Urban Planning and the Density Gradient Model

By C.P. Harris and Hong K. Sohn*

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I. Urban Models and Urban Planning

The major problems facing urban planners in growing urban centres may be grouped under three broad headings—the shape and structure of the urban centre, the spatial area of the urban centre, and land use patterns within the urban centre. These three elements are highly interdependent, and the urban plan reflects not only this interdependence but also the extent to which the solutions for one particular element are competitive with, or complementary to, the solutions for the other elements. For example the shape of the urban centre is in the first instance determined by the topography of the area available for urbanization, but it is also influenced by the quantity of land set aside for urban purposes and by the uses allocated to that land. Similarly, the area of land required for a particular sized urban population is a function of the quantities of land set aside for each kind of use of urban land and the quantity of land allocated to each kind of urban use is a function of the total quantity of land available and the projected size of the urban population.

Topography is a factor that does not readily lend itself to incorporation into theoretical models of urban growth and structure. Most of these models have assumed the problem of topography away by the premises that the

* C.P. Harris is Professor of Economics, James Cook University and Dr. Sohn is, on sabbatical leave of one year (1977) from the James Cook University in Australia, presently with the Economic Research Center, Department of Commerce, Government of Guam, Agana, Guam, U.S.A. The initial research of this paper was initiated and completed at the James Cook University, in Townsville, Australia.

land available for urban growth tends to be of a uniform or reasonably uniform character (in terms of physical features), and that the shape of the urban centre and land-use patterns are fairly regular, such as a series of concentric zones or star-shaped. The spatial patterns of actual cities are, of course, considerably more complex than those of the models. This divergence between actual cities and theoretical models does not invalidate the models as methods of urban research. Model building in complex cases is required to obtain an understanding of the range and variety of factors which in the real world can influence the growth of the urban centre.

The quantity of land required for a particular sized urban centre is not uniquely fixed by the number of urban dwellers and the kinds of economic activity that constitute the economic base of the city. This is so because the determinants of the uses of land in cities are not solely the decisions of private individuals and firms motivated by their own goals. That is, spatial patterns are not determined solely under market conditions in which private benefits and costs underlie the allocation. Uses of land are also affected by public instrumentalities, which in many cases are motivated by higher order social goals, reflecting political attitudes to the quality of urban life, and based on social benefits and costs rather than private benefits and costs.

Uses of land in cities may be divided into six types—residential, commercial, industrial public rights-of-way, institutional, and open space and recreation. Studies of United States cities, that have evolved over time under largely unplanned conditions, reveal the extent to which the proportion of urban land devoted to each of these uses varies in practice. For example, in Detroit, the home of the American automobile industry, 46 per cent of the land was found to be used for residential purposes and 32 per cent for public rights-of-way, indicating the high priority given in this city to the private automobile. In contrast in Philadelphia it was found that only 17 per cent of urban land was allocated to public rights-of-way. These choices are also reflected in other uses of land, particularly the proportion set aside for community use. Thus in Detroit it was found that only 6 per cent of the land was devoted to open space and recreation, while in New York the proportion was 28 per cent.(1)

(1) C. Abrams, "The Use of Land in Cities," *Scientific American*, September 1965, pp.154-5.

The attitudes of planners and city administrators to land use distribution, and the effect that these attitudes have on the demand for and supply of land, can be illustrated by reference to land use plans for new towns. Reston, a new town proposed in Virginia, was to have 56 per cent of its land given over to residential purposes, 15 per cent to open space and recreation, and 10 per cent to public rights-of-way. In contrast, the planners of Tapiola, a new town in Finland, proposed that only 24 per cent of the land be devoted to residential purposes, 9 per cent to public rights-of-way, and nearly 56 per cent to open space and recreation.(2)

Land use patterns, when related to a particular population size and quantity of urban land, may be summarized in a single statistic of average gross or net population density. Gross spatial density is computed by reference to all of the urban land irrespective of its actual use, while net spatial density is computed by dividing the population by the actual area available for residential purposes. Differences between the two statistics for a number of urban centres reveal divergences between land use patterns in those centres. For example, with 47 per cent of land set aside for residential purposes in Detroit, the net population density was 87 persons per hectare, compared with the gross population density of 40 persons per hectare. In contrast, the divergence between the two statistics in the proposed city of Tapiola was considerably greater, the net population density being 247 persons per hectare while the gross population density was only 59. The relatively greater net density figure for Tapiola results from the proportionately small amount of land set aside for housing and the large amount for open space and recreation.

