A MODEL TO PREDICT MARKET SHARE IN URBAN TRAVEL DEMAND

(大都市近郊의 交通行態論)

by

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1.1 Problem Definition and Objectives

The objective of this study is to provide a mdoel to predict modal split for urban journey to work.

The freedom offered by the motor vehicle is counteracted by growing congestion, parking deficiencies, rising number of accidents, decreasing patronage of public transport, and a changing environment.

Most transportation planners agree that to solve the above transportation problems, it is desirable to divert many automobile users onto some form of mass transportation. Whether such a diversion is possible depends on the answers to two questions:

(1) the degree to which the characteristics of public transportation can be improved relative to those of the automobile and,

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(2) the responsiveness of commuters to such changes.

This proposed study will attempt to provide a model for answering the second question: how responsive are people to changes in the characteristics of transportation system? How large a relative improvement is necessary to induce people onto transit and out of automobiles?

The study herein will be limited to home-to-work journey in urban areas because it is believed among most transportation planners that they are the main source of traffic congestion, and a highway or transit system adequate to deal with the journey to work will have capacity enough to deal with shopping and recreation trips, etc.

A salient characteristic of the urban work trip is in its demand inelasticity in the short run situation. People are not so free to change their jobs and residences in the short run situation, even though the mobility (to change jobs and residences) is relatively high in the long run situation.

On the other hand, inelasticity is a characteristic that simplifies the analysis. In addition, much of the available data are on the journey to work.

This study herein will also be limited to the prediction of choice of mode. The amount of trip generation will be assumed to be given. The ability to predict the relative share of travel market by each mode, when a number of alternative modes are given, is obviously of great interest, both in evaluating the probable success of proposed new systems, and in providing information to transportation planners about which system characteristics are most important, and hence which improvements commuters will be most responsive to.

In a nutshell, the objective of this study is to provide a model to predict modal split for urban journey to work.

1.2 Structure of the Model

The structure and outline of the model to be developed in this thesis will be described here briefly.

Travel cost, travel time, and relative comfort are proposed here as independent variables. The frequency of service, degree of privacy and independence, and decency of service of travel modes are incorporated into one variable in the name of relative comfort and convenience. Detailed discussions on the choice of independent variables will be in Section(2. 1).

The structure of the model is of two stages: at the first stage the abstract modal characteristics determine 'partial split ratios'. At the second stage, empirical procedures are designed to determine the weighting coefficients for those partial split ratios.

A partial split ratio for a given variable is defined as the ratio of trips by a particular mode to total trips if that variable were the only consideration. As three variables are considered in the model, there are three partial split ratios.

A final split ratio is defined as a weighted linear combination of the partial split ratios. The weighting coefficients are normalized fractional numbers which indicate the relative weights or relative importance that a given population group place on cost saving, time saving and comfort & convenience.

Thus, the model will represent modal split as the fractional areas of a *unit square*, a square with sides of unit length. The trips over all modes are represented by the total area of the unit square (area=1, or 100%), and particular mode trips are represented by a fractional area of the unit square.

Fractional areas, which represent particular mode trips, are formed by two components; horizontal and vertical. A partial split ratio determines the length of the vertical component and the weighting coefficient determines the length of the horizontal component. All quantities are so normalized that summation over all modes becomes one, i.e., the area of the unit square.

1.3 Potential Contributions of the Model

The contributions of this thesis may be thought of in terms of methodological establishments. The concept of partial split ratios, a methodological artifice to facilitate the analysis, which is based on the application of psychophysics to the analysis of travel demand behavior, may prove to be a useful tool in transportation study.

Another claim of contribution could be made of the exploration of possible analogical relationships between the electrical current split system and transportation modal split system. Limits to the analogy and, hence, the required modifications to the relationships, have been explored. The significance of this analogy study is that it has opened a door to a future study on the possibility of forecasting the travel volume split between mixed mode systems.

Currently modal split models are usually engaged in, and capable of, analysing modal split systems as in Figure 1.1. They are, however, almost incapable of predicting demand split among complicated systems of mixed modes as depicted in Figure 1.2.

System 1: A —— Mode$_1$ —— B
System 2: A —— Mode$_2$ —— B
System 3: A —— Mode$_3$ —— B

Three Systems connecting Node A and Node B.

Fig. 1.1

System 1: A —— Mode$_1$ —— B
System 2: A —— Mode$_2$ —— • —— Mode$_3$ —— B
System 3: A —— Mode$_4$ —— [Mode$_5$ / Mode$_6$] —— B

Fig. 1.2: Mixed mode systems

In the model, an abstract travel mode is represented by its impeding effects calculated from the mode's abstract modal characteristics. This impeding effect corresponds to resistance of a conductor which carries electric current. Hence, this electrical resistance analogy may serve as a starting point for future study on travel modal split among multimodal mixed systems by utilizing series and parallel network concepts.

Capitalization on the concept of entropy to determine weighting coefficients comprises another potential contribution of the thesis. The conventional way to determine the coefficients of a projection model is to estimate them against cross-sectional data. A cross-sectional relationship at time t is, however, not the same as one at time $t+\Delta t$. The maximum entropy technique employed in the model could be capable of long-range forecast through the incorporation of income variable which changes over time.

2. THE MODEL

2. 1. Variables

It is, of course, not possible to explicitly include in a travel demand model all of the many different variables which affect the travel demand, because of the complexities involved and lack of data with respect to some of them.

In view of these considerations and requirements, this model will choose travel cost, travel time and discomfort and inconvenience(to be abbreviated as D&I index) as independent variables. These three variables have been proved to be most important and significant ones by many previous studies. Furthermore, these three variables may be considered, from the standpoint of a transportation planner, as instrumental or control variables useful for implementing some normative goals.

As regards the travel cost and travel time, their concepts as accomodated usually by many previous studies will be utilized in the model. However, the concept of discomfort and inconvenience will mean in the model to include those arising from the infrequency of service, lack of privacy and independence, and lack of decency of service of a mode of travel. Each of these components is to be a discomfort sub-index. These sub-indices will be aggregated to form a final 'discomfort index'.

(a) Frequency of Service

Frequency of service is measured in terms of number of departures per unit time as determined from schedules or survey.

Obviously, the more frequent the service of a travel mode is, the more convenient it is to its users. Therefore, the frequency of service should be inverted in order to indicate the measure of inconvenience in using its mode. Hence, we choose to define 'frequency sub-index', denoted by I_f, as

$$I_f=\frac{1}{f}$$

where f stands for the frequency of service per day (or number of departures per day).

According to the above definition, the more frequent the service is, the smaller I_f becomes. In case of private passenger car, we assume that the frequency of service is completely at the user's disposal. This amounts to saying that the automobile always provide an indefinite frequency of service, so that its user is not inconvenienced as far as frequency of service is concerned.

Without any a priori reason, we choose the range of each sub-index to be 5. Hence, I_f will be multiplied by 5 so that it can vary between 0 and 5.

(b) Privacy and Independence

According to the University of Michigan Survey Research Center interviews, among the most frequent reasons given for prefering automobile travel (to public transportation modes) were privacy and independent selection of route.

We choose to define 'privacy' in terms of the ability to control selection of co-passenger(s). This definition include, naturally, the selection of *no* co-passenger.

We choose to define 'indepedence' in terms of the ability to control selection of travel route and destination.

