 average labor share income across these peripheral European countries during the sample period

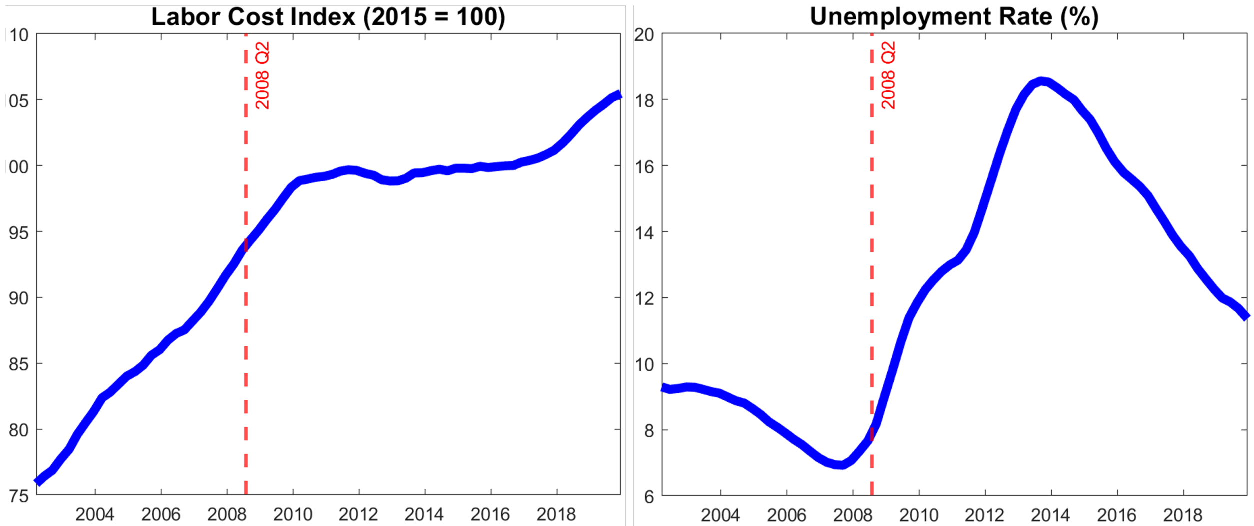

Figure 6: Unemployment Rate and Nominal hourly wage

Notes. Figure 6 presents the unemployment rate and nominal labor cost index from 2002 to 2019. I take a four-quarter average of each data and calculate weighted averages across PIGS countries (Portugal, Italy, Greece, and Spain) using population sizes. All calculations are from the Eurostat. The unemployment rate reached its lowest value in 2007Q4 and steadily trended upward until it peaked in 2013Q4. However, the nominal labor cost steadily increased or sluggishly decreased, indicating the existence of downward nominal wage rigidity.

Table 1: Unemployment rate (%) and implied value of γ

Country	2007Q4	2013Q4	Wage Growth (annualized)	γ
Greece	8.1	27.6	-2.63	0.9906
Spain	8.1	25.2	1.73	0.9987
Italy	6.3	12.3	2.40	1.0024
Portugal	8.0	15.6	0.28	0.9950

Table 2: Baseline Calibration

Variable	Values	Description
ν	0.1616	share of tradables[11]
ψ	0.001	parameter of debt-elastic rate
$\bar{d}$	1.1537	steady state debt level
ξ	0.44	elasticity of substitution between tradables and non-tradables
β	0.9940	subjective discount farctor
γ	0.99	degree of downward wage rigidity
σ	1	the curvature of utility
α	0.75	index of DRS to labor
φ	1.2743	curvature of labor utility
χ	1.0	curvature of labor utility
θ	0.83	Calvo index of price rigidity
ε	3.8	elasticity of substitution (nontradables)
$\bar{h}$	3	time endowment

