LETTER

Complexity-Reduced Channel Estimation in Spatially Correlated MIMO-OFDM Systems*

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SUMMARY The performance of a MIMO-OFDM system significantly depends upon the accuracy of the channel impulse response (CIR) estimates. In the presence of correlation between the CIRs of the transmit or receive antennas, it is desirable to exploit this correlation to improve the performance of CIR estimation. In this letter, we propose a low-complexity channel estimation filter composed of four concatenated one-dimensional Wiener filters which are optimized to the channel characteristics in the time and frequency domains, and the transmit and receiver antenna front ends, respectively. Finally, the performance of the proposed scheme is verified.

key words: *channel estimation, MIMO-OFDM, spatially correlated channel*

1. Introduction

Recently, orthogonal frequency division multiplexing (OFDM) has attracted much attention for next generation wireless access systems. To increase the throughput of OFDM, it is crucial to accurately measure the channel impulse response (CIR). For a single-input single-output (SISO) system, a number of 2-dimensional (2-D) channel estimation (CE) schemes have been studied, where the channel correlation in the time and frequency domain is widely considered [1], [2].

To further increase the transmission rate without increasing signal bandwidth, the use of multiple antennas, called multi-input multi-output (MIMO) technique, has also recently received large attentions [3]. When the CIR has no correlation among the antennas in MIMO systems, it is not necessary to consider signals from other antennas for the improvement of CE. In this case, conventional CE schemes for single antenna systems can directly be applied to MIMO systems. On the other hand, in the presence of CIR correlation among the antennas, it may be desirable to take advantage of the correlation. Although a number of studies have been reported on the CE in single-carrier correlated MIMO systems [4], [5], only a few results have been reported on the OFDM systems [6], [7]. A Kalman filtering technique was employed to estimate the CIR in a MIMO-OFDM system using a state-space model for spatial correlation between the antennas [6]. However, it may not practically be applicable due to a large amount of information for the channel modeling. Although the use of a 3-D Wiener channel estimation filter (CEF), which is an easy extension of 2-D Wiener CEFs in single antenna systems [1], can provide the optimum performance in the minimum mean square error sense, it may not be implementable in practice because of a large number of filter taps. Recently, the use of maximum likelihood (ML) in the frequency domain and 2-D Wiener methods in the time/space domain has been considered [7]. However, it results in performance degradation compared to the 3-D Wiener CEF and only considers the correlation between the receive antennas.

In this letter, extending the idea in [1], [2] of employing separate CEFs in each domain to MIMO-OFDM systems, we propose a low-complexity Wiener filter structure where Wiener filtering is performed independently in each domain with near optimum performance. Following Introduction, the system and channel model are described in Sect. 2. Section 3 describes the proposed CE scheme. The performance of the proposed scheme is verified in Sect. 4. Finally, conclusions are summarized in Sect. 5.

2. System and Channel Model

Consider an OFDM transmitter with multiple antennas, where K subcarrier data symbols, $X_m[i,k]$, $k = 0, 1, 2, \cdots, K-1$, $m = 1, 2, \cdots, M$, are converted into a time domain signal at the m-th transmit antenna and the i-th symbol time using inverse fast Fourier transform (FFT). Then, a cyclic prefix (CP) is inserted. In the receiver, the CP is removed before the FFT process. Assuming ideal synchronization in the receiver, the received symbol of the k-th subcarrier at the n-th receive antenna, $n = 1, 2, \cdots, N$, can be represented by

$$Y_n[i,k] = \sum_{m=1}^{M} H_{n,m}[i,k] X_m[i,k] + Z_n[i,k] \qquad (1)$$

where $H_{n,m}[i,k]$ is the frequency response of the CIR at the k-th subcarrier and the i-th symbol time for the (m,n)-th antenna pair, $Z_n[i,k]$ is the background noise plus interference term of the n-th receive antenna, which can be approximated as a zero mean additive white Gaussian noise (AWGN) with variance σ_Z^2.

