Estimating Best Response Functions with Strategic Substitutability

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This paper is concerned with bank's strategic behaviors when substitutability between banking services is assumed. Best response functions and Nash equilibria may be better described by non-linearity than by linearity. The nonlinearity is dependent on the nonlinearity of demand function, regardless of whether it is an individual or a market demand function. In the linear model, the dynamics and properties of a Nash equilibrium may be a priori, straight forward and trivial. However, nonlinearity contains the diverse possibility of dynamics, describing the game more realistically and carrying rich economic implications. Using nonlinear functions, our study investigates the game between banks with ATMs, telebanking and internet banking services, and discusses the existence of stable Nash equilibria and the possibility of collusion between players. It is also found that developing information technology accelerates the transformation of traditional banking services into electronic banking services.

Keywords: Linearity, Nonlinearity, Best response function, Nash equilibrium, Cournot competition, Bertrand competition, Collusion

JEL Classification: C30, D40, G21

I. Introduction

The objective of this study is to investigate bank's strategic behaviors when substitutability between banking services is assumed. For the

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purpose, we adopt best response functions approach that is useful to analyze strategic behaviors of players in the competitive markets. In particular, the best response function approach makes it possible to predict market prices and quantities through Nash equilibria, when players are assumed to have rational expectations.

Best response functions could have either linear or non-linear parameters, determined by the linearity or nonlinearity of the demand function. If the individual or market demand function is linear (nonlinear), the corresponding best response function is also linear (nonlinear). To handle the linear demand and best response function is relatively simple, however, the economic implications of a linear function may be straight forward and trivial. The game dynamics between competitive players are not appropriately analyzed even though parameter values may change. Different parameter values imply different dynamics of the game, however, the linear model fails to appropriately reflect such differences.

Nonlinear models may better describe the dynamics of competitions between players. Diverse structural parameter values imply diverse dynamics, and are appropriately described and estimated by the nonlinear model. Since all structural parameters are to be clustered at a constant term in the linear model, the meanings and roles of parameter values are ambiguous. Lacking clarity, the linear model may often result in inestimable nuisance parameter terms. When several parameter values are clustered at a term, it may not be possible to decompose a parameter estimator into individual estimators without strict restrictions.

Recently in Korea, IT technology has been applied to banking services in the form of electronic banking (*e.g.*, mobile banking, telebanking, internet banking and so on). As the IT based banking services develop, traditional banking services, such as window tellers, have been sharply reduced. As of the end of 2005, IT—based banking services represent around 80% of total banking services, while traditional window teller services declined dramatically during last several years (Figure 1).

The dramatic development of electronic banking raises questions on the future of banking services. Will the electronic banking services continue to develop? Are the electronic and traditional types of banking services complements or substitutes? How strong are the competitions between players if their services are strategic substitutes rather than strategic complements? If electronic banking service is a strategic substitute for traditional services, will it completely replace traditional banking? Might players collude when marginal costs decline as IT technology innovation creates economies of scale or economies of scope in banking services?

To investigate these curiosities, best response functions especially nonlinear functions would be very useful. Nonlinear best response functions describe well the properties and degree of competition between players. It is also very useful to investigate whether banking services are strategic complements or substitutes.

According to our study, IT technology creates substitutability rather than complementarity between banking services, even though electronic and traditional banking services may have complementary properties.[1] In particular, the degree of substitutability becomes stronger as the intensity of application of IT technology to banking services increases.

In estimating the best response functions, there is strong simultaneity between banking services as implied by the intrinsic properties of the game. To avoid biased results, simultaneity requires instrument variable (IV) estimation rather than ordinary least squares (OLS) estimation.

There are lots of studies on the new financial services created by IT technology. According to our brief review, most of the studies hint that IT creates fundamental changes in financial services and traditional banking services are being superceded by new IT-based financial services. Some results are carried as follows. Allen *et al.* (2002) define e-finance as "the provision of financial services and markets using electronic communications and computation," and perceive that financial services industry could be fundamentally changed by the new technology. McAndrews (1999) points out that recently new information-processing systems have been rapidly developing and supplanted old systems by which financial services were offered to consumers. Giannakoudi (1999) notices that electronic computerization creates new operational space for banking services and makes possible an marriage of banking services with internet resulting in 'internet banking.' With the marriage, banking services are to be offered beyond space and time borders even though the abolishment of face-to-face transactions increases the degree of uncertainty in security of banking services. Berger (2003) emphasizes that banks are intensive user of both IT and financial technologies contributing improvements in costs and lending capacities

[1] Traditional banking services with IT technology applications would be more competitive than pure traditional banking services without IT applications. However, such partial complementarity seems to be dominated by the substitutability of new banking services with full application of IT technology in our analysis and data. See Figure 1.

due to "back-office" technologies as well as consumer benefits due to "front-office" technologies. Kauffman *et al.* (2000) study network externality effects and find that the adoption of IT technology is dependent on the size of externality effects.

Kim (2006) finds a trendy phenomenon in Korea that banking services have been being transformed from the traditional paper- and face-based contracts such as Window services to paperless- and faceless-contracts such as Internet Banking services.

Lee *et al.* (2000) mentions that electronic financial services are totally different from the traditional financial services in sense that electronic services are based on the information technology. Lee (2001) studies competition models in the market of retail electronic payment services. Lee tries to investigate what conditions determine the possibility of survivals of certain types of banking services especially electronic retail banking services such as Smart Cards, T-money and so on. Lee suggests diverse nonlinear competition models that are dependent on the competition environments. KIF (2003) analyzes that market frictions imposed in the traditional banking service technologies are cleared by the electronic information technologies. Kim (2006) investigates to find that banking services have developed from paper- and face-based contracts such as Window services to paperless- and faceless-contracts such as Internet Banking services. This paper would contribute to enhance our understanding on the dynamic developments of banking services especially focusing on the substitutability rather than complementarity between banking services.

The paper consists of seven sections. Section 2 presents simplified models of Cournot and Bertrand competitions, and section 3 derives best response functions and Nash equilibrium based on the Cournot game. Section 4 discusses the properties of best response functions and equilibria when demand functions and corresponding best response functions are linear, while section 5 discusses when they are nonlinear. Section 6 investigates when players could collude, even though they are competitors in the market. Section 7 shows estimation results using OLS and IV estimations with discussion on estimated best response functions. Section 8 analyzes with the possibile outcomes of the game. Section 9 offers concluding remarks. The appendix includes the relative competitiveness of Cournot competition to Bertrand competition.

II. Model

Two banks $i=1, 2$ are in the market with cost functions c_i and banking services, x_i, $i=1, 2$, respectively. The total amount of banking services in the market is $x=x_1+x_2$. Market demand function is $x=D(p)$ and its inverse demand function is $p=P(x)$. The sets of strategies of two banks are $S_1=S_2=[0, \infty)$.