As privacy and independence defy explicit quantification, the model will employ proxy variables to deal with them. In accordance with the above definitions, the ability to control selection of co-passenger(s) will stand proxy for privacy, and the ability to control selection of travel route will stand proxy for independence. Furthermore, the nature of these proxies is such that we can limit each to a binary-valued description.

Let C_p stand for the controllability over selection of co-passenger, and C_r stand for the controllability over the selection of route. When a mode of travel allows one to have control over selection of co-passenger, C_p will take value of 0, otherwise, the value of 1. The same scheme is to be employed for C_r variable, i.e.,

	if controllable	if uncontrollable
C_p	0	1
C_r	0	1

Next, we choose to define the sub-index for privacy and independence of a mode of travel, denoted by I_p, as

$$I_p=(C_p+C_r)\times\frac{5}{2}$$

where $\frac{5}{2}$ is chosen arbitrarily such that the upper limit of the sub-index will be 5. In this way I_p will vary between 0 and 5.

(c) Decency of Service

Decent interior conditions (such as cleanness, quietness, controlled temperature, etc.) of the vehicle and sufficient attentive care from the conductors, stewardesses, or other crew members are intuitively very important in the choice of travel mode. We choose to call the aggregation of these factors 'decency of service'.

To deal with these unquantifiable factors, it would be desirable, as C.A. Lave suggested,[(1)] to have a scale of commuters' subjective valuations of comfort and corresponding list of objective characteristic such as cleanness, quietness, controlled temperature, etc. It would then be possible to use the subjective valuation as the dependent variable in a regression on the objective characteristics and hence produce an objectively measurable set of comfort indices.

(1) Lave, (1969)

Another suggestion could be to use a proxy variable. The expenditures spent on items such as temperature control, noise control, janitorial service, general interior design, and training of attentive personnels could be a proxy for the 'decency of service'.

Those sub-indices will be aggregated to form a final D&I index.

2. 2 Fundamental Assumptions

2. 2. 1 Electric Current Split versus Modal Split.

In a sense, our world is mischievous: where there is a motivation, there is a resistance against it. When an electric current is motivated (by a potential difference) to flow through a conductor, it is counteracted by the resistance in the conductor. This phenomenon, an omnipresence in the field of electrical engineering, is described in the name of Ohm's Law:

$$I=\frac{V}{R} \tag{2.1}$$

where I is the measure of current,
V is the potential difference between the nodes,
R is the resistance of the conductor.

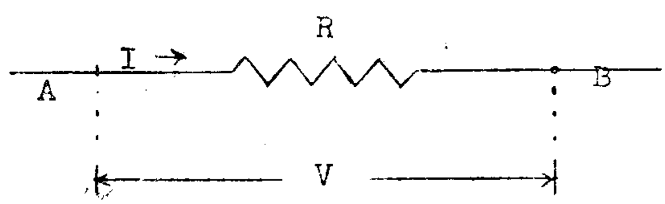


Fig. 2. 1

That is, the measure of current and the resistance of the conductor via which the current flows are in an antagonistic relation.

Analogically, the following may be said of the transportation phenomena: When an individual is motivated to travel via a certain travel mode, he is encountered by a certain degree of impeding force due to, for example, trip cost, travel time, or discomfort and inconvenience which he has to pay, spend or endure to make the intended trip.

Hence, the following analogical relationships are conceivable between electric current and transportation modal split systems:

Electric system	Transportation system
(1) Potential difference	Needs to make a trip
(2) Electric conductor	Mode of travel
(3) Resistance of conductor	Impeding effect from mode
(4) Current flow	Volume of travel.

Problems exist, however, in that no analogies are capable of complete one to one correspondence. Invariably, there is a limit where analogies break down. Hence, a considerable modification is required. The following

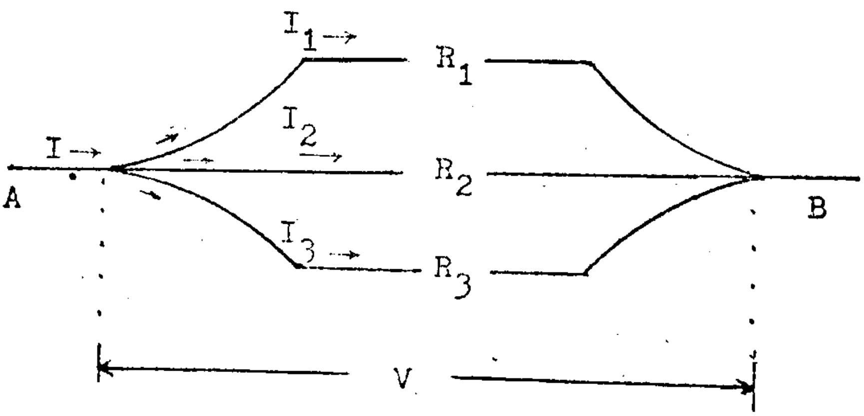


Fig. 2.2 Electric Current Split

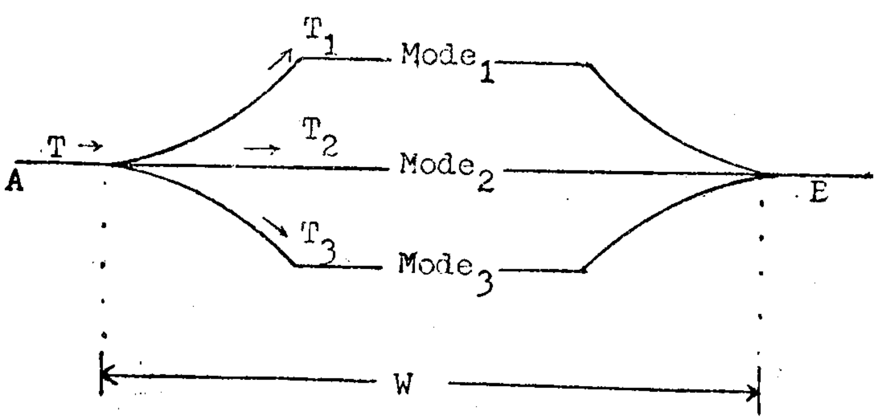


Fig. 2.3 Travel Modal Split

sections will explore this limit of the analogy and required modifications.

To have a clearer picture of the analogical relationships, the following two network systems will be compared.

Analogies between the two systems:

(1) A certain amount of electric current I (composed of electrons) is motivated by a potential difference V to flow (or 'travel') from node A to node B.

While, in the modal split system, a certain amount of generated trips (composed of traveling individuals) are motivated to 'flow' from node A to node B. Here, the motivation W corresponds, in case of urban work-trip model, to the willingness or desire to go to work.

(2) When the measure of potential difference remains constant, the amount of current flow varies inversely as the resistance of the conductor varies. In a similar way, when the degree of motivation remains constant, the volume of travel 'flow' varies inversely as the modal characteristics (=retarding effect) vary.

(3) On the other hand, when the resistance of the conductor remains constant, the electric current I can increase or decrease according as the potential difference increases or decreases, respectively. In a similar way, when the modal characteristics remain constant, the travel volume can increase or decrease according as the degree of motivation to travel becomes stronger or weaker, respectively.

(4) In the electric current split system, (Fig. 3.2), the sum of current split, that is, $I_1+I_2+I_3$, equals the total current I, i.e.,

$$I=\sum_j I_j.$$

On the other hand, in modal split system, the sum of trips over all modes is equal to the total trips generated (Fig. 3.3), i.e.,

$$T=\sum_j T_j.$$

So far, the analogical relationships between the two systems seem to be perfect. However, problems emerge next; the limit where analogies break down appears.

