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Development of an Interactive Graphics Software for Determining Geometrical Characteristics of Complex Shaped Bone

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= **Abstract** =A new graphic software written in PASCAL and TURBO GRAPHIC language has been developed for an IBM PC-XT machine. It determines the geometrical characteristics of complex shaped bone, e.g., area, centroid, area moment of inertia and principal axis of moment of inertia. This software can also determine three-dimensional pictures of bone from two-dimensional cross-sections of tomograph.

Key words: ***Geometrical characteristics of complex shaped bone, Interactive graphics software***

INTRODUCTION

It is very important to understand mechanical characteristics of complex bone cross-sections in designing artificial joint, analyzing stress of bone or musculoskeletal mechanics (Black and Dumbleton 1981). Therefore, many studies have been performed to investigate geometrical characteristics of complex and irregular bone corss-sections (Nagurka and Hayes 1980; Uhthoff and Jaworski 1978). However, in the most previous studies, many number of meshes were required to increase the accuracy of the methods. It is due to the approximation which simplifies the complex cross-sections as collection of triangles or circles (Martin 1975; Pizial 1976; Slatis 1978). Recently development of tomography scanning system enables us to obtain two-dimensional cross-sections of complex shaped bone from arbitrary angles.

In the present study, an interactive computer graphics software has been developed. This softward determines three-dimensional pictures of the bone projected from arbitrary angles and also determines geometrical characteristics of bone, i.e., 1. area, 2. centroid, 3. area moment of inertia, 4. principal moments of inertia and orientation of the axes.

MATERIALS AND METHODS

1. Analytical methods

An algorithm which derives geometrical characteristics of cross-section from perimeter coordinates of arbitrary cross-section is utilized to develop a new software packages (Nagurka and Hayes 1980; Wojeciechowski 1976).

An arbitrary cross-section is devided into a series of trapezoids. Segments between neighboring perimeter coordinates are assumed to be linear. The derivation of cross-sectional characteristics is as follows. Notations are given in Fig. 1.

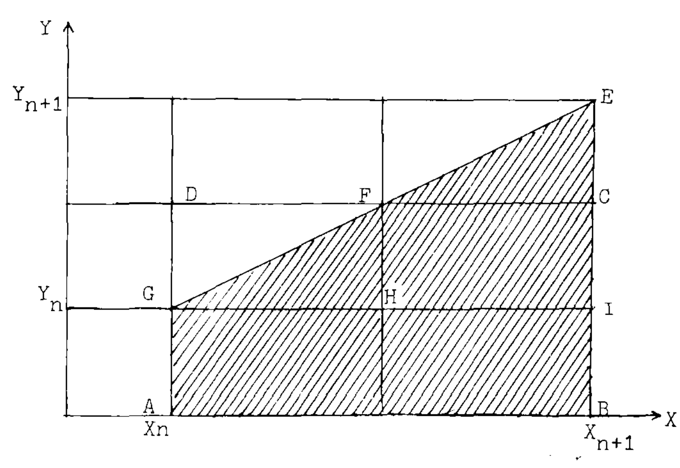

Fig. 1. Area section properties under line GE.

1) Area: A

The area of ▱ ABGE is $A = (X_{n+1}-X_n)(Y_{n+1}+Y_n)/2$.

2) Centroid: $\bar{X}$, $\bar{Y}$

The centroid of an area is the point of which the area might be considered to be concentrated and still leave unchanged the first moment of the area about any axis. Therefore, the centroid of an area is defined by the equations:

$$\bar{X}=\frac{\int_A x\,dA}{\int_A dA}=\frac{M_y}{A}, \qquad \bar{Y}=\frac{\int_A y\,dA}{\int_A dA}=\frac{M_x}{A}$$

where M_x, M_y is the first moment of inertia.

