# Orthogonalization Based Adaptive Interference Suppression for Direct-Sequence Code-Division Multiple-Access Systems

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Abstract— Rapid converging adaptive interference suppression algorithms for direct sequence code-division multiple-access (DS/CDMA) systems are presented in this letter after showing the limitations of conventional adaptive algorithms. The rapid converging algorithms are based on an orthogonal transformation preprocessing, and are adaptive implementations of the linear minimum mean square error (MMSE) receiver for interference suppression. These algorithms do not require a priori knowledge on interfering signal parameters such as spreading sequences and relative signal power levels.

### I. INTRODUCTION

INTERFERENCE cancellation schemes based on the minimum mean square error (MMSE) criterion were proposed in [1]–[4] for direct-sequence code-division multiple-access (DS/CDMA) systems. These schemes do not require a priori information regarding interfering signals. In [1]–[4], an adaptive algorithm implementation of the MMSE criterion for interference suppression was mentioned; however, the applicability and convergence rate of conventional adaptive algorithm for this application was not investigated. The purpose of this letter is to show problems with applying conventional adaptive algorithms to interference suppression, and to present rapid converging algorithms based on an orthogonal transformation.

### II. SYSTEM MODEL AND THE MMSE CRITERION

In a synchronous DS/CDMA system with L concurrent users, the received signal for the mth duration may be represented in a vector form in the baseband as follows:

$$R(m) = \sum_{k=1}^{L} g_k \ d_k(m) \cos(\theta_k) P_k + N_n(m)$$
 (1)

where  $g_k, d_k, \theta_k, P_k$  represent, respectively, the kth user received signal power level, data, carrier phase, and spreading sequence vector. Nn(m) is a zero mean Gaussian random noise vector. For simplicity of presentation, a synchronous system is considered in this letter. Adaptive algorithms proposed in this letter can easily be extended to an asynchronous system.

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# III. CONVENTIONAL ADAPTIVE ALGORITHMS AND LIMITATIONS

The convergence rate and suitability of adaptive signal processing algorithms are characterized by the properties of an input data correlation matrix [5]. An input correlation matrix  $\Theta$  in DS/CDMA systems, which is derived from (1), is described by

$$\Theta = \Theta_s + \Theta_n \tag{2}$$

where

$$\Theta_s = \sum_{k=1}^{L} g_k(\cos(\theta_k))^2 P_k P_k^T.$$
 (3)

In (3), the same symbols are used as in (1). The first term on the right-hand side of (2) is the correlation matrix  $\Theta_s$  of the signal part of input data, and the second term is the noise correlation matrix  $\Theta_n$ . The correlation matrix of an input data vector is nearly singular, when the number of transmitters is less than the number of chips and the noise variance is small compared to signal power. A signal power spread is closely related to a signal eigenvalue spread.

The least mean square (LMS) algorithm has been frequently used in adaptive signal processing for its simplicity. However, its convergence depends on the eigenvalue spread of an input data correlation matrix [5]. For an interference suppression case, the convergence of the LMS algorithm is found to depend mainly on the signal eigenvalue spread of the input correlation matrix  $\Theta_s$ , which is the ratio of the maximum nonzero eigenvalue of  $\Theta_s$  to the minimum nonzero eigenvalue. The LMS converges slowly when one signal power is greater than a desired signal power.

The use of the recursive least squares (RLS) algorithm in the near-far situation, which requires a nonsingular input correlation matrix, will result in numerical problems, because of the near-singularity of the input data correlation matrix. Hence, the RLS algorithm cannot be employed for interference cancellation in DS/CDMA systems.

## IV. RAPID CONVERGING ADAPTIVE ALGORITHMS

In mobile environments, channel characteristics change rapidly. Rapid converging adaptive algorithms are required to track quickly varying mobile channel characteristics. In this section, rapid converging adaptive algorithms based on an orthogonal transformation preprocessing are presented [6]. These algorithms consist of two parts: 1) orthogonal transformation preprocessing which orthogonalizes input data, and 2) desired signal estimation and updating tap weights using orthogonalized data from part 1).

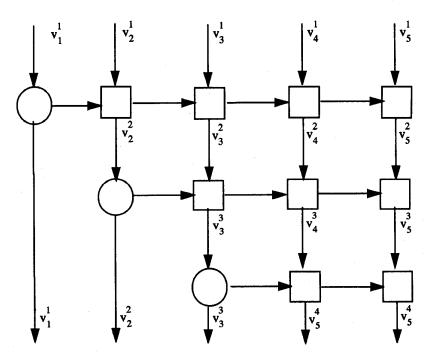


Fig. 1. Graphical representation of an orthogonal transformation.