The objective functions of two banks based on Cournot competition where quantity is strategic variable for the game are

$$\begin{aligned} \pi_1^C &= P(x_1+x_2)x_1 - c_1(x_1) \\ \pi_2^C &= P(x_1+x_2)x_2 - c_2(x_2) \end{aligned} \tag{1}$$

while the objective functions based on Bertrand competition where price is strategic variable are

$$\begin{aligned} \pi_1^B &= (p_1 - c)D_1(p_1, p_2) \\ \pi_2^B &= (p_2 - c)D_2(p_1, p_2) \end{aligned} \tag{2}$$

where c is constant marginal cost for the two competing banks.

The equilibrium of Cournot competition is not necessarily the same as the equilibrium of Bertrand competition. Since Bertrand competition implies more severe competition between players for the homogeneous banking services, the equilibrium may be more competitive than that of Cournot competition.[2] Because of asymptotic equivalence of Bertrand competition to perfect competition, the market price in the Bertrand competition may be asymptotically equivalent to marginal cost.

III. Nash Equilibrium

In this section, we like to investigate two banks' competition based on Cournot competition strategy for banking services x_1, x_2. The demands of the services are assumed to be highly sensitive to the relative prices even though the services are not completely homogeneous.

For the profit functions of Cournot competition, the first-order condition for Cournot-Nash equilibrium is

[2] Wang and Cho (2004), "Bertrand model produces more competitive result than Cournot model for homogeneous goods markets." p. 91, *Game Theory*. See appendix for more discussions.

$$\pi_i'(x_1, x_2)=0, \quad i=1, 2. \tag{3}$$

so that the best response functions (BR) are to be

$$\begin{aligned} &\pi_1'\{BR_1(x_2), x_2\}=0 \\ &x_1=BR_1(x_2) \\ \\ &\pi_2'\{x_1, BR(x_1)\}=0 \\ &x_2=BR_2(x_1) \end{aligned} \tag{4}$$

from which, Cournot-Nash equilibrium (x_1^*, x_2^*) could be derived. The slope of the best response function is

$$\frac{\partial BR_i(x_j)}{\partial x_j}=-\frac{\pi^{ij}}{\pi^{ii}} \tag{5}$$

where x_1, x_2 are strategic complements if $\pi^{ij}=\partial\pi^j/\partial x_i>0$, while strategic substitutes if $\pi^{ij}<0$. For the former, the best response function has upward-right slope and for the latter, the function has downward-right slope.

Since the profit function for player i is

$$\pi_i(x_1, x_2)=P(x_1+x_2)x_i-c_i(x_i), \tag{6}$$

the first-order condition is $P(x_1+x_2)+x_i[\partial P(x_1+x_2)/\partial x_i]-\partial c_i(x_i)/\partial x_i=0$ so that the best response function is available from

$$x_i=-\frac{P(x_1+x_2)-c_i'(x_i)}{P'(x_1+x_2)} \tag{7}$$

IV. Linear Best Response Function

Now suppose a special linear case $P(x_1+x_2)=a-b(x_1+x_2)$. It should be noticed that the special case of the linearity implies the perfect homogeneity between x_1 and x_2 since they may affect market price with equal weights. Then the best response function would be

$$x_i = -\frac{1}{2}x_j + \frac{1}{2}\frac{a-c}{b}$$
$$= -\frac{1}{2}x_j + \frac{1}{2}\theta \qquad (8)$$

where $\theta=(a-c)/b$, a is demand shifter and c is marginal cost.

Since the model in our study assumes symmetry in the two players' profit functions, $x_i^*=x_j^*$ in equilibrium so that Nash equilibrium is

$$(x_i^*,\ x_j^*) = (\frac{1}{3}\frac{a-c}{b},\ \frac{1}{3}\frac{a-c}{b})$$
$$= (\frac{1}{3}\theta,\ \frac{1}{3}\theta) \qquad (9)$$

and market equilibrium price and total banking services are

$$(p^*,\ x^*) = (\frac{a}{3} + \frac{2}{3}c,\ \frac{2}{3}\theta) \qquad (10)$$

and profits are

$$(\pi_1^*,\ \pi_2^*) = \{\frac{1}{9}(a-c)\theta,\ \frac{1}{9}(a-c)\theta\}. \qquad (11)$$

Due to the clustering of the unknown parameters at a constant term, prior information on the demand function parameters a, b is required. Otherwise, the linear best response function will result in inestimable terms.[3]

In particular, the linearity presupposes that the conjectural variation dx_j/dx_i is always negative and $-1/2$ regardless of the parameter values of a, b implying that market equilibrium price is always greater than the marginal cost. This may heavily limit in advance the dynamics and diversity of the game behaviors between two banks. The market price would be the same as marginal cost when the conjectural variation is -1 which is implausible in any linear case.

[3] If estimates of $\hat{a}$, $\hat{b}$ are available, then the indirect estimate for the marginal cost $\hat{c}$ also is available.

V. Nonlinear Best Response Function

To exploit the benefit of simplicity, a useful compact form of nonlinear response function with respect to strategic parameter γ could be suggested as follows:

$$x_i = BR_i(x_j) = (x_j)^{\gamma} \tag{12}$$

where, as Tirole (1988) points out, banking services of two banks would be strategic complements if $\gamma > 0$, and strategic substitutes if $\gamma < 0$.

To estimate the nonlinear best response functions, a useful method of variable change could be available as follows. First, take logarithm for the best response function $x_i = BR_i(x_j)$, then,

$$\log(x_i) = \log\{BR_i(x_j)\}. \tag{13}$$

Taking derivative with respect to time t, we get

$$\frac{dx_i}{dt}\frac{1}{x_i} = \frac{dBR_i}{dx_j}\frac{dx_j}{dt}\frac{1}{BR_i}. \tag{14}$$

Therefore, the following relation would be available allowing us linear model econometric estimations:

$$\frac{\dot{x}_i}{x_i} = \alpha + \gamma\frac{\dot{x}_j}{x_j} \tag{15}$$

where the growth rate of x_i is a function of the growth rate of x_j, and α is constant term.[4] The benefit of this expression is outstanding when the time series data is non-stationary with (near-) unit root property.

