송신 빔포밍과 결합한 MISO 릴레이 채널에서의 최적 전력 할당 기법

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Optimum Power Allocation with Transmit Beamforming in Correlated MISO Relay Channels

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Abstract: This paper proposes an optimum power allocation strategy to maximize the end-to-end capacity of regenerative relay channels with transmit beamforming. An achievable rate for this model is first derived, and is optimized in terms of spectral efficiency. This optimization is a *max-min* problem. A general technique for solving this *max-min* problem is applied to derive the optimum power allocation that satisfies the sum power constraints at the source and relay. The upper bound of the regenerative relay channels allocated by the proposed scheme is also derived. The analytic and simulation results show that the optimum power allocation highly depends on not only the average channel gain but also the achievable beamforming gain of the first hop and the second hop channel.

Keywords: beamforming, max-min problem, power allocation, regenerative relay system

Ⅰ. Introduction

Wireless relay has been obtained major attention in the wireless networks due to its various potentials for the enhancement of achievable rate and cell coverage [1]. In the relay systems, two types of protocols (i.e., regenerative and non-regenerative mode) have been often considered [2]. Especially, recent researches have focused on the regenerative relay mode since it does not suffer from the noise enhancement in low signal-to-noise ratio (SNR) compared to non-regenerative relay mode by using a decode-and forward (DF) scheme which decodes the noisy signal from the source and re-encodes /-transmits the signal to the destination.

In the regenerative relay mode, resource (e.g., bandwidth and power) allocation has widely been studied as an important issue to maximize the end-to-end capacity [3]-[6]. When single antenna is applied to relay systems, the optimum resource allocation to maximize the end-to-end capacity has been shown in [3], [4]. In this works, resources are optimally allocated for balancing the capacity of the hops based on SNR. Recently, resource allocation strategies for multiple-input multiple-output (MIMO) relay channels have been under consideration due to the ability of MIMO channels, which enables to increase the data rate or lower the transmission error rate for a given SNR [5], [6]. Although the previous works have considered the power allocation strategies based on the SNR of the hops with various transmission strategies (e.g., dirty-paper coding, distributed space-time coding), to author's best knowledge, resource allocation with transmit beamforming has not been considered.

In this paper, we consider the power allocation strategy for regenerative relay channels with multiple antennas. This work is differentiated from the previous works by proposing the optimum power allocation in terms of end-to-end capacity for regenerative relay channels with transmit beamforming. The proposed scheme can adaptively allocate the power of the source and the relay according to the effective channel gain to maximize the end-to-end capacity. And then it is applied to two types of relay systems with channel state information (CSI) feedback (i.e., short-term and long-term CSI). The analytic and numerical results show the proposed scheme highly depends on not only the average channel gain but also the achievable beamforming gain of the first hop and the second hop.

The remainder of this paper organized as follows. Section II describes the system model in consideration. In Section III, the optimum power allocation that maximizes the end-to-end capacity for regenerative relay system with transmit beamforming is proposed. Section IV verifies the performance of the proposed power allocation scheme in terms of the end-to-end capacity. Finally, conclusions are given in Section V.

Ⅱ. System model

We consider the regenerative MISO relay system with relay channels, where the source (S) transmits

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This research was supported by Seoul R&BD Program (10544)

to the relay (R) in one orthogonal channel (first hop), and the relay transmits to the destination (D) in other orthogonal channel (second hop). Assuming at total available bandwidth of w_0 Hertz, two orthogonal channels equally share the channel bandwidth. We ignore the direct link ($S \rightarrow D$) due to large path loss. We assume that the sum power is constant as p_0 . We draw the orthogonal MISO relay channels with solid and dashed line indicating the first hop and the second hop, respectively, in Fig. 1, with M_1 being the number of transmit antennas at the source, N_1 and M_2 being the number of receive and transmit antennas at the relay, and N_2 being the number of receive antennas at the destination.