Models appropriate for urban planning decisions have not as yet attempted to incorporate as variables the quantity of land set aside for each kind of urban use. This means of course, that the models are deficient in assuming away one of the most critical decisions facing urban planners. However, studies of such generalised models are still of great benefit to the planner whose task is to produce the specialised or particularised model of an actual centre, because behind the generalised models "there lies a core of truth sufficient to enable these models to enhance our understanding of even the most modern cities. The gap between the model and reality may

(2) *Ibid.*, p.155.

be a very wide one, but it can be bridged by relaxing some of the assumptions and taking account of new variables.(3)

The purpose of this paper is to examine one of these models of urban growth and structure, and to indicate its usefulness for urban planning decisions. The model examined is known as the urban gadient model, a model which, from the evidence of empirical analysis, hypothesises particular kinds of relationships between density ratios and distance from the central business district, and compares those relationships over time in a particular city as it grows, between different cities at a given moment of time, and between cities located in countries at different stages of economic and social development. The density gradient model is largely descriptive of actual ratios and trends, and to that extent is an objective and valuefree model. In particular it can not be used to examine the contentious question of the social consequences of high and rising density ratios. As Dyckman says "Density of settlement and the spacing of persons, according to persistent beliefs, materially affect the quality of life. In the early industrial era, when the close quarter of the industrial towns were accompanied by bad diet, poor sanitary engineering, little knowledge of the transmission of disease and the low immunity to these hazards of a recently rural labor force, the rapid increase of urban density nearly proved a total disaster. In the later stages of the growth of the city, density has acquired a connotation of urbanity, and has thereby come to be associated with social development and intellectual awareness…We still know little of the safely realizable densities of human congregation and of their effects on human performance…While human groups need not suffer as a group from being of a size that presses on a local food supply…one suspects the existence of some more subtle stresses on the individual under certain conditions…At the same time, there is evidence of strong congregating tendencies in human populations, including a tendency to congestion even where space is available, free, and riskless."(4)

It is true, as Dyckman says, that the early process of urbanization was nearly disastrous, because in the initial stages of the movement from

(3) H.W. Richardson, *Urban Economics*, Harmondsworth, Penguin Books, 1971, p.57.

(4) J. Dyckman, "The Changing Uses of the City" in D.W. Rasmussen and C.T. Haworth eds., *The Modern City: Readings in Urban Economics*, New York Harper & Row, 1973, p.35.

country to city induced by the industrial revolution urban death rates were consistently above urban birth rates. It was only in the twentieth century that medical and technological progress reversed this relationship, and thereby contributed to a more rapid growth of city population, a change of great social consequence because the numbers of rural inhabitants available as a source of migration to the cities were rapidly declining in developed economics. However, in recent years new and serious problems have arisen concerned with future life in cities, particularly problems associated with the quality of urban life as this is affected by social relationships and the quality of the physical environment. To this point in time it has generally been believed that the solution to these problems is one based on adjusting the urban environment to suit man. However, it may well be that the long-run solution will see a natural selection process in which the people who survive are those who adjust to this environment. As yet, of course, only science fiction writers suggest that a new kind of man and community relationships will have to evolve for mankind to survive. Examples include cities under the sea, cities "glassed in" and completely protected from the atmosphere, and a series of cities each inhabited by persons performing a specific and unique function. Of course, in the short-run, the professional urban planner may see little point in considering such fantasies, having to meet the current demands of local or State governments. However, in the long-run, when the future of urban planning is relevant, these matters are of great significance. To the planner drawing up a town plan for Townsville, for instance, it matters little that in 1900 only 6 per cont of the world's population lived in cities of 100,000 persons and over, while in 1975 this proportion had risen to 26 per cent, and on present trends the ratio in 2000 will be 40 per cent.(5) The fact that the planning profession on a worldwide basis has to face the task of designing large cities which over the next 25 years will grow to contain 4 persons out of every 10 in the world instead of the present 2.5 persons, may not be a matter of apparent immediate concern to planners in localised areas, but it is obviously of great concern to the profession as a whole, as well as to many municipal and central governments. Overall, the forecast growth of population in large cities of