The nonlinear response functions are corresponding with nonlinear demand functions rather than linear demand functions. In reality, actually, (market) demand function is not necessarily linear but it may be rather nonlinear for the parameters in many cases. Examples of nonlinear demand functions when competing banking services are homogeneous would be

[4] For the simplicity, the derivation of constant term is suppressed. However, $x_i = BR_i(x_j) = t^{\alpha}(x_j)^{\gamma}$ is adopted, constant term could be readily derived.

$$p=k-a(x_1+x_2)^b \tag{16}$$

or

$$p=k-a(x_1^b+x_2^b). \tag{17}$$

If parameter values for x_1, x_2 are allowed to be different, demand function could describe differentiated properties between banking services when, for example, x_1 and x_2 are partly homogeneous and partly differentiated:

$$p=k-a(x_1^b+x_2^d) \tag{18}$$

Of course, to allow certain amount of differentiation between banking services in the linearity, a linear form such as $p=a-bx_1-dx_2$ would be an alternative way. However, still economic implications in the linearity may not be rich as the nonlinear case since it a priori presuppose the degrees and directions of competitions of the game.

To study a nonlinear best response function, suppose the nonlinear demand function for homogeneous services is $p=k-a(x_1^b+x_2^b)$. Using the first-order condition for profit maximization, $x_i=-(p-c)/p'$,

$$x_i=-\frac{p-c}{p'}=-\frac{k-a(x_1^b+x_2^b)-c}{-abx_i^{b-1}} \tag{19}$$

so that

$$x_i^b=\frac{1}{1+b}(\frac{k-c}{a}-x_2^b) \tag{20}$$

then, the best response function is

$$x_i=(\frac{1}{1+b})^{1/b}(\frac{k-c}{a}-x_2^b)^{1/b}. \tag{21}$$

If $p=k-a(x_1^b+x_2^d)$ is adopted for demand function, the best response function would be

$$x_i=(\frac{1}{1+b})^{1/b}(\frac{k-c}{a}-x_j^d)^{1/b} \tag{22}$$

and

$$x_j=(\frac{1}{1+d})^{1/d}(\frac{k-c}{a}-x_i^b)^{1/d} \tag{23}$$

from which Nash equilibrium (x_i^*, x_j^*, p^*, π^*) could be derived.

For more differentiated banking services, nonlinear demand function of each banking service would be

$$P_i=k-a(x_i-x_j^b) \tag{24}$$

from which the best response function for the demand function may be

$$x_i=\frac{1}{2}(\frac{k}{a}-c)+\frac{d}{2}x_j^b. \tag{25}$$

Where k/a is demand shifter and c is marginal cost. If $k=ac>c$ is assumed,[5] the best response function would be simplified as

$$x_i=\frac{d}{2}x_j^b. \tag{26}$$

In particular, when the differentiation between banking services is relatively strong, that is, x_i has stronger effect on its own price than x_j has, an appropriate form of the nonlinear demand function may be

$$P_i=a\cdot exp(-\beta x_i t)+x_j^d \tag{27}$$

whose best response function will be

$$x_i=\frac{1}{a(1-\beta t)}exp(\beta x_1 t)(x_j^d-c). \tag{28}$$

[5] If substitutability is assumed, then this specific assumption does not make cost in building econometric model since exponential function with negative slope does not need constant terms.

The implied compact form of the best response function now may be

$$R_i = exp(\beta x_i t) \cdot (x_j)^{\gamma}, \tag{29}$$

from which we get the following linear functional relation for estimating the response function:

$$\frac{\dot{x}_i}{x_i} = \alpha + \beta x_i + \gamma \frac{\dot{x}_j}{x_j} \tag{30}$$

where the growth rate of x_i is function of itself and the growth rate of the competing banking service x_j.

VI. When is Equilibrium Collusive?

Is the Nash equilibrium collusive or competitive when the 'increasing returns to scale' phenomenon is taken place in the banking services? Collusion is referring to explicit or implicit cooperation or collaboration on (p, x) among rival players to exploit consumer surplus for their own profits.[6]

IT technology contributes to cost reduction making the best response function shift to the upward-right. Suppose there are IT innovations in both banks for their banking services. This implies that both best response functions of two banks are moving outward. In the case, a collusive situation can take place unless banks are eager to decrease prices for their services. If price adjustment takes time or is sticky, there would be also collusion-similar equilibrium.

The degree of collusion among players may be measured by the distance of collusive market price from competitive market price. Since the players collude to behave like a firm for their common interest, equilibrium under the collusion may be equivalent to the monopolistic equilibrium. Because of this, the degree of collusion may be alternatively measured by the inverse of the distance between collusive equilibrium and monopolistic equilibrium.

For the linear case, suppose marginal cost decreased from c to $c' < c$. Then the best response function would be

[6] Figure 11 shows a case when collusion can take place.

$$x_i = -\frac{1}{2}x_j + \frac{1}{2}\frac{a-c'}{b} \tag{31}$$

in the case, if the market is competitive,

$$(p^*,\ x^*) = (\frac{b}{3}\theta' + c',\ \frac{2}{3}\theta') \tag{32}$$

where $\theta' = (a-c')/b$ and

$$(\pi_1^*,\ \pi_2^*) = \{\frac{1}{9}(a-c')\theta',\ \frac{1}{9}(a-c')\theta'\}. \tag{33}$$

However, if the market is collusive, market price and Nash equilibrium possibly be, for example, the same as the Nash equilibrium before IT shocks happen so that the collusive benefits may be

$$\textit{collusive exploitation} = \frac{1}{9}(c\theta - c'\theta'). \tag{34}$$

VII. Estimating Best Response Functions

A. Data and Test Equations

In this section, we like to estimate the best response functions between banking services. The empirical studies utilize banking services data from transactions at teller windows, ATMs and via internet banking, during the period of January 1990 to April 2005 where the end period is confined since the data source institutions do not release after April 2005. Data sources are Bank of Korea, and KFTC (Korea Financial Telecommunication and Clearing Institute) that is a clearing house for retail banking services. Data is classified into three variables: Window services, ATM services and Internet Banking services where the services are measured by the number of transactions of provided services.

Figure 1 highlights diverse types of banking services such as tellers, ATMs, telebanking, internet banking, mobile banking service, *etc.* According to the figures, teller window and ATM services have long histories, while telebanking and internet banking services are relatively new.

According to the figures, there are dynamic interactions between banking services: teller window and ATM services grew together, until 2001. At that point, teller window services began to decrease while ATM services kept growing. As electronic banking services such as telebanking and internet banking services develop, ATM service is declining.

Since we will investigate three pairs of strategic banking service relations, there will be three sets of best response function equations for each of two nonlinear models. Henceforth, we deal with 6 estimation results.

Three sets of pairs are (Window service, ATM service), (ATM service, Internet Banking service) and (Telebanking service, Internet Banking service). Two econometric test equations are

$$\frac{\dot{x}_i}{x_i} = \alpha + \gamma \frac{\dot{x}_j}{x_j} \tag{35}$$

which is derived from $x_i = BR_i(x_j) = (x_j)^{\gamma}$, $i \neq j = 1, 2$ and

$$\frac{\dot{x}_i}{x_i} = \alpha + \beta x_i + \gamma \frac{\dot{x}_j}{x_j} \tag{36}$$

which is implied in the response function of $x_i = BR_i(x_j) = exp(\beta x_i t)(x_j)^{\gamma}$, $i \neq j = 1, 2$.

