Group Bargaining with Representation

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We study a strategic bargaining model where two groups of individuals first choose their representatives, who then bargain with each other using a standard alternating-offer protocol, and then the shares of the members of a group are determined by a similar *n*-person bargaining process within the group. We show that there exists a unique perfect equilibrium outcome of this three-stage game when the breakdown probabilities of both the inter-group bargaining and intra-group bargaining are small. In equilibrium, each group selects as its representative an individual who has the greatest marginal gain from increasing the group's share.

Keywords: Group bargaining, Nash bargaining solution, Representation, Delegation

JEL Classification: C71, C72, C78

I. Introduction

When groups of individuals bargain with each other, actual bargaining is typically carried out by their representatives. The literature on delegated bargaining studies how the relationship between the members of a group and its representative affects the outcome of the bargaining. In some models, the agreement reached by a representative should be approved by the members of the group. Perry and Samuelson (1994), Haller and Holden (1997), and Manzini and Mariotti (2005) investigate the effect of alternative approval processes on the outcome of bargaining. In some other models, representatives are elected. Segendorff (1998) and Cai (2000) study the effect of this election

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[**Seoul Journal of Economics** 2009, Vol. 22, No. 2]

on the bargaining outcome.

In this paper, we consider a strategic model of bargaining with three-stages: At stage 1, two groups choose their representatives. At stage 2, to be called the inter-group bargaining, the representatives of the two groups, chosen at stage 1, bargain over the split of a pie between the two groups. At stage 3, to be called the intra-group bargaining, the members of each group bargain over the division of the group's share, which was determined at stage 2, among the members.

Examples of bargaining situations where both inter-group and intra-group bargaining are present abound. To take a few examples, labor unions bargain over wages with management which represents shareholders; bankers and bondholders bargain over the assets of bankrupt companies; two neighborhoods bargain over public projects that affect them both.

For both the inter-group bargaining and intra-group bargaining, we use a Rubinstein-type (1982) alternating-offer protocol, where a rejection leads to a breakdown of bargaining with a positive probability. The main contribution of the paper is to show that despite complex feed-backs between the inter-group and intra-group bargaining, there exists a unique perfect equilibrium outcome of the game when the breakdown probabilities are small. Also, for a series of examples, we characterize the solutions in the limit as the breakdown probabilities become negligible.

We also demonstrate that an individual with the greatest marginal gain will be the best candidate to become a representative. The reason is that a group's share is larger if the marginal gain of the represen-tative from the group's share is larger. Such an individual will be actually chosen by the group in equilibrium if representatives are selected at the start of the inter-group bargaining, for there is no internal conflict among the members of a group regarding this choice.

The result on representation can be interpreted in two ways. First, one may say that an individual who has the most at stake should become the representative. Second, one may say that the player who is the toughest bargainer should become the representative. In the current setting, the two criteria actually coincide. The reason is that the toughest individual gets the most in the intra-group bargaining as well as in the inter-group bargaining. In general, the "toughest bargainer" cannot be defined in absolute terms, but only relative to his opponent in the two-person bargaining. But there are cases where an individual is uniformly "tougher" than the other members of her group at any level of the share and thus will be chosen as the group's representative, regardless of who represents the other group.

Note that once the inter-group bargaining determines the group shares, the intra-group bargaining that follows is a standard n-person bargaining problem. There are many different models extending Rubinstein's two-person alternating-offer model to the n-person case. It is well known that some of these n-person models lead to multiple equilibria.[1] Thus, in order to have a unique perfect equilibrium for the n-person case, we use the protocol used in Chae and Yang (1994).

The current paper adopts a strategic approach in investigating group bargaining. In an axiomatic approach, Chae and Heidhues (2004) characterize a group bargaining solution using axioms that include the four standard Nash (1950) axioms (efficiency, independence, invariance with respect to affine transformation, symmetry). They add a new axiom that essentially treats a group as one bargainer.[2] In their solution, the group's preferences reflect the average preferences of the members of the group, due to, among other things, symmetry. In contrast, in the strategic model of the current paper, the group's implied preferences are determined by those of the best bargainer in the group.