We consider the transmission over a wireless channel whose impulse response is represented as

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$$h_{n,m}(t,\tau) = \sum_{l=0}^{L_{n,m}-1} h_{l,n,m}(t)\delta(\tau - \tau_{l,n,m}) \quad (2)$$

where $L_{n,m}$ is the number of multipaths of the (m,n)-th antenna pair, $\delta(\cdot)$ is Kronecker delta function, and $\tau_{l,n,m}$ and $h_{l,n,m}(t)$ are the delay and complex-valued CIR at time t of the l-th path from the (m,n)-th antenna pair. Letting $H_{n,m}(t,f)$ be the frequency response of the time domain CIR $h_{n,m}(t,\tau)$, the frequency domain CIR with symbol time T_s and subcarrier spacing f_s can be represented in a discrete manner as

$$H_{n,m}[i,k] = H_{n,m}(iT_s, kf_s). \quad (3)$$

We consider a commonly used wide-sense stationary uncorrelated scattering (WSSUS) channel [3],

$$E\{H_{n,m}[i+\Delta i, k+\Delta k]H^*_{n,m}[i,k]\} = r_{t,f}(\Delta i, \Delta k) \quad (4)$$

where * denotes complex conjugate. Assuming each path has the same normalized time-correlation function $r_t(\Delta i)$ with different average power σ_l^2, the correlation can be decomposed as [1], [2]

$$r_{t,f}(\Delta i, \Delta k) = r_t(\Delta i) r_f(\Delta k) \quad (5)$$

where $r_f(\Delta k) = \sum_{l=0}^{L-1} \sigma_l^2 e^{-j2\pi\Delta k \tau_l}$. The channel correlation for a MIMO system can be represented as [6], [7]

$$\begin{aligned} &E\{H_{n_i,m_i}[i+\Delta i, k+\Delta k]H^*_{n_j,m_j}[i,k]\} \\ &\quad = r(\Delta i, \Delta k, n_i, n_j, m_i, m_j) \\ &\quad = r_t(\Delta i) r_f(\Delta k) r_s(n_i, n_j, m_i, m_j) \end{aligned} \quad (6)$$

where $n_i, n_j \in \{1, 2, \cdots, N\}$, $m_i, m_j \in \{1, 2, \cdots, M\}$ and $r_s(\cdot)$ denotes the spatial correlation function. Since the transmitter is far away from the receiver in general, the spatial correlation function can be decomposed into two correlations in the transmitter and receiver as [3], [7]

$$r_s(n_i, n_j, m_i, m_j) = r_{s_r}(n_i, n_j) r_{s_t}(m_i, m_j). \quad (7)$$

3. Proposed CEF

We consider pilot signals scattered in the time-frequency domain, as illustrated in Fig. 1 [1], where the pilot and data symbols are sent exclusively through each transmit antenna as

$$X_m[i,k] = \begin{cases} d_m[i,k], & \text{data} \\ p_m[i,k], & \text{pilot at } m\text{th transmit ant.} \\ 0, & \text{pilot at other transmit ant.} \end{cases} \quad (8)$$

where the pilot symbols for each transmit antenna are spaced by d_t and d_f symbols in the time and frequency domain, respectively, in a rectangular grid. The CIR from each transmit antenna can be estimated without any interference from other transmit antennas. When the m-th transmit antenna sends pilot symbol, we can estimate the instantaneous CIR of the (m,n)-th antenna pair by

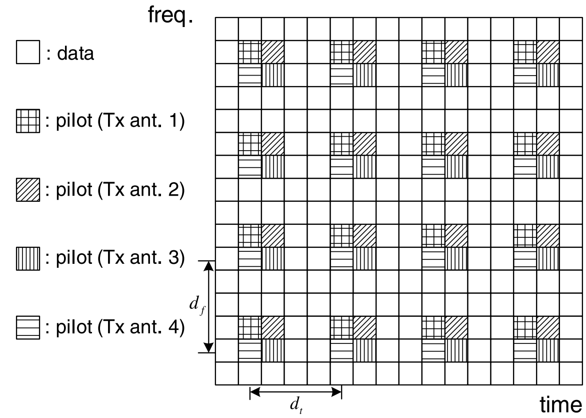

Fig. 1 Pilot pattern with the use of four transmit antennas.