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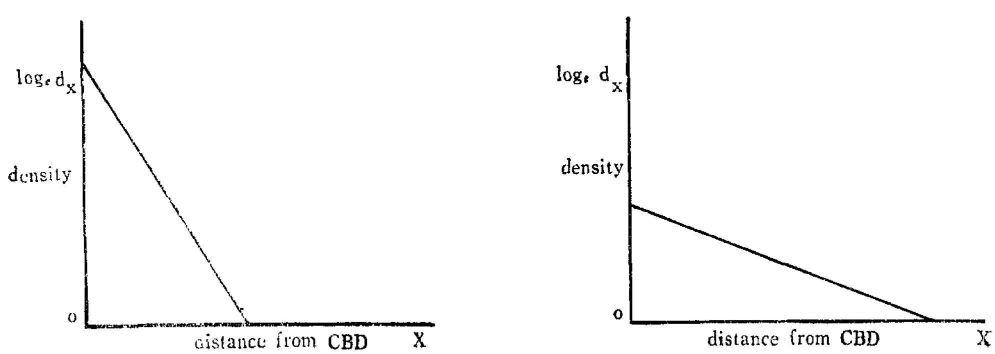
(5) Kingsley Davis, *World Urbanization 1950-1970*, (2 volumes) Berkeley, Institute of International Studies, University of California, 1969 & 1972.

100,000 persons and over means that in the next quarter of a century the average number of persons living in cities of this size in the world will rise by about 150,000 persons from the current level of 500,000 while the largest of these cities may well attain the staggering size of 100 million persons.

II. The Density Gradient Model

Colin Clark is generally credited with having evolved the density gradient model described below, although in his article he acknowledges the work done some 50 years previously by the French scientist, Meuriot and the American geographer, Jefferson.(6) On the basis of empirical data from 20 cities in Europe, U.S.A., Ceylon and Australia, Clark developed a model of the density of residential population in the form $d_x = d_c e^{-gx}$, where d_x is the population density at distance x from the CBD, d_c is the hypothetical population density in the CBD, and g (positive value) the density gradient indicating the rate of diminution of density with distance from the CBD. In this model the decline in density is an exponential function, and by measuring density as the natural logarithm of density, a linear equation $\log_e d_x = \log_e d_c - gx$ results.

Depending on the value of g, two limiting kinds of cities can be distinguished. The first, with a high value of g, is a city which economises on the use of land by having high density close to the centre, with density



(a) compact city (b) sprawling city

Fig. 1. Decline ot density gradients with distance.

(6) Colin Clark, "Urban Population Densities," *Journal of the Royal Statistical Society*, Series A, Vol. 114, 1951, pp.490-6.

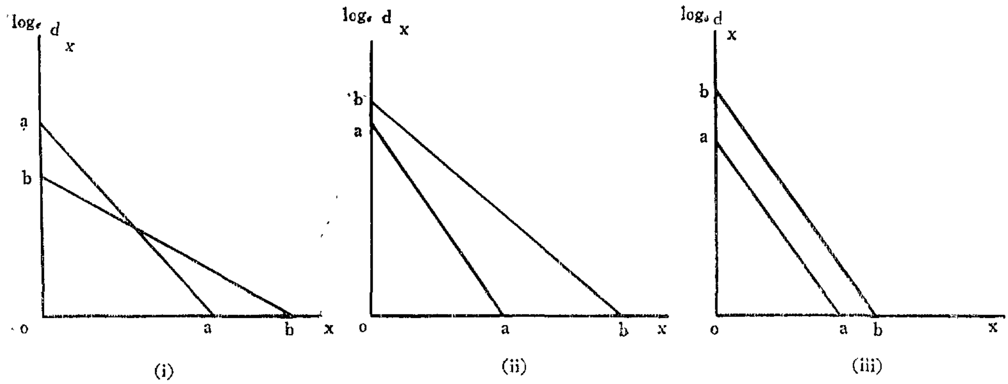


Fig. 2. Changes in density gradients over time (ⓐ *a*-function for initial year, ⓑ *b*-function for later year).

declining rapidly with distance from that centre (a compact city). The second case is one with a low value of *g*, where inner densities are relatively lower and where densities decline slowly with distance (a sprawling city). These two cases are illustrated in Figure 1.