II. Model and Results

We will first describe the stage-one game: Two non-overlapping groups of finite individuals bargain over the split of a pie. Group 1 and group 2 will be denoted G_1 and G_2. First, each group simultaneously selects one of its members as a representative. Here one may assume for simplicity that an arbitrary member of each group has the privilege of selecting the representative for the group. It turns out that in equilibrium the same representative will be chosen no matter who selects the representative, for it is in the interest of any member of the group to choose a representative that will maximize the group's pie.

Next, in the stage-two game, the two representatives bargain with each other over the split of the pie (X_1, X_2), where $X_1+X_2=\pi$, in a standard Rubinstein-type (1982) two-person alternating offer procedure

[1] See, for instance, Herrero (1985), Haller (1986), and Shaked's example in Sutton (1986).

[2] Chae and Moulin (forthcoming) generalizes the solution to a family of solutions with alternative axioms.

with a fixed probability of breakdown after each rejection. If the bargaining breaks down, the game ends and each individual receives its breakdown payoff d_i. We assume that $\sum_{i\in G_1\cup G_2} d_i < \pi$. The breakdown probability is denoted $1-p$.

Finally, we describe the stage-three game: Once the two representatives reach an agreement (X_1, X_2), each group $G_g(g=1, 2)$ immediately bargains over the split of X_g among its n_g members in a Chae-Yang-type (1994) n-person alternating-offer procedure with a fixed probability of breakdown after each rejection. The breakdown probability is denoted $1-q$. If the bargaining breaks down, the game ends and each individual receives its breakdown payoff d_i as before.

The intra-group bargaining game can be described as follows: First, one individual is chosen to be the initial proposer. Assume, for simplicity, that the representative of the group in the inter-group bargaining is the initial proposer in the intra-group bargaining.[3] She selects one responder and proposes that they sign a contingent contract stipulating that she pay him a certain share of the pie at the end of the bargaining process. If he accepts her proposal, he gives up his right to talk and waits on the sidelines until the end of the game, and she continues to be a proposer in the remaining game with $n-1$ active individuals who have the common knowledge of the contract. If he rejects her proposal, he becomes the initial proposer in a similar n-person game. The rules of the game in a subgame with $n-m$ active individuals who have common knowledge about m contracts are similar. The game ends in agreement if all individuals except one have given up their rights to talk. At this point, all contracts are executed and the individual who has not given up her right to talk keeps the residual share. (If any individual defaults on his or her debt, no individual receives any payment.)

Denote the von-Neumann Morgenstern (vN-M) utility function of an individual by u_i. We assume that u_i is smooth (that is, differentiable as many times as one wants), $u_i'>0$, and satisfies

***Assumption* 1.**

(strict log-concavity) $(d^2/dx_i^2)\ \log(u_i(x_i)-u_i(d_i))<0$ for any $x_i>d_i\geq 0$.

The assumption implies that the log of utility gain is strictly concave.

[3] It can be shown that the results of the paper do not depend on this simplifying assumption.

It is satisfied if $u_i'' \leq 0$. That is, it is satisfied by all risk-averse or risk-neutral preferences. But it is also satisfied by some risk-loving preferences. For instance, it is satisfied by all preferences that can be represented by vN-M utility functions with constant relative risk aversion, $u_i(x) = x^{1-r}$, where $r < 0$.[4]

Given a payoff x_i, a breakdown payoff d_i, and breakdown probability $1-p$, define an individual's certainty equivalent $c_i(p, x_i)$ as the payoff y such that $pu_i(x_i) + (1-p)u_i(d_i) = u_i(y)$.[5] The amount an individual is willing to pay in order to avoid an infinitesimal chance of breakdown will be called the *marginal risk concession (MRC)*. It is formally defined and denoted as

$$\mu_i(x_i) \equiv \lim_{p \to 1} \frac{x_i - c_i(p, x_i)}{1-p} = \frac{\partial}{\partial p} c_i(1, x_i) = \frac{u_i(x_i) - u_i(d_i)}{u_i'(x_i)}.$$

