LETTER *Wireless Communications Issue*

Fast Initialization of an MMSE Equalizer for Faster than Nyquist Signaling*

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SUMMARY We consider equalizer initialization problems when the transmitted symbol rate is higher than the available channel bandwidth. In this case, the coefficients of an adaptive equalizer in the receiver can be updated only once per a predefined symbol period, requiring unacceptably long training time. The training time can be reduced significantly if the equalizer begins the training process from a properly initialized condition. In this letter, a fast initialization method is analytically designed for a minimum mean squared error (MMSE) type equalizer. Finally, the initialization performance is verified by computer simulation.

key words: *equalization, MMSE, faster than Nyquist signaling*

1. Introduction

It is well known that the bandwidth W of the channel needs to be larger than or equal to $f_b/2$ to transmit the data at a symbol rate of f_b. However, when $f_b > 2W$, called *faster than Nyquist signaling* [1], it is still possible to transmit the user data by reducing the actual transmission rate lower than $2W$ symbols/sec [1]–[3]. For example, this can be done by transmitting sequences of N symbols comprised of M user data and $(N-M)$ zeros, where M and $N(\geq M)$ are integers such that $M/N \leq 2W/f_b$ [1].

The use of conventional equalizers cannot be applied to reception of the signal with $f_b > 2W$ because it will result in enhancement of signal components in the stop-band of the channel. To overcome this problem, a new type of zero-forcing equalizer was proposed in [4], where an LMS algorithm was applied to adaptation of the equalizer coefficients. Since the use of the LMS adaptation algorithm in [4] enables the update of the equalizer coefficients only once per received symbols, it requires a relatively long time to train the equalizer.

To reduce the equalizer training time, the use of a recursive least square (RLS) method can be considered. However, the RLS method may not be practical since it can be unstable under some channel conditions in addition to large implementation complexity. As an alternative, it may be practical to use an initialization method to reduce the equalizer training time. The equalizer can reach to the steady-state in a short time, if it can start the training from a properly initialized status. Although an LMS algorithm was applied to adaptation of an MMSE-type equalizer in [4], no result has been reported for fast training of the equalizer scheme.

We consider the use of a fast initialization method to reduce training time of the MMSE type equalizer. In this letter, a fast initialization method is analytically designed using the MMSE solution. Following Introduction, Sect. 2 describes the transceiver model. The proposed initialization method based on the MMSE criteria is described in Sect. 3. Some numerical results are discussed in Sect. 4. Finally, conclusions are summarized in Sect. 5.

2. System Model

For ease of description, we consider the case of uplink communications in the V.90 class modem [5]. Assume that we want to employ the PCM mode to increase the transmission throughput instead of using the V.34 mode [6]. Due to the use of anti-aliasing low pass filters and isolation transformers in the local loop, the effective available bandwidth is less than 3500 Hz, which is much smaller than the required bandwidth for transmitting the PCM formatted signal. A new equalizer scheme was proposed for application to this kind of *faster than Nyquist signals* [4]. Figure 1 depicts the structure of this equalizer, where $h(t)$ represents the impulse response of the channel and $c_m(t)$, $m = 0, 1, \cdots, (M-1)$, denotes the impulse response of the m-th sub-equalizer, Δ is the sampling time offset in the receiver, $T = NT_b$ and T_b is the PCM symbol rate equal to 1/8000 sec. To accommodate the available

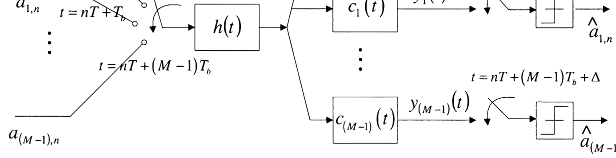

Fig. 1 Equalizer scheme for faster than Nyquist signaling.

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channel bandwidth, every N-PCM symbols comprises of M user data and $(N - M)$ consecutive zeros. As an example, when $M = 6$ and $N = 8$, the transmitter will sequentially send the data signal as

$$\cdots a_{0,n} a_{1,n} a_{2,n} \cdots a_{5,n} 00 a_{0,n+1} a_{1,n+1} \cdots a_{5,n+1} 00 \cdots$$

where $a_{m,n}$ denotes the user data at time $t = nT + mT_b$.