Clark also examined changes in the density function over time as the city grew. In general, he discovered that with growth the value of *g* declined, the city outgrowing its previous spatial boundaries. He discovered two different forms of this change, the first where inner densities declined with growth, so that the two density functions intersected; and second where inner densities increased with growth, so that the density function for the later year shifted upwards to the right. However, in some cases it appeared that the upward shift of the density function was not associated with any change in the value of *g*, the two density functions being parallel. These three cases are illustrated in Figure 2.

The major difference between case (i) and cases (ii) and (iii) in practice seems to be related to the stage at which central city functions are decentralised (i.e., shifted to the suburbs). If decentralisation does not occur, increasing overcrowding and rising densities are a feature of the central city. Once decentralisation begins to occur at time *t*, densities begin to fall. These processes are illustrated in Figure 3.

The model hypothesises that as a city grows, except in the extreme case of growth occuring within existing urban boundaries, the density function will shift upwards to the right, at first with a reduction in *g* but with

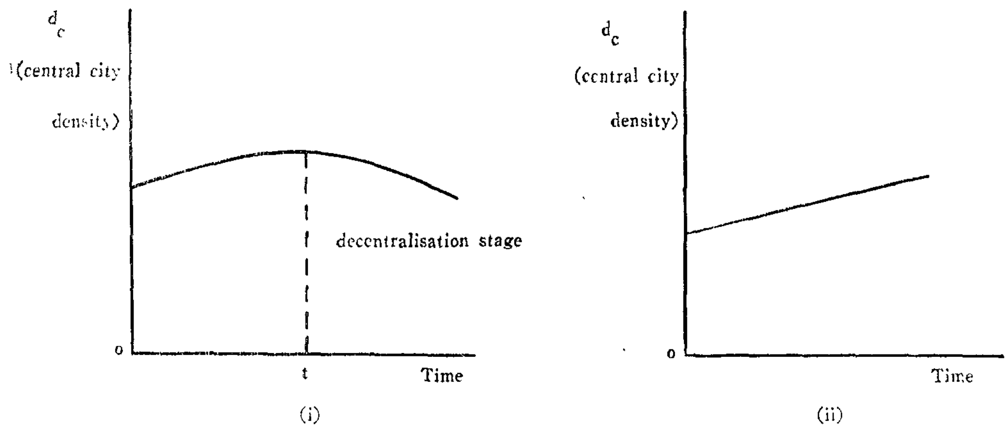


Fig. 3. Changes in central city density with urban population growth.

