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Short Note

Wave equation calculation of most energetic traveltimes and amplitudes for Kirchhoff prestack migration

Changsoo Shin*, Seungwon Ko*, Kurt J. Marfurt‡, and Dongwoo Yang*

INTRODUCTION

Because of its computational efficiency, Kirchhoff migration is the method of choice for 3D prestack depth migration, particularly in the initial velocity model-building stages where several iterations of migration are necessary. In the beginning, Kirchhoff migration used traveltimes calculated by either eikonal solvers or asymptotic ray theory, while amplitudes were calculated using relatively smooth geometrical spreading and obliquity considerations. More recently, Kirchhoff migration has been generalized to include more than one arrival time, with amplitudes calculated from geometrical optics, solution of the transport equations, or Gaussian beams (e.g., Hill, 2001).

Nevertheless, for reasons of computational efficiency and algorithmic simplicity (more important than ever on distributed memory machines), most Kirchhoff algorithms use only one traveltime. While there remains considerable debate whether the ray having the shortest path or most energy should be used, it is clear that the ray using the most energetic arrival time is superior to using the easier to calculate first-arrival time, which often carries little to no energy. Considerable progress has been made in calculating the most energetic arrival from the eikonal and transport equations.

However, our method is built on that of Nichols (1996), who proposes solving the wave equation within a window, which we call the wavefront, following the first arrival. As originally proposed, Nichols' (1996) algorithm has two drawbacks. The first, relatively minor drawback is that the first-arrival time is calculated using an algorithm independent of the wave equation solver, thereby requiring two rather than one traveltime calculation algorithm. The second, more serious drawback is that the traveltimes calculated within the short analysis window suffer from severe Fourier wraparound artifacts. Even though the waveform itself is corrupted by wraparound in the short analysis window, we are usually able to pick the traveltime of the most energetic event. However, situations arise where the most energetic event within the window is an unfortunate constructive interference of wrapped events.

We address both of these issues and show how we can reconstruct the waveform in the analysis window with minimal wraparound artifacts, thereby allowing us to unambiguously calculate the most energetic arrival time and amplitude. We then demonstrate the effectiveness of our algorithm on the 2D IFP Marmousi model.

THEORY

In discrete Fourier transforms, the Nyquist frequency, $f_N(= 1/2\Delta t)$, is determined by the temporal sampling interval Δt, while the frequency sample interval $\Delta f = 1/T$ is determined by the length of the seismic trace $T = n\Delta t$. If we were to double the frequency sample interval $(2\Delta f)$ and reconstructed the data within the window $(0, T/2)$, we would accurately reconstruct the data originally falling within the interval $(0, T/2)$. That part of the data that originally fell within the second interval $(T/2, T)$ would be wrapped around and added on top of the data within $(0, T/2)$. If, alternatively, we chose to reconstruct the data within the interval $(T/2, T)$, the converse would occur. Indeed, the data in the two intervals would be identical. If for reasons of wave propagation the first arrival t_0 were greater than $T/2$, there would be no real events in the window $(0, T/2)$ such that wraparound would not contaminate the second interval. The events that would arrive after time $t_0 > T/2$ would still wrap around into the first window, but we would know they are artifacts. The most energetic traveltime t_E would always arrive no earlier than the first arrival at time t_0.

Let us assume that this most energetic arrival appears within the interval $(t_0, t_0 + T_w)$, where T_w is a user-defined analysis window in which we reconstruct the seismic data. Our Fourier

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*Seoul National University, Division of Civil, Urban, and Geosystem Engineering, San 56-1 Shillim-dong, Kwanak-gu, Seoul, Korea. E-mail: css@model.snu.ac.kr; ko@gpl.snu.ac.kr; dwyang@gpl.snu.ac.kr.
‡University of Houston, Allied Geophysics Laboratories, Department of Geosciences, Houston, Texas, 77204-5006. E-mail: kmarfurt@uh.edu.

reconstruction of the seismic data (or in our case, the migration impulse response) $g(t)$ then becomes

$$g(t+t_0) = \sum_{j=1}^{J} G(\omega_j)\exp[i\omega_j(t+t_0)], \quad (1)$$

where $G(\omega_j)$ is the jth Fourier component of the wavefield and $J = f_N/\Delta f$. Equation (1) is essentially that presented by Nichols (1996), who notes that the number of frequencies decreases with the value of T_w. To suppress wraparound, we follow Marfurt and Shin (1989) and solve the wave equation to obtain the impulse response G—not at real frequencies ω_j but at complex frequencies $\omega_j + i\alpha$—to obtain a wraparound suppressed version of equation (1). To compensate for this suppression in the analysis window (t_0, t_0+T_w), we exploit the shifting theorem (Aki and Richards, 1980; Shin et al., 2003) and multiply the time domain solution by $\exp[+\alpha(t+t_0)]$ to obtain

$$g(t+t_0) = \left\{ \sum_{j=1}^{J} G(\omega_j + i\alpha)\exp[i\omega_j(t+t_0)] \right\} \times \exp[+\alpha(t+t_0)]. \quad (2)$$

