Separation of incident and reflected waves in wave-current flumes

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Abstract

A technique is developed to separate the incident and reflected waves propagating on a known current in a laboratory wave-current flume by analyzing wave records measured at two or more locations using a least squares method. It can be applied to both regular and irregular waves. To examine its performance, numerical tests are made for waves propagating on quiescent or flowing water. In some cases, to represent the signal noise and measurement error, white noise is superimposed on the numerically generated wave signal. For all the cases, good agreement is observed between target and estimation.

Keywords: Currents; Laboratory tests; Water waves; Wave-current flumes; Wave reflection

1. Introduction

Laboratory experiments provide a useful tool for various water wave problems along with numerical modeling and field observations. Especially used are laboratory experiments to investigate the phenomena that are too complex to be solved by mathematical or numerical means. In laboratory experiments involving wave reflection from sloping beaches or structures, one of the most fundamental and important task is to determine the characteristics of incident waves, because various experimental results are interpreted in terms of the incident wave parameters.

There are several ways to determine the incident waves in a laboratory wave flume. The easiest way is to divide the test section of the flume into two along the flume. The experimental beach or structure is installed in one of them, and the

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other is left empty, just having a wave absorber at the downwave side. The waves measured in the latter section are then considered as the incident waves. This method assumes that the re-reflection from the wave paddle of the waves reflected from the experimental beach or structure is small compared with the incident waves. To satisfy this assumption the width of the empty section should be several times larger than that of the experimental section so that a relatively wide flume is required.

Another way is to simultaneously measure the waves at several locations along the flume in front of the experimental beach or structure and separate the incident and reflected waves from the wave records (Goda and Suzuki, 1976; Gaillard et al., 1980; Mansard and Funke, 1980; Park et al., 1992). This method also assumes that the re-reflected wave energy from the wave paddle is small compared to the incident wave energy. Gravesen et al. (1974) and Kit et al. (1986) took into account the re-reflected waves from the wave paddle. Nowadays, however, most wave maker control system contains the so-called reflected wave absorbing filter that can suppress the re-reflection from the wave paddle. This is achieved by continuously sensing the reflected waves by a wave gauge attached at the front face of the paddle and correcting the input signal for the movement of the paddle. Therefore the methods of Goda and Suzuki (1976), Gaillard et al. (1980), Mansard and Funke (1980) or Park et al. (1992) can be applied with little errors. On the other hand, several investigators (Guza et al., 1984; Tatavarti et al., 1988; Kubota et al., 1990; Walton, 1992; Hughes, 1993) have used wave gauge and current meter co-located on the same vertical line rather than spatially-separated wave gauges. The co-located gauge method is useful in situations where there are spatial variations in the wavelength, such as on a mildly sloping bottom or in the region close to highly reflective structures. Recently Baldock and Simmonds (1999) presented a modification to the wave gauge array method to be used on an arbitrary bathymetry. They used linear shoaling to determine the amplitude and phase change between the measurement points.

The method of Goda and Suzuki (1976), which uses two wave gauges, does not give a solution when $\lambda / L = n / 2$ with $n = 0,1,2,\cdots$, where λ = distance between the wave gauges; and L = wavelength. Also, it is sensitive to secondary sloshing waves generated in the flume, nonlinear wave interactions, signal noise, measurement error, and so on. On the other hand, the method of Gaillard et al. (1980), Mansard and Funke (1980) or Park et al. (1992), which uses three wave gauges, is less sensitive to these phenomena and essentially there is no limitation in its application range of frequency (or wavelength).

The aforementioned methods have been applied to the situations in which only waves are present. A wave flume is often equipped with a current generating system to investigate wave-current interactions. In such a flume, sometimes we need to separate the incident and reflected waves propagating on an ambient current. In this study, we develop a technique to separate the incident and reflected waves propagating on a current by analyzing wave records measured at two or more locations using a least squares method. It can be applied to both regular and irregular waves. When the developed technique is used for the waves in quiescent water, it is the same as that of Park et al. (1992). In the next section, the separation technique is formulated. In the subsequent section, numerical experiments are described, which are carried out to examine the performance of the developed technique.