### A. Orthogonal Transformation Preprocessing

In DS-CDMA systems, the input vector, consisting of data received at the chip rate over one symbol period, is correlated. This correlated input vector may be transformed to a decorrelated vector by an orthogonal transformation. Fig. 1 shows a graphical representation of an orthogonal transformation based on the Gram-Schmidt transformation [7], [8] for an input vector of five elements. In this figure, the input vector, x, is fed in from the top and orthogonalized on an element-byelement basis. At the first level, transformation is applied to the input vector such that  $v_1^1$  is orthogonal to the elements,  $v_i^2$  for j from two through five. Note that the superscript and subscript denote the level of decorrelation and the position of a vector element, respectively. The same procedure continues at the following levels. The number of correlated vector elements decreases by one each step of decorrelation, as one more vector element becomes decorrelated from the rest of the vector elements.

The correlation between the element  $v_1^1$  and the elements  $v_j^2$  for j from two through five is mostly determined by the strongest interfering signal spreading sequence. The stronger the strongest interfering signal is, the closer the correlation is to the strongest interfering signal spreading sequence. The first level decorrelation may be viewed as a subtraction of the strongest signal from the received signal, since the portion correlated with  $v_1^1$  is subtracted from the received signal. The level-by-level decorrelation is similar to a successive interfering signal subtraction technique described in [9] and [10].

The signal eigenvalue spread of the correlation matrix formed by a vector consisting of correlated elements decreases with a level-by-level decorrelation, since strong interfering signals which contribute to large eigenvalue spread are subtracted through a level-by-level decorrelation. An input data vector becomes completely decorrelated after the L-1th level of decorrelation, where L is the number of transmitters. This will be referred to as a complete orthogonalization in this letter. The elements of the completely decorrelated vector may be divided into two parts: elements associated with signals and elements associated with noise. In estimating a desired signal, only signal vector elements are used. This requires the identification of signal and noise vector elements. The identification of signal and noise components may not be a trivial task.

A need for the identification of signal and noise components may be eliminated by using a partially, not completely, decorrelated output vector from a partial orthogonal transformation in estimating a desired signal and updating tap weights. Rapid convergence in the LMS adaptive interference suppression for DS/CDMA systems will be obtained using a partial decorrelated output vector, as long as strong interfering signals are decorrelated. In the determination of an optimal partial decorrelation level, *a priori* knowledge of a desired signal power level and/or an information-theoretical criteria, proposed by Akaike to estimate the order of a model for a stochastic process [11], may be used.

It is worth noting that the orthogonal transformation described in this section is similar to an orthogonal technique used in adaptive lattice filtering [5] and transform domain adaptive filtering such as frequency domain adaptive filtering [12]. However, there are some differences. In lattice filtering, the orthogonalization is serially applied to one input data element at a time, not a vector, through time and order updates. In transform domain processing, a complete orthogonalization, not partial, has been utilized.

SHADEATON I ARAMETERS			
Ratio of Interfering Signal Power to Desired Power	maximum eigenvalue	minimum eigenvalue	eigenvalue spread
10	71.88	4.00	17.97
225	1576.72	4.00	394.18
1000	7001.72	4 00	1750 43

TABLE I SIMULATION PARAMETERS

# B. Adaptive Desired Signal Estimation and Tap Update

With a complete orthogonalization preprocessing, both the LMS and RLS algorithms may be used in estimating a desired signal using only decorrelated vector signal elements. In estimating a desired signal with partially orthogonalized data which consist of both decorrelated and correlated vector elements, the LMS algorithm, not the RLS, has to be used. This is because of the near-singularity of the correlation matrix formed by partially decorrelated data. In using the LMS algorithm, step sizes optimized for each element may be used for decorrelated vector elements to improve convergence, whereas the same step size has to be used for correlated vector elements.

## V. EXPERIMENTAL RESULTS

Convergence of adaptive algorithms for interference suppression is investigated in this section by means of computer simulations. Ensemble-averaged learning curves are generated by averaging 100 independent simulation runs. In the simulations, five synchronous transmitters, one of which is a desired transmitter, are assumed to be present in additive white Gaussian noise (AWGN) channels. The five transmitters are assigned to unique Gold spreading sequences with period 7.

