Duopoly Competition Considering Waiting Cost

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1. Introduction

In this paper we consider two queueing systems that serve a large number of customers. Those two queueing systems compete for customers. Prices, (p_1, p_2), for each queueing system are announced, and each individual user makes a decision whether to join the queueing systems. In case a user decides to enter, he should determine which of the two queueing systems to join. Mendelson(1985) analyzes internal pricing scheme to control the job flow into a queueing system. Mendelson model considers the special case where all jobs are homogeneous in their time values and expected service requirements. But jobs usually have heterogeneous values specifying the gross value gained by system users per unit time. An important extension of Mendelson model was made by Mendelson and Whang. Mendelson and Whang(1990) consider an M/M/1 queueing system with multiple user classes. Each class is characterized by its delay cost per unit of time, its expected service time and its demand function. They derive a pricing mechanism which is optimal and incentive-compatible in the sense that the arrival rates and execution priorities jointly maximize the expected net value of the system while being determined, on a decentralized basis, by individual users.

In our paper, the number of queueing systems considered is generalized to two. In this case, each queueing system tries to maximize its own profit function. Therefore, unlike in the previous researches, we have to consider the game among two queueing systems and a group of customers, therefore a duopoly game. Each queueing system decides its price in competing with the other. Noting the price pair, each customer decides whether to enter a queueing system and get the service in return for paying the price. When he decides to enter, he should choose which queueing system to join. In choosing the queueing system, he compares the expected cost and chooses the one offering lower cost.

We show that under certain conditions there does not exist an asymmetric equilibrium. It may be suspected that one firm follows a high price strategy resulting in low market share and thus low waiting cost for a customer, and the other follows an opposite strategy.

2. Notations and Model Assumptions

We give some notations and assumptions used throughout the paper.
Let p_i is the price set by the server i.
c is the waiting cost of a customer per unit time.
λ is the potential customer arrival rate, which represents the highest possible customer arrivals per unit time.
μ is the service rate for each queueing system. We assume for simplicity each firm has the same service rate of μ, which is given exogenously.
We assume that $3\mu > \lambda$, which will be used in deriving the reaction function.
s_i denotes the market share of queueing system i.
V is a random variable representing a value gained by a system user. We assume that V follows a uniform distribution U[0,1].
Each of two firms are represented by an M/M/1 queueing system.

3. Reaction Functions and An Equilibrium

In the duopoly game, we can derive the following conditions for an equilibrium:

(1) $$p_1 + c\frac{1}{\mu - s_1\lambda} = p_2 + c\frac{1}{\mu - s_2\lambda}$$

(2) $$p_1 + c\frac{1}{\mu - s_1\lambda} = v(p_1, p_2)$$

(3) $$P[V \geq v(p_1, p_2)] = s_1 + s_2$$

The effective price of the service offered is the cost experienced by a customer in a queueing system and is the sum of service price and waiting cost. The first condition is that at equilibrium the effective prices for each firm are equival........ent. Otherwise, some customers in the more expensive queue would divert to the other queue. For an M/M/1 queueing system i, the mean throughput time is known to be $\frac{1}{\mu - s_i\lambda}$ and thus the waiting cost is $\frac{c}{\mu - s_i\lambda}$. The effective price at equilibrium given (p_1, p_2) is equivalent to the marginal value of a customer on the verge of entering the system, denoted by $v(p_1, p_2)$ in the second condition. The third condition is that the probability of a customer entering one of the queueing systems for service given (p_1, p_2) is the total fraction of customers in the market. In order to explain the random variable *V*, assume that there is a continuous distribution Φ of service valuations with probability density function ϕ. The probability that an arriving job values the service at *x* or higher is given by $\overline{\Phi}(x) = 1 - \Phi(x) = \int_x^\infty \phi(z)dz$. If all customers with values equal to or higher than *x* join the system, the arrival rate will be $\lambda\overline{\Phi}(x)$. Conversely, when the arrival rate is *a*, the marginal value *x*, i.e., the lowest service valuation among the jobs that choose to join the system, is equal to $\overline{\Phi}^{-1}(a/\lambda)$. There is thus a one-to-one mapping from the arrival rate *a* to the corresponding marginal value, $V'(a){:}V'(a) = \overline{\Phi}^{-1}(a/\lambda)$. Denoting $V'(a) = v(p_1, p_2), a = (s_1 + s_2)\lambda$, we get the third condition.