2. Theory

Consider a regular wave train that is formed by the superposition of the waves normally incident to an experimental beach or structure with the height H_i and the waves reflected from it with the height H_r. Also assume that these waves propagate on a steady, horizontally and vertically uniform current of the known velocity U; U is defined to be positive when the current flows with the incident waves, and negative when it opposes them. The incident waves are propagating in the positive x-direction with the surface elevation $\eta = 0$ being the still water level. The surface elevations of the incident and reflected waves can be expressed as

$$\eta_i(t) = \frac{H_i}{2}\cos(k_i x - \omega_a t + \phi_i) \tag{1}$$

$$\eta_r(t) = \frac{H_r}{2}\cos(k_r x + \omega_a t + \phi_r) \tag{2}$$

respectively, in which ϕ_i and ϕ_r = phase angles of the incident and reflected waves, respectively; and ω_a is the absolute angular frequency in the stationary frame of reference, which is related to the wave numbers of the incident and reflected waves, k_i and k_r, by the dispersion relationships:

$$\omega_a = k_i U + \sqrt{g k_i \tanh k_i h} \tag{3}$$

$$\omega_a = -k_r U + \sqrt{g k_r \tanh k_r h} \tag{4}$$

in which g = gravitational acceleration; and h = water depth. When there is no ambient current, the dispersion relationships reduce to a more familiar form: $\omega^2 = g k_0 \tanh k_0 h$, in which k_0 is the wave number in quiescent water. In a frame of reference moving along the wave orthogonal at velocity U, the angular frequency is ω_r, which is given by

$$\omega_r = \omega_a - k_i U \quad \text{or} \quad \omega_r = \omega_a + k_r U \tag{5}$$

Assume that wave measurements are made at N (≥ 2) locations in front of the structure, the coordinate of which from a reference point is denoted by x_n ($n = 1, \cdots, N$). The surface elevation at x_n is then given by

$$\eta_n(t) = \frac{H_i}{2} \cos(k_i x_n - \omega_a t + \phi_i) + \frac{H_r}{2} \cos(k_r x_n + \omega_a t + \phi_r) + e_n(t) \tag{6}$$

in which $e_n(t)$ = error due to signal noise, nonlinear wave interactions, etc. in the record of the wave gauge at x_n. The preceding equation can be written as

$$\begin{aligned} \eta_n(t) = {} & X_1 \cos(\omega_a t - k_i x_n) + X_2 \cos(\omega_a t + k_r x_n) \\ & + X_3 \sin(\omega_a t - k_i x_n) - X_4 \sin(\omega_a t + k_r x_n) + e_n(t) \end{aligned} \tag{7}$$

in which X_j's (j = 1 to 4) are unknowns which are expressed in terms of the height and phase of the incident and reflected waves as follows:

$$X_1 = \frac{H_i}{2} \cos\phi_i \tag{8}$$

$$X_2 = \frac{H_r}{2} \cos\phi_r \tag{9}$$

$$X_3 = \frac{H_i}{2}\sin\phi_i \qquad (10)$$

$$X_4 = \frac{H_r}{2}\sin\phi_r \qquad (11)$$

To find these unknowns, a least squares method is employed. Assume that the waves are simultaneously measured at the c_{ij} locations for duration of T_m. The total squared error involved in the measurement is defined as

$$\varepsilon^2 = \sum_{n=1}^{N}\int_0^{T_m}[e_n(t)]^2\,dt \qquad (12)$$

The heights and phases of the incident and reflected waves can be obtained by minimizing the total squared error with respect to X_j's, i.e.,

$$\frac{\partial\varepsilon^2}{\partial X_j} = 0 \quad , \quad j = 1 \quad \text{to} \quad 4$$

(13)

The minimization procedure results in a system of linear equations:

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ & c_{22} & c_{23} & c_{24} \\ & & c_{33} & c_{34} \\ & \text{symm.} & & c_{44} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \qquad (14)$$

Here c_{ij}'s are expressed in terms of the trigonometric functions on the right-hand side of (7), and F_j's in terms of these functions and the wave signal measured by each wave gauge. These coefficients are given in the appendix.

Once X_j's are calculated by (14), the phases of the incident and reflected waves are calculated by

$$\phi_i = \tan^{-1}\frac{X_3}{X_1} \tag{15}$$

$$\phi_r = \tan^{-1}\frac{X_4}{X_2} \tag{16}$$

and the wave heights are calculated by

$$H_i = \frac{2(X_1 + X_3)}{\cos\phi_i + \sin\phi_i} \tag{17}$$

$$H_r = \frac{2(X_2 + X_4)}{\cos\phi_r + \sin\phi_r} \tag{18}$$

The foregoing procedure developed for regular waves can be applied to irregular waves. The time series of the irregular surface profile measured by a wave gauge is Fourier transformed to obtain the amplitude and phase of each frequency component. For a particular frequency, the sinusoidal profile constructed using the corresponding amplitude and phase is considered to be the superposition of the regular incident and reflected waves propagating at that frequency. Note that two sinusoidal waves propagating collinearly with the same frequency but different amplitude and phase are superimposed to give another sinusoidal wave having different amplitude and phase but the same frequency as the individual waves. This procedure is repeated for each wave gauge. The thus constructed sinusoidal profiles for each of the wave gauges are then subjected to the foregoing separation analysis to give the heights and phases of the incident and reflected waves propagating at that frequency.