$$\tilde{H}_{n,m}[i,k] = Y_n[i,k]/X_m[i,k]. \quad (9)$$

We can estimate the CIR at the data symbol (i_d, k_d) of the (m,n)-th pair by interpolating adjacent $N_t N_f MN$ instantaneous CIRs (N_t and N_f pilot symbols in the time and frequency domains in every antenna pair) using a filter with coefficient $\mathbf{w}^H[i_d, k_d, n, m]$

$$\hat{H}_{n,m}[i_d, k_d] = \mathbf{w}^H[i_d, k_d, n, m]\tilde{\mathbf{H}} \quad (10)$$

where

$$\begin{aligned} \tilde{\mathbf{H}} &= [\tilde{\mathbf{H}}_{1,1}\ \tilde{\mathbf{H}}_{2,1} \cdots \tilde{\mathbf{H}}_{N,1}\tilde{\mathbf{H}}_{1,2}\ \tilde{\mathbf{H}}_{2,2} \cdots \tilde{\mathbf{H}}_{N,2} \cdots \\ &\qquad \cdots \tilde{\mathbf{H}}_{1,M}\ \tilde{\mathbf{H}}_{2,M}\ \tilde{\mathbf{H}}_{N,M}]^T, \end{aligned} \quad (11)$$

$$\begin{aligned} \tilde{\mathbf{H}}_{n,m} &= [\tilde{H}_{n,m}[i_0, k_0]\ \tilde{H}_{n,m}[i_1, k_1] \cdots \\ &\qquad \cdots \tilde{H}_{n,m}[i_{N_t N_f - 1}, k_{N_t N_f - 1}]]. \end{aligned} \quad (12)$$

The mean square error (MSE) of the estimated CIR can be minimized by employing a 3-D Wiener CEF

$$\hat{H}_{n,m}[i_d, k_d] = \mathbf{w}_o^H[i_d, k_d, n, m]\tilde{\mathbf{H}} \quad (13)$$

where $\mathbf{w}_o[i_d, k_d, n, m]$ is the coefficient of the Wiener CEF determined by

$$\mathbf{w}_o[i_d, k_d, n, m] = \mathbf{R}^{-1}\mathbf{p}[i_d, k_d, n, m]. \quad (14)$$

Here, $\mathbf{R}\left(= E\left\{\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\right\}\right)$ is the $(N_t N_f MN \times N_t N_f MN)$ autocovariance matrix of the instantaneous CIR and $\mathbf{p}[i_d, k_d]$ $\left(= E\left\{\tilde{\mathbf{H}}H^*_{n,m}[i_d, k_d]\right\}\right)$ is the $(N_t N_f MN \times 1)$ cross-covariance vector between the desired CIR and the instantaneous CIR. Then, the MSE of the estimated CIR is represented as [3]

$$\sigma_e^2 = \frac{1}{MNN_P}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{i_d,k_d \in P}\sigma_e^2[i_d, k_d, n, m] \quad (15)$$

where P denotes a set of considered symbols, N_P is the number of symbols in P and

$$\begin{aligned} \sigma_e^2[i_d, k_d, n, m] &= \sigma_d^2 - \mathbf{w}_o^H[i_d, k_d, n, m]\mathbf{p}[i_d, k_d, n, m] \\ &\quad - \mathbf{p}^H[i_d, k_d, n, m]\mathbf{w}_o[i_d, k_d, n, m] \\ &\quad + \mathbf{w}_o^H[i_d, k_d, n, m]\mathbf{R}\mathbf{w}_o[i_d, k_d, n, m] \\ &= \sigma_d^2 - \mathbf{p}^H[i_d, k_d, n, m]\mathbf{w}_o[i_d, k_d, n, m]. \end{aligned} \quad (16)$$

Here, $\sigma_d^2 = E[|H_{n,m}[i,k]|^2]$.

