Discrete Adjoint Approach for Aerodynamic Sensitivity Analysis and Shape Optimization on Overset Mesh System

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ABSTRACT

In the present talk, the strategies to apply the sensitivity analysis method to aerodynamic shape optimization problems of complex geometries are intensively discussed. To resolve the design of complicated aircraft geometries such as high-lift devices, wing/body configurations, overset mesh techniques are adopted. In addition, a noticeable sensitivity analysis method, adjoint approach, which shows very good efficiency and accuracy for aerodynamic design problems, is also introduced. For the incorporation of the adjoint method into the overset mesh system, adjoint formulations are derived for the overset boundary conditions based on linear interpolation. The feasibility of non-conservative adjoint overset boundary conditions for external flow applications is carefully investigated by comparison with a single block design result for the same geometry. Through the several design application problems for realistic aircraft geometries, the present design framework demonstrates its capability and applicability for aerodynamic design of complex geometries.

INTRODUCTION

Gradient-based design methods might have difficulties in dealing with highly non-linear design spaces, where a design solution can often be trapped in the local optimum. Nonetheless, the Gradient-Based Optimization Method (GBOM) is still very popular in Aerodynamic Shape Optimization (ASO), because the GBOM is very efficient in finding an optimal shape and it can be readily combined within the Multi-disciplinary Design Optimization (MDO) framework. The interests of ASO via GBOM have gradually moved into large-scale computations over complex geometries with the rapid progress of computational environment.[1] In order to deal with complex aircraft geometries, design optimization strategies based on various grid topologies are attracting more and more attentions.

Design on the multi-block grid system can be accomplished by extending the single block sensitivity module and successfully applied to various design problems. The multi-block grid technique generally secures good grid quality. However, in applications involving moving or deforming grids which result in a severe grid change or the change in grid topology, it is extremely challenging to realize the fully automatic design. In the case of the unstructured grid system, the automatic mesh generation is relatively amenable. Thus, unstructured sensitivity analysis codes with the discrete adjoint approach have been developed.[2] Compared with the structured mesh system, however, more grid points are generally required. In addition, memory overhead and computational cost are often inevitable.

On the other hand, the overset grid technique is very attractive in terms of computational accuracy and geometric modeling, which is beneficial to large-scale flow analysis and design optimization. Furthermore, these advantages can be fully exploited to drive the overall aerodynamic design optimization process into the final goal, i.e., “the fully automatic aerodynamic design from the CAD models”.

In order to implement the adjoint-based sensitivity on the overset mesh system, there are several problems to be resolved. First, convergence characteristics of the adjoint solver are seriously affected by the interpolation error at the overset boundary. Second, the objective function has to be fully hand-differentiated at the overlap surface by considering the reconstruction of surface mesh. This may cause a substantial difficulty in an adjoint solver. In addition, the overlap boundaries between the mesh blocks should be treated carefully during the mesh deforming process of design optimization. However, only a few researchers have investigated the studies on the ASO using the overset mesh system.[3]

In the present presentation, the pre- and post-processing methods as well as the adjoint boundary condition are carefully investigated on the overset mesh system to establish a practical three-dimensional aerodynamic shape design methodology based on the discrete adjoint approach. The performance of the overlap optimization technique[4] is investigated in terms of convergence of adjoint solver and accuracy of flow solver. The Spline-Boundary Interpolation Grid (S-BIG) scheme is proposed for efficiently evaluating cell differentiation in the adjoint solver. By exploiting these techniques, practical design optimization applications of aircraft configurations are successfully carried out on the overset grid system.

NUMERICAL BACKGROUNDS

A. Overlap Optimization for Overset Adjoint Solver

The capability of the adjoint-based ASO on the complex overset mesh system depends critically on the performance of pre-processor. One of the primary concerns in this field is developing a robust and efficient pre-processor. Several high-quality processors have been developed. Especially, PEGASUS[4] is regarded as one of the most efficient and robust pre-processors. It has been applied to high-fidelity flow simulations including the flow analyses over various full-body aircrafts, spacecrafts, and so on. In order to allow the complex overset mesh system with a huge number of blocks and complicated block connectivity, a process for finding hole-points and constructing block connectivity automatically is the key step, which is known as the overlap optimization. By the interpolation via Cell Difference Parameter (CDP), which considers the cell volume ratio and the cell aspect ratio between donor and fringe cells, overlap optimization can also contribute to the convergence characteristic of flow analysis codes. In addition, it can reduce numerical oscillation by minimizing the overlap computational domain. Based on this observation, overlap optimization is extended to sensitivity analysis module to improve the convergence and accuracy characteristics.