On 32-bit computers, a reasonable value of $\alpha \cong \ln(B/A)/T_w$ (where A is the picked amplitude of the first arrival and B is the picked amplitude of the most energetic arrival) would suppress any events arriving after time $t_0 + T_w$ and the events are folded back into the analysis window (t_0, t_0+T_w) by at least a factor of 100.

On the basis of the simple arithmetic described above, the most energetic traveltime can be calculated in the following six steps:

1) Calculate t_0 by solving the one-way wave equation for a single frequency as described by Shin et al. (2003). (Other methods such as eikonal solvers are acceptable but require additional software.)
2) Through experience or analysis of a few impulse responses in the most complex part of the model, estimate the length of T_w such that the most energetic event falls within (t_0, t_0+T_w). The value of T_w determines the frequency increment Δf and thereby the number of frequencies, $J = f_N/\Delta f = f_N T_w$.
3) Solve the one-way (or two-way) wave equation in the frequency domain at complex frequencies $\omega_j = 2\pi j\Delta f + i\alpha$ using your preferred propagator (paraxial, phase shift and interpolate, phase screen, hyperbolic, etc.).
4) Synthesize the wavefield within the analysis window (t_0, t_0+T_w) using equation (2).
5) Extract the amplitude a_E and traveltime T_E of the most energetic event. The picked most energetic traveltime will appear to fall within a computational analysis window $(0, T_w)$.
6) To properly map the picked T_E to the correct domain of the Fourier transform, use

$$t_E = \text{integer}\left(\frac{t_0}{T_w}\right) + T_E, \quad (3)$$

where t_E is the most energetic traveltime to be used in Kirchhoff migration.

NUMERICAL EXAMPLE

To demonstrate the robustness of our algorithm, we calculate the most energetic traveltimes and their corresponding amplitudes for the Marmousi model. In Figure 1, we display the most energetic traveltime (in white) obtained by using equations (2) and (3) by solving an 85° one-way wave equation for eight frequencies to reconstruct the data within a $T_w = 0.4$ s analysis window. To check the accuracy of our prediction, we generate results by solving the same one-way wave equation using 128 frequencies to reconstruct the data between 0.0 and 4.0 s. For both solutions, we pick the most energetic event at each depth point and plot t_E in Figure 1. In Figure 2a we plot a_E,

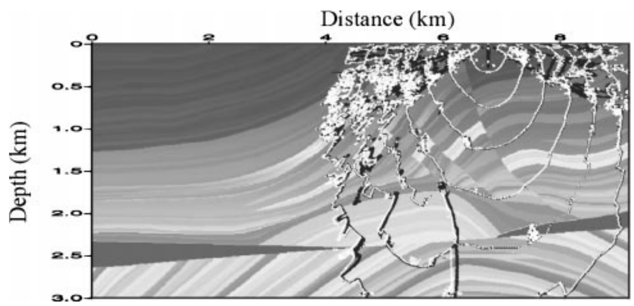

FIG. 1. Contours of the most energetic traveltime t_E for the Marmousi model. The white line indicates the traveltimes obtained using equations (2) and (3) and only eight frequencies, while the black line indicates the traveltimes obtained by solving the one-way wave equation for 128 frequencies, reconstructing the entire waveform for all time, and picking the most energetic event. The traveltime contours are superimposed on the velocity model.