As can be seen in Fig. 1, although the received samples are sequentially input to all the sub-equalizers, each sub-equalizer generates the output and updates its coefficients only once per every N received PCM symbols. Thus, it may require long training time to train the equalizer. The training time can be reduced by starting the training from a properly initialized status.

3. MMSE Equalization

An equivalent receiver model of the m-th sub-equalizer in additive white Gaussian noise (AWGN) channel is depicted in Fig. 2, where the channel $h(t)$ includes the transmit and receive filters as well as the local loop. The output $y_m(t)$ of the m-th sub-equalizer can be expressed as

$$y_m(t) = \sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{M-1} a_{\ell,n} w_m(t - nT + \ell T_b) + \nu_m(t), \quad \text{for } m = 0, 1, 2, \cdots, M{-}1 \tag{1}$$

where $\nu_m(t) \equiv \int n(t - \tau) c_m(\tau)\, d\tau$. Here, $n(t)$ is zero mean AWGN with a two-sided power spectral density of $N_0/2$.

The mean-squared error (MSE) of the m-th sub-equalizer output at time $t = kT$ is defined as

$$\varepsilon_m = E\{[y_m(kT) - a_{m,k}]^2\} \tag{2}$$

where $E\{\cdot\}$ denotes the expectation process. Since $\{a_{m,n}\}$ can be assumed statistically independent and identically distributed with the same power, (2) can be rewritten as [7]

$$\begin{aligned}\frac{\varepsilon_m}{P_a} = &\iint \left[\sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{M-1} h(nT - \ell T_b - t) h(nT - \ell T_b - \tau) + \sigma^2 \delta(t - \tau) \right] \\ &\cdot c_m(t) c_m(\tau) dt d\tau \\ &- 2\int h(mT_b - t) c_m(t) dt + 1, \end{aligned} \tag{3}$$

where P_a is the power of $a_{m,n}$ and $\sigma^2 \equiv N_0/P_a$. It can be easily shown that the coefficient of the m-th sub-equalizer minimizing the MSE ε_m is given by

$$\begin{aligned} c_m(t) &= \frac{1}{\sigma^2} \left[h(mT_b - t) - \sum_{n=-\infty}^{\infty} \sum_{\ell=0}^{M-1} w_m(nT - \ell T_b) h(nT - \ell T_b - t) \right] \\ &= \sum_{i=-\infty}^{\infty} \alpha_m(i) s(i) h(iT_b - t), \end{aligned} \tag{4}$$

where

$$\alpha_m(i) = \begin{cases} [1 - w_m(iT_b)]/\sigma^2, \\ \quad \text{for } i = m, m = 0, 1, \cdots, M-1 \\ 0, \quad \text{for } ((i))_N = 1, 2, \cdots, N-M \\ -w_m(iT_b)/\sigma^2, \quad \text{otherwise} \end{cases} \tag{5}$$

and

$$s(i) = \begin{cases} 0, & \text{for } ((i))_N = 1, 2, \cdots, N-M \\ 1, & \text{otherwise} \end{cases} \tag{6}$$

Here $((\cdot))_N$ denotes the modulo-N operation. Thus, $c_m(t)$ is determined by a weighted sum of the matched filter output with $(N - M)$ zeros in every N symbols. Since $\alpha_m(i)$ involves $f_m(iT_b)$ which is a function of $c_m(t)$ itself, it cannot be expressed in an explicit form. We consider the design of $c_m(t)$ (or equivalently the coefficients $c_m(i)$) as a function of the channel correlation and noise characteristics.