Notice that

$$\frac{d}{dx_i} \log(u_i(x_i) - u_i(d_i)) = \frac{u_i'(x_i)}{u_i(x_i) - u_i(d_i)} = \frac{1}{\mu_i(x_i)}$$

and thus Assumption 1 can be rewritten as

$$\mu_i'(x_i) > 0 \text{ for any } x_i > d_i, \tag{1}$$

for the derivative of $(d/dx_i)\log(u_i(x_i) - u_i(d_i))$ is negative if and only if the derivative of $\mu_i(x_i)$ is positive.

We will solve the bargaining game backward starting from stage 3, that is, from the intra-group bargaining between the members of group $g(=1, 2)$ over the division of given X_g. We will assume that $X_g > \sum_{i \in G_g} d_i$, for this will be the case in equilibrium. In the Appendix, we will prove[6]

[4] See Chae and Heidhues (1999).

[5] Throughout the paper the breakdown payoffs will be fixed. Thus we will not explicitly recognize them in our notation unless necessary. For instance, $c_i(p, x_i)$ would have been written as $c_i(p, x_i, d_i)$ if d_i were a variable in the course of our investigation. Similarly, $\mu_i(x_i)$ stands for $\mu_i(x_i, d_i)$.

[6] We prove the proposition by modifying the proof of a similar result in Chae and Yang (1994), where they study an n-person bargaining model in which the rejection of an offer leads to a time delay rather than the risk of a breakdown.

***Proposition* 1.**

Consider a subgame where group $G_g(g=1, 2)$ bargains over the split of $X_g(>\sum_{i\in G_g} d_i)$ among its n_g members after the representatives of the two groups have reached an agreement (X_1, X_2).

(i) There exists a unique subgame perfect equilibrium outcome of the subgame.
(ii) An individual's payoff in the equilibrium outcome increases as the pie, X_g, available for the group increases.
(iii) As the breakdown probability, $1-q$, goes to zero, the equilibrium outcome approaches the Nash bargaining solution.

The Nash bargaining solution, to which the unique perfect equilibrium outcome of the subgame of the above proposition converges, solves the maximization problem

$$\text{Maximize} \prod_{i\in G_g} \{u_i(x_i)-u_i(d_i)\}$$

$$\text{Subject to} \left(\begin{array}{l} \sum\limits_{i\in G_g} x_i \leq X_g \\ x_i \geq d_i \end{array}\right.$$

Thus it is a solution to the efficiency condition

$$\sum_{i\in G_g} x_i = X_g$$

and

$$\mu_i(x_i)=\mu_h(x_h) \text{ for any } i,\ h\in G_g, \tag{2}$$

which is the condition for balancing bargaining power.

Denote the share of individual i at the Nash bargaining solution as $x_i=\phi_i(X_g)$ and set

$$U_i(X_g)\equiv u_i(\phi_i(X_g)).$$

We will assume that

***Assumption* 2.**

$U_i(X_g)$ also satisfies the strict log-concavity condition in Assumption 1, *i.e.*, $(d^2/dX_g^2)\ \log(u_i(\phi_i(X_g)) - u_i(d_i)) < 0$ for any $X_g > \sum_{i\in G_g} d_i \geq 0$.

The assumption that $U_i(X_g)$ satisfies strict log-concavity is equivalent to

$$\frac{d}{dX_g}\{\frac{\mu_i(\phi_i(X_g))}{\phi_i'(X_g)}\} > 0 \tag{3}$$

A sufficient condition for (3) is

$$\phi_i''(X_g) \leq 0 \text{ for any } X_g.$$

For instance, if an individual's preferences exhibit constant absolute risk aversion, $\phi_i''(X_g) = 0$ for any $X_g > \sum_{i\in G_g} d_i$ and thus (3) is satisfied.

