INVENTORY ORDER QUANTITY

—A HEURISTIC APPROACH—

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I. Introduction

Ever since the scientific approach was introduced to business, management science has become the domain of scientific researchers. There is a proliferation of researches and articles which attempted to explain and answer management phenomena and problems by applying such scientific approaches as operations research, mathematics, statistics and so forth.

In reality, however, not all managers have enough backgrounds to understand these rigorous approaches. It is almost natural that they are irritated and frustrated by the numerous equations and formulas which most business consultants and books advise them to use for better management. Furthermore, most theories generated from these researches assume accurate and complete information about their topics. More often than not, this assumption is not realized.

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Under such circumstances, one of the serious problems facing us today is to bridge the gap between the ivory tower and the muddy business world. In this context, this paper attempts to help managers understand and utilize some basic concepts of management science in their daily practices.

Heuristic Approach to Decision Making

By definition, a heuristic is a way of finding a solution to a problem. In the present usage, the meaning is somewhat expanded as any device or procedure used to reduce problem-solving effort—in short, a rule of thumb used to solve a particular problem.(1)

Although the heuristic approach doesn't guarantee the optimal solution to a specific problem, a sub-optimal solution could be found through the heuristic approach without deviating too much from the optimal solution. Moreover, if the cost saving is greater than the difference between the benefits of optimal and sub-optimal solutions, the sub-optimal solution would turn out to be the real optimal solution.

There is no general way of heuristic programming, as this is its own nature. Yet Newell (1969) has attempted to identify and classify some methods of heuristic programming.

The first alternative is *generate-and-test.* All that is necessary is a generator of possible candidates for solutions and a test for the candidates to find if they are indeed solutions. Here, a generator is a process that takes information specifying a set and produces elements of that set one by one. The test is a process that determines whether some conduction or predicate is true of its input and behaves differentially as a result.(2)

The second alternative is *match.* It is the case of two sets of expressions, each capable of generating subparts of themselves and where one set X is

(1) Jerome D. Wiest, "Heuristic Programs for Decision Making," *Harvard Business Review,* Vol. 444 No. 5 (September-October, 1966), p. 130.

(2) Allen Newell, "Heuristic Programming: Ill-structured Problems," *Progress in Operations Research,* Vol. III, ed. Julius S. Aronofsky (New York: John Wiley & Sons, Inc., 1969), pp. 377-378.

a set defined by the other set.[3]

The third method is *hill climbing*. In hill climbing, possible solutions are generated, the solution is compared against the best solution generated previously and then is either discarded, or, if it is a better solution, used to replace the previously stored solution.[4]

Heuristic search is the most widely known method of heuristic programming, which casts the problem as a search through an exponentially expanding space of possibilities—as a search which must be controlled and focused by the application of heuristics.[5]

Finally, *induction* is classified also as a method of heuristic programming. There are two essential features of induction: The first essential feature of the method is revealed in the problem statement, which requires the problem to be cast as one of finding a function or mapping of the given data into the associated (or predicted) data. The space of functions is never provided by the problem poser. The second essential feature of the method is the use of a form or kernel for the function. This can be matched (in the sense of the match method) against the exempllrs. Evidence in the items then operates directly to specify the actual function from the kernel.[6]

These five methods have a number of common characteristics. First, the assumptions of the method need not be strong.[7] Other methods in operations research that use known processes have stronger assumptions that imply these processes. Second, the methods of heuristic programming are similar to each other. This quality allows flexible and creative combinations of these methods, without necessarily treating each method as a closed sub-routine with its output being tied to the input needed by the next

(3) George C. Michael, "A Review of Heuristic Programming," *Decision Sciences*, Vol. 3, No. 3 (July, 1972), p. 78.
(4) Ibid., p. 79.
(5) Ibid., p. 79.
(6) Ibid., p. 80.
(7) Ibid., p. 80.

method. Finally, the heuristic methods make much weaker demands for information than the more structured methods. This characteristic gives the heuristic methods a generality that cannot be matched.(8) These characteristics cannot be over-emphasized, since the problem to be investigated in the next chapter requires some combinations of the heuristic programming under uncertainty, and fewer assumptions will make a better program.

Now let us consider the advantages and disadvantages of heuristic programming. The greatest single advantage of heuristic programming is an inherent flexibility which allows it to be used on ill-structured and complex problems that do not fit the stringent conditions required for the application of most OR models. However, this flexibility can lead to misleading or even fraudulent manipulations.(9) Thus, it should be approached with careful manipulation and documentation. The next advantage is that heuristic programming often results in a saving of time and cost. Since a heuristic programming does not guarantee an optimal solution, this cost saving should be considered in relation with the marginal value of the solutions.

The heuristic approach is recommended in two cases. One is when problems have too many alternatives or variables for the present capacity of computing devices to test. The other is when problems are ill-structured so that they cannot be expressed in mathematical terms.(10)

The latter case is found in many situations. Sometimes qualitative criteria, such as happiness, beauty, and freedom which involve judgment, intuition, creativity or imagination can hardly be quantified and rationalized. Even when a problem is quantifiable, it is sometimes difficult to identify or predict the exact and precise value of each variable. In natural science, errors of specification and/or measurement can be minimized by careful preparation. In social science, there is no easy way of getting rid of these

(8) Ibid., p. 80.
(9) Ibid., p. 81.
(10) Wiest, "Hevristic Programs for Decision Making," Ibid., p. 131.

errors, because of the complexity of variables affecting the environment.

For example, consider the inventory system of a business firm. There is an abundance of alternative methods of inventory control which were developed during the past few decades. Once we know all the exact values of demand in the future, of ordering costs and setup costs, of inventory carrying costs, and of other variables involved, we can pinpoint the quantity and the date of ordering by plugging these values to a relevant model. But in the real world, how can we know the exact amount of these variables? How can we rely on prediction? Will the predicted value not be changed?

It would probably be better to predict each of these values in terms of approximation with a certain range. Then how can the solutions be found out of these ranges? The quantitative techniques which are already set up in the form of equations and formulas are not suitable. Here we need a new approach.

II. Review of Theories on Inventory Control

One of the several fields which are investigated and refined with precise and scientific results is that of inventory control. In almost every industry and firm, they incessantly confront questions of why, how much, and when to inventory. For the first question, the answer is that inventories are needed in a certain condition,(11) because without them they could not achieve smooth production flow, obtain reasonable utilization of machines and reasonable material handling costs, or expect to give reasonable services to customers on hundreds of items regarded as 'stock' items.(12) Thus, we hsould have inventories which can be classified as follows.

(11) For a detailed reference, see R.W. Fenske, "Non-stocking Criterion", *Management Science*, Vol. 14, No. 12 (August, 1968), B-705-14.

(12) Elwood S. Buffa, *Basic Production Management* (New York, John Wiley & Sons, Inc., 1961), pp. 464-465.

a) Movement inventories: which are needed because of the time required to move stocks from one place to another.(13)

b) Organization inventories: which include

i) Lot-size inventories, which are maintained in larger lots than are needed for the immediate purposes, because of expensive setup and ordering cost, or as a way to obtain quantity price discounts, etc.(14)

ii) Fluctuation stocks, which are held to cushion the stocks arising basically from unpredictable fluctuations in consumer demand.(15)

iii) Anticipation stocks, which are needed where goods or materials are consumed on a predictable but changing pattern through the year and where it is desirable to absorb some of these changes by building and depleting inventories rather than by changing production rates with attendant fluctuations in employment and additional capital capacity reqirement.(16)

For the next two questions concerning ordering quantity and time, numerous answers have been attempted and developed from the second decade of the twentieth century up to the present time. Among them, two basic devices of evaluating the economic potential of alternative ways of performing a specified task are break-even-point analysis and economic lot-size theory.(17)

First, according to the simplified break-even-point model, the total cost of each alternative is composed of fixed and variable costs. Thus,

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(13) John F. Magee, "Guides to Inventory Policy, I-Functions and Lot Sizes," *Harvard Business Review*, Vol. 34, No. 1 (January-February, 1956), p. 52.

(14) Ibid., p. 52.

(15) Ibid., p. 52.

(16) Ibid., p. 53.

(17) Wayland P. Smith, "An Investigation of Some Quantitative Relationships Between Break-even Point Analysis and Economic Lot-size Theory," eds. Robert H. Bock and William K. Holstein, *Production Planning and Control: Text and Readings* (Columbus, Ohio: Charles E. Merril Books, Inc., 1964), p. 242.

$$C_i = F_i + NV_i \tag{1}$$

where C_i: total cost of ith alternative

F_i: summation of all initial fixed cost for the ith alternative

N: the quantity to be produced

V_i: summation of all variable cost for the ith alternative.