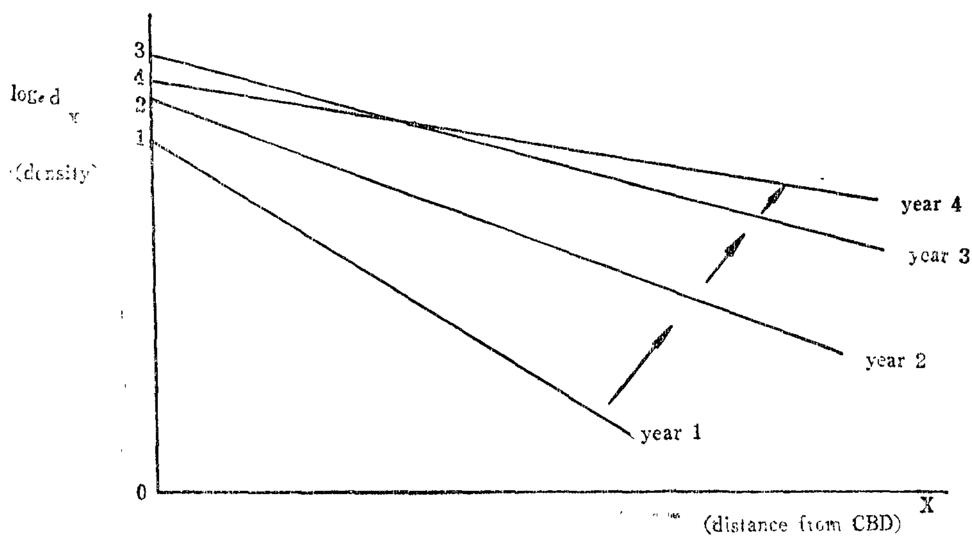


Fig. 4. Temporal comparison of density functions as the city grows.

increasing inner city densities; however, in the stage of decentralisation the new function will intersect the previous function, with a resulting decline in central city densities. Clark believed that these shifts were mainly associated with the nature and cost of intra-urban transport. This process of change is shown in Figure 4.

As the population increases from year 1 to year 2 the urban density function shifts upwards to the right with an increase of central density but a decline in g (the rate at which density decreases with distance). As the population increases further from year 2 to year 3 this process is repeated, but the rate of increase in central density is diminishing—see Figure 3(i). If decentralisation begins between year 3 and year 4, the central density decreases in absolute terms. The urbanization process does not in reality occur precisely in the above fashion, but in general, as cities grow, density gradients diminish and urban sprawl increases. Subsequent studies of the density function extended the work of Clark but generally his model was retained as the basis of the analysis. Muth undertook detailed studies of the factors affecting the value of g, including not only the factor stressed by Clark, the nature and cost of intra-urban transport, but also the intra-urban distribution of different kinds of industry and employment, and the relationship between residential location preferences and occupational status (or income). In the U.S. cities that he studied, Muth found that over two-thirds of the variance in the density gradients could be explained by such elements as the proportion of employment in manufacturing in the central city, the extent to which the central city offered residential accomodation, the generally low quality of housing in the central city, relative income levels, car registrations per head of population, and the proportion of Negroes in the central city population, a peculiarly U.S. factor.(7)

One of the implications of these studies is that the elasticity of the supply of housing with respect to distance from the CBD is positive and has a value greater than unity. Therefore, as the demand for housing increases with the growth of the city, the rate of increase in housing supply is greater than the rate of increase in distance from the CBD. This process will slow down the rate of density reduction as the city expands. There are three main reasons for this. First, the number of housing units per hectare tends to decline with distance from the centre; second, the number of persons per household tends to rise with distance from the centre; and third, the number of persons not residing in private household dwellings tends to be greatest near the centre.

Alonso approached locational choice from the viewpoint of the consumer motivated by the utility or satisfaction of living with more space, and the

(7) R. Muth, "The Spatial Structure of the Housing Market," *Papers and Proceedings of the Regional Science Association*, Vol. 7, 1961; R. Muth, "Urban Residential Land and Housing Markets" in H. Perloff and L. Wingo, eds., *Issues in Urban Economics*, Baltimore, Johns Hopkins Press, 1968; and R. Muth, *Cities and Housing*, Chicago, Chicago University Press, 1969.

disutility (cost) of commuting between place of residence and the central business district.(8) He claimed that the rich prefer to live at the periphery on relatively cheap land and they therefore consume more land at lower densities than the poor whose lower incomes give greater weight to the cost of travel in choosing locational sites. Thus nearness to the central city (accessibility) tends to be an inferior good (demanded mainly by those consumers with low incomes), and the demand for accessibility declines with rising incomes. These factors support the development of a sprawling city, with consequent reductions in the density gradient. Berry showed that the age of the city was highly correlated with central density, at first rising as the city grew over time, but subsequently declining.(9) Mohring developed a model that described the effect of the transportation system of land values through annual travel costs to the centre.(10) Other analysts have in contrast emphasised urban land use and land values rather than housing.(11)

Mills extended the empirical analysis of density functions, examing for 18 United States metropolitan areas the density functions not only for population but also for various kinds of employment (manufacturing, retailing, services, wholesaling).(12) His approach is based on the fact that the household affects land use patterns in two ways, first as a consumer (residence location) and second as a factor owner (employment location). Thus the intensity of use of land may be related either to the number of persons residing in particular zones or to the number of workers employed in particular kinds of economic activity in those zones. The latter approach produces evidence about the spatial distribution of industry within the city.