By (2), one has

$$\mu_i(\phi_i(X_g)) = \mu_h(\phi_h(X_g)) \text{ for any } i,\ h \in G_g. \tag{4}$$

Thus one can define, for $g = 1, 2$,

$$\mu_g(X_g) \equiv \mu_i(\phi_i(X_g)) \text{ for some (and all) } i \in G_g,$$

and call it the MRC for group g.

Using the intra-group result of Proposition 1, we can now analyze the inter-group bargaining game. We will prove in the Appendix

***Theorem* 1.**

Consider a subgame where individuals j and k have been chosen as the representatives of groups G_1 and G_2, respectively.

(i) There exist $\hat{p},\ \hat{q} \in (0, 1)$ such that for any $p > \hat{p}$, $q > \hat{q}$ there exists a unique subgame perfect equilibrium outcome of the game.

(ii) In the limit where the breakdown probabilities ($1-p$ and $1-q$) approach zero, the equilibrium outcome (X_1, X_2) of the inter-group bargaining game satisfies the efficiency condition $X_1 + X_2 = \pi$ and

$$\frac{\mu_1(X_1)}{\phi_j'(X_1)} = \frac{\mu_2(X_2)}{\phi_k'(X_2)}. \tag{5}$$

The theorem constitutes a new and significant result in that it shows that there exists a unique subgame perfect equilibrium outcome despite complex feedbacks between the intra-group bargaining and inter-group bargaining. But, unlike the standard results in the alternating-offer models of one-stage bargaining, the result holds only when the breakdown probabilities are small. Intuitively, as the breakdown probabilities become small, the model behaves like the limiting model, which yields the outcome described in (ii) of the above theorem.

Now consider the representation game where the two groups choose their representatives. By (ii) of Proposition 1, there is no intra-group conflict in choosing a group's representative. Every member of the group benefits from "the best bargainer" representing the group. We will prove in the Appendix

Theorem 2.

Consider the entire three-stage game.

(i) There exists a unique subgame perfect equilibrium outcome of the game if the breakdown probabilities $(1-p,\ 1-q)$ of both the intra-group bargaining and inter-group bargaining are sufficiently close to zero.

(ii) In equilibrium, the representative of a group is a member that has the greatest marginal gain from increasing the group's share.

In order to get an intuition for the above theorem, consider the representation game in the limit. Because (5) holds in equilibrium, it will be in the interest of a group to choose as its representative an individual with the greatest marginal gain, $\phi_i'(X_g)$, from increasing the group's share. "The best bargainer" for a group is one who has the most at stake.

For a formal description of the solution in the limit, define, for $g=1,\ 2$, $\varphi_g(Y_g) \equiv \mathrm{Max}_{i \in g} \phi_i'(Y_g)$. Then the function φ_g is continuous and $\mu_g(Y_g)/\varphi_g(Y_g)$ is increasing in $Y_g(\geq \Sigma_{i \in G_g} d_i)$. Thus there exists a unique solution X_1 to the equation

$$\frac{\mu_1(Y_1)}{\varphi_1(Y_1)} = \frac{\mu_2(\pi - Y_1)}{\varphi_2(\pi - Y_1)}. \tag{6}$$

Once X_1 and $X_2(\equiv \pi - X_1)$ are determined, the shares of individuals are determined as

$$x_i = \phi_i(X_g) \text{ for } i \in G_g.$$

This constitutes the unique equilibrium outcome of the game in the limit. The representatives chosen in equilibrium can be identified as individuals j and k such that $\varphi_1(X_1) = \phi_j'(X_1)$, $\varphi_2(\pi - X_1) = \phi_k'(\pi - X_1)$, *i.e.*,

$$\phi_j'(X_1) \geq \phi_h'(X_1) \text{ for any } h \in G_1,$$

$$\phi_k'(\pi - X_1) \geq \phi_l'(\pi - X_1) \text{ for any } l \in G_2.$$

Note that in general the best bargainer for a group depends on who represents the other group. In the special case where an individual is uniformly "tougher" than the other individuals in her group at any level of the share, the toughest player will be chosen regardless of who represents the other group.