Plotting this relationship, we can draw Fig. 1. F_i becomes the intercept and V_i becomes the slope of the straightline where C_i(dependent variable) varies along with the movement of N (independent variable).

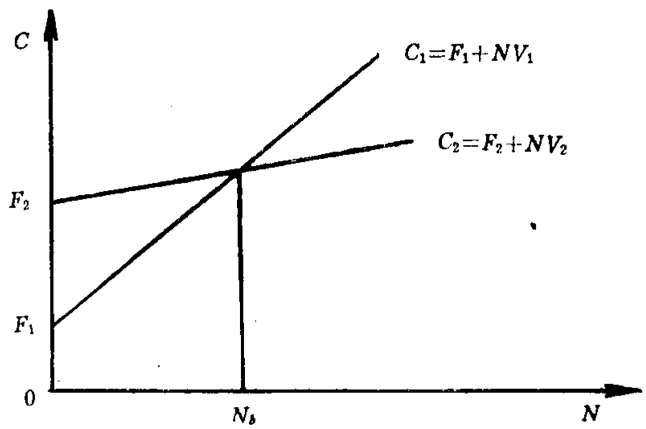

Fig. 1.

Then the decision rules would be,

if $N<N_b$, select 1

if $N>N_b$, select 2.[18]

The economic lot-size theory has been developed during the period 1910 through 1930. Here are some models based on this theory.

Case I: Assuming that backlog is allowed in a production-inventory situation.

$$Q^* = \sqrt{\frac{2RS(P+I)}{PI\left(1-\frac{R}{N}\right)}} \tag{2}$$

(18) Ibid., p. 245.

$$W^*=\sqrt{\frac{2RSI\left(1-\frac{R}{N}\right)}{P(P+I)}} \tag{3}$$

$$C^*=\sqrt{\frac{2RSIP}{(P+I)}\left(1-\frac{R}{N}\right)} \tag{4}$$

$$t_0^*=\frac{Y^*}{b} \tag{5}$$

Where

Q: Quantity of an order

W: Maximum shortage quantity

C: Total inventory cost per unit time

t_0: time per cycle

R: Demand quantity per unit time

N: Production quantity per unit time

S: Ordering cost (set-up cost) per order

I: Inventory carrying cost per unit time

P: Penalty cost per unit time

*: Optimal condition

Case II: Assuming no backlog in production-inventory system.

$$Q^*=\sqrt{\frac{2RS}{I\left(1-\frac{R}{N}\right)}} \tag{6}$$

$$C^*=\sqrt{2RSI\left(1-\frac{R}{N}\right)} \tag{7}$$

$$t_0^*=\frac{Q^*}{R}=\sqrt{\frac{2S}{RI\left(1-\frac{R}{N}\right)}} \tag{8}$$

Case III: Assuming that backlog is allowed in a purchasing-inventory model.

$$Q^*=\sqrt{2RS\left(\frac{1}{I}+\frac{1}{P}\right)} \tag{9}$$

$$W^*=\sqrt{\frac{2RSI}{P(P+I)}} \tag{10}$$

$$C^*=\sqrt{\frac{2RSIP}{(P+I)}} \tag{11}$$

$$t_0^* = \frac{Q^*}{R} \tag{12}$$

Case IV: Assuming no backlog in a purchasing-inventory model.

$$Q^* = \sqrt{\frac{2RS}{I}} \tag{13}$$

$$C^* = \sqrt{2RSI} \tag{14}$$

$$t_0^* = \frac{Q^*}{R} = \sqrt{\frac{2S}{RI}} \tag{15}$$

We can develop as many algebraic expressions for the ordering quantity as we need by changing assumptions and/or by adding constraints.

Still, neither the break-even-point nor the economic lot-size theory are complete. For one, break-even analysis ignores inventory cost and assumes a single set up, while economic lot-size theory ignores the possibility of different manufacturing methods with their accompanying differences and fixed variable cost patterns.[19]

From Theory to Practice

More often than not, these mathematically exquisite formulas are not convenient for ordinary managers to use. To avoid this headache, Welch (1956) suggested several ways of finding optimal ordering quantities using tables, linear charts, logarithmic charts, nomographs, and straight or circular slide rules.[20]

The simplest method he used is the tables which were developed with an assumption of only one variable while other variables are set to be constant (Table 1). The linear chart has an advantage of deciding Q^* values from a single page which might otherwise require many pages of tables (Fig. 2). The logarithmic chart has an advantage of having the same relative accuracy in reading at the low and the high end of the chart (Fig. 3). The nomograph form of chart is particularly useful and fairly easy to solve formulas of the order quantity type (Fig. 4). Straight or circular

(19) Ibid., p. 249.
(20) W. Evert Welch, *Scientific Inventory Control* (Greenwich, Conn.: Management Publishing Corp., 1956), Chap. 6.

rules are just variations of the nomograph(Fig. 5 and 6).

All these methods get the same answer in the end. Still, there are some problems applying these methods to the real world. First of all, some training is needed for operators before using these tools effectively. An even more critical problem concerns accuracy. The accuracy of the Q^* values depends upon the accuracy of the numbers estimated and inserted in the equations. Unfortunately, none of these values can be estimated with certainty.

Table 1. Typical Ordering Quantity Table

Annual Usage	(K=8)	Order Quantity(K=10)	(K=12.92)
$ 10	$ 26	$ 32	$ 41
25	40	50	65
50	57	71	92
75	70	87	112
100	80	100	129
250	126	158	204
500	179	224	289
750	219	274	354
1,000	253	316	408
2,500	400	500	646
5,000	566	707	914
7,500	689	866	1,120
10,000	800	1,000	1,290
25,000	1,260	1,580	2,040
50,000	1,790	2,240	2,890
75,000	2,190	2,740	3,540
100,000	2,530	3,160	4,080