B. Spline-Boundary intersecting Grid (S-BIG) Scheme

To calculate the aerodynamic coefficients in overset flow analysis, the zipper grid scheme is widely used.[5] This method consists of two steps: the blanking process of the overlap computational region, and the reconstruction process of the overlap region with a set of unstructured grids. The flow variables on the zipper grid are then interpolated from the donor cells of the original overlap region. In this case, the numerical differentiation of the flux terms on the zipper grid, which is necessary in the adjoint code, is quite difficult and inefficient. Thus, the Spline-Boundary Intersecting Grid (S-BIG) scheme has been newly devised, and it is applied to the post-processing and sensitivity analysis routines. The purpose of the S-BIG scheme is to evaluate aerodynamic coefficients without interpolating flow variables from donor cells. As a result, to numerically differentiate the flux terms or to evaluate aerodynamic coefficients, the S-BIG scheme does not require anything except the boundary information of the overlap region. The procedure of S-BIG scheme is prepared to eliminate the overlapped surface cells or to reform the edge cell on the basis of prescribed spline boundary. The reformed cells are represented by 8 triangles and 9 vertices form. And the aerodynamic coefficients can be evaluated using the area of surface meshes constructed by the reformed surface cells.

C. Adjoint Formulation for Overset Boundary

The sensitivity of an objective function with respect to a design variable from discrete adjoint formulation can be evaluated by Eq. (1), (2) and (3). The sensitivities can be acquired with the grid sensitivities of objective functions and residual equations, and the adjoint vector Λ as shown in Eq. (1).

$$\left\{\frac{dF}{dD}\right\} = \left\{\frac{\partial F}{\partial X}\right\}^T \left\{\frac{dX}{dD}\right\} + \left\{\frac{\partial F}{\partial D}\right\} + \Lambda^T \left(\left[\frac{\partial R}{\partial X}\right] \left\{\frac{dX}{dD}\right\} + \left\{\frac{\partial R}{\partial D}\right\} \right) \tag{1}$$

if and only if the adjoint vector Λ satisfies the following adjoint equation.

$$\left[\frac{\partial R}{\partial Q}\right]^T \Lambda + \left\{\frac{\partial F}{\partial Q}\right\}^T = \{0\}^T \tag{2}$$

The solution vector Λ is obtained by solving the Euler implicit method of Eq. (3) time-iteratively as

$$\left(\frac{I}{J\Delta t} + \left[\frac{\partial R}{\partial Q}\right]_{VL}^T \right) \Delta\Lambda = -\left[\frac{\partial R}{\partial Q}\right]^T \Lambda^m - \left\{\frac{\partial F}{\partial Q}\right\}^T \tag{3}$$

$\Lambda^{m+1} = \Lambda^m + \Delta\Lambda$ (update vector Λ of $(m+1)^{\text{th}}$ step)

where I is identity matrix, and J represents Jacobian matrix, and the subscript VL means the Van-Leer flux Jacobian.

Overset boundary conditions can be evaluated by a similar way to the conventional adjoint boundary conditions as like Eq. (4)-(7). These conditions are derived from 4 discrete residual equations on inner cells and fringe cells in a main grid and sub-grid, respectively.

$$\left[\frac{\partial R^M}{\partial Q^M}\right]^T \Lambda^M + \left[\frac{\partial R_F^S}{\partial Q^M}\right]^T \Lambda_F^S + \left\{\frac{\partial F^M}{\partial Q^M}\right\}^T = \{0\}^T \tag{4}$$

$$\left[\frac{\partial R^S}{\partial Q^S}\right]^T \Lambda^S + \left[\frac{\partial R_F^M}{\partial Q^S}\right]^T \Lambda_F^M + \left\{\frac{\partial F^S}{\partial Q^S}\right\}^T = \{0\}^T \tag{5}$$