3.2 Solution method

The baseline economy model features the DNWR occasionally binding constraint, which is difficult to solve, especially when the model involves many state variables and exogenous shocks. Thus, I take a detour and use the piecewise linear perturbation method developed by Guerrieri and Iacoviello [2015] rather than approximating full global solutions. This method regards an inequality constraint as two different endogenous regimes and applies the linear perturbation to each regime. Starting from an initial guess of the expected duration of each regime (slack vs. binding), it linearizes the equilibrium system under each regime. Then, it takes a guess-and-verify approach to determine how long the constraint will bind, iterating this until convergence. While each regime is solved linearly, the expected duration of each regime and endogenous regime switch induces a non-linearity of equilibrium dynamics, capturing the non-linearity caused by the occasionally binding constraint. It significantly simplifies solving the model and relieves numerical burdens, especially pertaining to large-scale models. However, this method postulates the nonstochastic steady-state in approximations and cannot capture any precautionary motive of economic agents[12].

3.3 Counterfactual analysis

In this section, I conduct a counterfactual experiment by feeding recovered shock sequences into the calibrated model without downward nominal wage rigidity[13]. The recovered shock sequences are constructed to match model implied output, interest rates, and inflation to the cyclical components of these variables in the data. Figure 7 compares the filtered variables of the baseline model with those from the model without DNWR constraint. During the European crisis, there was a huge spike in the real interest rate, which apparently had huge contradictory effects on employment, consumption, and output. From the mid-2000s to 2008, the real wage trended upwards, and employment remained stable around its steady-state level. These periods correspond to the boom equilibrium in Figure .

The stark difference between the two models comes out in the after-crisis dynamics of the real wage. Because of DNWR and the fixed exchange rate, the wage is decreasing sluggishly. Compared to its peak value in 2010, it decreased by about 10 percent, whereas the magnitude would be about 25 percent without downward nominal wage rigidity. During the crisis and recovery period, the cumulative output loss would be 11.52 % and the employment loss would be 18.38 % less without downward nominal wage rigidity. These results are comparable to the previous findings (Schaefer and Singleton [2022] Decressin et al. [2015]).

3.4 Downward nominal wage rigidity and Welfare in a Currency Union

The second order approximation of the representative household's average period utility losses is

$$\mathbf{L} \sim (\frac{1-\nu}{2})\mathbb{E}[(\hat{\pi}_t^N)^2 + \vartheta_h(\hat{h}_t)^2 + \vartheta_d \hat{d}_t] \tag{39}$$

where $\vartheta_h = \alpha(\frac{\bar{h}}{\bar{h}-h_s})(\frac{\lambda_p}{\mu_p})$ is the weight on employment, $\vartheta_d = \frac{2\psi(\beta\bar{d})^2}{(1-\nu)(1-(1-\beta)\bar{d})}(\frac{\lambda_p}{\mu_p c^{\frac{1-\xi}{\xi}}})$, and $\lambda_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p(\frac{\mu_p}{\alpha}+1-\mu_p)}$ is the slope of NKPC. See the appendix for a more detailed derivation of utility losses

[12]If households have the market power as in the standard New Keynesian model with the Calvo wage rigidity, these households strategically compress wage increases during the boom. This implies that the welfare cost of DNWR is overstated if the strategic wage compression played an important role during the boom periods in peripheral Europe. However, this is less likely as Schmitt-Grohé and Uribe [2016] point out

[13]I used the piecewise Kalman smoother to recover shocks while taking account occasionally binding constraint explicitly