Break-down Point of the Analogy:

One can reasonably accept that the volume of travel, T_j, via a certain travel mode j varies inversely as the impeding effect, Z_j, generated by the mode j (due to trip cost incurred, travel time required, etc.) varies, i.e.,

$$T_j=\frac{W}{Z_j} \tag{2.2}$$

where W is a parameter which is;

(i) related to the degree of willingness or motivation of the people to make trips between the nodes A and B.

(ii) independent of the mode since the 'needs' to make a trip(e.g., needs to go to work to earn income) is *a priori* to travel mode.

However, the problem is: What will be the functional form of the denominator? In case of electric conduction current, the denominator of the Ohm's Law is the resistance itself. In other words, the functional form there is an *identity* function of resistance, that is,

$$Z(R)=R.$$

Now the question is: Will this be true with our problem of travel modal split? In other words, will the denominator be travel cost itself, or travel time itself? If not, what will be the appropriate functional form?

2.2.2 Desiderata for Travel Impedance Function:

It is in order then to explore the functional form for Z_j (impeding effect) in the relationship (2.2) on the previous page.

Since it is apparent that trip cost will generate an impeding effect on the willingness of people to use a certain travel mode, the functional form of the impeding effect due to trip cost will be explored here first.

We shall specify criteria for the choice of the function. These criteria we shall call *desiderata* because they specify the features we desire to have in the function.

Desiderata:

(1) The graph of the function should pass through the origin of a rectangular coordinate system since the impeding effect due to cost should be zero when the trip cost incurred is zero, or

$$Z_c(0)=0.$$

(2) The function should be a monotonically increasing function since the impeding effect should increase as the trip cost increases, or

$$Z_c(c_1)<Z_c(c_2) \text{ if } c_1<c_2.$$

(3) The second derivative of the function with respect to its independent variable (cost) should be negative since the marginal utility of money is diminishing according to the theory of microeconomics. That is,

$$\frac{d^2Z(c)}{dc^2}<0.$$

From the third desideratum, it has become clear that the identity function is disqualified here. Hence, the analogy between the two impedance systems breaks down at this point.

Hence, it is in order now to find an appropriate functional form for the impeding effect which satisfies those three desiderata stated on the previous page. An assumption will be introduced for this purpose.

Assumption 1

One might reasonably assume that human perception of a 5¢ fare increase out of a 50¢ fare trip is approximately the same as that of a, say, 8¢ fare increase out of a 80¢ fare trip, or a 10¢ fare increase out of a $1.0 fare trip, etc. In other words, the incremental change in human being's psychological response to price change is proportional to the incremental change in price (δC) divided by the price level (C) at which the incremental change δC occurs; or

$$\delta Z=k\frac{\delta C}{C} \tag{2.3}$$

where δZ is the psychological response change to δC,

δC is the incremental change in price,

C is the price level at which the incremental change δC occurs, and

k is the proportionality constant.

Validity of the Assumption:

(1) The validity of the above assumption may fall into question as the price range becomes wider. For example, a fare increase of 5¢ at a 50¢ fare level may not be perceived as the same as a 10 dollar fare increase at a 100 dollar fare level,

However, with our model being intended for an urban work trip model, the price range won't be so big since work trips are normally within 30 or 40 mile range.

(2) On integration of the equation (2.3), we get a mathematically equivalent form to Eq. (2.3);

$$Z = k\log_e C \tag{2.4}$$

within the range of a constant of integration. This agrees to the belief, in microeconomics, that the utility of money, leisure time, and other resources is proportional to the logarithm of the amount of the resource.[2]

(3) Another supporter for our assumption comes from psychophysics. Psychophysics is the science that investigates the quantitative relationships between physical events and corresponding psychological events. A physical event (e.g., amplitude of sound wave) is called a stimulus, and the corresponding psychological event (e.g., sensory experience of loudness) is called response in psychophysics.

Weber's Law in psychophysics states that stimulus S must be changed in a certain ratio to produce equally perceptible increments in response R.[3]

In other words, for response R to increase by a certain amount δR, stimulus S must make a certain percentage increase. In the recent results on investigations of sensation, a few exceptions to Weber's Law have been

(2) Huber; Sahney; Ford, (1969) p. 484.
(3) Guilford, J.P. (1954); Chapter 2

found. Therefore, it certainly cannot be regarded as a universal law of differential sensitivity. It is, however, best regarded as the first important approximation to such a law.[4] Hence, our assumption gains another corroboration from psychophysics.

(4) The validity of Assumtion 1 suffers, however, from the following aspect:

Assumption 1 implicitly means that our perception of δC is a function of δC and C only; it is not a function of any other variables, such as travel time or D&I level, etc.

As a matter of fact, travelers' modal choice is a process of trade-offs between travel cost, travel time, etc. Hence, in a strict sense, commuters' perception of C could be a function not only of δC and C, but also of T (travel time) and D&I level.

However, for the simplicity of our analysis, we will incorporate the trade-off process when we determine weighting coefficients for the variables, and at this stage of partial split ratio formulation we will assume:

Assumption 2:

"Commuters' perception of a variable change is independent from other variables."

It should be noted here that Assumption 2 is not an independent one; it is implicitly implied in Assumption 1, and that the trade-off process, which is suppressed by Assumption 2, is incorporated in weighting coefficients for the variables.

2.3 Partial Split Ratio

2.3.1 Definition

In view of the discussions stated in the previous section, we choose the functional form for Z_c to be:

$$Z_c = k\log_e(C+1) \tag{2.5}$$

The only difference between Eq. (2.5) and Eq. (2.4) is that the inde-

(4) ibid.

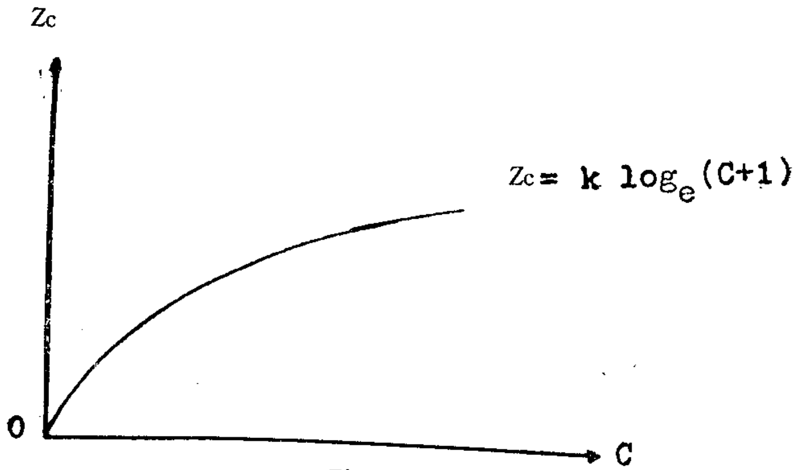


Fig. 2.4

pendent variable C has been translated by 1. This is due to Desideratum 1.

We will see whether Equation (2.5) satisfies the three desiderata:

(1) $Z_c(0)=k\ \ln\ (0+1)=0$,

(2) $Z_c(c_1)<Z_c(c_2)$ if $c_1<c_2$, $\forall C$,

(3) $\frac{d^2Z_c}{dc^2}=-\frac{1}{(c+1)^2}<0\ \forall C$.

Hence, it has been proved that function Z_c which was derived from Assumption 1 satisfies all of the three Desiderata.