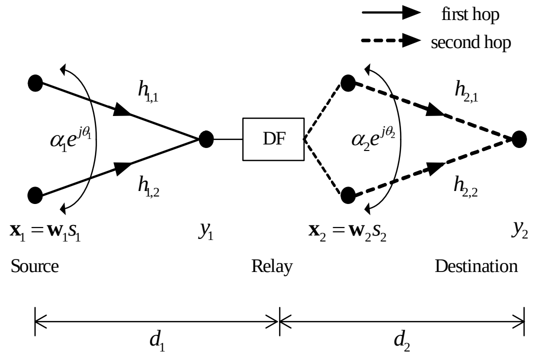

Fig 1. System model.

Let s_i be the transmitted signal, $\mathbf{w}_i$ be the beamforming vector, and $\mathbf{h}_i$ be fading for the ith hop. Then, the received signal of the first hop and the second hop can be respectively written as

$$S \rightarrow R : y_1 = \sqrt{\gamma_1}\mathbf{h}_1\mathbf{w}_1 s_1 + z_1 \tag{1}$$

$$R \rightarrow D : y_2 = \sqrt{\gamma_2}\mathbf{h}_2\mathbf{w}_2 s_2 + z_2 \tag{2}$$

where z_i denotes additive noise with variance σ_i^2 , and $\gamma_i\left(= p_i d_i^{-\tau}\right)$ denotes the received signal gain at ith hop. Here, p_i is an allocated power, d_i is the propagation distance, τ is the path loss exponent.

When each hop is a spatially correlated Rayleigh fading, the channel vector $\mathbf{h}_i$ can be generated using i.i.d. Rayleigh channel vector $\tilde{\mathbf{h}}_i$ by [7]

$$\mathbf{h}_i = \tilde{\mathbf{h}}_i \mathbf{R}_i^{1/2} \tag{3}$$

where $\mathbf{R}_i^{1/2}$ denotes the square root of channel correlation matrix $\mathbf{R}_i$ which is Hermitian and positive definite. If $M_i = 2$, $\mathbf{R}_i$ can be represented

$$\mathbf{R}_i = E\left[\mathbf{h}_i^* \mathbf{h}_i\right] = \begin{bmatrix} 1 & \rho_i \\ \rho_i^* & 1 \end{bmatrix} \tag{4}$$

where the superscript $(\mathbf{x})^*$ and $E[\mathbf{x}]$ denote the conjugate transpose and the expectation of $\mathbf{x}$, respectively. Furthermore, $\rho_i\left(=\alpha_i e^{j\theta_i}\right)$ denotes the transmit correlation coefficient, α_i and θ_i is its amplitude and phase, respectively.

Ⅲ. Optimum power allocation with transmit beamforming

1. *Problem Formulation*

The Shannon capacity C for time variant fading channel $\mathbf{h}$ with M transmit antennas and single receive antenna is derived in [8]

$$C = w_0 E\left[\log_2\left(1 + \frac{\gamma_0}{M\sigma_0^2}\|\mathbf{h}\|_F^2\right)\right] \tag{5}$$

where $\|\cdot\|_F$ denotes Frobenius norm.

If the source transmits the packet through the beamforming vector $\mathbf{w}$, the Shannon capacity C in (5) is represented as [8]

$$C = w_0 E\left[\log_2\left(1 + \frac{\gamma_0}{\sigma_0^2}|\mathbf{h}\mathbf{w}|^2\right)\right]. \tag{6}$$

In relay channels, the sum power p_0 should be allocated to each hop channel. Therefore, we have the achievable rate of the first hop and that of the second hop, respectively, by using the Shannon capacity formula in (6)

$$R_1(p) = \frac{w_0}{2} E\left[\log_2\left(1 + \frac{\gamma_1}{\sigma_1^2}|\mathbf{h}_1\mathbf{w}_1|^2\right)\right] \tag{7}$$

$$R_2(p) = \frac{w_0}{2} E\left[\log_2\left(1 + \frac{\gamma_2}{\sigma_2^2}|\mathbf{h}_2\mathbf{w}_2|^2\right)\right]. \tag{8}$$

When the regenerative relay is deployed, the achievable rate of the end-to-end link is dictated by the smallest R_i , i.e., $C = \min(R_1, R_2)$. Therefore, it is thus aim to find the fractional power $p_i(i=1,2)$ allocation for given power p_0 , such as to maximize the minimum achievable rate. Therefore, the optimum power allocation problem can now be formulated as

$$C = \max_{p=(p_1,p_2)} \min\left\{R_1(p), R_2(p)\right\} \text{ subject to } p_1 + p_2 = p_0 . \tag{9}$$