B. Hausman Test

Before going further advance toward estimation, our careful attention should be given to so-called simultaneity problem. Since the interactions between banking services x_i and x_j are presupposed, the simultaneity problem is presumed in the model.

To verify the simultaneity problem in the model, the Hausman test may prove useful. When the orthogonality condition between explanatory

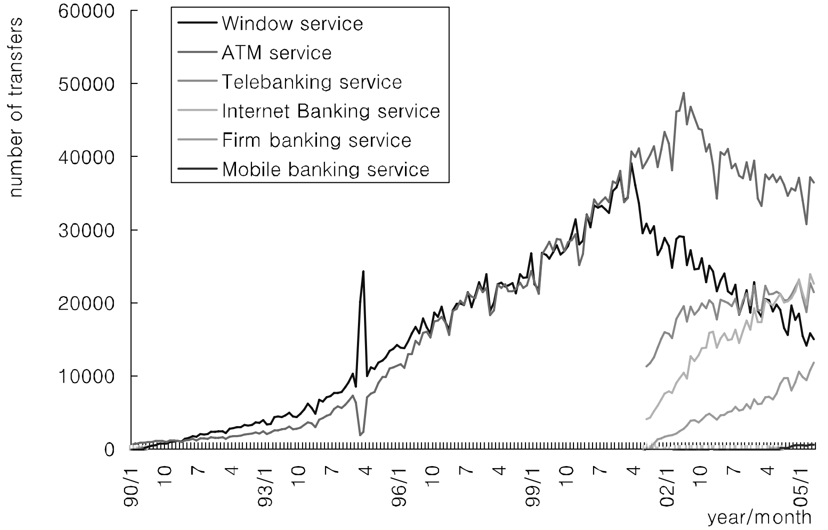

FIGURE 1
DIVERSE TYPES OF BANKING SERVICES

variables and error terms is not satisfied, ordinary least squares (OLS) estimation does not produce consistent results. However, instrument variable (IV) estimation produces consistent estimators as long as appropriate, high quality instrument variables are available.

The Hausman statistic is defiend as

$$m=(\gamma_{IV}-\gamma_{OLS})'\{var(\gamma_{IV})-var(\gamma_{OLS})\}^{-1}(\gamma_{IV}-\gamma_{OLS})\sim\chi^2 \tag{37}$$

which follows chi-square distribution. However, using the relationship between chi-square distribution and t-distribution, the Hausman statistic could be redefined as follows.

$$mt=(\gamma_{IV}-\gamma_{OLS})\{var(\gamma_{IV})-var(\gamma_{OLS})\}^{-1/2}\sim t_n \tag{38}$$

which follows t-distribution where n is the degree of freedom.

To show the effectiveness of IVs, the property of limit distribution of Hausman test can be utilized. It should be noted that the limit distribution of Hausman statistic may be degenerated to be around zero if the quality of instrument variables are poor since[7]

TABLE 1

COMPETITION BETWEEN WINDOW (x_1) AND ATM (x_2)

variables	For $\dot{x}_1/x_1$				For $\dot{x}_2/x_2$			
	OLS1	OLS2	IV1	IV2	OLS3	OLS4	IV3	IV4
constant	0.01*** (3.08)	0.19*** (5.52)	0.014*** (2.92)	0.18*** (4.97)	0.0004** (1.93)	0.016*** (11.5)	0.002** (2.72)	0.04*** (7.24)
x_1		-0.04*** (-5.14)		-0.04*** (-4.63)				
x_2						0.116*** (49.5)		-0.07 (-6.98)
$\dot{x}_1/x_1$					0.13*** (44.09)	-0.03*** (-11.3)	0.01 (1.08)	-0.014 (-1.48)
$\dot{x}_2/x_2$	-0.18*** (-2.49)	-0.21*** (-3.1)	-0.18*** (-2.38)	-0.212*** (-3.01)				
R^2	0.03	0.16	0.03	0.13	0.91	0.95	0.0006	0.22
$\bar{R}^2$	0.03	0.15	0.03	0.12	0.91	0.95	0.0009	0.21
F	6.18	16.7	5.65	13.85	1944	1711	1.17	25.1
H-statistic	OLS1 OLS2	vs. vs.	IV1: IV2:	0.00 0.11	OLS3 OLS4	vs. vs.	IV3: IV4:	-13.6 2.85

Note: values in ( ) are t-values. * implies significance in 90%, ** implies 95%, and *** implies 99%. H-statistic is Hausman statistic for simultaneity. If its value is beyond critical value 3.87, it implies simultaneity in the system.

$$plim(\gamma_{IV} - \gamma_{OLS}) \rightarrow 0. \tag{39}$$

Therefore, the effectiveness of instrument variables could be verified using Hausman test results. In other words, as long as Hausman test produces significant results, it implies the effectiveness of IVs.

Hausman test results in Table 1, 2, 3 produce significance showing that there are serious simultaneity problem in the system, and that the IVs exploited in the estimations are appropriate. Hausman H-statistics are all significant except when regressand is $\dot{x}_1/x_1$ in Table 1.

C. Games between Window and ATM services

In this subsection, for the first case of estimation, x_1 is supposed to be teller window service by bank 1 and x_2 is ATM service by bank 2. At a glance, teller window and ATM services are different. ATM service is

[7] The property is verified in Kim, J. (2005), "Asymptotic relationship between OLS and 2SLS with weak instruments." *Korean Economic Journal* 44(3-4), Seoul National University.

faceless and paperless, while teller window service is personal and requires a paper exchange. In these senses, the two services are differentiated. However, since service fees incite (or depress) demand, they are regarded as partly homogeneous, even though the services differ somewhat.[8]

For the estimations, OLS and IV estimates are suggested in Table 1. Instruments for IV are lagged own variables whose quality is verified so that a Hausman test will be meaningfully interpreted.

According to the estimation results, OLS 1, 2, 3, 4 and IV 1, 2, 3, 4 imply no simultaneity between teller window service and ATM service. Estimation results show that teller window service is gradually replaced with ATM services, while the reverse is not true. OLS 2 says that teller window service will declines at the rate of 0.21% if ATM service is increased 1%. ATM service decreases at the rate of 0.03% if teller window service is increased 1%. Therefore, the estimated best response functions are

$$\begin{aligned} \hat{x}_1^{window} &= x_2^{-0.18} \\ \hat{x}_1^{window} &= exp(-0.04tx_{1,-1})x_2^{-0.21} \end{aligned} \tag{40}$$

where $x_{1,-1}$ is lagged explanatory variable of x_1, and[9]

$$\begin{aligned} \hat{x}_2^{ATM} &= x_1^{-0.03} \\ \hat{x}_2^{ATM} &= exp(-0.07tx_{2,-1})x_1^{-0.014} \end{aligned} \tag{41}$$

This means that for the simple model, the speed of transformation from Window service to ATM service is 0.15% (=0.18%−0.03%) per month.