The MMSE coefficients of the m-th sub-equalizer can be obtained by transforming the above equations into the frequency domain. Setting the first variation of (3) with respect to $c_m(t)$ to zero and using (4), it can be shown that

$$\begin{aligned} \sum_{i=-\infty}^{\infty} &\left[\sum_{k=-\infty}^{\infty} \sum_{\ell=0}^{M-1} r_h(Nk - i - \ell) h(NkT_b - \ell T_b - t) + \sigma^2 h(iT_b - t) \right] \\ &\cdot \alpha_m(i) s(i) = h(mT_b - t), \end{aligned} \tag{7}$$

where $r_h(\cdot)$ is the channel correlation function defined by

$$r_h(Nk - i) \equiv \int h(NkT_b - \tau) h(iT_b - \tau) d\tau \tag{8}$$

Taking the Fourier transform of (7), we have

$$\begin{aligned} &\sum_{k=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} \sum_{\ell=0}^{M-1} r_h(Nk - i - \ell) \alpha_m(i) s(i) \\ &\quad \cdot e^{-j2\pi f(NkT_b - \ell T_b)} \\ &\quad + \sigma^2 \sum_{i=-\infty}^{\infty} \alpha_m(i) s(i) e^{-j2\pi f i T_b} = e^{-j2\pi f m T_b}. \end{aligned} \tag{9}$$

Here we assume that $H(f) \neq 0$ for all f. Note that, for any integer j

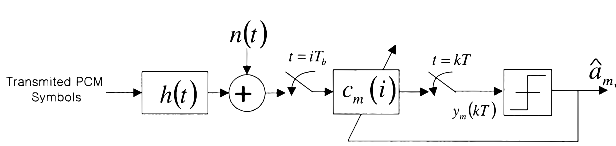

Fig. 2 An equivalent system model for the m-th sub-equalizer with $T = NT_b$.

$$\sum_{k=-\infty}^{\infty} r_h(Nk-i)e^{-j2\pi f(NkT_b-iT_b)} = \sum_{k=-\infty}^{\infty} r_h(Nk-(Nj+i))e^{-j2\pi f[NkT_b-(Nj+i)T_b]} \tag{10}$$

because (Nj)-sample shifted sequences of $r_h(Nk)$ have the same transformed output. Since $s(i)$ has $(N-M)$ zeros in every N positions, (9) can be rewritten as

$$\sum_{k=-\infty}^{\infty}\sum_{n=0}^{M-1}\left[\begin{array}{l} r_h(Nk-n)e^{-j2\pi f(NkT_b-nT_b)} \\ \times \sum_{i=-\infty}^{\infty}\alpha_m(i)p_n(i)e^{-j2\pi fiT_b} \end{array}\right] + \sigma^2\sum_{i=-\infty}^{\infty}\alpha_m(i)p_0(i)e^{-j2\pi fiT_b} = e^{-j2\pi fmT_b} \tag{11}$$

where $p_n(i) \equiv s(i)s(i-n)$, for $n=0,1,\cdots,N-1$. Note that $p_0(i)=s(i)s(i-0)=s(i)$. Defining $A_m(f), P_n(f)$ and $R_n(f)$ as

$$A_m(f) = \sum_{i=-\infty}^{\infty}\alpha_m(i)e^{-j2\pi fiT_b} \quad \text{for } m=0,1,\cdots,M-1,$$

$$R_n(f) = \sum_{k=-\infty}^{\infty} r_h(Nk-n)e^{-j2\pi f(Nk-n)T_b} \quad \text{for } n=0,1,\cdots,N-1,$$

$$P_n(f) = \frac{1}{N}\sum_{i=-\infty}^{\infty} p_n(i)e^{-j2\pi fiT_b} \quad \text{for } n=0,1,\cdots,M-1,$$

(11) can be rewritten as

$$\sum_{n=0}^{N-1} R_n(f)\left[P_n(f)\otimes A_m(f)\right] + \sigma^2 P_0(f)\otimes A_m(f) = e^{-j2\pi fmT_b} \tag{12}$$

where $\otimes$ denotes the convolution process and the term $\frac{1}{N}$ in $P_n(f)$ is used for normalization of the power. Note that $A_m(f)$ cannot be expressed in a closed form due to the convolution process. However, we only need discrete samples of $A_m(f)$ to calculate coefficients $\alpha_m(i)$ for ditital implementation.