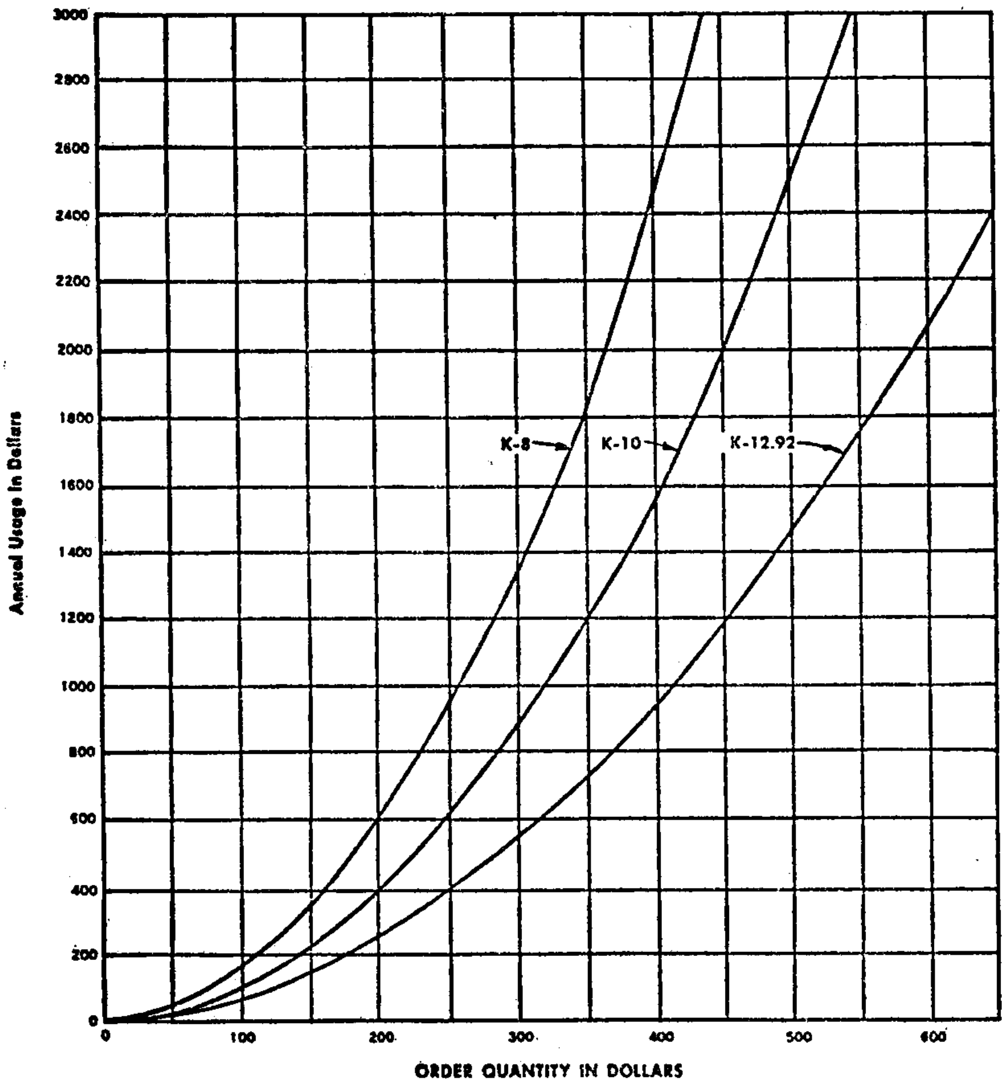

Fig. 2. Order Quantity Chart

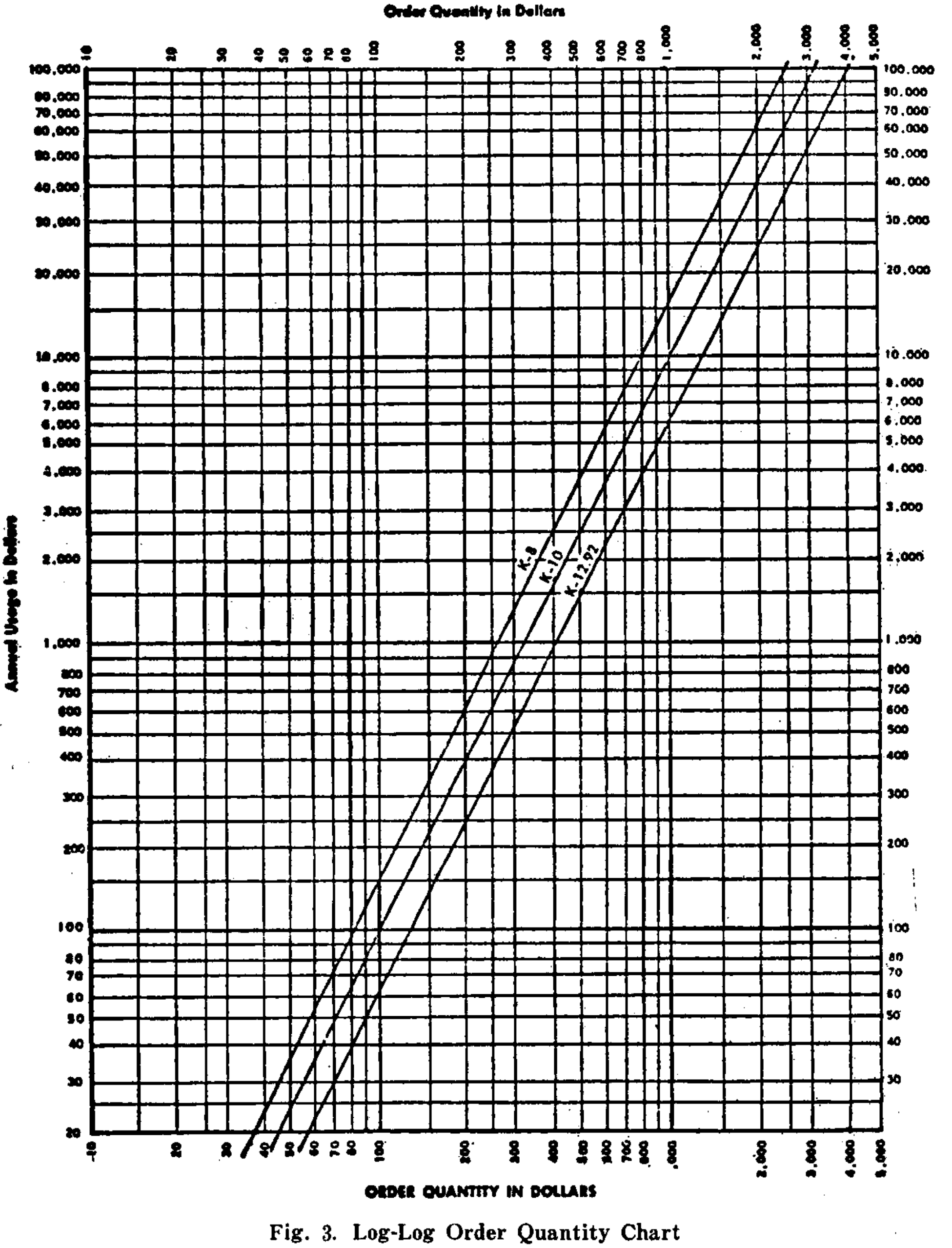

Fig. 3. Log-Log Order Quantity Chart

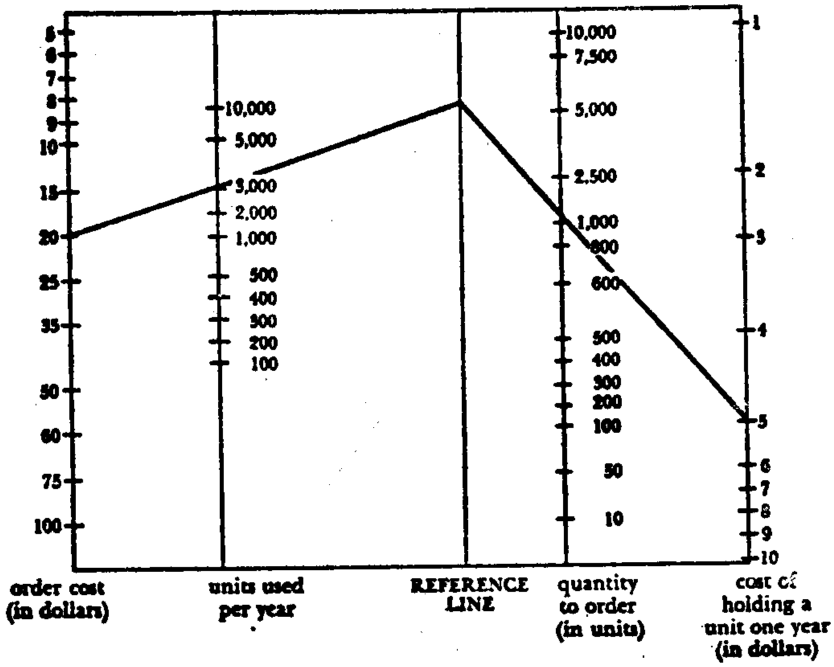

Fig. 4. Nomograph for Selecting Economic Ordering Quantity

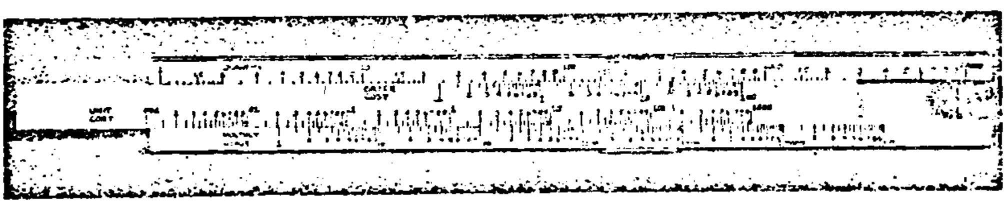

Fig. 5. Straight Rule

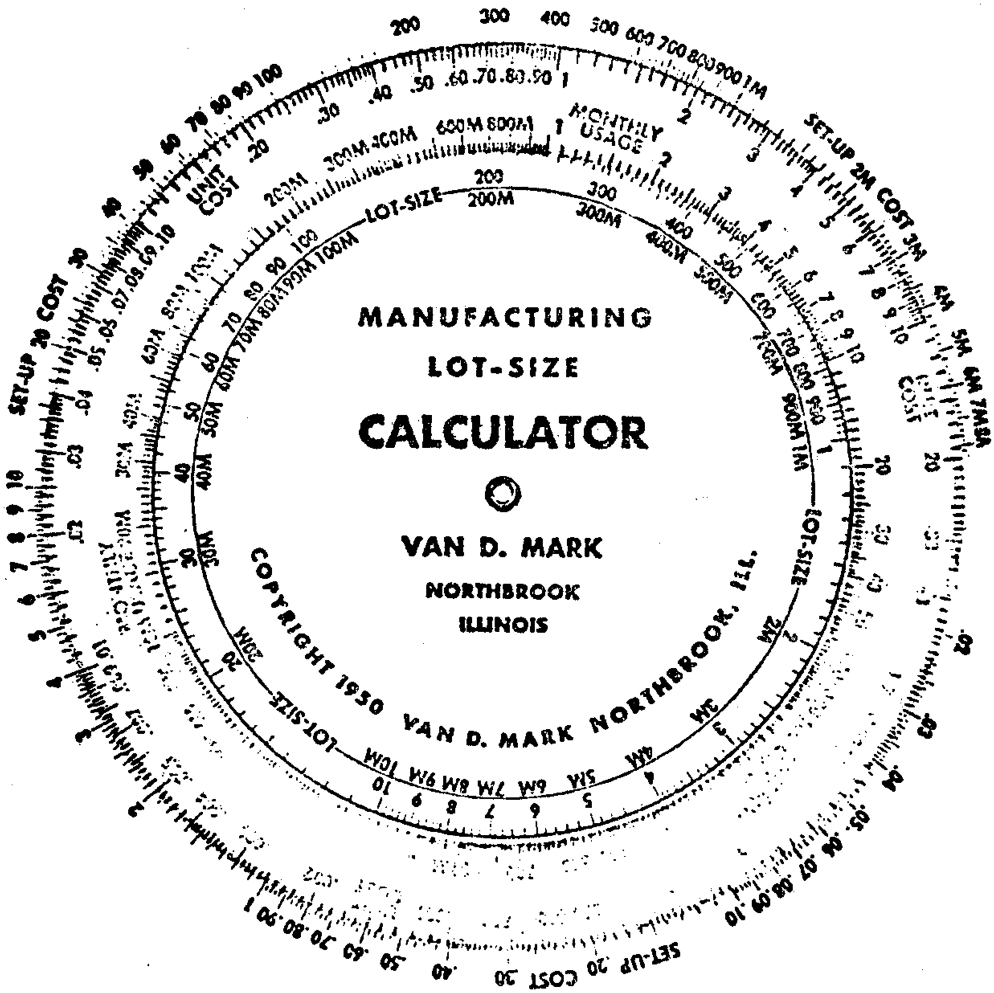

Fig. 6. Circular Rule

III. Development of the Model

Let us consider a situation on inventory control under the following assumptions.

a) Quantity of demand for a specific period is known and constant.

b) Ordering cost is constant throughout the period and inventory carrying cost is linear (i.e., it is proportional to the quantity of inventory).

c) Lead time is known and constant.

d) No stockout is allowed (i.e., the cost for the stockout is too great to be allowed).

e) Instantaneous replenishment is assumed.

f) No quantity discount suggested.

Then the total cost is the summation of ordering costs which are fixed per order regardless of quantity ordered; and inventory carrying costs which are variable according to the quantity ordered. Since the number of orders in a period is equal to the demand divided by the ordering quantity per order, the total ordering cost is found by multiplying the ordering cost per order by that number. And when the constant rate of demand or consumption is assumed, the average inventory level throughout a given period would be half the inventory quantity (Figure 7). Thus, the total inventory carrying cost for a given period is calculated by multiplying the inventory carrying cost by the average inventory level.

We can formulate these relations as follows using the same notations as in Chapter II of this paper:

$$C=IQ/2+RS/Q \tag{16}$$

Then by setting the first derivative of the function with respect to Q to

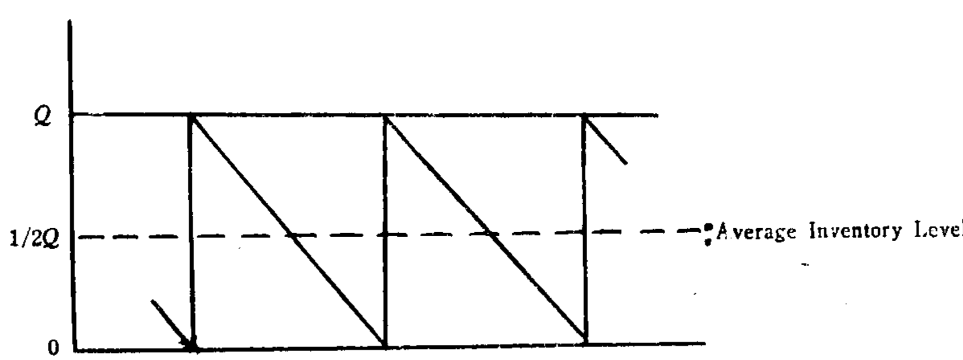

Fig. 7.

equal to zero, we can find the optimal quantity of Q (to be named Q^*).

$$\frac{dC}{dQ}=\frac{I}{2}-\frac{RS}{Q^2}\overset{(\text{set})}{=}0 \tag{17}$$

$$Q^{*2}=2RS/I \tag{18}$$

$$Q^*=\sqrt{2RS/I} \tag{19}$$

This result is the same as that of the Case IV in Chapter II.

The second derivative of the equation with respect to Q is:

$$\frac{d^2C}{dQ^2}=\frac{3RS}{Q^3}>0 \tag{20}$$

Equation (19) tells us the optimal quantity of Q while Equation (20) guarantees this point to be the minimum.

Now a problem arises when a manager is not able to pinpoint the exact value of each variable. Actually, collection and analysis of data leading to estimation of demand, ordering cost, holding cost and other variables are subject to error. In this context, it would be easier for mangers to predict the level of demand quantity with a certain range. Likewise, the ratio of setup cost or ordering cost to inventory carrying cost using a range would be a better approximation than a single value of respective variables.(21) Then the above EOQ model can be separated as follows.