Although the 3-D Wiener CEF in (13) can provide the optimum performance, it involves large complexity since it requires $N_t N_f MN$ multiplications and matrix inversion of a $(N_t N_f MN \times N_t N_f MN)$ matrix per symbol. Note that matrix inversion of a $N \times N$ matrix involves $O(N^3)$ complexity [8]. Although this costly complexity can be alleviated by approximation methods for matrix inversion [8], [9], they may be still impractical for real-time processing. As an alternative way to reduce the complexity without noticeable performance degradation, we consider the use of multiple low-dimensional Wiener filters by invoking the separation property of the channel correlation function.

The CIR of any antenna pair can be estimated by filtering in the frequency domain and then in the time domain using two 1-D Wiener CEFs with coefficient optimized to the correlation characteristics in the frequency and time domain, respectively. In this case, the filtering in the frequency domain is done once per d_t symbols in Fig. 1, requiring $(d_f + N_f/d_t)$ multiplications per symbol to obtain the CIR corresponding to the data symbol for each pair [2]. Finally, the CIR can be obtained by combining MN CIRs estimated from all antenna pairs. Similar to the filtering in the time and frequency domain, combining can be performed first among transmit antennas and then receive antennas, or vice versa. Consequently, we can replace a single 3-D Wiener CEF with four 1-D Wiener CEFs as

$$\hat{H}_{n,m}[i_d, k_d] = \mathbf{w}_p^H[i_d, k_d, n, m]\tilde{\mathbf{H}} \tag{17}$$

where

$$\mathbf{w}_p^H[i_d, k_d] = \mathbf{w}_{s,t}^H[m]\left(\mathbf{I}_M \otimes \mathbf{w}_{s,r}^H[n]\right)\left(\mathbf{I}_{MN} \otimes \mathbf{w}_t^H[i_d]\right) \times \left(\mathbf{I}_{N_t MN} \otimes \mathbf{w}_f^H[k_d]\right). \tag{18}$$

Here, $\otimes$ denotes the Kronecker product, $\mathbf{I}_A$ is a $(A \times A)$ identity matrix, and $\mathbf{w}_{s,t}$, $\mathbf{w}_{s,r}$, $\mathbf{w}_t$ and $\mathbf{w}_r$ are the coefficient vectors of 1-D Wiener CEF for the transmit antenna $(M \times 1)$, receive antenna $(N \times 1)$, time $(N_t \times 1)$ and frequency $(N_f \times 1)$ domain, respectively. Note that the final CIR calculation requires $(M+N)(d_f + N_f/d_t)$ multiplications and inversion of four small-size matrices. Resulting MSE is given by

$$\begin{aligned}\sigma_e^2[i_d, k_d, n, m] = \sigma_d^2 &- \mathbf{w}_p^H[i_d, k_d, n, m]\mathbf{p}[i_d, k_d, n, m] \\ &- \mathbf{p}^H[i_d, k_d, n, m]\mathbf{w}_p[i_d, k_d, n, m] \\ &+ \mathbf{w}_p^H[i_d, k_d, n, m]\mathbf{R}\mathbf{w}_p[i_d, k_d, n, m].\end{aligned} \tag{19}$$