***Example* 1.**

Suppose that an individual's preferences can be represented by vN-M utility functions with constant relative risk aversion, *i.e.*, $u_i(x) = x^{\alpha_i}$ where $0 < \alpha_i$ and that $d_i = 0$. (Notice here that if $\alpha > 1$ then individual i is actually risk loving.) Then

$$\mu_i(x_i) = \frac{x_i}{\alpha_i},$$

$$\phi_i(X_g) = \frac{\alpha_i X_g}{\sum_{h \in G_g} \alpha_h},$$

$$\phi_i'(X_g) = \frac{\alpha_i}{\sum_{h \in G_g} \alpha_h},$$

$$\mu_g(X_g) = \frac{X_g}{\sum_{h \in G_g} \alpha_h},$$

$$\varphi_g(X_g) = \frac{\alpha_g}{\sum_{h \in G_g} \alpha_h},$$

where $\alpha_g = \text{Max}_{i \in G_g} \alpha_i$ for $g = 1, 2$. Thus (6) becomes

$$\frac{X_1}{\alpha_1} = \frac{\pi - X_1}{\alpha_2}.$$

Thus

$$X_1 = \frac{\alpha_1}{\alpha_1 + \alpha_2} \pi,$$

$$X_2 = \frac{\alpha_2}{\alpha_1 + \alpha_2} \pi.$$

$$x_i = \frac{\alpha_i}{\sum_{h \in G_g} \alpha_h} \cdot \frac{\alpha_g}{\alpha_1 + \alpha_2} \pi \quad \text{if } i \text{ belongs to group } g.$$

***Example* 2.**

Suppose there are $n+1$ individuals with the same preferences and the same breakdown point, d. They are partitioned into two groups, one with n members and the other with a single member, individual $n+1$. In the multi-member group, denoted simply G, one arbitrary member, individual j, becomes the group's representative. Define $\mu_i^d(z_i) \equiv \mu_i(z_i + d, d)$ for $z_i \geq 0$ and let $Z_G = z_1 + \cdots + z_n$. Then (6) becomes

$$n\mu_j^d\left(\frac{Z_G}{n}\right) = \mu_{n+1}^d(\pi - (n+1)d - Z_G),$$

Looking at the left hand side of this equation, note that a homogeneous group being represented by one member does not get the same deal as what the member would get if she were the only member. In other words, X_G can be greater or smaller than $\pi/2$ (or equivalently, Z_G can be greater or smaller than $\{\pi - (n+1)d\}/2$). Whether a multi-member homogeneous group does better or worse than a single-member group depends on whether $\mu_j^d(z_i/n) < \mu_i^d(z_i)/n$ or $\mu_i^d(z_i/n) > \mu_i^d(z_i)/n$. Thus, if the function $\mu_i^d(z_i)$ is convex in z_i, the group does better, while if the function is concave in z_i, the individual does better.[7] The borderline case is the case of vN-M utilities with constant relative risk aversion and zero breakdown point studied in Example 1.

***Example* 3.**

Suppose there are two homogeneous groups, each consisting of n individuals. Group 1 consists of individuals of the "tough" type, denoted t, and group 2 consists of individuals of the "soft" type, denoted s. Then (6) becomes

$$n\mu_t(\frac{X_1}{n})=n\mu_s(\frac{\pi-X_1}{n}).$$

Thus group 1's share X_1 is the solution to

$$\mu_t(\frac{X_1}{n})=\mu_s(\frac{\pi}{n}-\frac{X_1}{n}). \tag{7}$$

Compare this with the tough individual's share X_1 when $n=1$, which satisfies

$$\mu_t(X_1)=\mu_s(\pi-X_1). \tag{8}$$

In general, solutions to (7) and (8) are different. When two individuals who represent their groups bargain, their perceived pie up for grabs is π/n. But when two individuals only represent themselves, they bargain over the whole π. In the latter case, they think "big." In the former case, they think "small." Only in special cases, such as Example 1, where the preferences are vN-M utilities with constant relative risk aversion and zero breakdown point, (7) and (8) will lead to the same solution.