Figures 1 and 4 show the simulated best response function using estimated parameter values for the simple and complicated nonlinear models.

[8] Of course, there are no perfectly homogeneous services in reality. Such a dichotomy would help explain how certain property would affect competition between players and as a result, market equilibrium.

[9] To draw the best response functions in figure 1~9, t is assumed to be 1 for simplicity. Since on the vertical line is x_2, the estimated equations should be inverted to draw best response functions if necessary. Note that when x_2 is both explained and explanatory variable, the one as explanatory variable is adopted to be lagged one to avoid complicatedness without hurting essential features.

TABLE 2

COMPETITION BETWEEN ATM (x_1) AND INTERNET BANKING (x_2)

variables	For $\dot{x}_1/x_1$				For $\dot{x}_2/x_2$			
	OLS1	OLS2	IV1	IV2	OLS3	OLS4	IV3	IV4
constant	-0.01*** (-2.86)	-0.17 (-0.41)	-0.00*** (-0.04)	-1.22** (-2.20)	-0.00 (-0.001)	-0.0002 (-0.26)	-0.0004 (-0.91)	-0.15* (-1.86)
x_1		0.03 (0.38)		0.266** (2.20)				
x_2								0.228* (1.86)
$\dot{x}_1/x_1$					0.095*** (747)	0.095*** (683.1)	-0.05** (-2.42)	-0.05** (-2.56)
$\dot{x}_2/x_2$	0.58*** (6.94)	0.56*** (5.87)	-0.34** (-2.40)	-0.39** (-2.82)				
R^2	0.53	0.53	0.15	0.26	0.99	0.99	0.15	0.23
$\bar{R}^2$	0.52	0.51	0.12	0.21	0.99	0.99	0.13	0.19
F	48.2	23.7	5.76	5.63	5582	2930	5.90	4.90
H-statistic	OLS1	vs.	IV1:	-8.07	OLS3	vs.	IV3:	-6.90
	OLS2	vs.	IV2:	-9.50	OLS4	vs.	IV4:	-7.63

Note: values in ( ) are t-values. * implies significance in 90%, ** implies 95%, and *** implies 99%. H-statistic is Hausman statistic for simultaneity. If its value is beyond critical value 3.87, it implies simultaneity in the system.

It should be noted that the condition for the stability of Nash equilibrium is

$$\left|\frac{dBR_2(x_1)}{dx_1}\right| < \left|\frac{1}{\frac{dBR_1(x_2)}{dx_2}}\right| \tag{42}$$

i.e.,

$$\left|\frac{dBR_2(x_1)}{dx_1}\right| \cdot \left|\frac{dBR_1(x_2)}{dx_2}\right| < 1 \tag{43}$$

whose value for the game between Window and ATM services is 0.0054 $(=0.018\times|-0.03|)<1$ that satisfies the condition.

D. Games between ATM and Internet Banking services

$$\begin{aligned} \hat{x}_1^{ATM} &= x_2^{-0.34} \\ \hat{x}_1^{ATM} &= exp(0.266\, tx_{1,-1})x_2^{-0.39} \end{aligned} \tag{44}$$

TABLE 3
COMPETITION BETWEEN TELEBANKING (x_1) AND INTERNET BANKING (x_2)

variables	For $\dot{x}_1/x_1$				For $\dot{x}_2/x_2$			
	OLS1	OLS2	IV1	IV2	OLS3	OLS4	IV3	IV4
constant	-0.0007*** (-2.97)	-0.28** (-1.98)	0.01** (2.20)	0.17 (0.43)	0.00* (1.80)	0.003*** (5.81)	0.001** (1.65)	0.0002 (0.004)
x_1		0.06* (1.94)		-0.03 (-0.40)				
x_2						-0.005*** (-5.78)		0.001 (0.01)
$\dot{x}_1/x_1$					0.10*** (491)	0.101*** (651)	-0.047** (-2.49)	-0.047** (-2.43)
$\dot{x}_2/x_2$	0.80*** (14.5)	0.80*** (15.1)	-0.38 (-2.77)	-0.40 (-2.73)				
R^2	0.83	0.84	0.15	0.16	0.99	0.99	0.13	0.13
$\bar{R}^2$	0.83	0.84	0.13	0.12	0.99	0.99	0.11	0.09
F	210	113.9	7.69	3.85	2419	2123	6.19	3.02
H-statistic	OLS1	vs.	IV1:	4.06	OLS3	vs.	IV3:	-7.74
	OLS2	vs.	IV2:	3.87	OLS4	vs.	IV4:	-7.79

Note: values in ( ) are t-values. * implies significance in 90%, ** implies 95%, and *** implies 99%. H-statistic is Hausman statistic for simultaneity. If its value is beyond critical value 3.87, it implies simultaneity in the system.

and

$$\hat{x}_2^{Internet\ \ Banking} = x_1^{-0.05} \qquad (45)$$
$$\hat{x}_2^{Internet\ \ Banking} = exp(0.228tx_{2,-1})x_1^{-0.05}$$

This means that for the simple model, the speed of transformation from ATM service to Internet Banking service is 0.29% (=0.34%−0.05%) per month.

Figures 2 and 5 show the simulated best response function for the simple and complicated nonlinear models, and the stability condition of Nash equilibrium is satisfied.

E. Games between Internet Banking and Telebanking services

$$\hat{x}_1^{Telebanking} = x_2^{-0.38} \qquad (46)$$
$$\hat{x}_1^{Telebanking} = exp(-0.03\,tx_{1,-1})\,x_2^{-0.40}$$

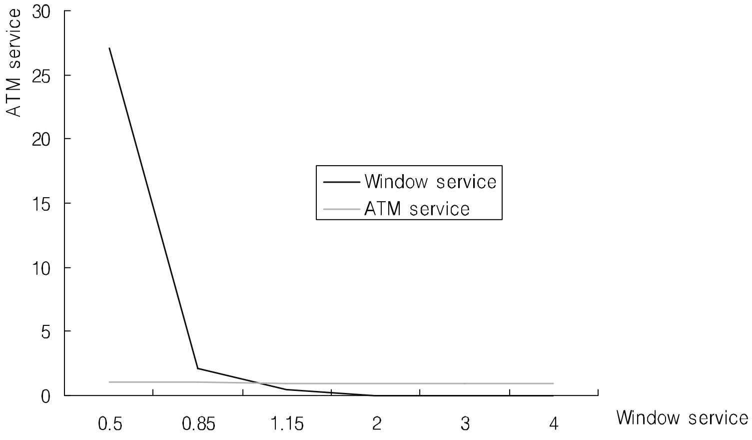

FIGURE 2
SIMULATED BEST RESPONSE FUNCTION: WINDOW AND ATM SERVICE

and

$$\hat{x}_2^{Internet\ \ Banking} = x_1^{-0.047}$$
$$\hat{x}_2^{Internet\ \ Banking} = exp(0.001\,tx_{2,-1})x_1^{-0.047} \tag{47}$$

This means that for the simple model, the speed of transformation from Telebanking service to Internet Banking service is 0.333% (=0.38%−0.047%) per month.