4. Performance Evaluation

Figure 2 depicts the MSE of the estimate CIR as a function

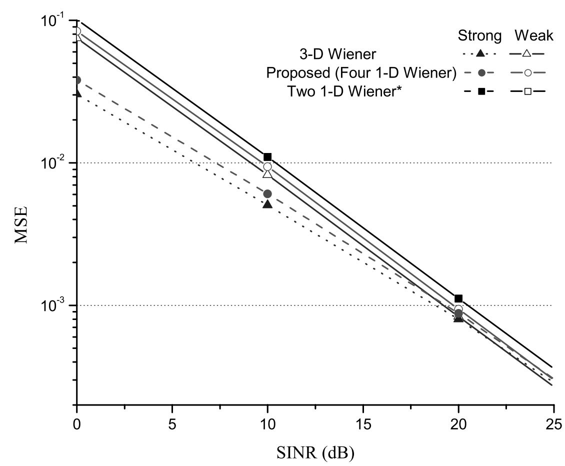

Fig. 2 MSE performance of the proposed CE scheme. The use of two 1-D Wiener CEFs has the same MSE irrespective of presence of spatial correlation.

Table 1 System parameters.

Parameters	Values
Carrier frequency/Bandwidth	5.8 GHz/100 MHz
Symbol duration/Guard interval	20.48 μsec/5 μsec
Number of subcarriers	2048
Pilot interval (d_t,d_f)	(4,4)
Fading	Rayleigh (Jakes spectrum, Maximum Doppler: 16Hz (3km/h))
Power-delay profile	Exponential with -3.5 dB exponent(rms delay: 70 nsec) delay spacing: 200 nsec, 10 paths
Correlation matrix (strong)	$R_T = \begin{bmatrix} 1.00 & -0.74+0.68i & 0.08-0.99i & 0.60+0.78i \\ -0.74-0.68i & 1.00 & -0.74+0.68i & 0.08-0.99i \\ 0.08+0.99i & -0.74-0.68i & 1.00 & -0.74+0.68i \\ 0.60-0.78i & 0.08+0.99i & -0.74-0.68i & 1.00 \end{bmatrix}$ $R_R = \begin{bmatrix} 1.00 & -0.70+0.35i & 0.49-0.37i & -0.38+0.35i \\ -0.70-0.35i & 1.00 & -0.70+0.35i & 0.49-0.37i \\ 0.50+0.37i & -0.70-0.35i & 1.00 & -0.70+0.35i \\ -0.38-0.35i & 0.49+0.37i & -0.70-0.35i & 1.00 \end{bmatrix}$
Correlation matrix (weak)	$R_T = \begin{bmatrix} 1.00 & -0.29-0.40i & -0.08+0.18i & 0.09-0.01i \\ -0.29+0.40i & 1.00 & -0.29-0.40i & -0.07+0.18i \\ -0.08-0.18i & -0.29+0.40i & 1.00 & -0.29-0.40i \\ 0.09+0.01i & -0.08-0.18i & 0.29+0.40i & 1.00 \end{bmatrix}$ $R_R = \begin{bmatrix} 1.00 & 0.09+0.43i & -0.07+0.01i & -0.08+0.02i \\ 0.09-0.43i & 1.00 & 0.09+0.43i & -0.07+0.01i \\ -0.07-0.01i & 0.09-0.43i & 1.00 & 0.09+0.43i \\ -0.08-0.02i & -0.07-0.01i & 0.09-0.43i & 1.00 \end{bmatrix}$

Table 2 Comparison of computational complexity. (N_t=N_f=3, M=N=4, d_t=d_f=4)

Type	Multi./sym./ant. pair (example)	Matrix inversion (example)
3-D Wiener	$N_t N_f MN$ (144)	$N_t N_f MN \times N_t N_f MN$ (144×144)
Two 1-D Wieners (No spatial filtering)	$N_t + N_f / d_t$(3.75)	$N_t \times N_t, N_f \times N_f$(3×3)
Proposed (Four 1-D Wieners)	$\left(N_t + N_f / d_t\right)(M + N)$(30)	$N_t \times N_t, N_f \times N_f, M \times M, N \times N$(3×3,4×4)

of the signal to interference and noise power ratio (SINR), where the simulation parameters are summarized in Table 1 [10]. We consider the use of four element uniform linear dipole array antennas both at the transmitter and receiver, and two kinds of spatial channel correlation; high and low correlation in the transmitter and receiver antennas [11]. The degree of spatial correlation can differ according to antenna spacing in the same fading environment, e.g., antenna spacing of 0.2λ and 0.4λ for high and low correlated channel where λ is the wavelength of the modulated signal [12]. Due to the effect of mutual coupling, highly correlated antenna generally lowers the received power compared to slightly correlated antenna if they are evaluated in the same fading environment [12], [13]. In this letter, however, the same average received signal power is assumed irrespective of spatial correlation in order to evaluate the effect of spatial correlation on channel estimation under the same SINR.