When waves travel from one current region to another there will be no change in the apparent frequency, ω_a. However, there will be changes in the relative wave frequency, ω_r, in the wavelength and in the wave height. When the developed technique is applied to the waves propagating on currents, the incident and reflected wave heights on currents are calculated. However, sometimes it is necessary to define the reflection coefficient as the ratio of the reflected wave height to the incident wave height in quiescent water. In order to convert the wave heights on currents to those in quiescent water, the principle of wave action conservation (Bretherton and Garrett, 1968) is employed:

$$\frac{\partial}{\partial x}\left[\frac{E(U + C_{gr})}{\omega_r}\right] = 0 \tag{19}$$

in which E $(= \rho g H^2/8)$ = wave energy density; C_{gr} = relative group velocity of the waves; and ρ = water density. For waves traveling from quiescent water onto a current, an equivalent form of (19) is

$$\frac{E_0 C_{g0}}{\omega_a} = \frac{E(U + C_{gr})}{\omega_r} \tag{20}$$

in which the subscript 0 refers to quantities in zero-current area; and C_{g0} = wave group velocity in this area. According to linear wave theory

$$C_{g0} = \frac{1}{2}\left(1 + \frac{2k_0 h}{\sinh 2k_0 h}\right)\frac{\omega_a}{k_0} \tag{21}$$

and

$$C_{gr} = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)\frac{\omega_r}{k} \tag{22}$$

Substituting these into (20) gives

$$H_0 = H\left\{\frac{2k_0\left[U + \frac{\omega_r}{2k}\left(1 + \frac{2kh}{\sinh 2kh}\right)\right]}{\omega_r\left(1 + \frac{2k_0 h}{\sinh 2k_0 h}\right)}\right\}^{\frac{1}{2}} \tag{23}$$

In addition to predicting the change in the wave height of regular waves, (20) may also be used to describe the effects of currents on irregular waves. In this case, the value of ω_a of each component of the irregular waves will remain constant as the waves cross from the quiescent area into the current region. Consequently, the spectral density of free-surface displacement in the quiescent area, $S_0(\omega_a)$, is related to the value in the current region, $S(\omega_a, U)$, by

$$S_0(\omega_a) = S(\omega_a, U) \frac{2k_0 \left[U + \frac{\omega_r}{2k} \left(1 + \frac{2kh}{\sinh\ 2kh} \right) \right]}{\omega_r \left(1 + \frac{2k_0 h}{\sinh\ 2k_0 h} \right)} \tag{24}$$

Note that U is defined to be positive when the current flows with the incident waves and negative when it opposes them. Therefore (22) to (24) can be used just with $k = k_i$ for the incident waves, but for the reflected waves they should be used with $k = k_r$ and the opposite sign of U.

The foregoing theory assumes a vertically uniform current, U, but in most cases the actual current generated in a laboratory flume is a depth-varying shear current. To the first approximation, the depth-averaged current velocity could be used in place of U. A more reasonable approach may be to use the so-called equivalent uniform current, U_e, proposed by Hedges and Lee (1992), which is defined as the uniform current that produces the same wavelength, L, as the actual depth-varying current for a given wave period and water depth. The equivalent uniform current, U_e, is given by

$$U_e = \frac{1}{h_e} \int_{h-h_e}^{h} u(z) dz \tag{25}$$

in which z = vertical coordinate measured upwards from the bed surface; u = depth-varying horizontal current velocity; and h_e is given by

$$h_e = \frac{\tanh\ kh}{k} \tag{26}$$

Note that h_e becomes smaller as the wave number becomes larger. This means that a short wave feels the current only in the upper part of the water column. As the wave becomes longer, h_e approximates to h so that U_e approaches the depth-averaged current velocity.

