LETTER

A Novel Prefilter-Type Beamformer Robust to Directional Error

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SUMMARY Some conventional beamformers require the direction of the desired signal. The performance of such beamformers can substantially be degraded even in the presence of small error on the directional information. In this letter, we propose a prefilter-type beamforming scheme robust to directional error by employing a simple compensator. The performance of the proposed scheme is verified by computer simulation.

key words: *array signal processing, interference cancellation, directional error, robust beamforming*

1. Introduction

Exploding demand for wireless communications has led to the development of advanced transceiver techniques to increase the transmission capacity without increasing the bandwidth. The use of beamforming techniques has been considered to mitigate multipath fading and co-channel interference [1]–[4], improving the signal to interference plus noise ratio (SINR) of the received signal. One of the most advanced techniques for increasing the capacity is spatial division multiple access (SDMA) technique [3], [4].

Beamformers can be designed by using data independent or statistical methods [5]. In a data independent beamformer, the weight vector is designed independently of the input signal [6]–[8]. In a statistically optimized beamformer, the weight vector is determined to optimize some criteria using the statistics of the data signal [4], [9]. Statistically optimized beamformers have a zero gain in the direction of interferences, while maximizing the signal to noise power ratio (SNR) at the beamformer output. They can be designed using a multiple sidelobe canceller (MSC) [9], using a reference signal [4], maximizing the SNR [4], [9], or minimizing the variance [10]–[12]. When the strength of the desired signal is unknown and/or the reference signal is not available, the MSC method may cancel out the desired signal [9]. Moreover, it may not be easy to estimate the signal and noise covariance matrix. These problems can be alleviated by imposing a linear constraint to the weight vector [10]. The weight vector of the beamformer can be optimized to minimize the variance of the output, while making the desired signal have a prescribed gain. The resulting beamformer can minimize the interference from direction other than the direction of interest, while preserving the desired signal.

Assuming that the direction of the desired signal is known at the receiver, a minimum variance (MV) type beamforming scheme was proposed by employing a spatial prefilter whose coefficient is determined by the direction of the desired signal [13]. However, the SINR performance of this beamformer is susceptible to directional error in an interference dominant environment. In this letter, we consider the improvement of this prefilter-based beamformer. The beamformer can be robust to the directional error by using an adaptive signal canceller [14], [15].

Following Introduction, Sect. 2 describes the structure of the prefilter beamformer. In Sect. 3, we propose a beamforming scheme robust to the directional error by using an adaptive signal canceller. The performance of the proposed beamformer is verified by computer simulation. Finally, conclusions are summarized in Sect. 4.

2. System Model and Prefilter Beamformer

We consider the use of an N-element uniformly spaced linear array (ULA) for the purpose of beamforming. Assume that there are L active users and that the number of interferences is not larger than the number of antennas. Assuming the transmission with a line-of-sight (LOS), the signal of each user can be modeled as a plane wave arriving to the ULA.

A baseband equivalent received signal through an N-element ULA can be represented as

$$\mathbf{x}[k] = \sum_{l=1}^{L} \beta_l[k] s_l[k] \mathbf{a}(\theta_l) + \mathbf{n}[k] \tag{1}$$

where $\beta_l(t)$ is the channel gain of the l-th user, $\mathbf{n}(t) = [n_1(t),\ n_2(t),\ \cdots, n_N(t)]$ denotes additive noise vector, $s_1[k]$ is the desired signal and other $s_l[k]$'s, $l = 2,\ 3, \cdots, L$, are interfering signals. $\mathbf{a}(\theta_l)$ is the steering vector given by

$$\mathbf{a}(\theta_l) = [1,\ e^{-j2\pi\frac{d}{\lambda}\sin(\theta_l)}, \cdots, e^{-j(N-1)2\pi\frac{d}{\lambda}\sin(\theta_l)}]^T \tag{2}$$

where d is the distance between the antennas, λ is the wavelength of the carrier frequency, θ_l is the direction of the l-th user and the superscript T denotes the transpose of a vector. We assume that the channel is unchanged during a packet interval.