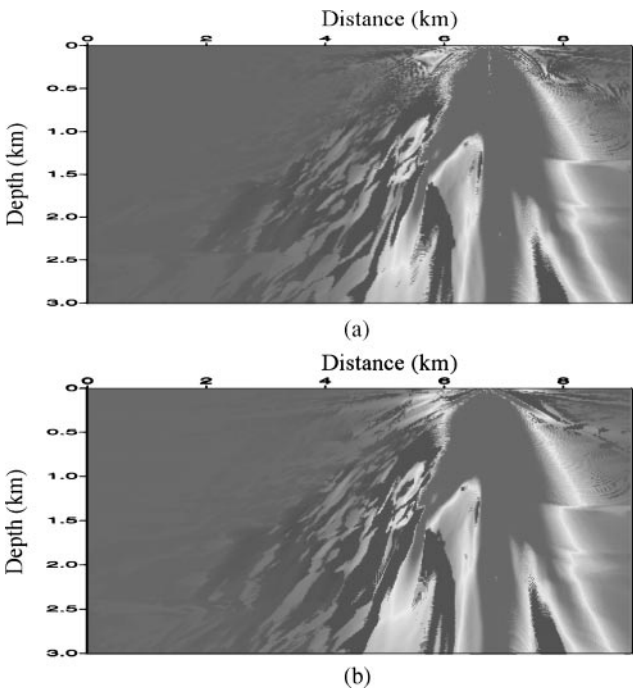

FIG. 2. The amplitude a_E corresponding to the traveltimes shown in Figure 1. (a) Amplitude obtained using equations (2) and (3). (b) Amplitude obtained by solving the one-way wave equation for all 128 frequencies. The poor correlation of traveltime picks between the two methods seen in Figure 1 corresponds to zones of low amplitude here.

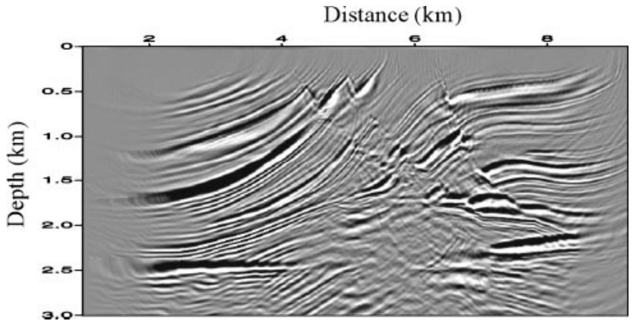

FIG. 3. Prestack depth-migrated image obtained by using first-arrival traveltime and amplitudes described by Bleistein (1987).

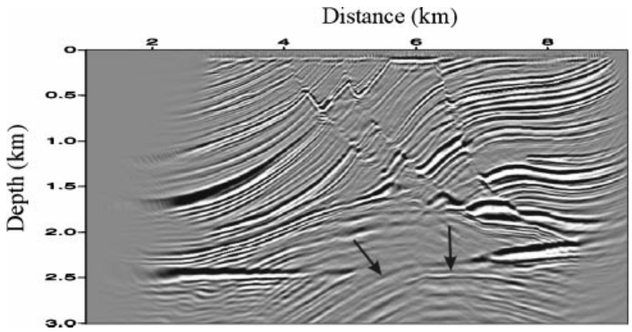

FIG. 4. Prestack depth-migrated image obtained by using the most energetic traveltime and amplitude computed by this algorithm. Note the improved clarity of the fault faces and turtle (marl) structure. Arrows indicate zones where the image is improved.

corresponding to t_E, for our new algorithm using eight frequencies. In Figure 2b we plot a_E, for the complete 128-frequency reconstruction. The agreement between the two solutions is excellent. Considerable energy corresponds to a negative rather than a positive amplitude. We interpret this amplitude reversal in polarity as attributable to caustics.

In Figures 3 and 4 we display the prestack Kirchhoff depth-migrated image of the Marmousi model obtained by using the first-arrival time and amplitudes described by Bleistein (1987) and t_E and a_E obtained by using equations (2) and (3). As others (e.g., Hill, 2001) have shown using alternative most energetic or multiarrival algorithms, the images of both the shallow steeply dipping events in the center of the model and the deeper target (indicated by arrows) are improved by using the most energetic arrival time.

CONCLUSIONS

We have described a means of extending Nichols' (1996) traveltime estimation technique to provide accurate, unambiguous most energetic arrival times. In addition to accurate most energetic arrival times, we obtain accurate amplitudes that account for all wave propagation effects, including caustics, accurately modeled by wave equation methods. Such a change in amplitude polarity can have a very strong impact on the quality of seismic images generated using Kirchhoff migration. Our algorithm is easily implemented using currently existing and frequency-domain wave extrapolation and modeling software used in more expensive wave equation migration algorithms. Combined with the first-arrival traveltime algorithm described by Shin et al. (2003), our algorithm is accurate, and easy to implement, and easy to maintain, and it provides images consistent with more accurate wave equation images that may follow iterative Kirchhoff imaging for velocity analysis. Extending this algorithm to use the entire waveform within the analysis window, thereby generating a multiple-arrival wavefront migration algorithm, is straightforward but computationally more intensive.

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