3. Numerical tests

In order to examine the performance of the developed technique, numerical tests are made for regular and irregular waves propagating on quiescent or flowing water. When two wave gauges are used, the developed technique gives large errors at the points where $\lambda / L = n / 2$ with $n = 0,1,2,\cdots$, as in the Goda and Suzuki's (1976) method. Therefore, in the following only the results using three wave gauges are presented. In some cases, to represent the signal noise and measurement error, white noise, whose frequency band extends from 0.1 to 2.1 Hz and whose significant wave height is 10% of the incident wave height, is superimposed on the numerically generated wave signal. All the tests are made in water of 0.5 m depth.

3.1. *Regular waves in quiescent water*

The wave parameters used in the test of regular waves are as follows: wave period T = 1.0 s, H_i = 0.2 m, and H_r = 0.1 m. Letting the spacing between the first and second wave gauges be λ_1, that between the second and third gauges was taken as $\lambda_2 = 0.6\lambda_1$.

Fig. 1 shows the error of the developed technique in terms of λ_1 / L. The error is defined as $[(H_{ie} - H_{it}) / H_{it}] \times 100\%$, in which H_{ie} and H_{it} = estimated and target incident wave height, respectively. When there is no noise, the result is almost error-free, and even when the noise is included, the error is less than two percent for all the tested values of λ_1 / L.

3.2. *Regular waves on currents*

The incident and reflected wave heights in quiescent water region are the same as before, i.e., H_{0i} = 0.2 m, and H_{0r} = 0.1 m. Two different wave periods were tested, i.e., T = 0.6 and 1.0 s. The vertical velocity profile in the current region was assumed as (Coleman, 1981)

$$u(z) = u_{max} + u_* Z \qquad (27)$$

with

$$Z = \frac{1}{\kappa}\ln\left(\frac{z}{h}\right) - \frac{2P}{\kappa}\left[1 - \sin^2\left(\frac{\pi z}{2h}\right)\right] \tag{28}$$

in which u_{max} = maximum velocity at water surface; u_* = shear velocity; κ = von Karman constant (= 0.4 for clear water); P = a constant related to the turbidity of fluid (= 0.19 for clear water). In this test, u_{max} = 0.2 m/s and u_* = 0.006 m/s were used, which give the equivalent uniform current velocity U_e = 0.194 and 0.196 m/s for following and opposing current, respectively. Tests were made for both following (with respect to the incident wave direction) and opposing currents. In order to examine only the effect of currents, white noise was not included.

Fig. 2 shows the error in terms of λ_1 / L_0 in which L_0 is the wavelength in quiescent water region. The error is defined as $[(H_{0_{ie}} - H_{0_{it}})/H_{0_{it}}]\times 100\%$, in which $H_{0_{ie}}$ and $H_{0_{it}}$ = estimated and target incident wave height, respectively, in quiescent water. For longer waves with T = 1.0 s, the result is almost error-free, and for shorter waves with T = 0.6 s, the error is less than two percent for all the tested values of λ_1 / L_0.

3.3. *Irregular waves in quiescent water*

The Bretschneider-Mitsuyasu spectrum (Goda, 1985) was used as the input target spectrum:

$$S(f) = 0.205 H_{1/3}^2 T_{1/3} (T_{1/3} f)^{-5} \exp[-0.75 (T_{1/3} f)^{-4}] \tag{29}$$

in which f = wave frequency; $H_{1/3}$ and $T_{1/3}$ = significant wave height and period, respectively. $H_{1/3}$ = 0.2 m and $T_{1/3}$ = 1.0 s were used and the gauge spacing was taken as λ_1 = 0.5 m and λ_2 = 0.3 m. Assuming that the incident waves are reflected from a plane slope of 1:5 ($\tan\beta$ = 0.2 in which β is the slope angle), Battjes' (1974) formula was used to provide the reflection coefficient for each frequency component of the numerically generated wave signal:

$$R_m = \begin{cases} 0.1\xi_m^2 & \text{if this is less than 1} \\ 1 & \text{otherwise} \end{cases} \quad (30)$$

with

$$\xi_m = \frac{\tan\beta}{\sqrt{H_m / L_m^0}} \quad (31)$$

in which the subscript m denotes the mth frequency component and the superscript 0 stands for deep water. The aforementioned white noise, whose significant wave height is 10% of the incident significant wave height, was superimposed on the numerically generated wave signal.

Fig. 3 shows a comparison between the target and the estimated spectra of the incident and reflected waves. The thick and thin lines indicate the incident and reflected wave spectra, respectively, while the solid and dashed lines target and estimated spectra, respectively. The comparison is satisfactory. Perfect reflection occurs in the low frequency region where the reflection coefficient is calculated to be unity by (30).