3) the first moments of inertia of an element □ABEG are:

$$\Delta M_x=(M_{ABEG})_x=(M_{ABCD})_x+(M_{FCE})_x-(M_{DGF})_x$$
$$=1/8(X_{n+1}-X_n)[(Y_{n+1}+Y_n)^2+1/3(Y_{n+1}-Y_n)^2]$$

$$\Delta M_y=(M_{ABEG})_y=(M_{ABIG})_y+(M_{GIE})_y$$
$$=1/6(X_{n+1}-X_n)[Y_{n+1}(2X_{n+1}+X_n)+Y_n(X_{n+1}+2X_n)].$$

4) Area moments of inertia: I_{xx}, I_{yy}

The axial moment of inertia of an element of area about an axis in its plane is the product of the area of the element and square of its distance from the axis. For the element of ABEG, the area of moments of inertia with respect to each axes are,

$$(I_{ABEG})_{xx}=\int y^2 dA=(I_{ABIG})_{xx}+(I_{GIE})_{xx}$$
$$=1/24(X_{n+1}-X_n)(Y_{n+1}+Y_n)[(Y_n+Y_{n+1})^2+(Y_n-Y_{n+1})^2]$$

$$(I_{ABEG})_{yy}=\int x^2 dA=(I_{ABIG})_{yy}+(I_{GIE})_{yy}$$
$$=1/12(X_{n+1}-X_n[(3X^2_n+2X_nX_{n+1}+X^2_{n+1})Y_n+(X^2_n+2X_nX_{n+1}+3X^2_{n+1})Y_{n+1}].$$

5) Product moment of inertia: I_{xy}

The product moment of inertia of an element □ABEG with respect to the x-y axes in the plane of the area is given by:

$$(I_{ABEG})_{xy}=\int xy\,dA=\frac{1}{(X_{n+1}-X_n)}[1/8(Y_{n+1}-Y_n)^2(X^2_{n+1}+X^2_n)(X_{n+1}+X_n)+1/3(Y_{n+1}-Y_n)(X_{n+1}Y_n-X_nY_{n+1})(X^2_{n+1}+X_nX_{n+1}+X^2_n)+1/4(X_{n+1}Y_n-X_nY_{n+1})^2(X_{n+1}+X_n)].$$

6) Rotated set of axes

The moments of inertia of any area with respect to a rotated set of axes (x′, y′) as shown in Fig. 2 may be expressed in terms of the moments and product of inertia with respect to the (x,y) axes as follows:

$$I_{x'}=\tfrac{1}{2}(I_x+I_y)+\tfrac{1}{2}(I_x-I_y)\cos 2\theta-I_{xy}\sin 2\theta$$
$$I_{y'}=\tfrac{1}{2}(I_x+I_y)-\tfrac{1}{2}(I_x-I_y)\cos 2\theta+I_{xy}\sin 2\theta$$
$$I_{x'y'}=\tfrac{1}{2}(I_x-I_y)\sin 2\theta+I_{xy}\cos 2\theta.$$