By substituting Formula (2.5) into Equation (2.2), we obtain the modified Ohm's Law for travel modal split system as follows:

$$Tc_j=\frac{W}{k\ \log_e(C_j+1)} \tag{2.6}$$

where 'c' stands for trip cost, and

'j' stands for mode j.

Now we define 'Partial Split Ratio'.

Definition

Partial split ratio for cost for mode j. denoted by M_{cj}, will be defined as follows:

$$M_{cj}=\frac{T_{cj}}{\sum_i T_{ci}} \tag{2.7}$$

where $i=1, 2, \cdots, n$, and n is the total number of competing modes between node A and node B.

Discussions on PSR:

(1) Partial split ratio for cost for mode j (abbreviated as $PSR(cj)$) has been defined here as a methodological artifice to facilitate our analysis.

(2) Therefore, it is a conceptual being which has no physical meaning unless it is combine with a weighting coefficient (to be discussed in Section 3) and linked to final split ratio (to be discussed later in this Section).

(3) It is an imaginary trip split ratio to mode j (to the total trips generated), were the trip cost the only impeding factor.

Substituting Equation (2.6) into Equation (2.7), we get

$$M_{cj}=\frac{W}{k\log_e(c_j+1)}\Big/\sum_i\frac{W}{k\log_e(c_i+1)}$$

$$=\frac{\dfrac{W}{k}\,\dfrac{1}{\log_e(c_j+1)}}{\dfrac{W}{k}\sum_i\dfrac{1}{\log_e(c_i+1)}}$$

$$=\frac{1}{\log_e(c_j+1)\sum_i\dfrac{1}{\log_e(c_i+1)}} \tag{2.8}$$

In a system with three competing modes, i.e., $n=3$, Eq. (2.8) reduces to the following relationships:

$$M_{c1}=\frac{\log_e(c_2+1)\log_e(c_3+1)}{D_c} \tag{2.8a}$$

$$M_{c2}=\frac{\log_e(c_1+1)\log_e(c_3+1)}{D_c} \tag{2.8b}$$

$$M_{c3}=\frac{\log_e(c_1+1)\log_e(c_2+1)}{D_c} \tag{2.8c}$$

where $D_c=\log_e(c_2+1)\log_e(c_3+1)+\log_e(c_1+1)\log_e(c_3+1)+\log_e(c_1+1)\log_e(c_2+1)$.

2.3.2 Characteristics of $PSR(cj)$

Observing (2.8), one can easily notice the following charateristics of partial split ratio for cost:

(1) The partial split ratio for travel cost for mode j, M_{cj}, is a function not only of the travel cost of its own, but also of those of all other com-

peting modes, i.e.,

$$M_{cj}=f(c_1, c_2, \ldots, c_j, \ldots, c_n), \tag{2.9}$$

where n is the total number of competing modes.

(2) The sum of M_{cj}'s over all competing modes is equal to one, i.e.,

$$\sum_j M_{cj}=1. \tag{2.10}$$

(3) When $c_j=0$, (trip cost of mode $j=0$)

$$M_{cj}=1.$$

That is, when the trip cost with mode j is zero, $PSR(cj)$ becomes 1, i.e., 100% of travelers use mode j, were the trip cost the only one factor.

On the other hand, it is easy to see that

when $c_j=\infty$,

$$M_{cj}=0.$$

2.3.3 PSR for travel time

It is in order now to discuss the partial split ratio for travel time for mode j, denoted by M_{tj}, where t stands for travel time and j stands for mode j.

To begin with, it should be reminded that the *PSR* for travel cost was based on three Desiderata and Assumption 1. Therefore, let's examine those three desiderata and Assumption 1 in relation to travel time.

(1) The impeding effect due to travel time required should be zero when the required travel time is zero. Hence,

$$Z_t(0)=0$$

where Z_t is the assumed functional form for the impeding effect due to travel time.

(2) $Z_t(t)$ should be a monotonically increasing function since the impeding effect should increase as the travel time required increases, or

$$Z_t(t_1)<Z_t(t_2) \text{ if } t_1<t_2.$$

(3) The second derivative of $Z_t(t)$ with respect to t should be negative since the marginal utility of leisure time is diminishing according to microeconomic theory. That is,

$$\frac{d^2Z_t(t)}{dt^2}<0$$

On the other hand, we assume here the same kind of assumption as Assumption 1 with regard to travel time that human perception of a 5 minute speed increase out of a 30 minute trip is approximately the same as that of a, say, 10 minute speed increase out of a 60 minute trip, etc. When mathematically expressed, we get a similar equation to Eq. (2.3) except for the proportionality constant, i.e.,

$$\delta Z_t = m\frac{\delta_t}{t} \tag{2.11}$$

where δZ_t is the change in response to δ_t,

δ_t is the incremental change in required travel time.

t is the absolute value of travel time at which δ_t is taken place, and

m is the proportionality constant.

It should be noted here that the proportionality constant associated with travel time is different from that with travel cost. This is why we use here 'm' instead of 'k' which was the proportionality constant for cost. The rationale for the different constant is this: People's sensitivity to speed change may be different from that to cost change. This means that even though the response curve for speed change follows the same pattern as that for fare change (i.e., both are logarithmic curves), the slopes of both curves are different from each other.

On examination of the three desiderata and Assumption 1, we have found that the functional form for Z_t is the same as that for Z_c except for the different proportionality constant, so that we will get:

$$Z_t(t) = m\log_e(t+1) \tag{2.12}$$

By substituting Eq. (2.12) into Eq. (2.2), we get:

$$T_{tj} = \frac{W}{m\log_e(t_j+1)} \tag{2.13}$$

where j stands for mode j.

Definition:

In exactly the same way as in *PSR(cj)*, partial split ratio for travel time for mode j, denoted by M_{tj}, will be defined as follows:

$$M_{tj}=\frac{T_{tj}}{\sum_i T_t} \tag{2.14}$$

The same discussions as those on *PSR(cj)* will apply to *PSR(tj)*. That is, *PSR(tj)* is an artifice to facilitate our analysis, a conceptual being which has no physical meaning unless it is combined with a weighting coefficient, and it is an imaginary trip split ratio to mode j (to the total trips generated), were the travel time required the only impeding factor.

Substituting Eq. (2.13) into Eq. (2.14), we get:

$$M_{tj}=\frac{\frac{W}{m}\frac{1}{\log_e(t_j+1)}}{\frac{W}{m}\sum_i\frac{1}{\log_e(t_i+1)}}$$

$$=\frac{1}{\log_e(t_j+1)\sum_i\frac{1}{\log_e(t_i+1)}} \tag{2.15}$$

where $i=1,2,\dots j,\dots,n$, and

n is the total number of competing modes.

It should be noted here also that both W (which is a parameter related to the degree of 'needs' of the people to make a trip between nodes A and B) and m (a proportionality constant related to the sensitivity of people to an abstract characteristic speed) are independent of a specific travel mode. Hence, they were canceled out in the above ratio formula.

2.3.4 PSR for Discomfort & Inconvenience

Partial split ratio for discomfort and inconvenience for mode j, denoted by M_{dj}, will be discussed following the same line of reasoning as with the former two PSR's.

We will examine the three desiderata and Assumption 1 in relation to discomfort and inconvenience. Discomfort and inconvenience, being treated as a conglomerated impedance factor in this model, will be abbreviated to 'D&I'.

(1) The impeding effect due to D&I associated with trips should be zero when the measure of D&I is zero. Hence,

$$Z_d(0)=0$$

where Z_d is the assumed functional form for the impeding effect due to D&I index.