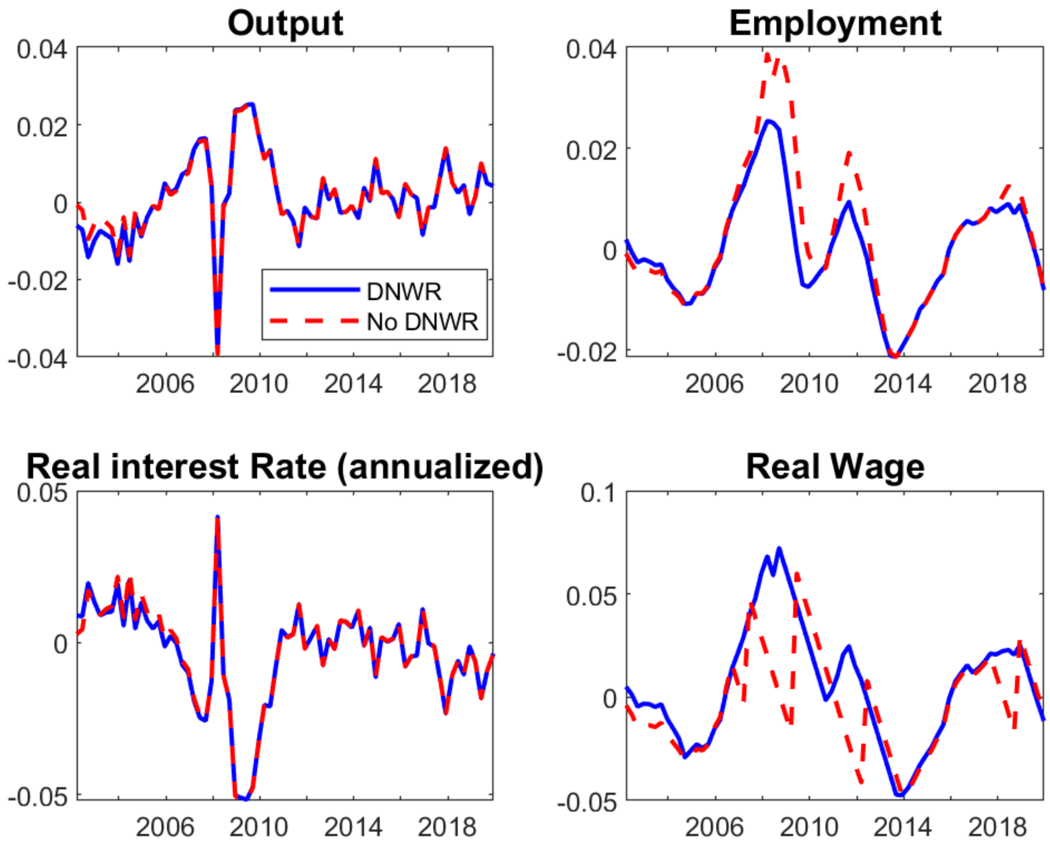

Figure 7: Baseline vs. Counterfactual without DNWR

Notes. Figure 7 presents the counterfactual analysis comparing baseline results with ones without downward nominal wage rigidity. I use GDP, aggregate hours, and net external debt from 2002:Q2 to 2019:Q4 from the Eurostat and apply the piecewise Kalman smoother to recover smoothed shock sequences. The counterfactual results are obtained by feeding recovered shocks into the model without downward nominal wage rigidity. All calculations are converged within ten iterations.

[14]. To get ergodic moments of inflation, employment, and debt levels, I simulate the model for 15,000 periods with the same randomly drawn shocks following Amano and Gnocchi [2020][15]. Note that the approximation (39) is scaled by steady state level $(\partial U/\partial c)c$ and measures welfare in terms of consumption equivalent. Therefore, the expected losses measures how much the representative households are willing to pay, in terms of fraction of steady state consumption, in order to live in the frictionless economy.