According to the Parallel axis theorem, the area

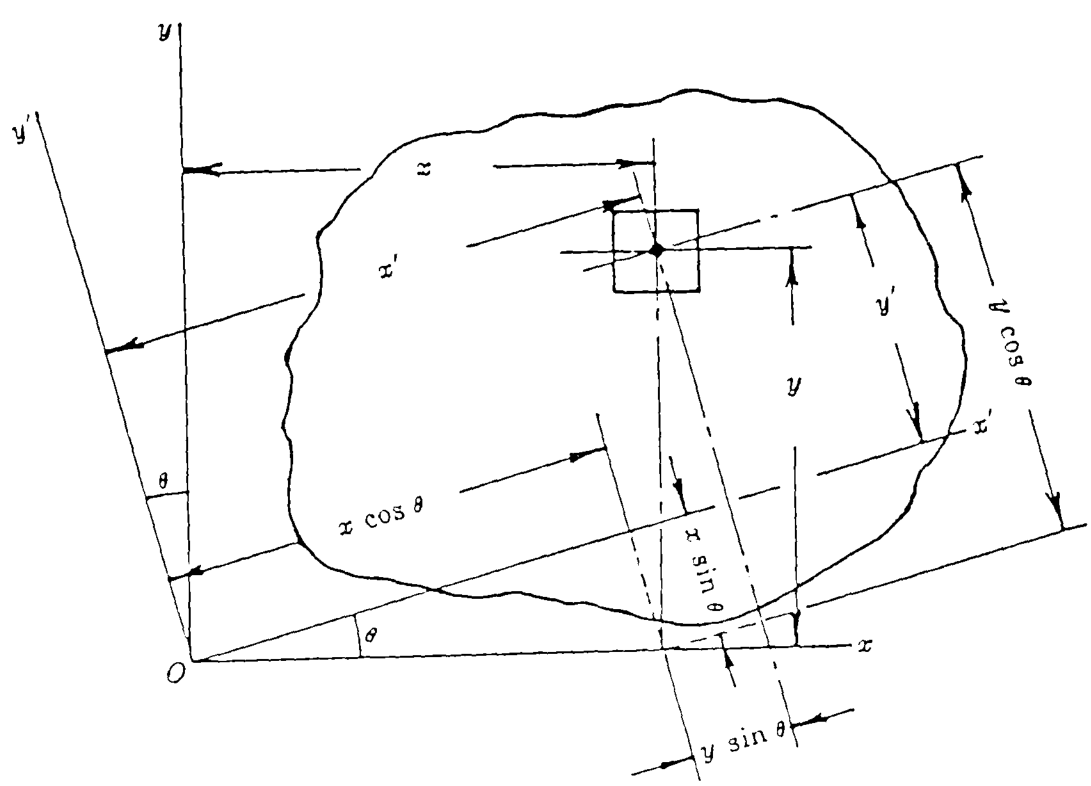

Fig. 2. Moment of inertia with respect to a rotated set of axes (x′,y′).

moment of inertia about any axis is equal to the moment of inertia about a parallel axis through the centroid of the area plus the product of the area and the square of the perpendicular distance between the two axes.

The I_{xx} and I_{yy} are given by:

$$I_{xx}=(I_{xx})G+A(y_1)^2$$
$$I_{yy}=(I_{yy})G+A(x_1)^2$$

7) Principal moments of inertia: I_p

At any point in the plane of an area, there exist two perpendicular axes (principal axes) about which the moments of inertia of the area are maximum and minimum for that point. These *max* and *min* values of moment of inertia are termed principal moments of inertia. The magnitudes of these moments are,

$$(I_p)_{\substack{max\\min}}=\left(\frac{I_{xx}+I_{yy}}{2}\right)\pm\sqrt{\left(\frac{I_{xx}-I_{yy}}{2}\right)^2+(I_{xy})^2}$$

and the orientation of these axes is

$$\phi=\tfrac{1}{2}\tan^{-1}\left(\frac{2I_{xy}}{I_{xx}-I_{yy}}\right).$$

The product of inertia vanishes if the axes are principal axes.

2. Computer program

In this research, the graphic software package was developed by means of the PASCAL and TURBO GRAPHICS language, and executed on IBM PC-XT computer.

The output variables are as follows:

AREA: area

Xbar, Ybar: the X and Y coordinates of the centroid

Ixx, Iyy: the moments of inertia about x and y axes

Ixy: the product of inertia

Ixxbar, Iyybar: the moments of inertia about the x and y axes translated to the centroid

Ixybar: the product of inertia about the translated axes

Phi: the angle between the translated axis and principal axis

Ixxbar-P, Iyybar-P: the moments of inertia about the translated and rotated principal x and y axes

Theta: the angle between the translated axis and arbitrarily desired axis

Ixxbar-R, Iyybar-R: the moments of inertia about the translated and arbitrarily rotated x and y axes

Ixybar-R: the product moment of inertia about the translated and arbitrarily rotated axes.

Fig. 3 shows the flowchart of the developed computer program.