(2) $Z_d(d)$ should be a monotonically increasing function since the impeding effect should increase as the measure of D&I increases, or

$$Z_d(d_1)<Z_d(d_2) \text{ if } d_1<d_2.$$

(3) When we are reminded that comfort or discomfort is rather closely related to psychological variables which are usually subject to Weber's Law (which apparently satisfies Desideratum 3), we find that Assumption 1, which was based on Weber's Law, may be applicable as well to D&I.

Hence, we get a similar equation to Eq. (2.3) except for a different proportionality constant, i.e.,

$$\delta Z_d=p\frac{\delta d}{d} \tag{2.16}$$

where d stands for the measure of D&I, and p is the proportionality constant. The adoption of a different constant is based on the same rationale as with travel time in the previous section.

Following the same line of reasoning as in the previous section with travel time, we get

$$Z_d(d)=p \log_e (d+1) \tag{2.17}$$

and

$$T_{dj}=\frac{W}{p\log_e(d_j+1)} \tag{2.18}$$

where j stands for mode j.

Definition

In exactly the same way as in the previous two PSR's, partial split ratio for D&I for mode j, denoted by M_{dj}, will be defined as follows:

$$M_{dj}=\frac{T_{dj}}{\sum_i T_{di}} \tag{2.19}$$

Substituting Eq. (2.18) into Eq, (2.19), we get:

$$M_{dj}=\frac{\frac{W}{p}\frac{1}{\log(d_j+1)}}{\frac{W}{p}\sum_i\frac{1}{\log(d_i+1)}}$$

$$=\frac{1}{\log(d_j+1)\sum_i\frac{1}{\log(d_i+1)}} \tag{2.20}$$

It is easy to see that the partial split ratio for travel time and the same for D&I show the same characteristics discussed in Section (2.3.2).

2.4 Final Split Ratio

2.4.1 Weighting Coefficients

When we derived partial split ratios, it was assumed (by Assumption 2) that commuter's perception of a certain variable change is independent from the other variables. This assumption was made for the simplicity of analysis.

Modal choice, however, is believed to be a process of trade-offs between variables, such as travel cost, travel time, convenience, etc. This trade-off process, which was suppressed by Assumption 2 in the formulation of partial split ratios, will be incorporated into the weighting coefficients for those partial split ratios.

Hence, a final split ratio will be defined as a weighted linear combination of the partial split ratios. In other words, it will be of the following form:

$$Q_j=w_cM_{cj}+w_tM_{tj}+w_dM_{dj} \tag{2.21}$$

where Q_j=final split ratio for mode j,

M_{ij}=partial split ratio for attribute i and for mode j,

w_i=weighting coefficient for M_{ij}.

$i=c$, t, and d; $j=1, 2, \ldots, n$.

Partial split ratios were designed to incorporate the quality of modal characteristics. Weighting coefficients are to incorporate the relative importance or weight with which those modal characteristics are felt by individual travelers.

Hence, the weighting coefficients will represent the socioeconomic characteristics of the travelers since relative weights they put on different factors will vary according as their economical, social, or intellectual status vary.

If we normalize the weighting coefficients, they will satisfy the following conditions:

$$\sum_i w_i = 1, \tag{2.22}$$

$$\text{and } 0 < w_i < 1 \text{ for all i's.} \tag{2.23}$$

This normalization is needed since the summation of split ratios over all competing modes should be equal to one. On the other hands, this normalization makes it possible to express our model graphically. This graphical representation of the model will be done in section (2.4.3).

Section 3 of this paper will be devoted to a new methodology for determining the weighting coefficients.

2.4.2 Proof that $\sum_j Q_j = 1$.

For a relative share model it is required that the summation of split ratios over all competing modes should be equal to one, i.e., $\sum_j Q_j = 1$. The following is to prove that for the model.

From Equation (2.21),

$$\begin{aligned}\sum_j Q &= \sum_j (w_c M_{cj} + w_t M_{tj} + w_d M_{dj}) \\ &= w_c \sum_j M_{cj} + w_t \sum_j M_{tj} + w_d \sum_j M_{dj}.\end{aligned}$$

From the second characteristics of the partial split ratio, i.e., Equation (2.10), we know that

$$\sum_j M_{ct} = \sum_j M_{tj} = \sum_j M_{dj} = 1.$$

Inserting these relationships into the above equation, we get

$$\begin{aligned}\sum_j Q_j &= w_c 1 + w_t 1 + w_d 1 \\ &= w_c + w_t + w_d\end{aligned}$$

$=1$ [according to Eq. (2.22)].

2.4.3 Final Split Ratio Diagram

Our model will represent modal split as fractional areas of a unit square. Figure (2.5) diagrams modal split as it might occur in a three mode system. The total area of the unit square represents the total trips, while particular mode trips are represented by fractional areas of the unit square.

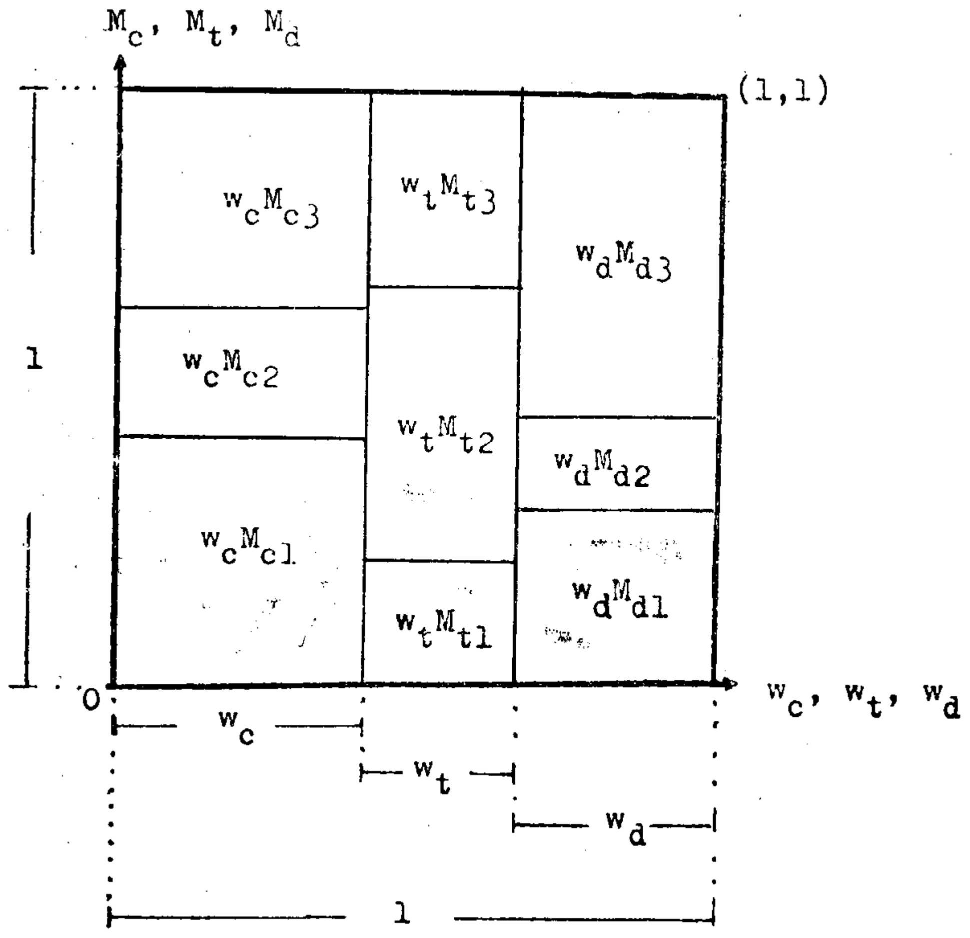


Fig. (2.5)