Letting L be the number of discrete sample points, (12) can be expressed as

$$\sum_{n=0}^{N-1} R_n(k)\left[P_n(k)\otimes A_m(k)\right] + \sigma^2 P_0(k)\otimes A_m(k) = e^{-j2\pi fkm/L}, \; k=0,1,\cdots,L-1 \tag{13}$$

where $R_n(k), A_m(k), P_n(k)$ and $e^{-j2\pi fkm/L}$ are obtained by sampling $R_n(f)$, $A_m(f)$, $P_n(f)$ and $e^{-j2\pi fmT_b}$ at frequency $f = k/(LT_b)$, respectively. Thus, (13) can be represented in a matrix form of size $(L\times L)$ as

$$\begin{pmatrix} K(0,0) & K(0,-1) & \cdots & K(0,-L+1) \\ K(1,1) & K(1,0) & \cdots & K(1,-L+2) \\ \vdots & \vdots & \ddots & \vdots \\ K(L-1,L-1) & K(L-1,L-2) & \cdots & K(L-1,0) \end{pmatrix} \times \begin{pmatrix} A_m(0) \\ A_m(1) \\ \vdots \\ A_m(L-1) \end{pmatrix} = \begin{pmatrix} 1 \\ e^{-j2\pi m/L} \\ \vdots \\ e^{-j2\pi(L-1)m/L} \end{pmatrix} \tag{14}$$

where $K(i,j) = \sum_{n=0}^{N-1} R_n(i)P_n(j) + \sigma^2 P_0(j)$. For example, the span of the impulse response of the local loops in the telephone network is less than $128\,T_b$ in most of practical cases. In this case, the use of $L=256$ sample points is large enough to represent the autocorrelation of the channel impulse response. The use of (14) with $L=256$ can uniquely determine the MMSE solution.

The computational burden to solve L equations can be significantly reduced by noting that $A_m(k)$ has a conjugate symmetric property and that $P_n(k)$ is nonzero only at N positions among L discrete frequency bins. If $L=\mu N$, where μ is an even integer, it is possible to break an $(L\times L)$ matrix of (14) into $\mu/2$ matrices of size $(N\times N)$, where the factor 2 comes from conjugate symmetry of $A_m(k)$. Thus, the reduced matrix for the m-th sub-equalizer can be represented as

$$\mathbf{M}_k\mathbf{A}_{m,k} = \mathbf{E}_{m,k}, \; k=0,1,2,\cdots,\mu/2-1 \tag{15}$$

where $\mathbf{M}_k$ is an $(N\times N)$ matrix whose (i,j)-th element is

$$M_k(i,j) = \left[\begin{array}{l} \sum_{n=0}^{N-1} R_n((i-1)\mu+k)P_n((i-j)\mu) \\ +\sigma^2 P_0((i-j)\mu) \end{array}\right], \quad 1\le i,j\le N, \tag{16}$$

$\mathbf{A}_{m,k}$ is an N-dimensional vector of unknowns whose i-th element is

$$A_{m,k}(i) = A_m((i-1)\mu+k), \; i=1,2,\cdots,N \tag{17}$$

and $\mathbf{E}_{m,k}$ is an N-dimensional unit rotation vector whose i-th element is

$$E_{m,k}(i) = e^{-j2\pi[(i-1)\mu+k]mT_b}, \; i=1,2,\cdots,N. \tag{18}$$

Note that the rank of $\mathbf{M}_k$ is M since the transmitted sequence has only M independent data symbols among every N symbols. Thus, (15) can be solved by setting $(N-M)$ unknowns in $\mathbf{A}_{m,k}$ to arbitrary constants.

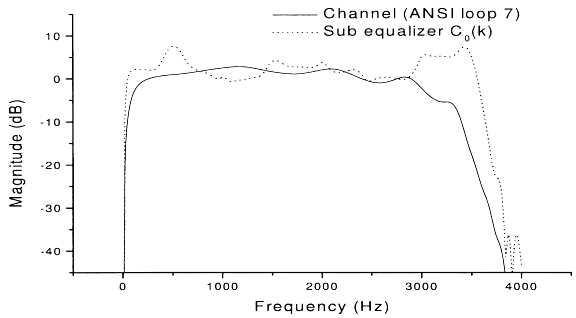

Fig. 3 Magnitude response of the channel (loop7) and the sub equalizer $C_0(k)$.