In order to prove the non-existence of an asymmetric equilibrium, we consider the case of $p_1 \geq p_2$ without loss of generality. From the assumption that V follows a uniform $U[0,1]$, we get

$1 - v = s_1 + s_2 \equiv t$

$v = 1 - t.$

For the upper limit of uniform distribution of V, we assumed it to be 1. This unitized value of 1 is without loss of generality since we can adjust the units of p and c.

Using this, the second condition now becomes:

$$p_1 = 1 - s_1 - s_2 - \frac{c}{\mu - s_1\lambda},$$

$$p_2 = 1 - s_1 - s_2 - \frac{c}{\mu - s_2\lambda}.$$

Utilizing the one-to-one correspondence between (p_1, p_2) and (s_1, s_2), we take (s_1, s_2) as a decision variable for convenience instead of (p_1, p_2). The profit of firm 1, denoted by Π_1, now becomes

$$\Pi_1 = s_1\lambda p_1$$

$$= s_1\lambda(1 - s_1 - s_2 - \frac{c}{\mu - s_1\lambda}).$$

Since λ is a constant, we derive the first order partial derivative of Π_1 / λ with respect to s_1.

$$\frac{\partial(\Pi_1/\lambda)}{\partial s_1} = 1 - s_1 - s_2 - \frac{c}{\mu - s_1\lambda} - s_1[1 + \frac{c\lambda}{(\mu - s_1\lambda)^2}]$$

$$= \frac{(1 - s_2 - 2s_1)(\mu - s_1\lambda)^2 - c\mu}{(\mu - s_1\lambda)^2},$$

$$\frac{\partial^2(\Pi_1/\lambda)}{\partial s_1^{\ 2}} = -2 - \frac{2c\lambda}{(\mu - s_1\lambda)^2} - \frac{2c\lambda^2 s_1}{(\mu - s_1\lambda)^3} < 0.$$

We now study the sign of the above partial derivative. From the second order partial derivative above, we know that Π_1 is strictly concave and thus we can get the optimal solution of s_1 from the first order necessary condition. From the nominator being zero(first order necessary condition), we get the graph

$$s_2 = 1 - 2s_1 - \frac{c\mu}{(\mu - s_1\lambda)^2}.$$

And this is the reaction function of firm 1 in response to s_2 of firm 2. Considering the derivative of s_2 with respect to s_1, we get

$$\frac{ds_2}{ds_1} = -2 - \frac{2c\mu\lambda}{(\mu - s_1\lambda)^3} < 0,$$

$$\frac{d^2 s_2}{ds_1^{\ 2}} < 0$$

From this we know that s_2 graph above has steeper slope than -2 and gets steeper as s_1 becomes larger. That is, for the reaction function of firm 1, s_2 is concave decreasing with respect to s_1. Likewise, we can get the reaction function of firm 2 by exchanging s_1 and s_2.

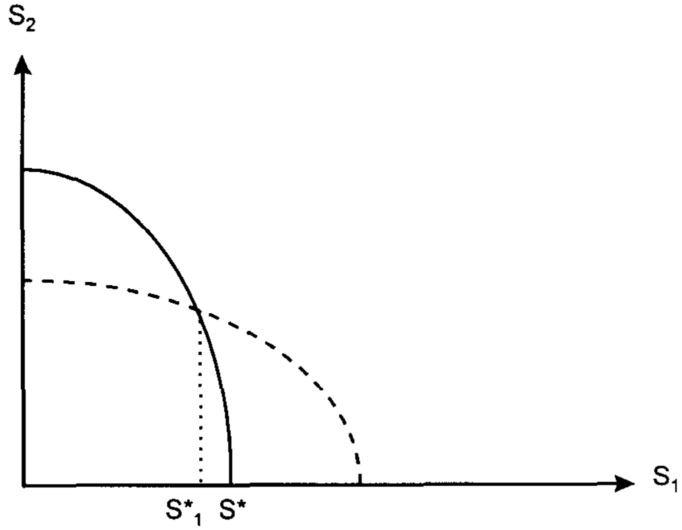

Figure 1

The intersection of those two reaction functions is unique and can be derived from the third order equation (Figure 1):

$(1-3s_1)(\mu - s_1\lambda)^2 = c\mu, s_1 = s_2$

Clearly we can see that there cannot exist an asymmetric equilibrium.