$$Q^*=\sqrt{2}\cdot\sqrt{R}\cdot\sqrt{S/I} \tag{21}$$

It is assumed that a typical manager can afford expenses up to $100x\%$ of the estimated optimal inventory costs. Then:

$$C/C^*\leq x\ :\ (x>1) \tag{22}$$

Since $C/C^*=1/2(Q/Q^*+Q^*/Q)$ (23)

$$\frac{1}{2}\left(\frac{Q}{Q^*}+\frac{Q^*}{Q}\right)\leq x \tag{24}$$

$$\frac{Q^2+Q^{*2}}{Q^*Q}\leq 2x \tag{25}$$

(21) It can never be overemphasized because this ratio of setup cost to inventory carrying cost can reduce these two variables to one, thus eliminating the range of errors. Furthermore this way enables us to develop two dimensional tables out of three variables. Prichard and Eagle (1965) developed the tables using the range of R given S and I to be constant. Their approach was proper to the heuristic nature, but had limitations regarding variations of S and I.

$$Q^2+Q^{*2}\leq 2xQ^*Q \qquad (Q^*Q>0) \tag{26}$$

$$Q^2-2xQ^*Q+Q^{*2}\leq 0 \tag{27}$$

$$(Q^2-2xQ^*Q+x^2Q^{*2})-x^2Q^{*2}+Q^{*2}\leq 0 \tag{28}$$

$$(Q-xQ^*)^2-x^2Q^{*2}+Q^{*2}\leq 0 \tag{29}$$

$$(Q-xQ^*)^2\leq Q^{*2}\ (x^2-1) \tag{30}$$

$$-\sqrt{Q^{*2}(x^2-1)}\leq(Q-xQ^*)\leq+\sqrt{Q^{*2}(x^2-1)} \tag{31}$$

$$xQ^*-\sqrt{Q^{*2}(x^2-1)}\leq Q\leq xQ^*+\sqrt{Q^{*2}(x^2-1)} \tag{32}$$

$$(x-\sqrt{x^2-1})Q^*\leq Q\leq(x+\sqrt{x^2-1})Q^* \tag{33}$$

Let $Q=\sqrt{2R'}\sqrt{\left(\frac{S'}{I}\right)}$ (34)

Substituting Equation (34) for Q in Equation (33):

$$(x-\sqrt{x^2-1})Q^*\leq\sqrt{2R'}\sqrt{\left(\frac{S}{I}\right)'}\leq(x+\sqrt{x^2-1})Q^* \tag{35}$$

Substituting Equation (21) for Q^* in Equation (35):

$$(x-\sqrt{x^2-1})\sqrt{2R}\sqrt{\frac{S}{I}}\leq\sqrt{2R'}\sqrt{\left(\frac{S}{I}\right)'}\ (x+\sqrt{x^2-1})\sqrt{2R}\sqrt{\frac{S}{I}} \tag{36}$$

For the purpose of arranging the rows and the columns of the basic table with the same sensitivity range (this supposition is arbitrary and can vary with different purposes), we can divide Equation (36) as:

$$\sqrt{x-\sqrt{x^2-1}}\sqrt{2R}\leq\sqrt{2R'}\leq\sqrt{x+\sqrt{x^2-1}}\sqrt{2R} \tag{37}$$

$$\sqrt{x-\sqrt{x^2-1}}\sqrt{\frac{S}{I}}\leq\sqrt{\left(\frac{S}{I}\right)'}\leq\sqrt{x+\sqrt{x^2-1}}\sqrt{\frac{S}{I}} \tag{38}$$

Still, we may reach the same sensitivity. By multiplying Equations (37) and (38), we get:

$$(x-\sqrt{x^2-1})\sqrt{\frac{2RS}{I}}\leq\sqrt{2R}\sqrt{\left(\frac{S}{I}\right)}\leq(x+\sqrt{x^2-1})\sqrt{\frac{2RS}{I}} \tag{36}$$

Which is identical with Equation (36).