The MMSE coefficients of the m-th sub-equalizer can be obtained in the frequency domain using (4)

$$C_m(k) = [P_0(k) \otimes A_m(k)]\, H^*(k), \quad m = 0, 1, \cdots, M-1 \tag{19}$$

where the superscript $*$ denotes the complex conjugate. Finally, the coefficients $c_m(i)$ are obtained by taking the inverse discrete Fourier transform of $C_m(k)$.

4. Numerical Results

The initialization performance of the proposed method is verified by computer simulation when the PCM formatted signal of $f_b = 8$ kHz is transmitted over band limited AWGN channels with $W \leq 3.5$ kHz. Figure 3 shows the magnitude response of the channel and the sub-equalizer $C_0(k)$ just after initialization when the PCM formatted signal is sent over the ANSI loop 7, with $N = 8$, $M = 6$ and $L = 256$ at an SNR of 45 dB. Although the transmitted signal has spectrum component of up to 4 kHz, each sub-equalizer dose not need to compensate all the channel distortions unlike a general MMSE equalizer that enhances the signal components attenuated by the channel. Since each of the M sub-equalizers has to recover only non-zero symbols of every N-th transmitted symbol of the corresponding phase, it dose not need to recover the signal components in the stop-band. This process can be easily seen in the overall impulse response of the channel and sub equalizer as illustrated in Fig. 4, where every N-th symbol of $\{a_{0,k}\}$ are passed through without any interference. The overall impulse response has a magnitude of one at index 0 and has magnitude zero otherwise, except when zero symbols are transmitted.

The MMSE solution can be obtained by using the estimated channel response. The channel response can be estimated by using six periods of a periodic sequences of length 128 as in [8], corresponding to a time interval of approximately 800 PCM symbols. Table 1 summarizes the initialization performance by the proposed method on the ANSI local loop 1 through 7 when the channel SNR is 40 dB, 45 dB and 50 dB. To illustrate the initialization performance, Fig. 5 depicts

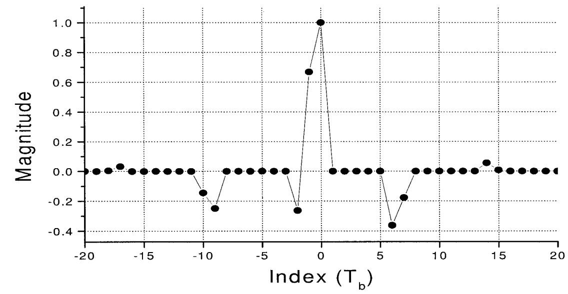

Fig. 4 Overall impulse response of the channel (loop7) and the sub-equalizer $c_0(t)$.

Table 1 Equalizer output SNR (dB) after initialization.

Channel SNR	loop 1	loop 2	loop 3	loop 4	loop 5	loop 6	loop 7
40 dB	40.2	40.5	40.5	40.2	40.3	38.1	38.4
45 dB	44.6	45.5	45.5	44.6	45.2	43.0	43.1
50 dB	48.0	50.4	50.3	48.0	50.1	50.1	50.1

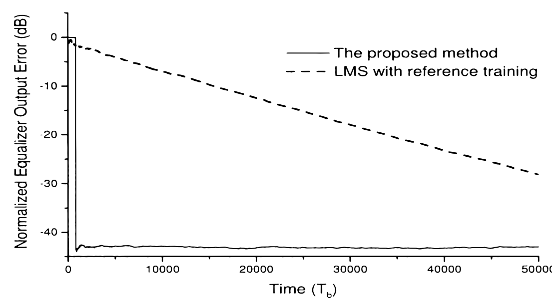

Fig. 5 Convergence characteristics of the equalizer.

the normalized equalization error when the proposed method is employed in the loop 7 at a channel SNR of 45 dB. For simple comparison, the equalization error is also depicted when the conventional LMS adaptive training method of [4] is employed. It can be seen that the use of the proposed method significantly reduces the training time at the expense of small additional complexity for the initialization.

5. Conclusion

In this letter, we have proposed a fast initialization method for an MMSE equalizer applicable to faster than Nyquist signaling. The proposed initialization method can be applied to real environment by estimating the channel response in real-time. Simulation results show that the MMSE equalizer can be fast trained by the proposed method.

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