***Example* 4.**

Consider a special case of Example 3 where $n=2$. Compare its outcome with that of an alternative situation where there exist two identically composed groups. In each group, there are two individuals,

[7] Chae and Heidhues (1999) use the convexity of the marginal risk concession function in showing the advantage of forming an alliance.

one tough type and one soft type. Since each group will select the tough type as its representatives, $X_1=X_2=\pi/2$, and the intra-group bargaining leads to

$$\mu_t(x_t)=\mu_s(\frac{\pi}{2}-x_t),$$

which yields the same solution x_t as in (7), where $X_t/2=x_t$. Thus a tough individual's share is the same in the two situations. In the two situations, the representatives' perceived pie for bargaining is at the same level, *i.e.*, $\pi/2$.

III. Concluding Remarks

In this paper, we studied a strategic model of group bargaining where both inter-group bargaining and intra-group bargaining are carried out according to well-known alternating-offer procedures. We showed that there exists a unique subgame perfect equilibrium outcome when the breakdown probabilities are sufficiently small. In our model, an arbitrary member of each group chooses a representative for the group. We showed that each group will choose as its representative an individual whose marginal gain from increasing the group's share is the greatest.

The model can be applied to a variety of bargaining situations where groups such as households, labor unions, firms, and countries bargain with each other. Even though we only studied some basic theoretical issues in this paper, the future research may yield richer implications specific to applications using variations of the model presented here.

(Received 14 January 2009; Revised 24 March 2009)

Appendix

***Proof of Proposition* 1:** We will modify the proof of a similar result in Chae and Yang (1994). In their model, the rejection of an offer leads to a time delay rather than the risk of a breakdown. Steps that are obvious from Chae and Yang (1994) will be omitted for simplicity. Denote the members of group g simply by $1, \cdots, n$. In equilibrium, the game ends immediately. The outcome depends on the identity of the initial proposer, for the initial proposer has some advantage. We will denote the equilibrium payoff vector in the case where $i\,(=1, \cdots, n)$ is the initial proposer in the intra-group bargaining game as $(x_1^i, \cdots, x_n^i)$. It turns out that

$$x_j^i = c_j(q, x_j^j) \text{ for any } j \neq i.$$

What an individual receives in equilibrium when he is not the initial proposer is the certainty equivalent of the consequences of his rejection. After his rejection, with probability q he will become the initial proposer in the next round, and with probability $1-q$ the game will break down. In particular, individual j's payoff when he is not an initiator does not depend on who is the initiator. Denote $y_i \equiv x_i^i$. Then $(y_1, \cdots, y_n)$ is the unique solution to the following simultaneous system of equations:

$$y_1 + c_2(q, y_2) + \cdots + c_n(q, y_n) = X_g,$$

$$c_1(q, y_1) + y_2 + \cdots + c_n(q, y_n) = X_g,$$

$$\cdots$$

$$c_1(q, y_1) + c_2(q, y_2) + \cdots + y_n = X_g.$$

The premium of an individual i when he is the initiator is $y_i - c_i(q, y)$. From the above system of equations, one has

$$y_1 - c_1(q, y_1) = \cdots = y_n - c_n(q, y_n). \tag{A1}$$

Since $y_i - c_j(q, y_i)$ is an increasing function of $y_i(\geq d_i)$ by Assumption 1, $y_1, \cdots, y_n$ increase or decrease together as group's share X_g increases or decreases.

Denote $y_i = \phi_i^q(X_g)$ and $\phi_i(X_g) \equiv \lim_{q \to 1} \phi_i^q(X_g)$. Then $x_i \equiv \phi_i(X_g)$ is i's share of X_g according to the Nash bargaining solution. In order to see this, observe that from (A1), one has

$$\lim_{p \to 1} \frac{y_1 - c_1(p, y_1)}{1-p} = \cdots = \lim_{p \to 1} \frac{y_n - c_n(p, y_n)}{1-p},$$