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(8) W. Alonso, "A Theory of the Urban Land Market," *Papers and Proceedings of the Regional Science Association*, Vol. 6, 1960, pp.149-57.

(9) B. Berry, "Research Frontiers in Urban Geography" in P. M. Hauser & L.F. Schnore, eds., *The Study of Urbanization*, New York, Wiley, 1965, pp.403-30; B. Berry, J. Simmons & R. Tennant, "Urban Population Densities: Structure and Change," *Geographical Review*, Vol. 53, No. 3, 1963, pp.389-405.

(10) H. Mohring, "Land Values and Measurement of Highway Benefits," *Journal of Political Economy*, Vol. LXIX No. 3, 1961, pp.236-49.

(11) For example: P. Wendt, "Theory of Urban Land Values," *Land Economics*, Vol. 33, 1957, pp.228-240; C. Kramer, "Population Density Patterns," *CATS Research News*, Vol. 2, 1958, pp.3-10; L. Wingo, "An Economic Model of the Utilization of Urban Land for Residential Purposes," *Papers and Proceedings of the Regional Science Association*, Vol. 7, 1961, pp.191-205; E. Cassetti, "Urban Population Density Patterns: an Alternative Explanation," Vol. XI, No. 2, 1967, pp.96-100.

(12) E. S. Mills, "Urban Density Functions," *Urban Studies*, Vol. 7, No. 1, 1970, pp.5-20.

For all variables examined, Mills in general found that g declined as the cities grew over time, and he stressed the following relationships:

(i) size of city—d_c (central density) an increasing function of size, and g (rate of decline of density with distance) a decreasing function of size,

(ii) income of urban dwellers—d_c and g both decreasing functions of income per household or family unit,

(iii) cost of transportation (including money and opportunity costs)—d_c and g both decreasing functions of transport prices.

The conclusions of Mills except for the first are in accord with the preceding discussion. It has previously been contended that the first relationship he postulates is valid only in the non-decentralisation stage of growth and that once the decentralisation stage is reached, both d_c and g will be decreasing functions of size.

III. Urban Planning and the Density Gradient Model

The density gradient model is largely a descriptive technique designed to establish empirically the relationship between density and distance from the central city. This empirical analysis produces evidence from which certain hypotheses are formulated about changes in the structure of cities as they grow over time, changes that take place largely in a market (unplanned) urban area. In this sense, the model is of value to urban planners, in that it stresses some of the "natural" (market) forces at work within the city, forces which might either support or hinder proposed urban plans.

Of course, in the form described above, the gradient model suffers from many deficiencies as a general model applicable to all urban centres. In particular, it is constrained by its assumption of a single nucleus of influence or activity (the CBD) thus making it applicable to urban centres which conform largely to the concentric zone pattern. Similarly, the assumption of linearity (constant rate of decline of density with distance) does not allow for discontinuities in the gradient function, nor for changes in the rates at which densities decline with distance. However, these are matters that do not detract in any essential way from the use of the model as an aid to urban planning, for where necessary the model can be extended to analyse a multi-nuclei city and the case of curvilinear relationships between the rate of decline of density and distance. In particular, the simple model may be directly applicable to the planning of medium and small cities, say with populations not exceeding 100,000 to 150,000, cities which are particularly relevant in Queensland.

It appears that urban planning in these smaller cities in Queensland can be improved by increasing the information available about existing density gradients in these cities for both population and particular kinds of employment, given that for many Queensland cities a regular shape is not a completely invalid assumption. The increased information will highlight the options open to the planner in accommodating the forecast population growth. In particular, the model stresses the inverse relationship between densities and the quantity of land required. Thus if the land available for urban expansion is valuable in its existing use, it might be cheaper in the long run to plan for a more compact city, economising on the urbanization of new land. In this case, which applies in such sugar cities as Cairns, Mackay and Bundaberg, the plan will require substantial redevelopment in the inner and older parts of the city to raise densities, and impose general restrictions on the size of urban allotments.