4. Comparison with Other Cases

We first consider the monopoly case. Applying the same method, we get the following equation for deriving optimal market share:

$(1-2s)(\mu - s\lambda)^2 = c\mu.$

As we can see in the graph, we note that the optimal market share, s^*, is greater than that in the duopoly case:

$\Delta \equiv s^* - s_1^* > 0.$

We can say that Δ is the loss in market share due to duopoly competition.

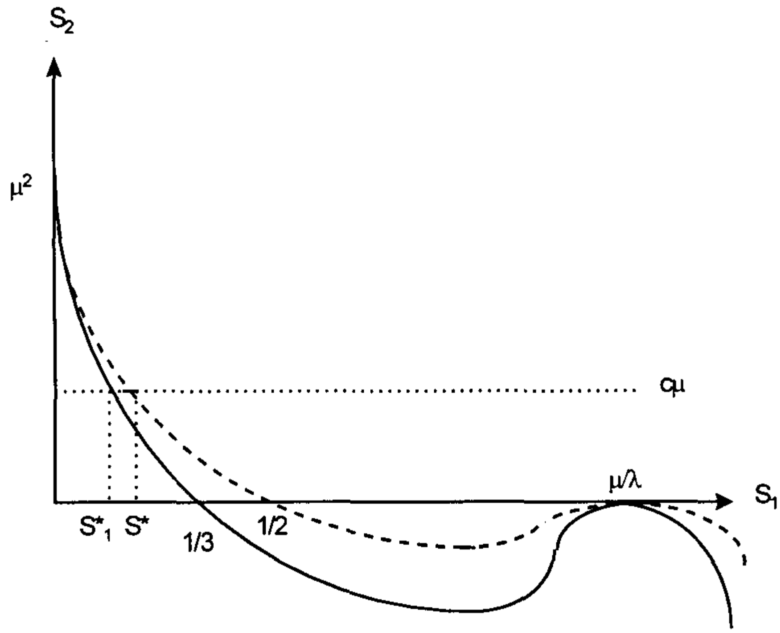

Figure 2

Now we deal with the traditional case where waiting time is not considered. This is the case where c=0. In this case, we get

$$s_1^* = \frac{1}{3}, s^* = \frac{1}{2},$$

$$p_1^* = \frac{1}{3}, p^* = \frac{1}{2}.$$

This result implies that we should enlarge market share when the waiting time does not affect the purchase decision.

5. Concluding Remarks

In this paper we considered mean waiting time of a customer in purchasing decisions. A customer would want to buy service from a seller offering less waiting time if other things are equal. This is because customers suffer opportunity cost of waiting in addition to the price paid to the seller. Therefore a seller should consider the mean waiting time of a customer in choosing a price. In a duopoly competition, we showed that there cannot exist an asymmetric equilibrium under the assumption that time value of customers follow a uniform distribution.

In the future study, we should consider other than uniform distributions for the customers value function. When we relax one or both of the uniform distribution condition and an M/M/1 queueing system assumption, we may have an asymmetric equilibrium which exists in the real business world. For the special case as in the following graph (Figure 3), we can get an asymmetric equilibrium. The real line and the dotted line represent the reaction functions of firm 1 and firm 2 respectively.

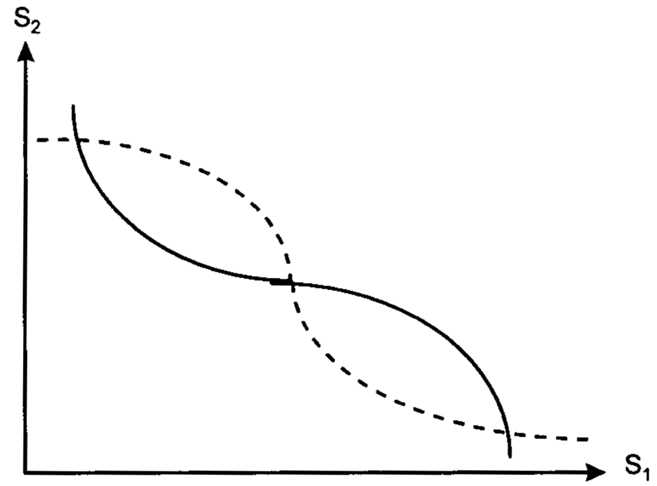

Figure 3

Reference

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