Figure 8 shows a graphical representation of the approximated welfare losses evaluated at different degree of DNWR (γ). It shows that as nominal wage becomes more downwardly rigid, welfare losses from employment volatility (black short-dashed line) goes rise and inflation volatility (red long-dashed line) decreases. Losses from foreign debt level is also increasing, but the order of magnitude is relatively low compared to other two components. OVerall, the unconditional welfare losses shows a non-monotonic U-shaped pattern in relation to the degree of DNWR (γ). When

[14] As Amano and Gnocchi [2020] point out, a second-order Taylor approximation is well-defined despite a non-differentiability of wage equation. This is because the DNWR inequality constraint is not used in the approximation

[15] An employment subsidy τ neutralizes a steady state distortion from the product mark-up and steady state resource allocation becomes efficient from the viewpoint of small open economy

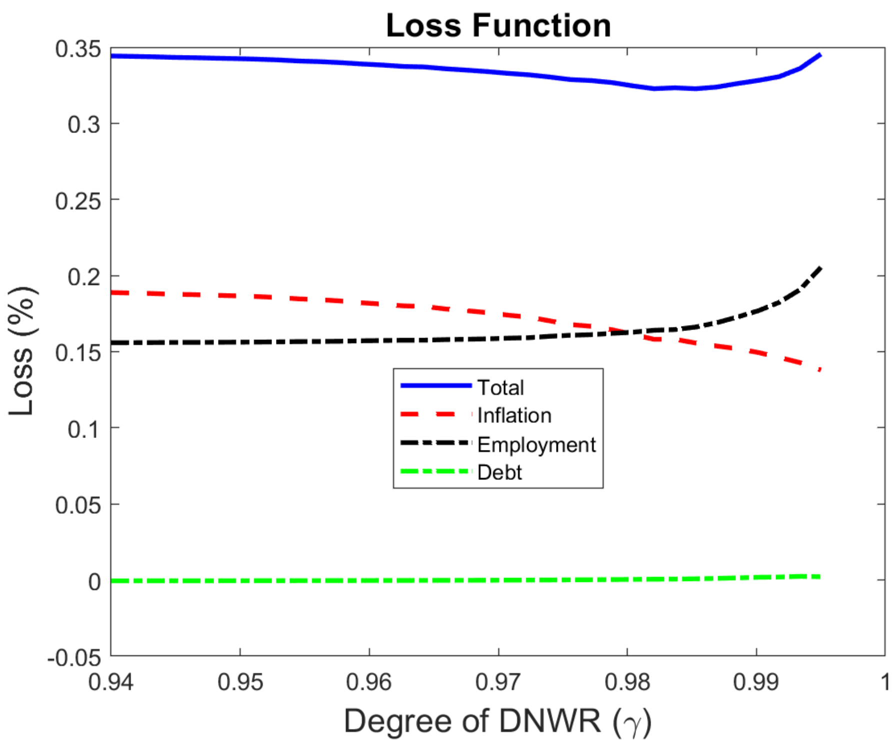

Figure 8: Downward nominal wage rigidity and Welfare loss components in a currency union

the degree of DNWR remains low, the gains from inflation stability dominates the losses from employment volatility. However, the losses from employment volatility dominates once the degree of DNWR trespasses a certain threshold value of 0.99. After all, the unconditional welfare losses exponentially trend upwards, supporting the conventional wisdom about the necessity of internal devaluation. This result is a mixture of conclusions of both Galí and Monacelli [2016] and Schmitt-Grohé and Uribe [2016]. Under the modest degree of DNWR (γ), the economy gains from higher (γ) and inflation stability (paradox of flexibility).

4 Conclusion

This study incorporates the downward nominal wage rigidity into the small open economy New Keynesian model with a fixed exchange rate. It aims to evaluate the effectiveness of conventional wisdom calling for wage flexibility regarding countries under the currency union. Two results stand out. First, In counterfactual analysis, without DNWR, employment and output would have been 2.56% and 1.92% higher on average, respectively, and inflation would have been -8.6 basis points lower during 2008:2014. During the crisis, wage moderation can play a role in insulating employment and output. This result is consistent with the conventional wisdom during the European Crisis. Second, DNWR has a non-monotonic impact on unconditional welfare loss. When the degree of DNWR (γ) takes a higher value than around 0.99, the welfare loss from involuntary unemployment dominates the gain from inflation stability. This result is a mixture of conclusions of both Galí and Monacelli [2016] and Schmitt-Grohé and Uribe [2016]. With the modest degree of DNWR (γ), the economy gains more from higher (γ). This is because gains from inflation stability dominate losses from inefficient employment fluctuations (paradox of flexibility). However, as wages become significantly rigid downwards (γ is sufficiently high), the downward nominal wage rigidity starts raising the welfare losses.