Since all terms in Equations (37) and (38) are positive, these can be squared without changing signs:

$$(x-\sqrt{x^2-1})2R\leq 2R'\leq(x+\sqrt{x^2-1})2R \tag{40}$$

$$(x-\sqrt{x^2-1})\frac{S}{I}\leq\left(\frac{S}{I}\right)\leq(x+\sqrt{x^2-1})\frac{S}{I} \tag{41}$$

For convenience, let us assume that $x=1.1$, (42)

or 10% allowance. Then:

$$(1.1-\sqrt{.21})2R\leq 2R'\leq(1.1+\sqrt{.21})2R \quad (43)$$

$$(1.1-\sqrt{.21})\frac{S}{I}\leq\left(\frac{S}{I}\right)'\leq(1.1-\sqrt{.21})\frac{S}{I} \quad (44)$$

Setting the starting point of R and S/I as 10,000 and 100 respectively, we get the following sets of ranges using Equations(43) and (44).(22) (See Tables 2 and 3). Using these ranges we can develop Table 4.

Likewise, by assuming $x=1.01$, or 1% allowance, we have the following ranges.

$$(1.01-\sqrt{.0201})2R\leq 2R^2\leq(1.01+\sqrt{.0201})2R \quad (45)$$

$$(1.01-\sqrt{.0201})\frac{S}{I}\leq\left(\frac{S}{I}\right)'\leq(1.01+\sqrt{.0201})\frac{S}{I} \quad (46)$$

Thus, Table 6 shows the economic ordering quantity with the allowance of less than 1%.

Table 4 is too numerical for day-to-day managerial purposes. For the benefit of the realistic application, we may set the range of rows and columns with simpler variations. Since $(1.1+\sqrt{.21})/(1.1-\sqrt{.21})=(1.1+\sqrt{.21})^2=2.428$, let us use 2 and 2.5 as ranges allowing small variation from a 10% allowance. By using 2 twice and 2.5 once, we have the range of R as:

……100…200…500…1,000…2,000…5,000…10,000…20,000…50,000…100,000……

and the range of (S/I) as:

……0.01…0.02…0.05…0.1…0.2…0.5…1…2…5…10…20…100…200…500…
……1,000……

Using these ranges, Q is to be calculated as follows resulting in Table 7.

$$Q=\sqrt{2\cdot\sqrt{R_L R_U}\cdot\sqrt{(S/I)_L(S/I)_U}} \quad (48)$$

where R_L: lower limit of R

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(22) For the logarithmic approach, see Y.H. Rutenberg, "Calculation of Economical Order Quantities Using Ranges of Setup Cost," The Journal of Industrial Engineering, Vol. XV, No. 1 (January-February, 1964), pp. 44-46.

Table 2. The Range of R with the same Relative Sensitivity

R_L	R	R_U
—	—	—
—	—	—
—	—	—
—	—	$10,000(1.1-\sqrt{.21})^3$
$10,000(1.1-\sqrt{.21})^3$	$10,000(1.1-\sqrt{.21})^2$	$10,000(1.1+\sqrt{.21})^1$
$10,000(1.1-\sqrt{.21})^1$	$10,000(1.1\pm\sqrt{.21})^{0*}$	$10,000(1.1+\sqrt{.21})^1$
$10,000(1.1+\sqrt{.21})^1$	$10,000(1.1+\sqrt{.21})^2$	$10,000(1.1+\sqrt{.21})^3$
$10,000(1.1+\sqrt{.21})^3$	—	—
—	—	—
—	—	—
—	—	—

$$* \ (1.1\pm\sqrt{.21})^n=(1.1\mp\sqrt{.21})^{-n} \tag{47}$$

Table 3. The Range of $\left(\frac{S}{I}\right)$ with the same Relative Sensitivity

$(S/L)_L$	(S/I)	$(S/I)_U$
—	—	—
—	—	—
—	—	—
—	—	$100(1.1-\sqrt{.21})^3$
$100(1.1-\sqrt{.21})^3$	$100(1.1-\sqrt{.21})^2$	$100(1(1-\sqrt{.21})^1$
$100(1.1-\sqrt{.21})^1$	$100(1.1\pm\sqrt{.21})^0$	$100(1.1+\sqrt{.21})^1$
$100(1.1+\sqrt{.21})^1$	$100(1.1+\sqrt{.21})^2$	$100(1.1+\sqrt{.21})^3$
$100(1.1+\sqrt{.21})^3$	—	—
—	—	—
—	—	—
—	—	—

R_U: upper limit of R

$(S/I)_L$: lower limit of (S/I).

$(S/I)_U$: upper limit of (S/I).

In consideration of reality, we may be satisfied with the following constraint.

$$AR \leq Q \leq BR \tag{49}$$

It indicates that there can be a certain limit of Q in regards with the value of R. For example, let

Table 4. Economic Ordering Quantity with the Sensitivity Range of 10%

$(\frac{S}{I})_L$	$(\frac{S}{I})$	$(\frac{S}{I})_U$											
		R_L	—	76	185	449	1,089	2,644	6,418	15,580	37,818	91,799	—
	R_U	R	—	119	288	699	1,697	4,120	10,000	24,274	58,921	143,023	—
			—	185	449	1,089	2,644	6,418	15,580	37,818	91,799	222,829	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.009	0.014	0.022	—	2	3	4	7	11	17	26	41	63	—
0.022	0.034	0.053	—	3	4	7	11	17	26	41	63	99	—
0.053	0.083	0.129	—	4	7	11	17	26	41	63	99	154	—
0.129	9.201	0.314	—	7	11	17	26	41	63	99	154	240	—
0.314	0.489	0.762	—	11	17	26	41	63	99	154	240	374	—
0.762	1.19	1.85	—	17	26	41	63	99	154	240	374	583	—
1.85	2.88	4.49	—	26	41	63	99	154	240	374	583	908	—
0.49	6.99	10.89	—	41	63	99	154	240	374	583	908	1,414	—
10.89	16.97	26.44	—	63	99	154	240	374	583	908	1,414	2,203	—
26.44	41.20	64.18	—	99	154	240	374	583	908	1,414	2,203	3,433	—
64.18	100.	155.9	—	154	240	374	583	908	1,414	2,203	3,433	5,348	—
155.9	242.7	378.2	—	240	374	583	908	1,414	2,203	3,433	5,348	8,333	—
378.2	589.2	918.0	—	374	583	908	1,414	2,203	3,433	5.348	8,333	12,982	—
918.0	1430.	2288.	—	583	908	1,414	2,203	3,433	5,348	8,333	12,982	20,226	—
2288.	3472.	5409.	—	908	1,414	2,203	3,433	5,348	8,333	12,982	20,226	31,512	—
5409.	8427.	13130.	—	1,414	2,203	3,433	5,348	8,333	12,982	20,226	31,512	49,097	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—

Table 5. Economic Ordering Quantity with the Sensitivity Range of 5%

$(\frac{S}{I})_L$	$(\frac{S}{I})$	$(\frac{S}{I})_U$								
		R_L	59	110	207	389	730	1,370	2,572	4,829	9,065
	R_U	R	81	151	284	533	1,000	1,877	3,524	6,616	12,421
			110	207	389	730	1,370	2,572	4,829	9,065	17,019
.59	.81	1.10	11	16	21	29	40	55	75	103	141
1.10	1.51	2.07	16	21	29	40	55	75	103	141	194
2.07	2.94	3.89	21	29	40	55	75	103	141	194	265
3.89	5.33	7.30	29	40	55	75	103	141	194	265	364
7.30	10.00	13.70	40	55	75	103	141	194	265	364	494
13.70	18.77	25.72	55	75	103	141	194	265	364	494	683
25.72	35.24	48.29	75	103	141	194	265	364	494	683	936
48.29	66.16	90.65	103	141	194	265	364	494	683	936	1,282
90.65	124.21	170.19	141	194	265	364	494	683	936	1,282	1,757