Fig. 4 shows a comparison between the target and the estimated temporal variations of surface elevation of the incident and reflected waves at the location of one of the gauges. Though the estimation shows more irregular pattern than the target, overall agreement is acceptable.

3.4. *Irregular waves on currents*

The waves in quiescent water region are again described by (29) to (31), and the vertical velocity profile in the current region is given by (27) and (28). All the relevant wave and current parameters are the same as before. The same white noise was also included.

In Fig. 5 is shown a comparison between the target and the estimated spectra of the incident and reflected waves for the case of following currents. The result is comparable to the no-current case (cf. Fig. 3) except for the appearance of sharp peaks at the high-frequency tail of the estimated spectra, which might be attributed to lower coherence between wave gauges (Mansard and Funke, 1980). Fig. 6

shows a comparison between the target and the estimated temporal variations of surface elevation of the incident and reflected waves at the location of one of the gauges. Again the estimation shows more irregular pattern than the target, but overall agreement is acceptable. Figs. 7 and 8 show the results for opposing currents, which are comparable to those for following currents.

Finally, in Table 1 are given the significant wave heights and periods calculated by the zero-crossing method for the surface profiles shown in Figs. 4, 6 and 8. Only a fraction of the surface profiles was shown in the figures, but in the zero-crossing analysis the whole profiles were used. The estimated significant wave periods are a little shorter than the target values because the estimated profile shows more irregular pattern than the target. The periods of the reflected waves are longer than the incident wave periods because the longer waves are perfectly reflected while partial reflection occurs for shorter waves.

4. Conclusions

We have developed a technique to separate the incident and reflected waves propagating on a known current by analyzing wave records measured at two or more locations using a least squares method. It can be applied to both regular and irregular waves. In order to examine the performance of the developed technique, numerical tests have been made for regular and irregular waves propagating on quiescent or flowing water. In some cases, to represent the signal noise and measurement error, white noise has been superimposed on the numerically generated wave signal. For regular waves, when there is no noise, the result is almost error-free, and even when the noise is included, the error is less than a few percent. For irregular waves propagating on a current, sharp peaks appear at the high-frequency tail of the spectra where the wave energy is negligibly small. Hydraulic experiments may be necessary to demonstrate the practical usage of the developed technique.

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Appendix. Coefficients c_{ij} and F_j

The coefficients c_{ij} 's in (13) are expressed in terms of the trigonometric functions on the right-hand side of (6), which can be integrated as follows:

$$c_{11} = \sum_{n=1}^{N} \int_0^{T_m} \cos^2(\omega_a t - k_i x_n) dt$$

$$= \sum_{n=1}^{N} \left[\frac{T_m}{2} + \frac{1}{4\omega_a} \left\{ \sin 2(\omega_a T_m - k_i x_n) + \sin 2k_i x_n \right\} \right] \quad (32)$$

$$c_{12} = \sum_{n=1}^{N} \int_0^{T_m} \cos(\omega_a t - k_i x_n) \cos(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \left[\frac{T_m}{2} \cos(k_i x_n + k_r x_n) + \frac{\sin 2\omega_a T_m}{4\omega_a} \cos(k_i x_n - k_r x_n) \right.$$

$$\left. + \frac{1 - \cos 2\omega_a T_m}{4\omega_a} \sin(k_i x_n - k_r x_n) \right] \quad (33)$$

$$c_{13} = \sum_{n=1}^{N} \int_0^{T_m} \cos(\omega_a t - k_i x_n) \sin(\omega_a t - k_i x_n) dt$$

$$= \sum_{n=1}^{N} \frac{1}{4\omega_a} \left[\cos 2(\omega_a T_m - k_i x_n) + \cos 2k_i x_n \right] \quad (34)$$

$$c_{14} = \sum_{n=1}^{N} \int_0^{T_m} -\cos(\omega_a t - k_i x_n) \sin(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \left[-\frac{T_m}{2} \sin(k_i x_n + k_r x_n) - \frac{\sin 2\omega_a T_m}{4\omega_a} \sin(k_i x_n - k_r x_n) \right.$$

$$\left. - \frac{1 - \cos 2\omega_a T_m}{4\omega_a} \cos(k_i x_n - k_r x_n) \right] \quad (35)$$