A fractional area is formed by two components, horizontal and vertical. The horizontal component is determined by a weighting coefficient w, while the vertical component is determined by a partial split ratio M. Summation of fractional areas with subscript j (where $j=1$, 2, or 3 in the diagram) gives the final split ratio for mode j. For instance, the shaded area in Fig. (2.5) represents the final split ratio for mode 1, i.e.,

$$Q_1 = w_c M_{cL} + w_t M_{tL} + w_d M_{dL}.$$

3. WEIGHTING COEFFICIENTS:

Our objective in this section is to develop an appropriate methodological procedure to determine the weighting coeffiients for the partial split ratios of the model. Within the framework of the model, the weighting coefficients could be viewed as normalized fractional numbers which indicate the relative importance which a given society as a whole puts on cost saving, time saving, or convenience. Maximum entropy technique adds up to a general partitioning theory in the sense that it presents measures for the way in which some set is divided into subsets. Hence, our problem here is to take advantage of the Maximum entropy technique to find the weighting coefficients for the model so that it may be capable of long-range forecast.

Assumption 3:

It will be assumed here that lower income group tends to cost saving, while as their income goes up, they can afford to choose time saving or comfort-oriented mode for their everyday work trip.

3.1. Corroborations for the Assumption

3.1.1 Backward-bending Supply Curve of Human Labor

In the theory of micro-economics, one of the interesting findings about human nature is that of the backward-bending supply curve of total hours that a group of people will want to work at each different wage. Figure (3.1) shows the curve.(5)

This seems to indicate that with a higher wage rate, man tends to want more leisure time; i.e., beyond a certain level of income, man is apt to

(5) Samuelson, P.A. p. 580.

prefer time to money.

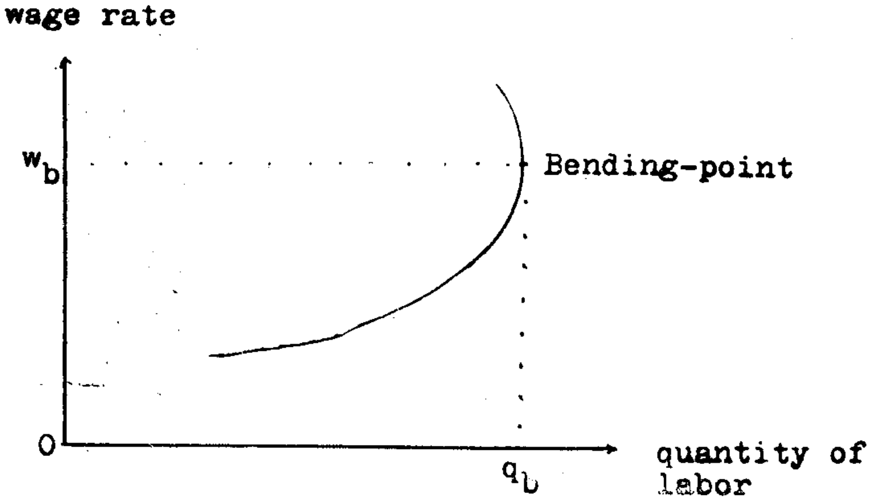


Fig. 3.1

When translated in terms of transportation demand behavior, this amounts to saying that, while people with low income will put more weight on cost-saving when considering the selection of travel mode, the weight will be shifted toward time-saving as their income goes up.

3.1.2 Hierarchy of Human Needs

There are at least five sets of goals which psychologists call basic needs of human beings. There is general agreement among the psychologists that these basic human needs are organized into a hierarchy of relative prepotency.

Most of the people with whom psychologists have worked seemed to have a hierarchical order of basic needs as follows:[6]

(1) Physiological needs; hunger, thirst, etc.,

(2) Safety needs; desire for safety in general,

(3) Love needs; affectionate relationships with family, friends etc. (Love is not synonymous with sex here.),

(6) Maslow, A.H. (1964) pp. 6-24.

(4) Esteem needs; desire for a firmly based self-esteem, independence, freedom, prestige, reputation, recognition, etc.,

(5) Self-actualization; desire for self-fulfilment, namely, the tendency for one to become actualized in what one is potentially.

Undoubtedly physiological needs are the most prepotent of all needs. But when the physiological needs are adequately satisfied, the next prepotent (or 'higher') need emerges in turn to dominate the conscious life and to serve as the center of organization of behavior. When this need in turn is satisfied, again new (and still 'higher') need emerges and so on. This is what psychologists mean by saying that the basic human needs are organized into a hierarchy of relative prepotency.

There are, of course, some people in whom self-esteem, for instance, seems to be more important than any other. Most of the people, however, with whom psychologists have worked have seemed to have the basic needs in about the order that have been indicated.

Thus man is a perpetually wanting animal. The hierarchy principle could be observed in people's transportation modal choice behavior.

If we admit that, with increasing personal income, the 'higher' hierarchical human needs (such as safety, love, or self-esteem) tend to emerge to dominate the human behavior, and if we admit that the private passenger car offers *safety* (since the hoodlums, thieves, or pickpockets in the big city have threatened the safety of public areas such as subways, streets, or stations, etc.), *love-protection* (because the private car can protect the privacy of family or friends as a traveling unit), and *self-esteem* (because private cars can be considered to be social goods, a symbol of prestige, independence, freedom, etc.), then the boom of private cars since World War II can be accounted for as a result of growing affluence of industrialized countries. In other words, if we define transportational comfort in terms of convenience (due to frequency of service), privacy and independence during the trip, and decency or prestige of the travel mode, then the theory of hierarchical human needs says that the comfort-oriented mode will dominate the transportation system as a result of economic affluence.

On the basis of these corroborations the model will make use of Assumption 3 and calls;

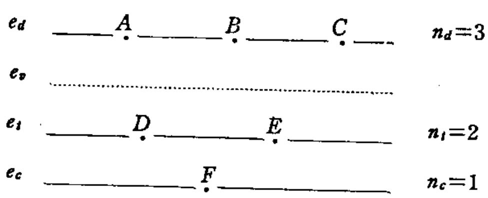
$$\left.\begin{array}{l}\text{the median income of cost-saving oriented group} \ldots\ldots\ldots\ldots e_c,\\ \text{that of time-saving oriented group} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots e_t,\\ \text{and that of comfort-oriented group} \ldots\ldots\ldots\ldots\ldots\ldots\ldots e_d.\end{array}\right\} (3.1)$$

Now the maximum entropy technique will be used here to determine the most probable or minimally prejudiced partition of the total population of the area in concern into three groups, that is, cost-saving oriented group, time-saving oriented group, and comfort-oriented group.

No one will hardly prefer only one factor of cost, time, or comfort. However, it is assumed here that this minimally prejudiced partition of people into those three groups will approximate the relative importance which the society as a whole puts on cost saving, time saving, or convenience.

3.2 Derivation of the Formula

First, let's consider Figure (3.2) in which e_c, e_i, and e_d are as defined in section (3.1.2), and e_v stands for the per capita income of the region in concern.