Figures 3 and 6 show the simulated best response function for the simple and complicated nonlinear models, and the stability condition of Nash equilibrium is satisfied.

F. Speed of Transformation

It is found in the estimation that the speed of transformation between banking services becomes faster along the degree of application of IT technology. For the simple form of estimation equation, the speed of transformation from Window service to ATM service is estimated as 0.15% while the speed from ATM service to Internet Banking service is estimated 0.29%. For the complicated form, the speed of transformation from Window service to ATM service is estimated as 0.18% while the speed from ATM service to Internet Banking service is estimated 0.34%.

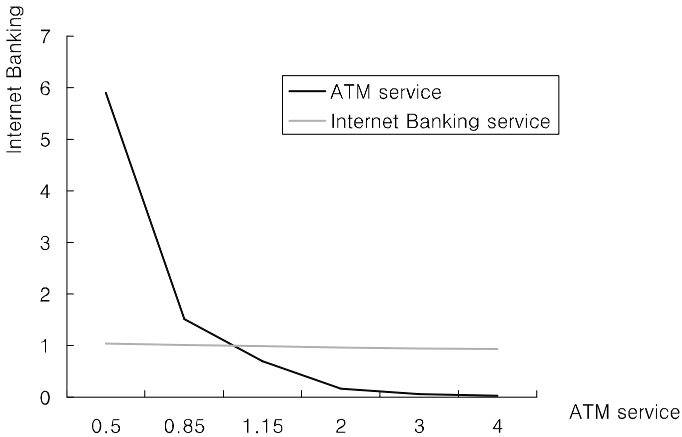

FIGURE 3
SIMULATED BEST RESPONSE FUNCTION: ATM AND INTERNET BANKING

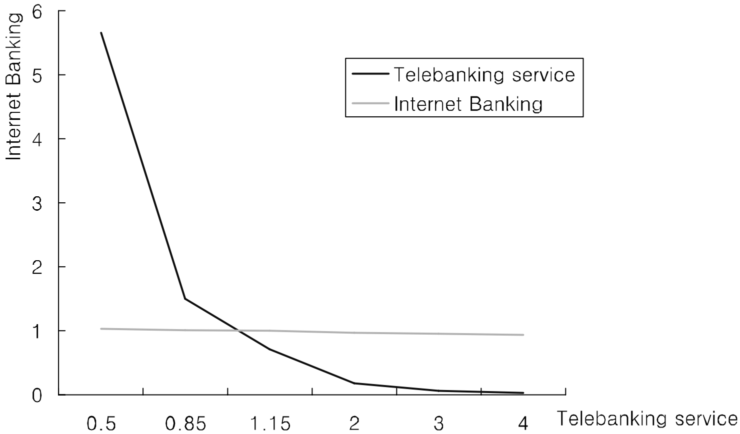

FIGURE 4
SIMULATED BEST RESPONSE FUNCTION: TELEBANKING AND INTERNET BANKING

These results imply that substitute effects is stronger as banking services utilize IT technology.

The speed of transformation from Telebanking service to Internet Banking service is 0.333% for the simple model and is 0.353% for the complicated model.

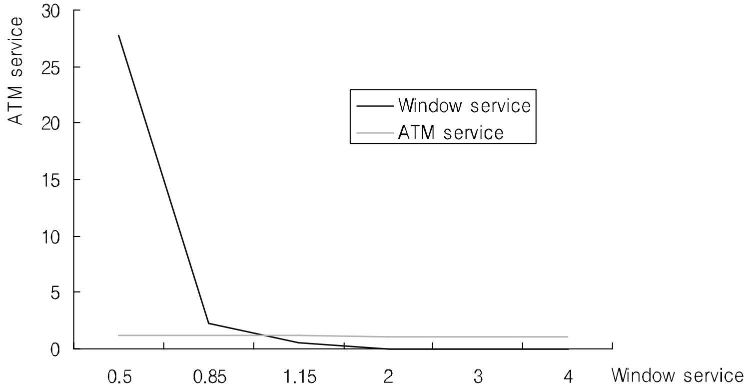

FIGURE 5

SIMULATED BEST RESPONSE FUNCTIONS: WINDOW AND ATM SERVICES

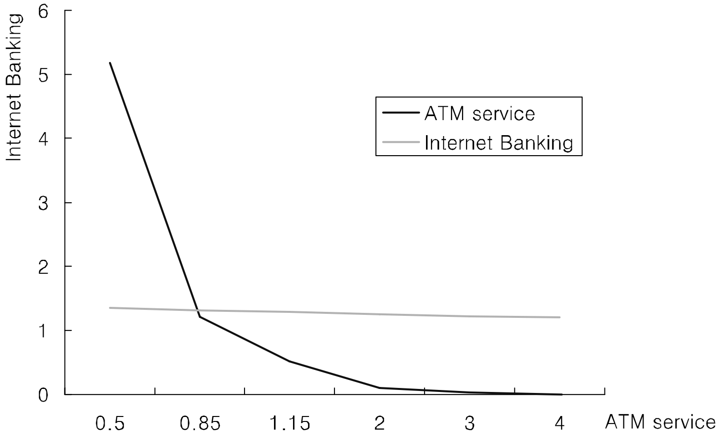

FIGURE 6

SIMULATED BEST RESPONSE FUNCTIONS: ATM AND INTERNET BANKING

VIII. Possibility of Collusion

Figures 7 to 9 show certain possibility of collusion between players. Figure 7 contains two best response functions for ATM and telebanking services against internet banking service, where the best response function of telebanking service is at the outer position. This position of the response function argues that marginal cost to produce telebanking service is lower compared to the marginal cost to produce ATM service.