Table 6. Economic Ordering Quantity with the Sensitivity Range of 1%

$\left(\frac{S}{I}\right)_L$	$\left(\frac{S}{I}\right)$	$\left(\frac{S}{I}\right)_U$																	
		R_L	91	120	159	211	280	372	493	654	868	1, 152	1, 528	2, 027	2, 689	3, 567	4, 732	5, 277	8, 327
		R	104	138	183	243	323	428	568	754	1, 000	1, 327	1, 760	2, 335	3, 097	4, 108	5, 450	7, 230	9, 591
		R_U	120	159	211	280	372	493	654	868	1, 152	1, 528	2, 027	2, 689	3, 567	4, 732	5, 277	8, 327	11, 046
0. 91	1. 04	1. 20	15	17	20	23	26	30	34	40	46	53	60	70	80	93	107	123	141
1. 20	1. 38	1. 59	17	20	23	26	30	34	40	46	53	60	70	80	93	107	123	141	163
1. 59	1. 83	2. 11	20	23	26	30	34	40	46	53	60	70	80	93	107	123	141	163	188
2. 11	2. 43	2. 80	23	26	30	34	40	46	53	60	70	80	93	107	123	141	163	188	216
2. 80	3. 23	3. 72	26	30	34	40	46	53	60	70	80	93	107	123	141	163	188	216	249
3. 72	4. 28	4. 93	30	34	40	46	53	60	70	80	93	107	123	141	163	188	216	249	287
4. 93	5. 68	6. 54	34	40	46	53	60	70	80	93	107	312	141	163	188	216	249	287	330
6. 54	7. 54	8. 68	40	46	53	60	70	80	93	107	123	141	163	188	216	249	287	330	380
8. 68	10. 00	11. 52	46	53	60	70	80	93	107	123	141	163	188	216	249	287	330	380	438
11. 52	13. 27	15. 28	53	60	70	80	93	107	123	141	163	188	216	249	287	330	380	438	505
15. 28	17. 60	20. 27	60	70	80	93	107	123	141	163	188	216	249	287	330	380	438	505	581
20. 27	23. 35	26. 89	70	80	93	107	123	141	163	188	216	249	287	330	380	438	505	581	669
26. 89	30. 97	35. 67	80	93	107	123	141	163	188	216	249	287	330	380	438	505	581	669	771
35. 67	41. 08	47. 32	93	107	123	141	163	188	216	249	287	330	380	438	505	581	669	771	888
47. 32	54. 50	52. 77	107	123	141	163	188	216	249	287	330	380	438	505	581	669	771	888	1, 022
52. 77	72. 30	83. 27	123	141	163	188	216	249	287	330	380	438	505	581	669	771	888	1, 022	1, 178
83. 27	95. 91	110. 46	141	163	188	216	249	287	330	380	438	505	581	669	771	888	1, 022	1, 178	1, 356

Table 7. Economic Ordering Quantity with the Maximum Sensitivity Range of 11.2%

$(\frac{S}{I})_L$	$(\frac{S}{I})$	$(\frac{S}{I})_U$ \ R_L / R / R_U	—	100	200	500	1,000	2,000	5,000	10,000	20,000	50,000	—
		R_U	—	200	500	1,000	2,000	5,000	10,000	20,000	50,000	100,000	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.01		0.02	—	2	3	5	6	10	14	20	30	45	—
0.02		0.05	—	3	5	7	10	14	21	30	45	67	—
0.05		0.1	—	5	7	10	14	21	32	45	67	100	—
0.1		0.2	—	6	10	14	20	30	45	63	95	140	—
0.2		0.5	—	10	14	21	30	45	67	95	140	210	—
0.5		1	—	14	21	32	45	67	100	140	210	320	—
1		2	—	20	30	45	63	95	140	200	300	450	—
2		5	—	30	45	67	95	140	210	300	450	670	—
5		10	—	45	67	100	140	210	320	450	670	1,000	—
10		20	—	63	95	140	200	300	450	630	950	1,400	—
20		50	—	95	140	210	300	450	670	950	1,400	2,100	—
50		100	—	140	210	320	450	670	1,000	1,400	2,100	3,200	—
100		200	—	200	300	450	630	950	1,400	2,000	3,000	4,500	—
200		500	—	300	450	670	950	1,400	2,100	3,000	4,500	6,700	—
500		1,000	—	450	670	1,000	1,400	2,100	3,200	4,500	6,700	10,000	—
1,000		2,000	—	630	950	1,400	2,000	3,000	4,500	6,300	9,500	14,000	—
2,000		5,000	—	950	1,400	2,100	3,000	4,500	6,700	9,500	14,000	21,000	—
5,000		10,000	—	1,400	2,100	3,200	4,500	6,700	10,000	14,000	21,000	32,000	—
10,000		20,000	—	2,000	3,000	4,500	6,300	9,000	14,000	20,000	30,000	45,000	—
20,000		50,000	—	3,000	4,500	6,700	9,500	14,000	21,000	30,000	45,000	67,000	—
50,000		100,000	—	—	—	—	—	—	—	—	—	—	—

$$A=\frac{1}{52} \tag{50}$$

and

$$B=1 \tag{51}$$

where unit time is a year, thus making R an annual demand quantity. By substituting Equations (50) and (51) for A and B respectively in Equation (49):

$$R/52 \leq Q \leq R \tag{52}$$

That is, if the quantity of an order is less than a weekly demand, we may order at least to that amount, and if the quanity of an order is greater than a yearly demand, we may order at most to that amount, regardless

Table 8. Economic Ordering Quantity with the Maximum Sensitivity Range of 11.2% with Restrictions on *Q*

$\left(\frac{S}{I}\right)_L$	$\left(\frac{S}{I}\right)$	$\left(\frac{S}{I}\right)_U$ \ R_L / R / R_U	—	100 / 200	200 / 500	500 / 100	1,000 / 200	2,000 / 5,000	5,000 / 10,000	10,000 / 20,000	20,000 / 50,000	50,000 / 100,000	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.01		0.02	—	2									
0.02		0.05	—	3	5				$Q=\frac{R}{52}$				—
0.05		0.1	—	5	7	10							—
0.1		0.2	—	6	10	14	20						—
0.2		0.5	—	10	14	21	30	45					—
0.5		1	—	14	21	32	45	67	100				—
1		2	—	20	30	45	63	95	140	200			—
2		5	—	30	45	67	95	140	210	300	450		—
5		10	—	45	67	100	140	210	320	450	670	1,000	—
10		20	—	63	95	140	200	300	450	630	950	1,400	*
20		50	—	95	140	210	300	450	670	950	1,400	2,100	—
50		100	—	140	210	320	450	670	1,000	1,400	2,100	3,200	—
100		200	—		300	450	630	950	1,400	2,000	3,000	4,500	—
200		500	—			670	950	1,400	2,100	3,000	4,500	6,700	—
500		1,000	—				1,400	2,100	3,200	4,500	6,700	10,000	—
1,000		2,000	—					3,000	4,500	6,300	9,500	14,000	—
2,000		5,000	—						6,700	9,500	14,000	21,000	—
5,000		10,000	—		$Q=R$					14,000	21,000	32,000	—
10,000		20,000	—								30,000	45,000	—
20,000		50,000	—									67,000	**
—	—	—	—										—

* If $Q \leq \frac{R}{52}$, then let $Q=\frac{R}{52}$ ** If $Q \geq R$, then let $Q=R$.

of the number in the table. Then the table will be set up as in Table 7. In the table, Q is rounded to make the effective number of integers two except the values which are less than 10.

Evaluation of Table 7

Since we used 2 and 2.5 instead of $(1.1+\sqrt{.21})^2$ in table 7, and since we rounded the number Q to make Table 7 simple and neat, it is necessary to evaluate the sensitivity of the table.

From table 7, let us choose an element $Q=140$ (53)

where $2,000 \leq R \leq 5,000$ (54)

and $2 \leq (S/I) \leq 5.$ (55)

Multiplying Equations (54) and (55):

$$8,000 \leq 2RS/I \leq 50,000 \quad (56)$$

Since every term is positive, taking the square root of each term results in:

$$40\sqrt{5} \leq \sqrt{2RS/I} \leq 100\sqrt{5} \quad (57)$$

Substituting Equation (19) for $\sqrt{2RS/I}$ in Equation (57):

$$40\sqrt{5} \leq Q^* \leq 100\sqrt{5} \quad (58)$$

Then $LQ^* = 40\sqrt{5}$ (59)

$UQ^* = 100\sqrt{5}$ (60)

where LQ^*:lower limit of Q^*

and UQ^*: upper limit of Q^*.

The sensitivity will be greatest where $Q = LQ^*$ and (or) $Q = UQ^*$.