In contrast, if the land into which the urban centre can expand is not very valuable in its present use, the cheapest way to accommodate growth may be to accept a more sprawling city. This will apply to Rockhampton and Toowoomba. In the case of smaller cities, however, this may still require a rise in inner densities, the planned density function shifting upwards but not intersecting the existing function—Figure 2 (ii) above.

A third case is where the topography of the land on the outskirts of the city is unfavourable for conversion to urbanization. Either of two solutions may apply in such cases, which have some relevance to Gladstone and Townsville. First, the topography may be accepted as imposing a constraint on the outward sprawl of the city, and emphasis will be placed on the planning of a compact city. Alternatively, however, the plan may develop a discontinuous system of urbanization, utilising for habitation favourable land further distant from the city. This will, of course, increase the need for transportation facilities, and raise the cost of physical infrastructure. A discontinuous urban system will generally require that the decentralisation

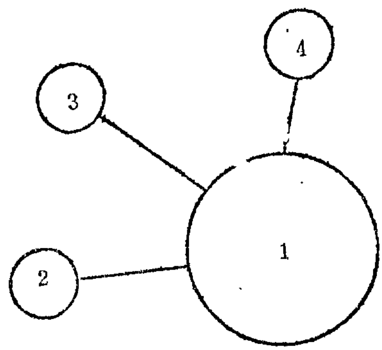

Fig. 5. Growth of urban centre—old urban area (1) and 3 satellite centres (2, 3, 4).

stage be implemented, with the development of minor nuclei (business districts) in the more remote urban areas. This latter form of urban structure, which is illustrated in Figure 5, appears appropriate for the Townsville urban area.

IV. Conclusion

Theoretical models of urban spatial structure and land-use patterns reveal some of the significant relationships underlying the complexity of urban structure and growth in the real world. Empirical analysis based on these models, such as that required for the derivation of density gradients, increases the analyst's understanding of the factors at work within a particular city. This greater understanding will assist first in the specification of the objectives of the proposed urban planning scheme, and second in the identification of the most appropriate strategy to achieve those objectives.

With respect to physical elements, the goals of planning may be designed to achieve an orderly and compatible spatial arrangement of different uses of land. With land zoned residential densities are normally specified but plans have not generally incorporated the spatial density of the labour force. Rather, zoning has been concerned with such broad classifications as heavy industry, light industry, commercial enterprise and the like. Yet the proposed densities of the labour force in particular locations will be a significant factor determining the volume of commuting traffic between residential areas and work areas, and land-use patterns to accommodate forecast traffic flows are a basic element in any plan.

All elements included in the plan do not contribute to the same objective, and in practice the urban plan must be multi-objective, with trade-offs between different objectives, many of which are competitive with each other. The system of priorities choice underlying the plan will, of course, affect the final shape and form of the urban centre. The trade-offs may in theory represent community preferences, but in practice they are too often based on the values of the professional planner and/or the local politician. Community participation in town planning is not well developed, mainly because the education of urban residents as to the aims and strategy of urban planning is at a very low level. Participation goes hand in hand with understanding, and there is a tendency for the general public either to believe, or to be led to believe, that it is only the professional planners who can make the best decisions and choices. These choices or trade-offs are not explicit in any town plan, which merely presents one choice, not a series of options. Without evidence before him of the consequences of different kinds of plans, the citizen can not make a choice between lower residential densities or shorter trips to the central city; between more open space or higher residential densities; between a city based on people or one based on the private automobile. These are the kinds of choices which affect the quantity of urban land (compact or sprawling city), the intensity of its use (density), and the allocation of land to various kinds of uses, including open space and recreation, historical and cultural preservation, and ecological conservation. The density gradient model is one theoretical model of urban form and growth which generates information that makes these kinds of choices explicit and more meaningful to the average citizen.