A block diagram of the prefilter beamformer in [13] is depicted in the dotted rectangular box in Fig. 1. The $(m,\ n)$-th coefficient of the prefilter $\mathbf{B}$ is given by

$$b_{m,n} = \begin{cases} 1, & m = n \\ -\alpha_1, & m = n+1 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

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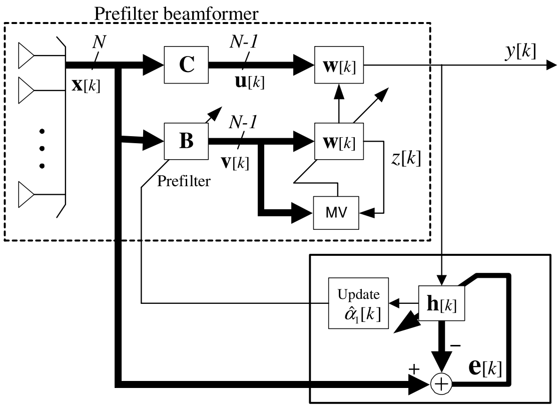

Fig. 1 Block diagram of the proposed beamformer.

where $\alpha_1 = e^{-j2\pi(d/\lambda)\sin\theta_1}$, $m = 1, 2, \ldots, N$ and $n = 1, 2, \ldots, N-1$. The output of the prefilter is

$$\mathbf{v}[k] = \mathbf{B}^H \mathbf{x}[k] \tag{4}$$

where the superscript H denotes complex conjugate and transpose of a matrix. Thus, the element of $\mathbf{v}[k]$ is obtained by

$$v_n[k] = x_n[k] - \alpha_1^* x_{n+1}[k], \quad n = 1, 2, \ldots, N-1 \tag{5}$$

where the superscript * denotes complex conjugate. The prefilter $\mathbf{B}$ behaves as a spatial filter since the desired signal is removed at its output.

Let $\mathbf{w}[k]$ be the weighting vector of the beamformer. Note that both $\mathbf{v}[k]$ and $\mathbf{w}[k]$ have the same dimension equal to $(N-1)$. The output gain in direction θ is

$$\begin{aligned} G_z[\theta, k] &= \mathbf{w}[k]^H \mathbf{B}^H \mathbf{a}(\theta) \\ &= \left(1 - \alpha_1^* e^{-j2\pi(d/\lambda)\sin\theta}\right) \\ &\quad \cdot \sum_{n=1}^{N-1} w_n^*[k] e^{-j(n-1)2\pi(d/\lambda)\sin\theta} \\ &= \left(1 - e^{-j2\pi(d/\lambda)(\sin\theta_1 - \sin\theta)}\right) \\ &\quad \cdot \sum_{n=1}^{N-1} w_n^*[k] e^{-j(n-1)2\pi(d/\lambda)\sin\theta} \end{aligned} \tag{6}$$

where $-\pi/2 \le \theta < +\pi/2$. Since any signal from direction θ_1 is filtered out by the prefilter, the output $z[k]$ does not contain any desired signal component. Therefore, the optimum least mean square (LMS) weight $\mathbf{w}[k]$ can be obtained by minimizing $|z[k]|^2$, i.e.,

$$\mathbf{w}[k+1] = \mathbf{w}[k] - \mu_1 z^*[k] \mathbf{B}^H \mathbf{x}[k] \tag{7}$$

where μ_1 is the step size for adaptation. Here, we assume $|y[k]| = 1$ to prevent $\mathbf{w}[k]$ from converging to zero.

Since the dimension of $\mathbf{w}[k]$ is less than that of $\mathbf{x}[k]$ by one, the input $\mathbf{x}[k]$ needs to be preprocessed by $\mathbf{C}$ whose (m, n)-th element is given by

$$c_{m,n} = \begin{cases} 1, & m = n \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

where $m = 1, 2, \ldots, N$, and $n = 1, 2, \ldots, N-1$. The LMS weight vector $\mathbf{w}[k]$ is applied to the original received signal vector $\mathbf{x}[k]$, yielding the output

$$y[k] = \mathbf{w}[k]^H \mathbf{C}^H \mathbf{x}[k]. \tag{9}$$

The spatial gain of the output $y[k]$ in direction θ is

$$\begin{aligned} G_y[\theta, k] &= \mathbf{w}[k]^H \mathbf{C}^H \mathbf{a}(\theta) \\ &= \sum_{n=1}^{N-1} w_n^*[k] e^{-j(n-1)2\pi(d/\lambda)\sin\theta}. \end{aligned} \tag{10}$$

3. The Proposed Beamformer Robust to Directional Error

When there exists a directional error $\Delta\theta$, the coefficient α_1 of the prefilter becomes

$$\alpha_1 = \exp\left[-j2\pi(d/\lambda)\sin(\theta_1 + \Delta\theta)\right]. \tag{11}$$

Then, the desired signal can partially be included in $z[k]$. Since the beamformer weight is determined to minimize the power of $z[k]$, the directional error $\Delta\theta$ results in degradation of the output SINR. This problem can be alleviated by correcting the coefficient α_1.