For comparison, Fig. 2 also depicts the MSE with the use of the optimum 3-D Wiener CEF, and two 1-D Wiener CEFs optimized for a SISO scheme in the time and frequency domain [1], [2]. It can be seen that the proposed scheme can provide near optimum CE performance, while the use of conventional two 1-D Wiener CEFs suffers from performance degradation especially when the channel has strong spatial correlation and the SINR is low. As another measure, Table 2 compares the required computational complexity. It can be seen that the proposed scheme can significantly reduce the implementation complexity over the 3-D Wiener CEF without noticeable performance degradation.

5. Conclusions

In this letter, we have proposed a cascaded four 1-D Wiener CEFs to reduce the implementation complexity of 3-D Wiener CEFs by exploiting the separation properties of the channel correlation in the time, frequency and spatial domain. The proposed scheme provides near optimum performance, while significantly reducing the implementation complexity. The same separation approach can be applied to other CEFs.

References

[1] P. Hoeher, S. Kaiser, and P. Robertson, "Two-dimensional pilot-symbol aided channel estimation by Wiener filtering," Proc. IEEE ICASSP, pp.1845–1848, April 1997.
[2] R. Nilsson, O. Edfors, M. Sandell, and P.O. Borjesson, "An analysis of two-dimensional pilot-symbol assisted modulation for OFDM," Proc. IEEE Conf. Personal Wireless Commun., pp.71–74, Dec. 1997.
[3] A. Paulraj, R. Nabar, and D. Gore, Introduction to space-time wireless communications, Cambridge University Press, 2003.
[4] M. Klessling, J. Speidel, and Y. Chen, "MIMO channel estimation in correlated fading environments," Proc. IEEE VTC Fall, pp.1187–1191, Oct. 2003.
[5] Y.-C. Chen and Y.T. Su, "MIMO channel estimation in spatially correlated environments," Proc. IEEE PIMRC, pp.498–502, Sept. 2004.
[6] M. Enescu, T. Roman, and V. Koivunen, "Channel estimation and tracking in spatially correlated MIMO OFDM systems," Proc. IEEE Workshop Statistical Signal Processing, pp.347–350, Sept. 2003.
[7] H. Miao and M.J. Juntti, "Space-time MMSE channel estimation for MIMO-OFDM system with spatial correlation," Proc. IEEE VTC Spring, pp.1806–1810, May 2004.
[8] Y.-H. Chan and X. Yu, "A reduced-rank MMSE-DFE receiver for space-time coded DS-CDMA systems," Proc. IEEE VTC Fall, pp.3659–3663, Sept. 2004.
[9] M.A. Hasan and M.R. Azimi-Sadjadi, "Fast algorithms for computing full and reduced rank Wiener filters," Proc. IEEE ISCAS, pp.IV-41–IV-44, May 2003.
[10] J. Moon, J.-Y. Ko, and Y.-H. Lee, "A framework design for the next-generation radio access system," IEEE J. Sel. Areas Commun., vol.24, no.3, pp.554–564, March 2006.
[11] 3GPP TR 25.996, Spatial channel model for Multiple Input Multiple Output (MIMO) simulations, Sept. 2003.
[12] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wirel. Pers. Commun., vol.6, no.3, pp.311–335, March 1998.
[13] M.K. Ozdemir, H. Arslan, and E. Arvas, "Mutual coupling effect in multi-antenna wireless communication systems," Proc. IEEE Globecom, pp.829–833, Dec. 2003.