Evaluating the sensitivity at the point $Q = LQ$:

$C/C^* = 1/2(Q/Q^* + Q^*/Q)$: from Equation(23)

$= 1/2\ (Q/LQ^* = LQ^*/Q)$

$= 1/2\ (140/40\sqrt{5} + 40\sqrt{5}/140)$: from Equations (53) and (59)

$= 69\sqrt{5}/140$

< 1.1021 (61)

And at the point $Q = UQ^*$:

$C/C^* = 1/2(Q/Q^* + Q^*/Q)$

$= 1/2\ (Q/UQ^* + UQ^*/Q)$

$= 1/2(140/100\sqrt{5} + 100\sqrt{5}/140)$

$= 87\sqrt{5}/175$

< 1.1117 (62)

From Equations (61) and (62):

$$C/C^* < 1.1117 \quad (63)$$

All the other elements are to be evaluated using the same procedures. In this way, we find that the maximum sensitivity of Table 7 is 11.2%. Thus, we can say that Table 7 may be used under the assumption of maximum inventory cost allowance of 11.2%. Furthermore, the expected sensitivity of the total cost would be 1.8%, much less than maximum allowance. (See Appendix.)

IV. Conclusion

The problems of inventory control analysis lie on its characteristics that prediction of variables can never be absolutely correct. Still, most approaches to inventory control assume that managers know exact values of these variables. The farther the prediction of any variable is from the real value, the poorer would be the result of these approaches.

Since the total inventory cost is not sensitive within a certain range of economic ordering quantity, there would be a controllable range of ordering quantity without causing too much difference from the optimal cost. This range can be controlled by setting ranges of R and S/I in a table. This ratio of S/I enables the variation of three variables of a simple EOQ model on two dimensional table. This table guarantees the deviation of total inventory cost within $100(x-1)\%$ of the optimal cost. Because the magnitude of error is inversely related to the time and money spent in collecting data, this table is recommended as an optimal answer if the cost for refining the input data is greater than the amount of reduction in total inventory cost by refining. For the inventory items with low dollar values and large demand quantities,(23) this table might be sufficient for finding EOQ.

This table is made for illustrative purpose. Certainly, various tables with different degrees of sensitivity such as 2.5%, 5%, or 20% and so on, and those tables with different sensitivity ratios between R and S/I can be developed without problems.

Using these simple and more realistic tables, managers would find it easy to be scientific in inventory control.

(23) *C* items of the *ABC* analysis might be included in this category.

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APPENDIX: Expected Sensitivity of The Total Inventory Cost

It is safe to assume that the distribution of real R and S/I is uniformly distributed over the range of $(A-\sqrt{A^2-1})R \sim (A+\sqrt{A^2-1})R$ and $(A-\sqrt{A^2-1})\ S/I \sim (A+\sqrt{A^2-1})\ S/I$ respectively.

For convenience, let's use the following notations.

$$A+\sqrt{A^2-1}=K \tag{64}$$

$$A-\sqrt{A^2-1}=K^{-1} \tag{65}$$

$$R=X \tag{66}$$

$$S/I=Y \tag{67}$$

$$R'=X' \tag{68}$$

$$(S/I)'=Y' \tag{69}$$

$$R'/R=X'/X=X_0 \tag{70}$$

$$(S/I)'/(S/I)=Y'/Y=Y_0 \tag{71}$$

Given the assumption of uniform distribution, the probability function of X_0 and Y_0 is;

$$P(X_0,\ Y_0)\begin{cases} =C: \text{for } K^{-1}\leq X_0\leq K, \text{ and } K^{-1}\leq Y_0\leq K \\ =O: \text{otherwise.}\end{cases} \tag{72}$$

To find the value of C:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x_0,\ y_0)\ dx_0 dy_0=\int_{K^{-1}}^{K} dY_0 \int_{K^{-1}}^{K} C\ dX_0 = C\int_{K^{-1}}^{K} dY_0 \left[X_0 \Big|_{K^{-1}}^{K} \right]$$

$$=C\int_{K^{-1}}^{K} dY_0(K-K^{-1})=C(K-K^{-1})\int_{K^{-1}}^{K} dY_0$$

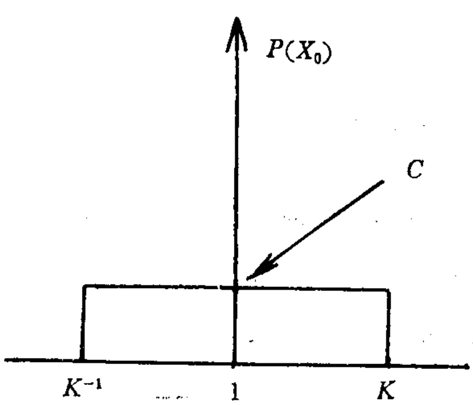


Fig. 8. Probability distribution of X_0

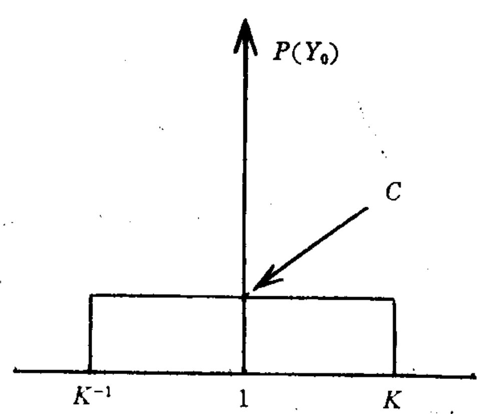


Fig. 9. Probability distribution of Y_0

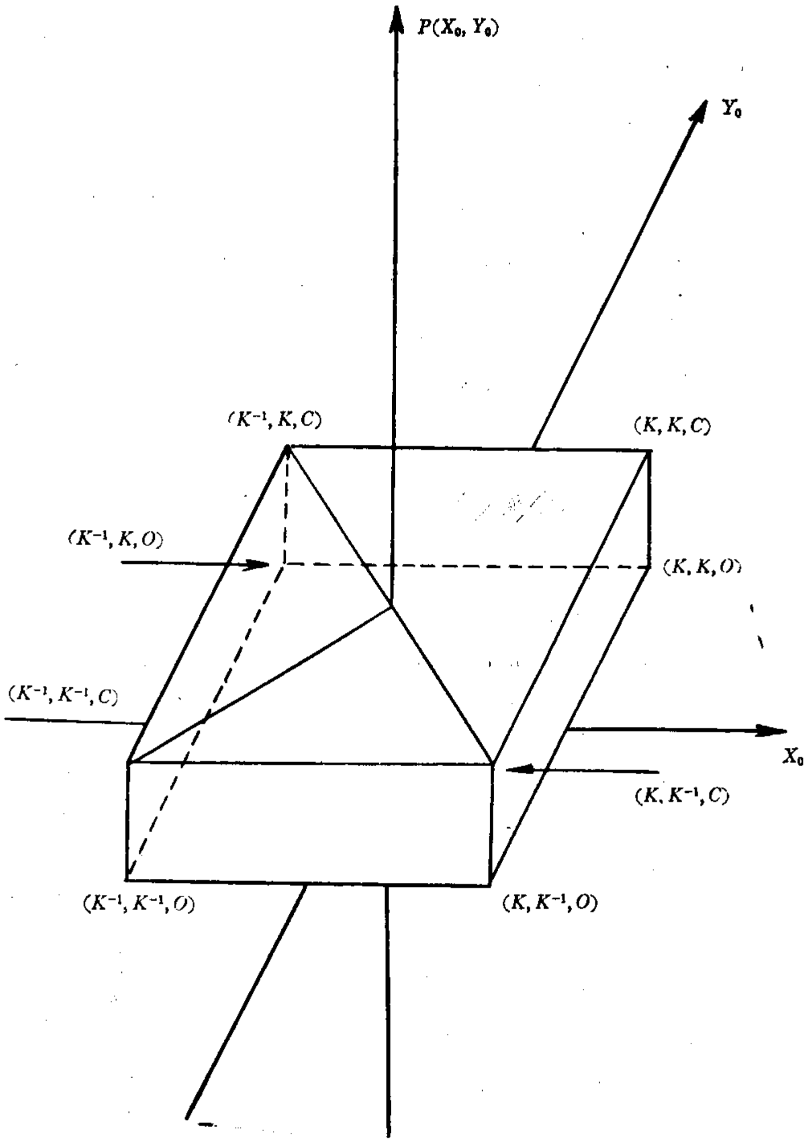


Fig. 10. Probability distribution of X_0 and Y_0

$$=C(K-K^{-1})\left[\,Y_0\,\Big|_{K^{-1}}^{K}\right]=C(K-K^{-1})^2=1 \quad (73)$$

Thus, $\quad C=(K-K^{-1})^{-2}$ (74)

The expected value of sensitivity is to be calculated as follows:

Since $\quad Q^*=\sqrt{2R(S/I)}=\sqrt{2XY}$ (75)

$\quad Q=\sqrt{2R'(S/I)'}=\sqrt{2X'Y'}$ (76)