The directional error can be adjusted by estimating α_1. Define an N-dimensional error vector by

$$\mathbf{e}[k] = \mathbf{x}[k] - \mathbf{h}[k] y[k] \tag{12}$$

where $\mathbf{h}[k]$ represents the coefficient of the estimator. The coefficient $\mathbf{h}[k]$ can be obtained by minimizing the power of $\mathbf{e}[k]$, as shown in the solid box in Fig. 1,

$$\begin{aligned} &h_n[k+1] = h_n[k] - \frac{\mu_2}{2} \cdot \frac{\partial E\left\{|e_n[k]|^2\right\}}{\partial h_n}, \\ &n = 1, 2, \cdots, N \end{aligned} \tag{13}$$

where μ_2 is the step size for adaptation and $E\{x\}$ denotes the ensemble average of x. This estimator was proposed as a signal canceller in [14] and [15].

Assuming that the desired signal and interference are independent, it can be shown that the optimum MMSE coefficient $\hat{\mathbf{h}}$ is determined as

$$\begin{aligned} &\hat{h}_n = \frac{\sigma_{s_1}^2}{\sigma_y^2} \exp\left[-j(n-1)2\pi\left(\frac{d}{\lambda}\right)\sin\theta_1\right], \\ &n = 1, 2, \cdots, N \end{aligned} \tag{14}$$

where $\sigma_{s_1}^2$ is the power of $s_1[k]$ and σ_y^2 is the variance of $y[k]$. In practice, the coefficient can be obtained by

$$\mathbf{h}[k+1] = \mathbf{h}[k] + \mu_2 \mathbf{e}[k] y[k]^*. \tag{15}$$

When $\mathbf{h}[k]$ converges to $\hat{\mathbf{h}}$, it can be seen that

$$\begin{aligned} \lim_{h \to \hat{h}} \frac{h_n[k]}{h_{n-1}[k]} = \frac{\hat{h}_n}{\hat{h}_{n-1}} &= e^{-j2\pi(d/\lambda)\sin\theta_1} \\ &= \alpha_1, \end{aligned} \tag{16}$$

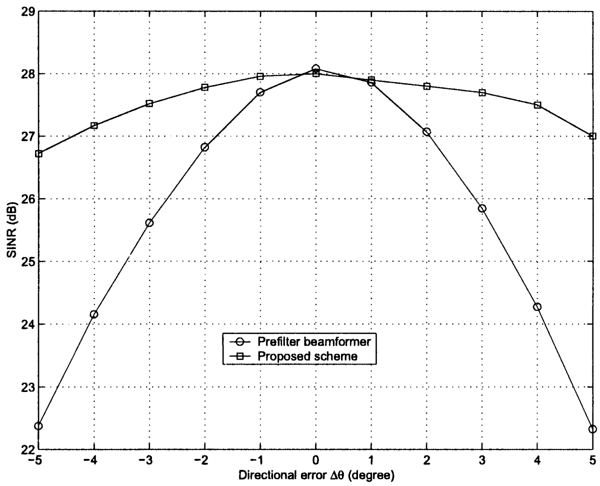

Fig. 2 SINR performance due to directional error.

for $n = 2, 3, \cdots, N$. Since there are $(N-1)$ ratio terms, the prefilter coefficient can be adjusted by averaging the ratio terms,

$$\hat{\alpha}_1[k] = \frac{1}{N-1}\sum_{n=1}^{N-1}\frac{h_{n+1}[k]}{h_n[k]}\cdot\frac{|h_n[k]|}{|h_{n+1}[k]|} \tag{17}$$

where the magnitude of $\hat{\alpha}_1[k]$ is normalized to the unity.

To evaluate the performance of the proposed beamformer, we consider the transmission of QPSK signal over 2.4 GHz fixed wireless channel. We assume that the channel is unchanged during a packet interval. Assume that the desired signal is received from direction 0° and four interferences are received from 60°, −40°, 30° and 45°. The desired signal and interferences are modulated with the same power at a symbol rate of 160 Kbaud at 20 dB SNR. Figure 2 depicts the SINR performance of the beamformer with the use of an 8-element ULA with one half wave-length spacing, i.e., $d = \lambda/2$. It shows that the prefilter beamformer has good SINR performance when there is no directional error. However, the SINR performance of the prefilter beamformer is degraded by small directional error. It can be seen that the proposed scheme can provide nearly the same SINR performance as the prefilter beamformer when there is no directional error and is quite robust to a wide range of directional error by simply adjusting the coefficient of the prefilter.

4. Conclusions

When the direction of the desired signal is exactly known, the use of an MV type prefilter-based beamformer is very efficient. However, the performance of the prefilter beamformer can be degraded substantially by a small directional error. We have proposed a prefilter-based beamformer robust to directional error. The coefficient of the prefilter is adjusted by estimating the directional error using a simple estimator. Simulation results show that the proposed beamformer can provide SINR performance quite robust to a wide range of directional error.

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