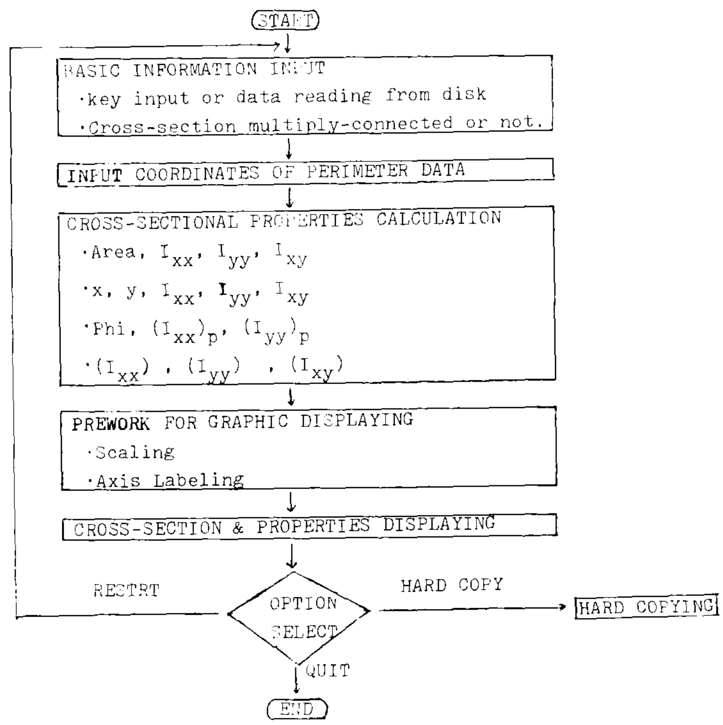

Fig. 3. Flowchart of the program.

To use this program, the reference coordinate frame should be selected to satisfy the all perimeter coordinates of the interest cross-section positive. The cross-section may be multiply-connected, defined by an outer perimeter and inner perimeters, denoting "holes" or cross-section voids. Outer perimeter coordinates must be input sequentially in a clockwise path around the boundary. Coordinate points for inner perimeter must be input in a counter-clockwise.

RESULTS

A number of simple geometric shapes of known geometrical characteristics, as shown in Fig. 4, have been used as test examples to validate the program and to verify its high degree of accuracy. The program attains accuracy to five significant figures between analytical and numerical results. Therefore, the numerical method applied to the more complex shaped bone to get all the area properties. Fig. 5 shows the geometric characteristics of the cross-sectional picture of the proximal part of femoral shaft of the twenty-seven aged man. By means of the digitizer, the perimeter--coordinates of the cross-section of a complex shaped bone are obtained from the CT scanning. All the digital data of a series of the cross-sectional area are filed up along the Z-axis. Corrdinate trans-