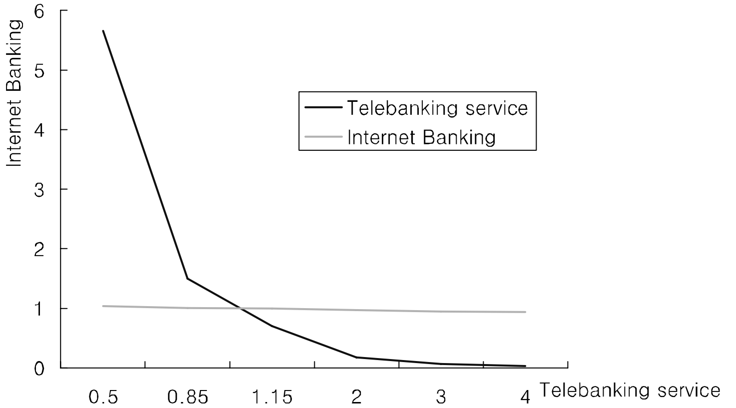

FIGURE 7
SIMULATED BEST RESPONSE FUNCTIONS: TELEBANKING AND INTERNET BANKING SERVICES

Figure 8 shows three best response functions where the best response function of internet banking service is added to figure 7. Nash equilibrium A is an intersection between the best response function of ATM service and the best response function of internet banking service. Nash equilibrium B is an intersection between the best response function of telebanking service and the best response function of internet banking service. Since the marginal cost of telebanking service is less than that of ATM service, service fees for telebanking and internet banking services should be less than the service fees for telebanking alone. Otherwise, there may be a collusion between players for telebanking and internet banking services.

Figure 9 adds an internet banking service best response function to figure 8 which describes interactions between internet banking and telebanking service. Nash equilibrium A is an interaction between ATM and internet banking services (internet banking best response function 1), and Nash equilibrium B is an interaction between telebanking and internet banking services (internet banking best response function 2). The Nash equilibria imply that the marginal cost of internet banking service competing with telebanking service is larger than when competing ATM service. The best response function for internet banking service competing with telebanking service is lower than that when competing with ATM service.

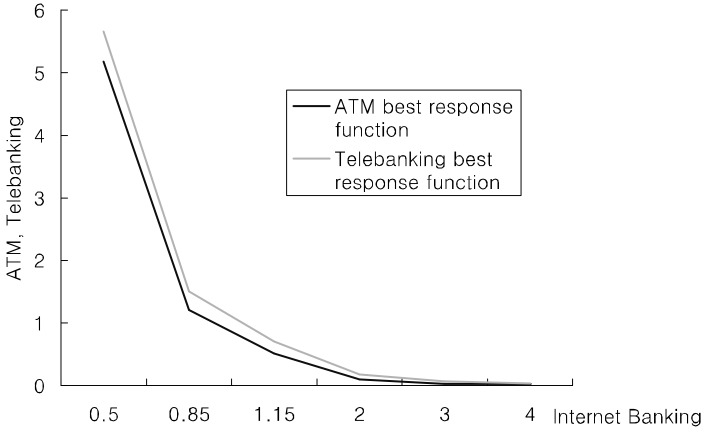

FIGURE 8
COLLUSIVE OR COMPETITIVE

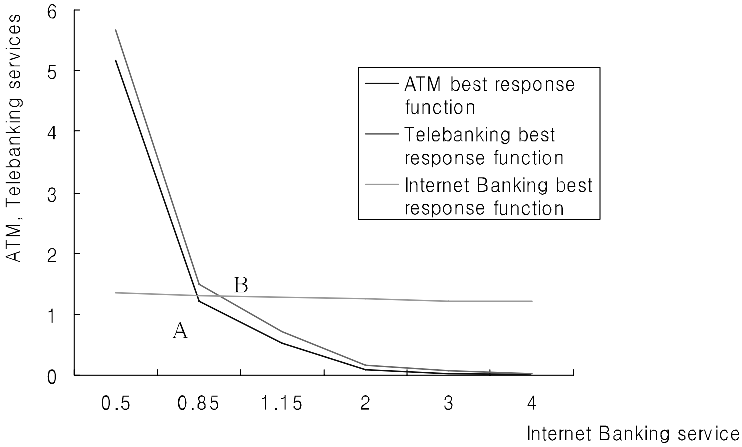

FIGURE 9
COLLUSIVE OR COMPETITIVE

These dynamics imply the service fees in telebanking are lower than those for ATM service. The service fees for internet banking are larger when competing with telebanking than when competing with ATM service. Therefore, if obvserved fee structures are not similar or equivalent to the implied ones, there may be collusion between players.

Figure 10 shows the case where players collude. Nash equilibrium B

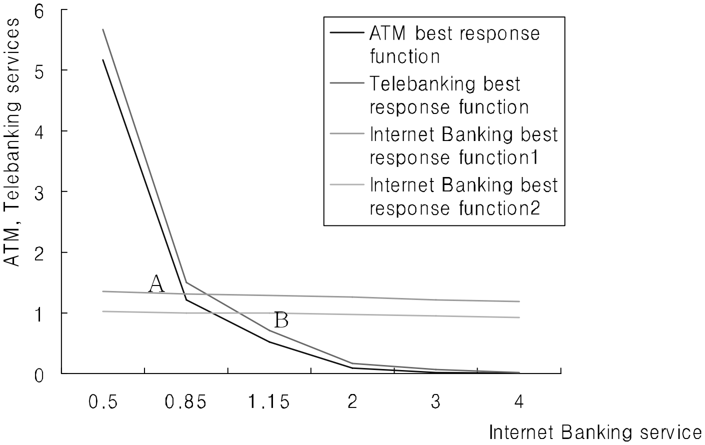

FIGURE 10
COLLUSIVE OR COMPETITIVE

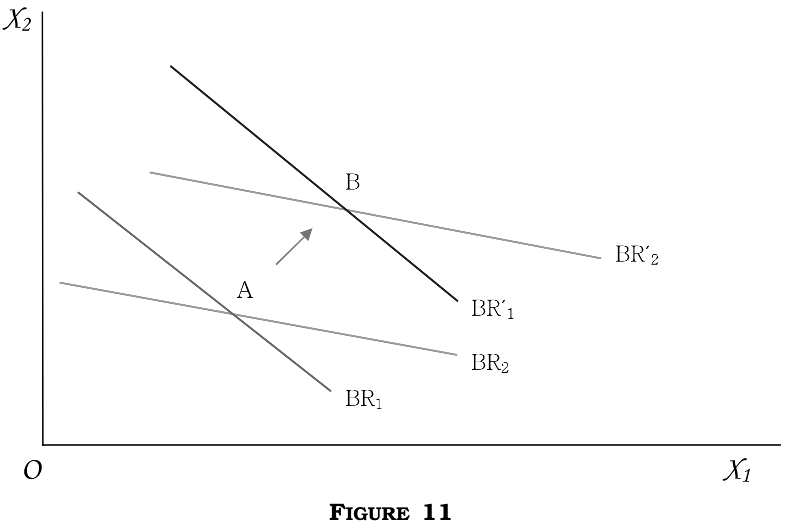

FIGURE 11
POSSIBILITY OF COLLUSION

is the result of reduced marginal costs between interacting players so that the fees in equilibrium B are smaller than those in A. Therefore, if the fees for B are the same as those for A, there may be a collusion between players.