# A. Convergence of Adaptive Algorithms Based on Complete Orthogonalization

Convergence of rapid algorithms based on a complete orthogonal preprocessing is investigated under three different conditions which are summarized in Table I.

Received power from one of the interfering transmitters is varied to simulate different eigenvalue spread conditions, while received signal levels from four other transmitters including a desired transmitter are assumed to be equal. The AWGN power is set to 30 dB lower than the desired signal power. Fig. 2 shows three ensemble-averaged learning curves of the conventional LMS algorithms without an orthogonalization. These learning curves are associated with three different eigenvalue spreads: 17.97, 394.18, and 1750.43. In these simulations, a step size, which is used in updating tap weights, is set to one quarter of the inverse of the maximum eigenvalue for each simulation condition. Fig. 2 confirms that an increase in eigenvalue spread slows down the rate of convergence.

The results of simulations using the LMS updating equations with an orthogonal transformation preprocessing are shown in

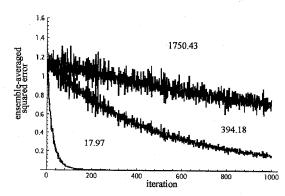


Fig. 2. Learning curves of the conventional LMS algorithm for eigenvalues spreads: 17.97, 394.18, and 1750.43.

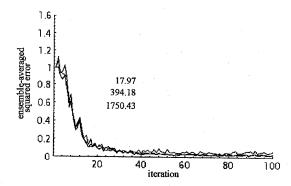


Fig. 3. Learning curves of the LMS algorithm with an complete orthogonalization for eigenvalues spreads: 17.97, 394.18, and 1750.43.

Fig. 3. In the simulations, the number of signals is assumed to be known, and step sizes are individually set to one quarter of the inverse of the power levels of the signal vector elements after an orthogonalization. A comparison of Figs. 2 and 3 indicates that the use of optimal step sizes for each vector element after an orthogonal transformation significantly speeds up the convergence rate. The overlapped learning curves in Fig. 3 for different eigenvalue spread cases confirm that the rate of convergence of the LMS algorithm with an orthogonalization does not depend on the eigenvalue spread.

# B. Convergence of Adaptive Algorithms Based on Partial Orthogonalization

Various level decorrelated data from a partial decorrelation are employed in estimating a desired signal and updating coefficients to investigate decorrelation level effects on convergence. In the simulations, five synchronous transmitters, one of which is a desired transmitter, are assumed to be present. The ratios of four interfering signal powers to the desired signal power are 1000, 10, 1, and 1. The AWGN power is set to 30 dB lower than the desired signal power.

A step size for decorrelated elements, used in LMS tap updating equations, is optimized for each element and set to one quarter of the inverse of the power of each element. For correlated elements, a step size is set to one quarter of the inverse of the maximum eigenvalue of the correlation matrix of correlated elements. Fig. 4 shows five learning curves of

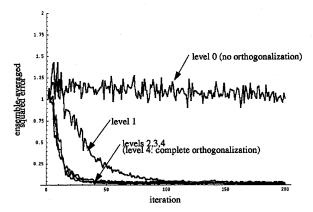


Fig. 4. Learning curves of the LMS algorithm with a partial orthogonalization, Interfering signals power wrt. desired: 30 dB, 10 dB, 0 dB, 0 dB.

the LMS algorithm with different levels of decorrelation. The LMS with a level-one decorrelation, where most of the strongest interfering signal are decorrelated, converges much faster than the LMS without any decorrelation. The LMS with level-two and level-three decorrelation performs as well as the LMS with a complete decorrelation. The reason is that the eigenvalue spread of the correlation matrix after level-two and level-three decorrelations is close to one.

### VI. CONCLUSION

In this letter, we have presented rapid converging adaptive equalization algorithms based on both complete and partial orthogonal transformations for interference suppression, after showing a convergence problem with the LMS algorithm and a near-singularity problem with the RLS algorithm. The use of a partial orthogonalization eliminates the need for signal

and noise identifications after an orthogonalization. Simulation results confirmed that the proposed algorithms with an orthogonalization converge much faster than the conventional LMS algorithm without an orthogonalization. The LMS with a partial orthogonalization was found to converge as fast as the LMS with a complete orthogonalization as long as strong interfering signals are decorrelated.

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