$$c_{22} = \sum_{n=1}^{N} \int_0^{T_m} \cos^2(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \left[\frac{T_m}{2} + \frac{1}{4\omega_a} \left\{ \sin 2(\omega_a T_m + k_r x_n) - \sin 2 k_r x_n \right\} \right] \tag{36}$$

$$c_{23} = \sum_{n=1}^{N} \int_0^{T_m} \cos(\omega_a t + k_r x_n) \sin(\omega_a t - k_i x_n) dt$$

$$= \sum_{n=1}^{N} \left[-\frac{T_m}{2} \sin(k_i x_n + k_r x_n) - \frac{\sin 2\omega_a T_m}{4\omega_a} \sin(k_i x_n - k_r x_n) \right.$$

$$\left. + \frac{1 - \cos 2\omega_a T_m}{4\omega_a} \cos(k_i x_n - k_r x_n) \right] \tag{37}$$

$$c_{24} = \sum_{n=1}^{N} \int_0^{T_m} - \cos(\omega_a t + k_r x_n) \sin(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \frac{1}{4\omega_a} \left[\cos 2(\omega_a T_m + k_r x_n) - \cos 2 k_r x_n \right] \tag{38}$$

$$c_{33} = \sum_{n=1}^{N} \int_0^{T_m} \sin^2(\omega_a t - k_i x_n) dt$$

$$= \sum_{n=1}^{N} \left[\frac{T_m}{2} - \frac{1}{4\omega_a} \left\{ \sin 2(\omega_a T_m - k_i x_n) + \sin 2 k_i x_n \right\} \right] \tag{39}$$

$$c_{34} = \sum_{n=1}^{N} \int_0^{T_m} - \sin(\omega_a t - k_i x_n) \sin(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \left[-\frac{T_m}{2} \cos(k_i x_n + k_r x_n) + \frac{\sin 2\omega_a T_m}{4\omega_a} \cos(k_i x_n - k_r x_n) \right.$$

$$\left. + \frac{1 - \cos 2\omega_a T_m}{4\omega_a} \sin(k_i x_n - k_r x_n) \right] \tag{40}$$

$$c_{44} = \sum_{n=1}^{N} \int_0^{T_m} \sin^2(\omega_a t + k_r x_n) dt$$

$$= \sum_{n=1}^{N} \left[\frac{T_m}{2} - \frac{1}{4\omega_a} \left\{ \sin 2(\omega_a T_m + k_r x_n) - \sin 2 k_r x_n \right\} \right] \tag{41}$$

On the other hand, the coefficients F_j's are defined as follows, which can be calculated numerically:

$$F_1 = \sum_{n=1}^{N} \int_0^{T_m} \cos(\omega_a t - k_i x_n)\eta_n(t)dt \tag{42}$$

$$F_2 = \sum_{n=1}^{N} \int_0^{T_m} \cos(\omega_a t + k_r x_n)\eta_n(t)dt \tag{43}$$

$$F_3 = \sum_{n=1}^{N} \int_0^{T_m} \sin(\omega_a t - k_i x_n)\eta_n(t)dt \tag{44}$$

$$F_4 = \sum_{n=1}^{N} \int_0^{T_m} -\sin(\omega_a t + k_r x_n)\eta_n(t)dt \tag{45}$$

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Table 1
Comparison of significant wave height and period between target and estimation

Fig. No.	Target		Estimation	
	$H_{1/3}$ (m)	$T_{1/3}$ (s)	$H_{1/3}$ (m)	$T_{1/3}$ (s)
4(a)	0.217	1.034	0.194	1.005
4(b)	0.101	1.249	0.105	1.194
6(a)	0.213	1.009	0.219	0.978
6(b)	0.095	1.204	0.092	1.180
8(a)	0.217	0.978	0.216	0.952
8(b)	0.112	1.190	0.090	1.108

Captions of figures

1. Errors between target and estimated incident wave heights in terms of λ_1 / L.
2. Errors between target and estimated incident wave heights in terms of λ_1 / L_0: (a) T = 0.6 s; (b) T = 1.0 s.
3. Comparison between target and estimated spectra of incident and reflected waves in quiescent water.
4. Comparison between target and estimated surface profiles of incident and reflected waves in quiescent water.
5. Comparison between target and estimated spectra of incident and reflected waves on following currents.
6. Comparison between target and estimated surface profiles of incident and reflected waves on following currents.
7. Comparison between target and estimated spectra of incident and reflected waves on opposing currents.
8. Comparison between target and estimated surface profiles of incident and reflected waves on opposing currents.