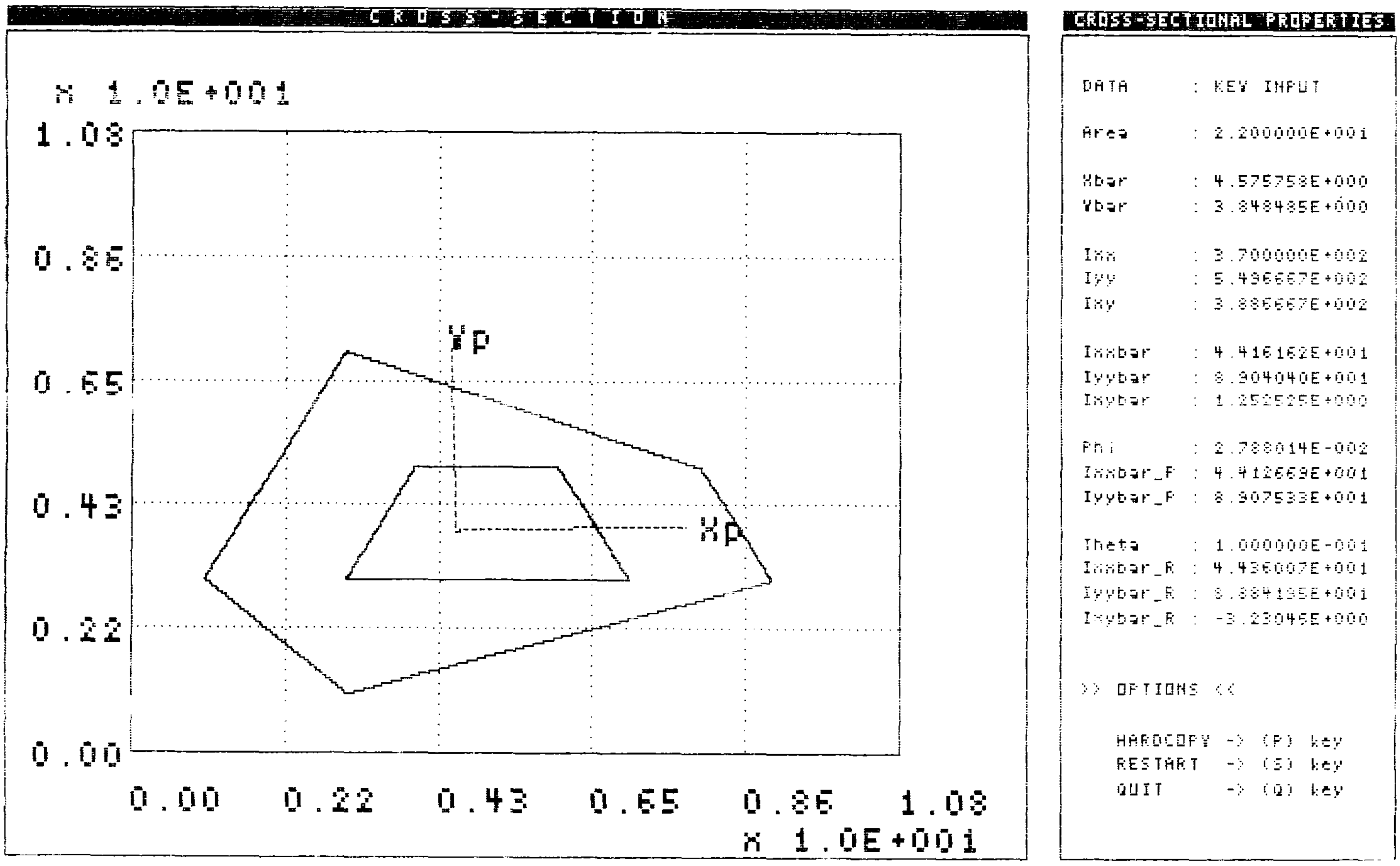

Fig. 4. Test example for program verification.