[A macro-state in which $n_c=1$, $n_t=2$, $n_d=3$]

Fig. (3.2)

A specification of the number of individuals belonging to each level of income (i.e., n_c, n_i, and n_d in the above Fig.) is said to define a *macrostate*. When we just exchange individuals without changing the macrostate, we say *microstate* is changing.(7) For example, suppose that we identify the

(7) Constant, F.W., p. 77.

individuals as $A, B, C,$ etc.; then Fig. (3.2) shows a certain microstate corresponding to the macrostate for which $n_c=1$, $n_t=2$, and $n_d=3$. If we interchange any two individuals from different cells, say A and D, we will have a different microstate but the same macrostate. If we interchange two individuals in the same cell, say A and B, we will have the same microstate as well as the same macrostate, because here we are not considering any subdivision of an income level.

It is, therefore, possible for a macrostate to have large number of microstates. A fundamental hypothesis is that all microstates are equally probable. The number of microstate corresponding to any given macrostate is called the *thermodynamic probability* of the macrostate and is represented by W.[8] A macrostate which has more microstates is said to be more probable to occur than one which has fewer microstates.

Our problem is to find the most probable macrostate, which occurs, according to the maximum entropy principle, when the entropy S of the system, where S is defined as

$$S=k\ln W, \tag{3.2}$$

is at its maximum.

From probability theory, we know that the number of microstates corresponding to a given macrostate $[n_c,\ n_t,\ n_d]$ of a system of population size N is given by

$$W=\frac{n!}{n_c!n_t!n_d!} \tag{3.3}$$

where n_c, n_t, n_d are the number of individuals belonging to e_c, e_t, e_d levels of income, respectively.

Since N and n_i ($i=c, t,$ and d) are very large numbers, Sterling's approximation formula,

$$\ln x! = x\ln x - x,$$

can be utilized.

(8) Sears, F.W., p. 280.

Taking logarithms on both sides of Eq. (3.3) and using Sterling's formula, we get

$$\begin{aligned}\ln W &= \ln N! - \sum_i \ln n_i! \\ &= N \ln N - N - \sum_i n_i \ln n_i + \sum_i n_i \\ &= N \ln N - \sum_i n_i \ln n_i \end{aligned} \tag{3.4}$$

where $\sum n_i = N$, and $i = c, t$, and d.

If W is at its maximum value W^*, then the first variation of W^* arising from the variations in n_i's is zero.

Therefore, the condition for W to be maximum is

$$\delta \ln W^* = -\sum_i n_i \, \delta \ln n_i - \sum_i \ln n_i \, \delta n_i = 0 \tag{3.5}$$

Since the total number of people is constant (at time t),

$$\delta N = \sum \delta n_i = 0. \tag{3.6}$$

Hence, $\sum n_i \, \delta \ln n_i = \sum n_i \frac{1}{n_i} \delta n_i = 0.$ (3.7)

Therefore, (3.5) becomes

$$\sum \ln n_i \, \delta n_i = 0. \tag{3.8}$$

Since the total income of the given society (per capita income times population) is constant (at time t),

$$\sum e_i n_i = E, \tag{3.9}$$

where $E = e_v N =$ constant. ($e_v =$ per capita income).

Hence, $\delta E = \sum e_i \, \delta n_i = 0.$ (3.10)

Multiplying Eq. (3.6) by an arbitrary constant a, and multiplying Eq. (3.10) by another arbitrary constant b, and adding these to Eq. (3.8), we get

$$\sum (\ln n_i + a + b e_i) \delta n_i = 0,$$

$$\text{or } n_i = \exp(-a - b e_i). \tag{3.11}$$

The constant a can be expressed in terms of the total population of the society, i.e.,

$$\begin{aligned} N &= n_c + n_t + n_d \\ &= \exp(-a) \sum \exp(-b e_i) \\ &= \exp(-a) Z, \end{aligned}$$

where $Z = \sum \exp(-b e_i)$ is called 'partition function'.

Thus, we may write

$$\exp(-a)=\frac{N}{Z}. \qquad (3.12)$$

Inserting Eq. (4.12), into Eq. (4.11), we get

$$n_i=\frac{N}{Z}\exp(-be_i) \qquad (3.13)$$

or

$$n_c=\frac{N}{Z}\exp(-be_c) \qquad (3.13a)$$

$$n_t=\frac{N}{Z}\exp(-be_t) \qquad (3.13b)$$

$$n_d=\frac{N}{Z}\exp(-be_d). \qquad (3.13c)$$

n_c, n_t, and n_d express the most probable partition of a set (the total population of size N) into three subsets; cost saving preference subset, time saving preference subset, and comfort-seeking subset.

The weighting coefficients can be expressed, according to the definition, by

$$w_c=\frac{n_c}{N}=\frac{1}{Z}\exp(-be_c) \qquad (3.14a)$$

$$w_t=\frac{n_t}{N}=\frac{1}{Z}\exp(-be_t) \qquad (3.14b)$$

$$w_d=\frac{n_d}{N}=\frac{1}{Z}\exp(-be_d) \qquad (3.14c)$$

where $Z=\sum\exp(-be_i)$.

The Relationship between 'b' and Per Capita Income

Combining Eq. (3.9) and Eq. (3.13), the constant b can be obtained in terms of per capita income.

From Eq. (3.9),

$$\sum e_i n_i=e_v N. \qquad (3.15)$$

Hence,

$$e_c\exp(-be_c)+e_t\exp(-be_t)+e_d\exp(-be_d)$$
$$=e_v[\exp(-be_c)+\exp(-be_t)+\exp(-be_d)]. \qquad (3.16)$$

Now, when we obtain e_c, e_t, and e_d values by sample survey for the area, and if we are given the per capita income e_v and the population size of the region, we can calculate the weighting coefficients we need.

4. SUMMARY AND DISCUSSIONS

A modal split model on the journey to work was developed herein. The journey to work may be very important for urban transportation planning since capacity is strained at this period and the idea that traffic can be satisfied the rest of the day, if the peaks can be satisfied, is a popular notion.

According to this model, the market share for mode j is determined, in one dimension, by partial split ratios, and, in the other dimenion, by weighting coefficients for the partial split ratios, as was shown in Figure 2.5 which is reproduced below for the benefit of convenience.

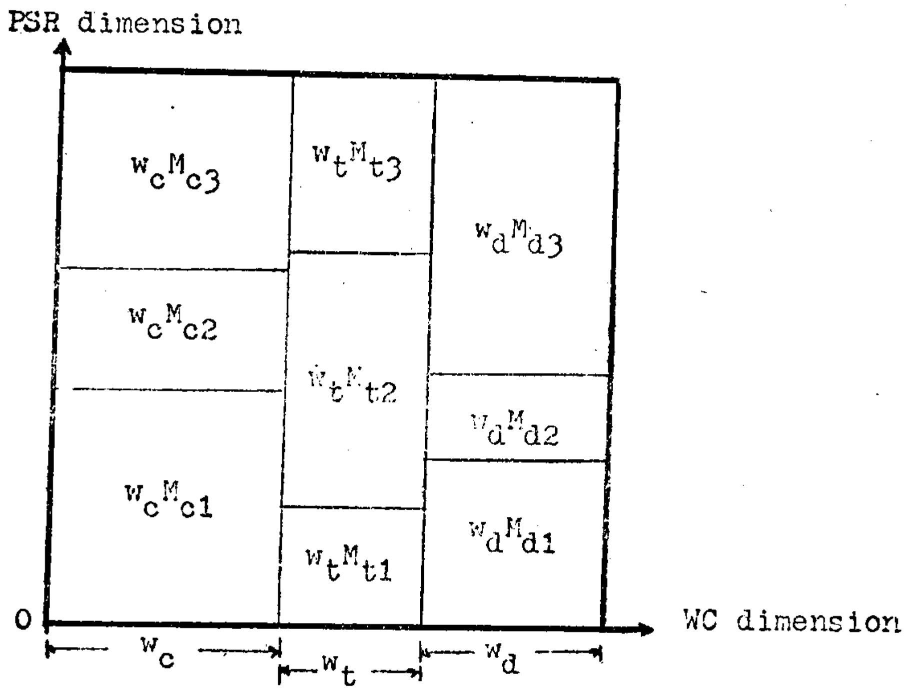


Fig. (2.5)

For example, in Figure 2.5, the final split ratio for mode 1, in a three mode competing travel market, is represented by the summation of three sub-rectangles, that is, w_cM_{c1}, w_tM_{t1}, and w_dM_{d1}. The horizontal component of each sub-rectangle is determined by a weighting coefficient w, while the vertical component is determined by a partial split ratio M.