$$\left[\frac{\partial R^M}{\partial Q_F^M}\right]^T \Lambda^M + \left[\frac{\partial R_F^M}{\partial Q_F^M}\right]^T \Lambda_F^M + \left\{\frac{\partial F^M}{\partial Q_F^M}\right\}^T = \{0\}^T \tag{6}$$

$$\left[\frac{\partial R^S}{\partial Q_F^S}\right]^T \Lambda^S + \left[\frac{\partial R_F^S}{\partial Q_F^S}\right]^T \Lambda_F^S + \left\{\frac{\partial F^S}{\partial Q_F^S}\right\}^T = \{0\}^T \tag{7}$$

where the subscript F represent fringe cells and the superscript M and S represent the main grid and sub-grid domain respectively. Through these 4 system equations, each overset boundary values can be updated to inner adjoint variables of the next time step. Inner values of sub-grid domain are evaluated by Eq. (6), (5) orderly. And for the main-grid domain calculations are carried out from (7) to (4).

DESIGN OPTIMIZATION

The present overset design approach is applied to optimization of DLR-F4 W/B configurations. Optimization is performed using the Broydon-Fletcher-Goldfarb-Shanno (BFGS) variable metric method which is a kind of non-constrained optimization technique. As an aerodynamic shape optimization problem with the overset GBOM tool, drag minimization with a constant lift coefficient is performed in inviscid and viscous flow region. The total number of design variables is 200 at 10 different sections of the wing surface. The objective function is defined by Eq. (9) with the constraint of Eq. (8). To balance the variation of the objective and penalty functions, the weighting factor of the lift constraint is given by the ratio of the lift sensitivity to that of the drag with respect to the angle of attack.

$$\textit{Minimize: } C_D$$
$$\textit{Subject to } C_L \geq C_{L_0}$$
$$C_{L_0} = \textit{(Lift Coefficient of the Baseline Model)} \tag{8}$$
$$F \textit{ (Objective Function)} = C_D + Wt \times \min[0, C_L - C_{L_0}]$$
$$Wt = \frac{\partial C_D}{\partial \alpha} \bigg/ \frac{\partial C_L}{\partial \alpha} \tag{9}$$

As a test application of the present design tool, inviscid design of DLR-F4 is firstly performed. The drag coefficient decreases from 0.0227 to 0.0202 (12% reduction) after 10 design iterations. Taking into account the drag portion of the fuselage, this seems to be quite reasonable since the drag reduction for the wing only is about 17%. The L/D increases from 32.26 to 36.25 (12.3%). It can be observed in Figure 1 that the shock strength on the wing surface is remarkably diminished after the design.

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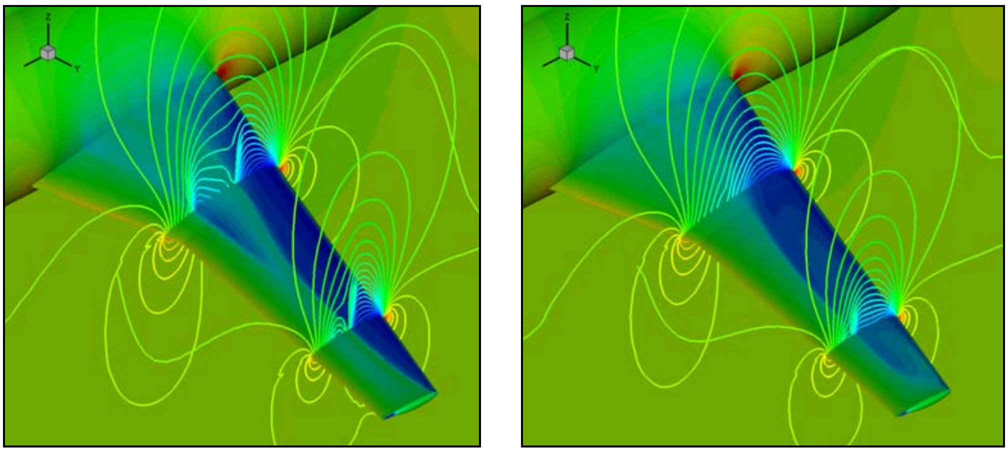

(a) Baseline model (b) Designed model

Figure 1 – Design Results with Adjoint Method on Overset Mesh System