5 Appendix

In this section, I derive the welfare losses as to the model studied in this paper up to a second-order approximation. With the assumption of an efficient steady state and $\sigma = 1$, the second-order approximation of period utility is[16]:

$$u_t \approxeq u + u_c c\hat{c}_t + \frac{1}{2}u_c c(1 + \frac{u_{cc}c}{u_c})\hat{c}_t^2 + (u_h h)\hat{h}_t + (u_h h)(\frac{1 + u_{hh}h/u_h}{2})\hat{h}_t^{\,2} \tag{40}$$

Putting $\hat{c}_t \approxeq \nu\hat{c}_t^T + (1-\nu)\hat{c}_t^N$ and imposing the market clearing condition $c_t^N = y_t^N$ give (ignoring u):

$$V_t \approxeq \mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t(\nu\hat{c}_t^T + (1-\nu)\hat{y}_t^N - \varphi(\bar{h}-h_s)^{-\chi}h_s\{\hat{h}_t + (\frac{1+\chi(h_s/(\bar{h}-h_s))}{2})\hat{h}_t^{\,2}\})] \tag{41}$$

since $u_h = -\varphi(\bar{h}-h_s)^{-\chi}$ and $\frac{u_{hh}h_s}{u_h} = \chi(\frac{h_s}{\bar{h}-h_s})$. Substituting $\hat{y}_t^N = \alpha(\hat{h}_t - \log(v_t^P)) + \hat{A}_t$ where v_t^P is the price dispersion and up to a second-order approximation, it is known that[17]:

$$\alpha\log(v_t^P) \approxeq \frac{\mu_p(\alpha + (1-\alpha)\mu_p)}{2\alpha}Var_i\{p_t^N(i)\} \tag{42}$$

in a neighborhood of a symmetric steady state, and additionally

$$\sum_{t=0}^{\infty}\beta^t Var_i\{p_t^N(i)\} = \frac{\theta_p}{(1-\beta\theta_p)(1-\theta_p)}\sum_{t=0}^{\infty}\beta^t(\pi_t^N)^2 \tag{43}$$

Ignoring the higher order terms and putting things together gives[18]:

$$\begin{aligned} V_t \approxeq \mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t(\nu\hat{c}_t^T + (1-\nu)\alpha\hat{h}_t - (1-\nu)\frac{\mu_p}{2\lambda_p}(\pi_t^N)^2 \\ - \varphi(\bar{h}-h_s)^{-\chi}h_s\{\hat{h}_t + (\frac{1+\chi(h_s/(\bar{h}-h_s))}{2})\hat{h}_t^2\} + (1-\nu)\hat{A}_t)] \end{aligned} \tag{44}$$

Setting $\varphi = \frac{\alpha(1-\nu)(\bar{h}-h_s)}{h_s^{1-\alpha+\alpha/\xi}}$ normalizes $h_s = 1$. Under the baseline calibration ($\chi = 1$), it follows that $\varphi(\bar{h}-h_s)^{-\chi}h_s = (1-\nu)\alpha$. Substituting $\varphi(\bar{h}-h_s)^{-\chi}h_s = (1-\nu)\alpha$ and imposing $\chi = 1$ gives the final expression for V_t:

$$\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t(\nu\hat{c}_t^T - (1-\nu)\{\frac{\mu_p}{2\lambda_p}(\pi_t^N)^2 - \hat{A}_t + \alpha(\frac{1+(h_s/(\bar{h}-h_s))}{2})\hat{h}_t^2\})] + t.i.p.s + O(\|\ \xi^3\ \|) \tag{45}$$