A partial split ratio for attribute i is, as it were, an imaginary trip split ratio for a certain mode to total personal trips, if the attribute i were the only consideration. It is a conceptual being, devised to facilitate the analysis, which has no physical meaning unless it is combined with a weighting coefficient and linked to final split ratio.

Methodologically, PSR's were derived using deductive logic from two assumptions. Assumption 2 implies that PSR's are independent from each other. Hence, trade-offs between variables were suppressed by this assumption. However, the trade-offs were later incorporated into weighting coefficients for those partial split ratios.

Assumption 1 was concerned with commuters' perception of variable changes which affect their travel modal choice behavior.

Assumption 1 was corroborated by:

(1) our common sense about the perception of fare increase or speed change,

(2) a micro-economic theory, that is, the diminishing marginal utility of money, leisure time, etc.,

(3) a well-known law by the name of Weber's Law in psychophysics.

A partial split ratio for mode j is a function of the attributes, not only of the mode j, but also of all other competing modes. This is a very desirable characteristic of a modal split model since due to this characteristic, in combination with the normalization characteristic, i.e., Equation (3.10), the impact of a newly introduced mode, characterized by its own system attributes, on the demand for every other existing mode, can be calculated straight-forward from the mathematical formula for partial split ratios.

Weighting coefficients for PSR's represent individual trip maker's trade-offs between cost-saving, time-saving, and comfort & convenience. Macroscopically, they indicate the relative weight or importance which a given population group places on time saving, cost saving, and comfort & convenience.

Maximum entropy technique is currently being accepted as a useful tool in decision making when uncertainties are involved. The maximum entropy technique was used here to determine the most probable or minimally prejudiced partition of people into three groups; cost saving preference group, time saving preference group, and comfort-seeking group.

In this model analogical relationships between electrical current split system and travel modal split system were explored and employed. An abstract travel mode is represented by its impeding effects calculated from the mode's abstract modal characteristics. This impeding effect corresponds to resistance of a conductor which carries electric current.

By virtue of this new concepts in viewing travel impeding factors as electrical resistance analogy, this model could, with further study, be used to predict demand split among complicated systems of mixed modes as depicted in Figure 1. 2. which most current models are incapable of.

More discussions on these points are as follows.

When electrical conductors are connected in series, as in Figure 5. 3, the overall resistance, denoted by R_s, of the connected system, is given by

$R_s=\sum_i R_i$

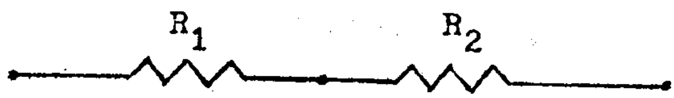


Fig. (5. 3)

On the other hand, when they are connected in parallel, as in Figure 5. 4, the overall system resistance, R_s, is given by

$R_s=\left[\sum_i \frac{1}{R_i}\right]^{-1}$

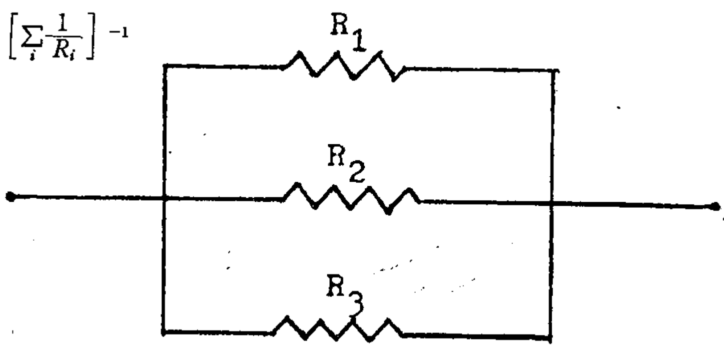


Fig. (5. 4)

Keeping the above discussion in mind, we will compare System 2 and System 3 in Figure 1.2.

Suppose both System 2 and System 3 connect nodes A and B. For the benefit of clearer comparison, suppose every mode, comprising the systems, i.e., $Mode_2$ through $Mode_6$, had numerically the same value of impeding effect, say 10.

Then, the overall system impeding effect, Z_s, for System 2 is

$$Z_s = 10 + 10 = 20,$$

and for System 3, it is

$$Z_s = 10 + \left[\frac{1}{10} + \frac{1}{10}\right]^{-1}$$
$$= 15$$

This means that System 3 will give lower impeding effect than System 2 with a result that more people will use System 3 than System 2.

Will this theoretical result reconcile with real world situations? The answer may be 'Yes' on the basis of the following arguments:

(1) The insertion of the dual system, $Mode_5$ and $Mode_6$ in parallel in Figure 1.2, increases the overall system reliability due to redundancy, since in case one mode fails, commuters still have the other mode. This feeling of increased system reliability will make people favor System 3 to System 2.

(2) The insertion of the dual system, in most cases, will decrease the waiting time for transfer. Since the waiting time for transfer is onerous to travelers, the decrease in this waiting time will cause people favor System 3 to System 2.

(3) The insertion of the dual system could increase the capacity. This increase in capacity will increase the probability for a traveler to get a seat in the vehicle, which will be considered as a factor in favor of System 3.

(4) The insertion of the dual system can satisfy people when they feel like diversity or a change of travel mode. This also could be considered as a factor in favor of System 3.

Currently-used models are mainly concerned with modal split among *unimodal* alternatives. (as in Figure 1. 1). However, because of increasing complexities and requirements to be met in urban and regional planning, planners may need to analyze and forecast modal split among *multimodal* alternatives. (as System 2 or 3 in Figure 1. 2).

This electrical resistance analogy model may serve as a good starting point for future study on travel modal split among multimodal complex systems.

Limitations to this Model:

(1) The model developed in this thesis may be limited to urban journey to work. This limitation comes from the differential sensitivity assumption (Assumption 1). It was stated in the pertaining chapter that this Assumption 1 may not hold valid for a long distance trip.

(2) One of the shortcomings of this model may be the lack of equilibrium of the transport system. This model assigns traffic to various modes based on assumptions of certain characteristics of those modes, e.g., travel time. However, if the actual time on the mode is greater than the time assumed (as the result of, say, congestion), a certain number of people who desired this mode at the assumed time, would be expected to divert to another mode, if possible, or stop making trips altogether (in the case of elastic trip demand). Basically, one cannot load a mode without affecting speed, comfort, cost, etc., the variables which in turn influence demand.

(3) This model leaves out the car ownership variable. This is a very popular variable among macro-level models.

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