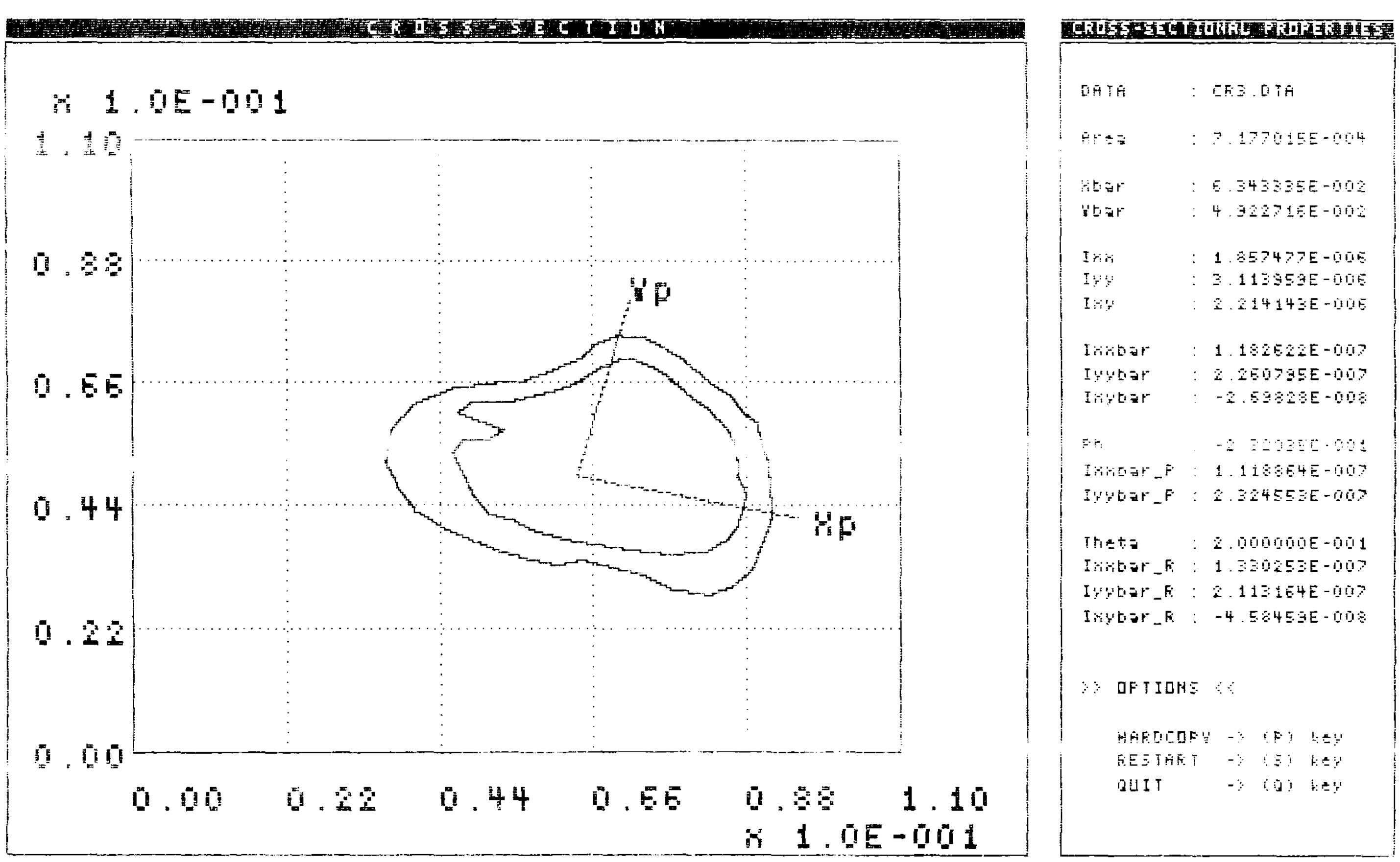

Fig. 5. Cross-section of femur at arbitrary position.

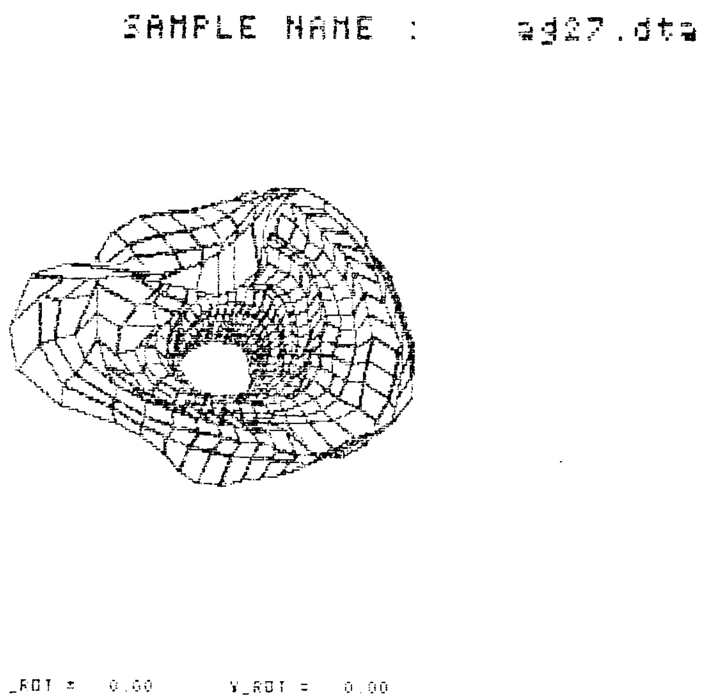

Fig. 6. Front view of femur in (0, 0, 0) rotation.