i.e.,

$$\mu_1(x_1) = \cdots = \mu_n(x_n),$$

which is condition (2). *Q.E.D.*

***Proof of Theorem* 1:** We will only sketch the proof omitting obvious steps. Denote the equilibrium payoff vector in the case where group g's representative is the initial proposer in the inter-group bargaining game by (X_1^g, X_2^g) $(g=1, 2)$. Also, denote the inverse of the function of ϕ_i^q by $\psi_i^q (i=j, k)$. Then

$$X_1^2 = \psi_j^q(c_j(p, \phi_j^q(X_1^1))),$$

$$X_2^1 = \psi_k^q(c_k(p, \phi_k^q(X_2^2))).$$

Denote $Y_1 \equiv X_1^1$ and $Y_2 \equiv X_2^2$. Then (Y_1, Y_2) is the solution to the following system of equations:

$$\begin{aligned} &Y_1 + \psi_k^q(c_k(p, \phi_k^q(Y_2))) = \pi, \\ &\psi_j^q(c_j(p, \phi_j^q(Y_1))) + Y_2 = \pi \end{aligned} \tag{A2}$$

From equation system (A2), one has

$$Y_1 - \psi_j^q(c_j(p, \phi_j^q(Y_1))) = Y_2 - \psi_k^q(c_k(p, \phi_k^q(Y_2))), \tag{A3}$$

i.e.,

$$\frac{Y_1 - \psi_j^q(c_j(p, \phi_j(Y_1)))}{1-p} = \frac{Y_2 - \psi_k^q(c_k(p, \phi_k(Y_2)))}{1-p}.$$

Each side of the above equation is an increasing function of Y_g $(g=1, 2)$ if p and q are sufficiently close to 1 as can be seen as follows: Denote each side of the above equation by $F_i(Y_g)$, where $i=j, k$ for groups 1 and 2, respectively. Then

$$F_i'(Y_g) = \frac{1}{1-p}\{1 - \psi_i^{q\prime}(c_i(p,\ \phi_i^q(Y_g))) \cdot \frac{\partial}{\partial x} c_i(p,\ \phi_i^q(Y_g)) \cdot \phi_i^{q\prime}(Y_g)\}$$

$$= \frac{1}{1-p}\{1 - \{\psi_i^{q\prime}(c_i(p,\ \phi_i^q(Y_g))) \cdot \phi_i^{q\prime}(Y_g)\} \cdot \frac{\partial}{\partial x} c_i(p,\ \phi_i^q(Y_g))\}$$

By L'Hospital's rule, noticing that ψ_i^q is the inverse function of ϕ_i^q,

$$\lim_{p,q\to 1} F_i'(Y_g) = \mu_i'(\phi_i(Y_g)) - \frac{\mu_i\{\phi_i(Y_g))\,\phi_i''(Y_g)}{\{\phi_i'(Y_g)\}^2},$$

which is positive for any $Y_g > \sum_{i\in G_g} d_i$ by Assumption 2. Thus, $F_i'(Y_g) > 0$ for p and q sufficiently close to 1. Since $F_i'(Y_g)$ is continuous in p and q, it is uniformly continuous on a compact interval. This statement does not depend on Y_g, that is, for any small $\varepsilon > 0$, there exist $\hat{p}$, $\hat{q} \in (0,1)$ such that for any $p > \hat{p}$, $q > \hat{q}$, and $Y_g \in [\varepsilon + \sum_{i\in G_g} d_i,\ \pi]$, one has $F_i'(Y_g) > 0$.

Now assume that $p > \hat{p}$, $q > \hat{q}$, and denote each side of Equation (A3) by $f_i(Y_g)$. Then $f_i'(Y_g) > 0$. To the extent that either $f_j(Y_1)$ or $f_k(Y_2)$ can be bounded from above, assume, without loss of generality, that $f_j(Y_1)$ has the smaller least upper bound. (If $f_j(Y_1)$ is not bounded from above, its least upper bound is ∞.) Then the function $Y_2 = f_k^{-1} \circ f_j(Y_1)$ is well defined. Using this, one can rewrite the first equation of the equation system (A2) as

$$Y_1 + \psi_k^q(c_k(p,\ \phi_k^q(f_k^{-1} \circ f_j(Y_1)))) = \pi. \tag{A4}$$

The left hand side of this equation increases continuously from 0 to infinity as $Y_1 (> \sum_{i\in G_1} d_i)$ increases. Thus there exists a unique solution Y_1 of Equation (A4).