To express the average loss in terms of debt, inflation, and employment, consider the following lifetime budget constraint

[16] $u_c c = \frac{1}{c} \times c = 1$ since $\sigma (= \frac{u_{cc}c}{u_c}) = 1$. Also, $(1 + \frac{u_{cc}c}{u_c}) = 0$

[17] See Galí [2015]

[18] $\lambda_p = (\frac{\alpha}{(\alpha+(1-\alpha)\mu_p)})(\frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p})$ is the slope of NKPC

$$\sum_{t=0}^{\infty}\frac{c_T}{R^t}\hat{c}_t^T+\bar{d}\hat{d}_{-1}=\sum_{t=0}^{\infty}\frac{y_T}{R^t}\hat{y}_t^T-\sum_{t=0}^{\infty}\frac{\bar{d}/R}{R^t}\hat{R}_t+\underbrace{\lim_{T\to\infty}\frac{\bar{d}/R}{R^T}\hat{d}_{T+1}}_{=0\ \textbf{by no Ponzi condition}} \tag{46}$$

Rearranging the terms to get an expression for lifetime tradables consumption stream and substituting into the lifetime utility gives:

$$\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t(\frac{u_t-u}{u_c c})]=\mathbb{E}_t[\sum_{t=0}^{\infty}\beta^t\{\nu(\frac{c^T}{c})^{\frac{\xi-1}{\xi}}((\frac{y^T}{c^T})\hat{y}_t^T-\frac{\bar{d}(1-\beta)}{c^T}\hat{d}_{-1}-\frac{\bar{d}}{Rc^T}\hat{R}_t)$$
$$-\alpha(1-\nu)c^{\frac{1-\xi}{xi}}(\frac{1+\varphi}{2})\hat{h}_t^2-(1-\nu)c^{\frac{1-\xi}{xi}}\frac{\mu_p}{2\lambda_p}(\hat{\pi}^N)^2\}\}]+t.i.p.s+O(||\ \xi^3\ ||) \tag{47}$$

Note that $\hat{R}_t=\hat{R}_t^*+\psi(\frac{\bar{d}}{R})\hat{d}_t$ and $\beta R=1$. Putting this external debt elastic interest rate (EDEIR) equation into the above gives the expression for the lifetime welfare in terms of debt, employment, and inflation:

$$\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t(\frac{u_t-u}{u_c c})]=-\mathbb{E}_0[\sum_{t=0}^{\infty}\beta^t\{\frac{\psi\bar{d}^2}{R(R-(R-1)\bar{d})}\hat{d}_t+\alpha(1-\nu)c^{\frac{1-\xi}{xi}}(\frac{1+\varphi}{2})\hat{h}_t^2+$$
$$(1-\nu)c^{\frac{1-\xi}{xi}}\frac{\mu_p}{2\lambda_p}(\hat{\pi}^N)^2\}]+t.i.p.s+O(||\ \xi^3\ ||) \tag{48}$$

Thus, period average loss is (after ignoring higher order terms and the terms independent of policy and the degree of downward nominal wage rigidity (γ)):

$$\mathbf{L}\sim(\frac{1-\nu}{2})\mathbb{E}[(\hat{\pi}_t^N)^2+\vartheta_h(\hat{h}_t)^2+\vartheta_d\hat{d}_t] \tag{49}$$

where $\vartheta_h=\alpha(\frac{\bar{h}}{\bar{h}-h_s})(\frac{\lambda_p}{\mu_p})$ is the weight on employment, $\vartheta_d=\frac{2\psi(\beta\bar{d})^2}{(1-\nu)(1-(1-\beta)\bar{d})}(\frac{\lambda_p}{\mu_p c^{\frac{1-\xi}{\xi}}})$, and $\lambda_p=\frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p(\frac{\mu_p}{\alpha}+1-\mu_p)}$ is the slope of NKPC.

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