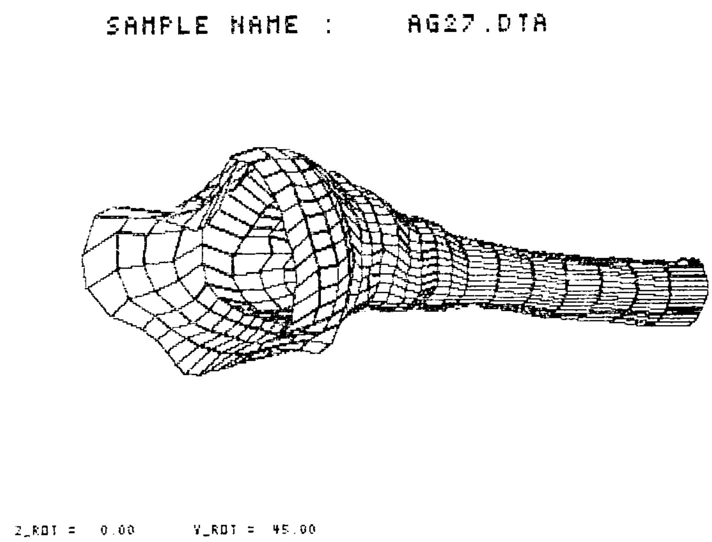

Fig. 7. Three-dimensional view of femur in (0, 0, 0) rotation.

formations are carried out to the 3-dimensional picture with the arbitrary view angle. In this procedure, hidden line treatment is also executed. Fig. 6 and Fig. 7 show the three-dimensional pictures of the femur for different points of view, that is, (0°, 0°, 0°) and (0°, 45°, 0°) respectively.

DISCUSSION

A simple algorithm for computing the cross-sectional properties of complex geometric shaped bone has been programed for automatic data acquisition and analysis. This program can be used to analyze the area properties of multiple, complex biological cross-sections and to display the three-dimensional pictures of the bone from the arbitrarily desired point of view.

Especially, biological changes in cross-sectional geometries can be quantified to better understand changes due to aging, metabolic bone disease, and internal fixation devices.

As shown in Fig. 7, three-dimensional pictures of femur can be directly used in the computer aided design of femoral prothesis. Besides, these pictures make possible to better understand spine fracture and acetabular fracture which are not clearly observed when a simple radiography (a two-dimensional tomography) is used.

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＝국문초록＝

뼈의 기하학적 특성에 관한 그래픽소프트웨어의 개발

서울대학교 의과대학 정형외과학교실

이 춘 기

생체역학적으로 중요한, 복잡하고 다양한 형상을 가진 뼈 단면의 기하학적 특성—면적, 도심, 면적관성모멘트, 관성주축—등을 구하고, 또한 단층촬영으로부터 얻은 뼈의 2차원 단면들의 형상을 이용하여 실제 뼈의 형상을 3차원으로 재현하는 그래픽소프트웨어를 PASCAL과 TURBO GRAPHIC 언어를 이용하여 IBM PC-XT에 맞도록 개발하였다.

이 소프트웨어로서 응력해석(stress analysis), 인공관절의 설계, 내고정 및 노화와 대사성 질환등에 의한 뼈의 변화등을 이해하는 데 도움을 줄 수 있으며 2차원으로 나타나는 단층촬영을 3차원으로 재합성함으로써 2차원 단층촬영으로는 알기 힘든 복잡한 골절등의 뼈의 변화를 보다 쉽게 해석할 수 있다.