Then, $$\frac{Q}{Q^*}=\frac{\sqrt{2X'Y'}}{\sqrt{2XY}}=\sqrt{\frac{X'}{X}}\cdot\sqrt{\frac{Y'}{Y}}=\sqrt{X_0Y_0} \tag{77}$$

Now let $f(X_0,\ Y_0)$ be a function of sensitivity;

$$f(X_0,\ Y_0)=\frac{TC}{TC^*}-1=\frac{1}{2}\left(\frac{Q}{Q^*}+\frac{Q^*}{Q}\right)-1$$
$$=\frac{1}{2}\left(\sqrt{X_0Y_0}+\frac{1}{\sqrt{X_0Y_0}}-2\right) \tag{78}$$

Then:

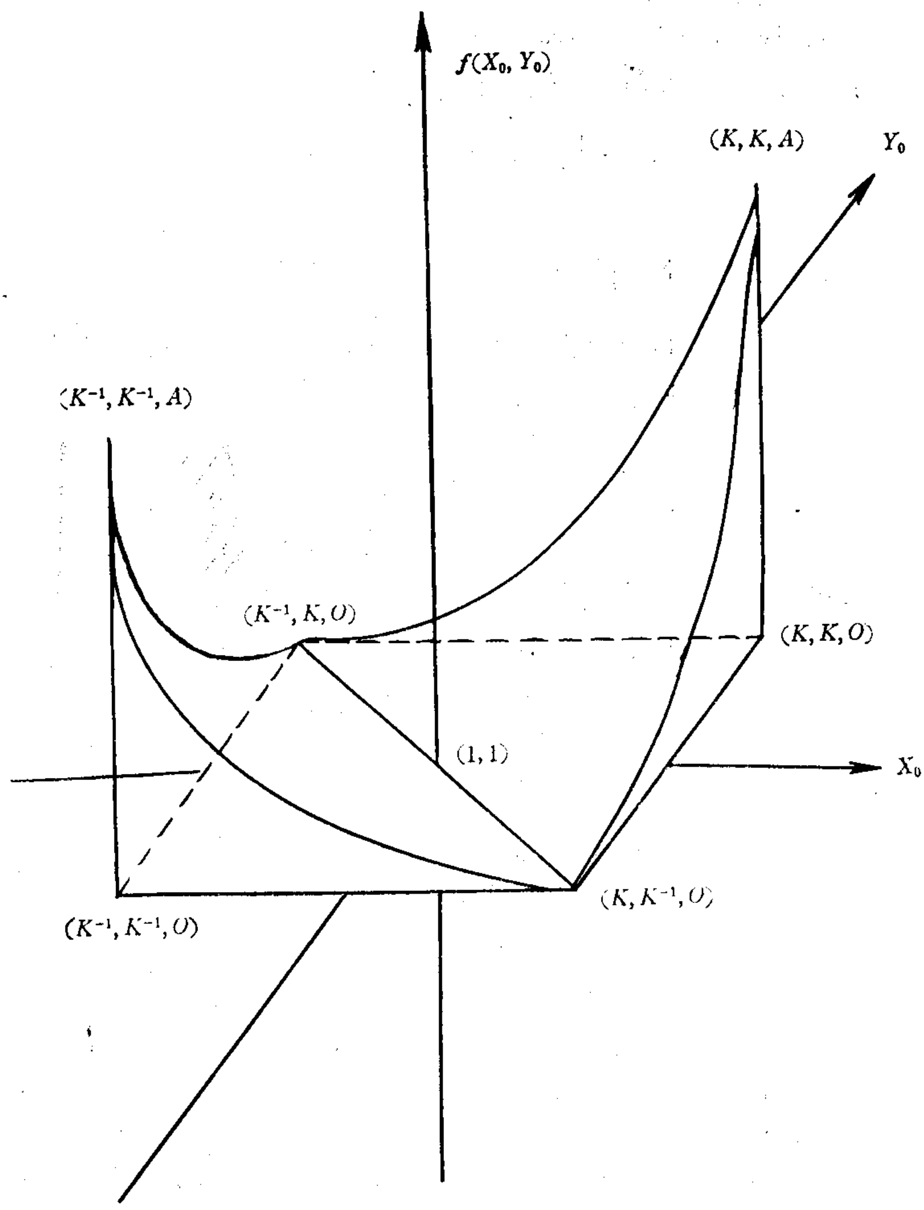
$$E[f(X_0,\ Y_0)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}P(X_0,\ Y_0)\ f(X_0,\ Y_0)dX_0dY_0$$
$$=\int_{K^{-1}}^{K}dY_0\int_{K^{-1}}^{K}(K-K^{-1})^{-2}\frac{1}{2}\left(\sqrt{X_0Y_0}+\frac{1}{\sqrt{X_0Y_0}}-2\ dX_0\right)$$
$$=\frac{1}{2}(K-K^{-1})^{-2}\int_{K^{-1}}^{K}dY_0\left[\frac{2}{3}X_0^{\frac{3}{2}}Y_0^{\frac{1}{2}}\right.$$
$$\left.+2X_0^{\frac{1}{2}}Y^{-\frac{1}{2}}-2X\Big|_{K^{-1}}^{K}\right]$$
$$=\frac{1}{2}(K-K^{-1})^{-2}\int_{K^{-1}}^{K}dY_0\left[\frac{2}{3}K^{\frac{3}{2}}Y_0^{\frac{1}{2}}+2K^{\frac{1}{2}}Y^{-\frac{1}{2}}\right.$$
$$\left.-2K-\frac{2}{3}K^{-\frac{3}{2}}Y_0^{\frac{1}{2}}-2K^{-\frac{1}{2}}Y_0^{-\frac{1}{2}}+2K^{-1}\right]$$
$$=\frac{1}{2}(K-K^{-1})^{-2}\left[\frac{4}{9}K^{\frac{3}{2}}Y_0^{\frac{3}{2}}+4K^{\frac{1}{2}}Y_0^{\frac{1}{2}}-2KY_0\right.$$
$$\left.-\frac{4}{9}K^{-\frac{3}{2}}Y_0^{\frac{3}{2}}-4K^{-\frac{1}{2}}Y_0^{\frac{1}{2}}+2K^{-1}Y_0\Big|_{K^{-1}}^{K}\right]$$
$$=\frac{1}{2}(K-K^{-1})^{-2}\left[\frac{4}{9}K^3+4K-2K^2-\frac{4}{9}-4+2\right.$$
$$\left.-\frac{4}{9}-4+2+\frac{4}{9}K^{-3}+4K^{-1}-2K^{-2}\right]$$
$$=\frac{1}{2}(K-K^{-1})^{-2}\left[\frac{4}{9}(K^3+K^{-3})-2(K^2+K^{-2})\right.$$
$$\left.+4(K+K^{-1})-\frac{44}{9}\right] \tag{79}$$

Since $$K=(A+\sqrt{A^2-1}) \tag{80}$$
$$K^2=(2A^2+2A\sqrt{A^1-1}-1) \tag{81}$$
$$K^3=(4A^3+4A^2\sqrt{A^2-1}-3A-\sqrt{A^2-1}) \tag{82}$$
$$K^{-1}=(A-\sqrt{A^2-1}) \tag{83}$$
$$K^{-2}=(2A^2-2A\sqrt{A^2-1}-1) \tag{84}$$
$$K^{-3}=(4A^3-4A^2\sqrt{A^2-1}-3A+\sqrt{A^2-1}) \tag{85}$$

Substituting Equations (80)~(85) into Equation (79);

$$E[f(X_0,\ Y_0)]=\frac{1}{2}\left[\frac{1}{4}(A^2-1)^{-1}\right]\left[\frac{4}{9}(8A^3-6A)\right.$$
$$\left.-2(4A^2-2)+4(2A)-\frac{44}{9}\right]$$

Fig. 11. Sensitivity range of X_0 and Y_0



$$=\frac{1}{8}(A^2-1)^{-1}\left(\frac{32}{9}A^3-8A^2+\frac{48}{9}A-\frac{8}{9}\right)$$

$$=\frac{1}{9}(A^2-1)^{-1}(4A^3-9A^2+6A-1)$$

$$=\frac{1}{9}(A^2-1)^{-1}(A-1)^2(4A-1)$$

$$=\frac{1}{9}(A-1)(4A-1)(A+1)^{-1}$$

$$=\frac{(A-1)(4A-1)}{9(A+1)} \tag{86}$$

When $A=1.1$, (87)

$$E[f(X_0,\ Y_0)]=\frac{(1.1-1)\ (4.4-1)}{9(1.1+1)}$$

$$=\frac{0.34}{18.9}$$

$$=0.017989 < 1.8\% \tag{88}$$