IX. Concluding Remarks

Best response functions and Nash equilibria may be better described with nonlinearity than with linearity. The nonlinearity of best response functions is dependent on the nonlinearity of the demand function, regardless of whether it is an individual or market demand function. If market demand functions in the Cournot model or individual demand functions in the Bertrand model are linear (nonlinear), then the best response functions will be linear (nonlinear).

In the linear model, the dynamics and properties of Nash equilibrium are a priori, straight forward regardless of the diverse parameter values. That is, the dynamics of the game are stipulated by a limited value of $-1/2$ implying that no further investigation makes a contribution to understanding the dynamics of the game.

However, nonlinearity describes the game more realistically and contains rich implications allowing diverse possibilities of dynamics. Our investigation shows that the pair games between banks with ATMs, telebanking and internet banking services carries the existence of stable Nash equilibria with possibility of collusion between players. It is also found that the transformation of traditional banking services into electronic banking services accelerates with the help of the information technology.

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Appendix: Proof of dominance of Cournot competition over Bertrand competition

Since non-zero profits would be preferred to zero profits, Bertrand competition may be less desirable than Cournot competition. If this is true, players would try to choose quantity as strategic variables for competition regardless of the degree of homogeneity or differentiation of their competing services. In particular, since information technology-based services are readily possible to be increased in its capacities, banking services may not be strongly constrained by the 'time to build' even in the short-run unlikely the doubts raised by Bertrand (1883).

For the special case of linear inverse demand function $p=a-b(x_1+x_2)$ and cost functions cx_i, $i=1$, 2, the dominance of Cournot competition over Bertrand competition in the homogeneous banking services could be readily shown. In this sense, Cournot model would be more relevant in analyzing competitions than Bertrand model would even when there are certain differentiations.

Let's take an example for the dominance of Cournot competition over Bertrand competition in the homogeneous banking services. For the special case of linear inverse demand function $p=a-b(x_1+x_2)$ and cost functions cx_i, $i=1, 2$, the best response functions of two banks based on Cournot competition strategies are respectively

$$x_1^C=BR_1(x_2)^C=\frac{a-c-bx_2}{2b}$$
$$x_2^C=BR_2(x_1)^C=\frac{a-c-bx_1}{2b} \tag{48}$$

and Nash equilibrium is

$$x_1^{*C}=x_2^{*C}=\frac{a-c}{3b} \tag{49}$$

where the profit is

$$\pi_1^{*C}=\pi_2^{*C}=\frac{(a-c)^2}{9b} \tag{50}$$

greater than zero as long as $a>c$, which is always plausible, while the profit based on Bertrand competition is zero. This result tells that even myopic behaviors of competing banks in the Cournot Competition guarantee non-zero profits.

Now lets investigate the relative dominance when the services are differentiated. Bertrand models for the differentiated services are

$$D_1(P_1,\ P_2)=a-P_1+bP_2$$
$$D_2(P_1,\ P_2)=a-P_2+bP_1 \tag{51}$$

where two services are substitute if $b>0$, while complements if $b<0$. Assume marginal cost is zero for simplicity. Then, the best response functions are

$$p_i = \frac{a+bp_j}{2}, \quad i \neq j = 1,\ 2 \tag{52}$$

and the market price is $p_i = a/(2-b)$, $i=1,\ 2$, and profits are

$$\pi_i^* = (\frac{a}{2-b})^2, \quad i=1,\ 2 \tag{53}$$

However, if the competition is severe between players, market price would be the same as marginal cost assumed to be zero, henceforth, the profit would be degenerated into zero since

$$\pi_i^* = (p_i)^2 \rightarrow 0, \quad i=1,\ 2 \tag{54}$$

Cournot models for the differentiated services are

$$\begin{aligned} P_1 &= g - q_1 + hq_2 \\ P_2 &= g - P_2 + hP_1 \end{aligned} \tag{55}$$

the best response functions are

$$q_i = \frac{g+hq_j}{2},\ i \neq j = 1,\ 2 \tag{56}$$

and the market price is $p_i = 1/2\{(2+\mathrm{h})^2/(4-h)\}g$, $i=1,\ 2$, and the profits with Cournot model are

$$\pi_i^* = 2(1+\mathrm{h})(\frac{2+h}{4-h})^2\ g^2,\ i=1,\ 2. \tag{57}$$

that is greater than zero as long as $h > -1$ and $g > 0$, in which Cournot competition may dominate Bertrand competition in the perspective of profits.

References

Allen, Franklin, McAndrews, James J., and Strahan, Philip. "E-Finance: an introduction." *Journal of Financial Services Research*

22 (No. 1 2002): 5-27.

Berger, Allen N. "The economic effects of technological progress: evidence from the banking industry." *Journal of Money, Credit and Banking* 35 (No. 2 2003): 141-172.

Bertrand, J. "Theorie Mathematique de la Richesse Sociale." *Journal des Savants* 67 (Septembre 1883): 499-508.

Giannakoudi, Sofia. "Internet banking: the digital voyage of banking and money in cyberspace." *Information & Communication Technology Law* 8 (No. 3 1999): 205-43.

Hausman, Jerry A. "Specification tests in econometrics." *Econometrica* 46 (No. 5 1978): 1251-71.

Kauffman, Robert J., McAndrews, James, and Wang, Yu-Ming. "Opening the 'black box' of network externalities in network adoption." *Information Systems Research* 11 (No. 1 2000): 61-82.

Kim, Jabonn. "Asymptotic relationship between OLS and 2SLS wih weak instruments." *Korean Economic Journal* 44 (Nos. 3-4 2005): 587-605.

__________. *Recent Development of Electronic Finance in Korea*. Korea Institute of Finance, 2006 (in Korean).

Korea Institute of Finance. *Understanding on the Digital Finance*. Seoul, Korea: KIF Press, 2003 (in Korean).

Lee, Choong-Lyul, Kim, Jae-Pil, and Lee, Young Soo. *Digital Banking*. Seoul, Korea: Hong Press, 2000 (in Korean).

Lee, Sanche. "The survival and competition of retail electronic payment instruments." *Review of Financial Economics* 15 (No. 2 2001): 61-85 (in Korean).

McAndrews, James J. "E-money and payment system risks." *Comtemporary Economic Policy* 17 (No. 3 1999): 348-57.

Tirole, Jean. *The Theory of Industrial Organization*. USA: MIT Press, 1988.

Wang, Kyu Ho, and Cho, In-Koo. *Game Theory*. Seoul, Korea: ParkYoungSa, 2004 (in Korean).