Now, we will look at the outcome of the inter-group bargaining game in the limit. Let $\psi_i(X_g) \equiv \lim_{q\to 1} \psi_i^q(X_g)$. As q goes to 1, Equation (A3) can be rewritten in the limit as

$$Y_1 - \psi_j(c_j(p,\ \phi_j(Y_1))) = Y_2 - \psi_k(c_k(p,\ \phi_k(Y_2))),$$

Thus

$$\lim_{p\to 1}\frac{Y_1-\psi_j(c_j(p,\ \phi_j(Y_1)))}{1-p}=\lim_{p\to 1}\frac{Y_2-\psi_k(c_k(p,\ \phi_k(Y_2)))}{1-p}.$$

By L'Hopital's rule,

$$\psi_j'(\phi_j(Y_1))\cdot\frac{\partial}{\partial p}c_j(1,\ \phi_j(Y_1))=\psi_k'(\phi_k(Y_2))\cdot\frac{\partial}{\partial p}c_k(1,\ \phi_k(Y_2)),$$

i.e.,

$$\frac{\mu_j(\phi_j(Y_1))}{\phi_j'(Y_1)}=\frac{\mu_k(\phi_k(Y_2))}{\phi_k'(Y_2)},$$

which is the same as (5).

Proof of Theorem 2: For $g=1,\ 2$, define $f_g(Y_g)\equiv\min_{i\in G_g}f_i(Y_g)$. Then the function $f_g(Y_g)$ is continuous and increasing in Y_g for p sufficiently close to 1. (The identity of the best bargainer changes as Y_g changes.) To the extent that either $f_1(Y_1)$ or $f_2(Y_2)$ can be bounded from above, assume, without loss of generality, that $f_1(Y_1)$ has the smaller least upper bound. Then the function $Y_2=f_2^{-1}\circ f_1(Y_1)$ is well defined. Feasibility implies

$$\{Y_1-f_1(Y_1)\}+f_2^{-1}\circ f_1(Y_1)=\pi.$$

The left hand side of this equation increases continuously from 0 to infinity as $Y_1(>\sum_{i\in G_1}d_i)$ increases. Thus there exists a unique solution X_1 of the above equation when p and q are sufficiently close to 1. Once X_1 is determined, the shares of individuals are determined as

$$x_h=\phi_h^q(X_1)\ \text{for any member}\ h\ \text{of group 1},$$

$$x_l=\phi_l^q(\pi-X_1)\ \text{for any member}\ l\ \text{of group 2}.$$

This constitutes the unique equilibrium outcome of the representation game. The representatives chosen in equilibrium can be identified as follows: Let j and k be the members of groups 1 and 2 such that $f_1(X_1)=f_j(X_1)$ and $f_1(\pi-X_1)=f_k(\pi-X_1)$ (at X_1 in equilibrium), *i.e.*,

$$f_j(X_1) \leq f_h(X_1) \text{ for any } h \in G_1,$$

$$f_k(\pi - X_1) \leq f_l(\pi - X_1) \text{ for any } l \in G_2.$$

Then groups 1 and 2 will choose such individuals j and k as their representatives in equilibrium. In order to see this, suppose to the contrary that there exists some $h \in G_1$ such that $f_j(X_1) > f_h(X_1)$. Let $\tilde{X}_1$ be the solution to the following equation:

$$f_h(Y_1) = f_k(\pi - Y_1).$$

Then since $f_h(X_1) < f_k(\pi - X_1)$ and both sides of the above equation are monotonic, one has $\tilde{X}_1 > X_1$. Thus, given that group 2 chooses k as its representative, it is not optimal for group 1 to choose j as its representative. *Q.E.D.*

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