We refer to this problem as *max-min* optimization and the optimum power allocation p^{opt} maximizing the achievable rate C overall possible power allocation rules p as a *max-min rule.*

2. *Proposed Power Allocation Strategy*

A general technique to solve this *max-min problem* is known in [9]. Applying this solution to the regenerative relay system, we can obtain the conditions as

$$R_1\left(p^{\text{opt}}\right) = R_2\left(p^{\text{opt}}\right). \tag{10}$$

This means that p^{opt} makes the achievable rates of two hops equal.

We apply the *max-min* solution (10) to the regenerative relay system with transmit beamforming, and derive the optimum power allocation p^{opt} to maximize the achievable rate C.

From the Jensen's inequality [8], an upper bound of rate (7) and (8) can be represented as

$$R_1(p) \le \frac{w_0}{2}\log_2\left(1+\frac{\gamma_1}{\sigma_1^2}\kappa_1\right) \tag{11}$$

$$R_2(p) \le \frac{w_0}{2}\log_2\left(1+\frac{\gamma_2}{\sigma_2^2}\kappa_2\right) \tag{12}$$

where $\kappa_i = E\left[\left|\mathbf{h}_i\mathbf{w}_i\right|^2\right]$ denotes the beamforming gain of ith hop.

By the relationship (10) at the optimum power allocation rule p^{opt}, the average SNRs of the first hop and the second hop should be equal, i.e.,

$$\frac{\gamma_1}{\sigma_1^2}\kappa_1 = \frac{\gamma_2}{\sigma_2^2}\kappa_2\,. \tag{13}$$

Since $\gamma_i = p_i d_i^{-\tau}$ for $i = 1,2$, we have

$$p_2 = \frac{\kappa_1}{\kappa_2}\left(\frac{d_1}{d_2}\right)^{-\tau} p_1 \tag{14}$$

where we assume that $\sigma_1^2 = \sigma_2^2$.

By inserting (14) into the sum power constraints in (9), thus we have the optimum power allocation rule p^{opt} as

$$p_1^{\text{opt}} = \frac{1}{1+\frac{\kappa_1}{\kappa_2}\left(\frac{d_1}{d_2}\right)^{-\tau}} p_0 \tag{15}$$

$$p_2^{\text{opt}} = \frac{1}{1+\frac{\kappa_2}{\kappa_1}\left(\frac{d_2}{d_1}\right)^{-\tau}} p_0\,. \tag{16}$$

(15) and (16) show that the optimum power allocation rule p^{opt} maximizing the achievable rate C depends on not only the average channel gain (i.e., $d_1^{-\tau}$ and $d_2^{-\tau}$) but also the achievable beamforming gain (i.e., κ_1 and κ_2). For example, if the achievable gain of the first hop is larger than that of the second hop, the allocated power p_1 for the source is smaller than the allocated power p_2 at the relay, and vice versa.

If the coherent beamforming based on short term CSI applies at the source and the relay, the optimum power allocation rule $p_{\text{Coh.BF}}^{\text{opt}}$ is represented as

$$p_{1,\text{Coh.BF}}^{\text{opt}} = \frac{1}{1+\frac{M_1}{M_2}\left(\frac{d_1}{d_2}\right)^{-\tau}} p_0 \tag{17}$$

$$p_{2,\text{Coh.BF}}^{\text{opt}} = \frac{1}{1+\frac{M_2}{M_1}\left(\frac{d_2}{d_1}\right)^{-\tau}} p_0 \tag{18}$$

(17) and (18) show that the optimum power allocation rule $p_{\text{Coh.BF}}^{\text{opt}}$ depends on the full beamforming gain (i.e., M_1 and M_2) as well as the average channel gain. This is because the coherent beamforming can fully achieve the beamforming gain by using the instantaneous CSI.

If the eigen beamforming based on long term CSI applies at the source and the relay, the optimum power allocation rule $p_{\text{Eig.BF}}^{\text{opt}}$ is represented as

$$p_{1,\text{Eig.BF}}^{\text{opt}} = \frac{1}{1+\left(\frac{1+\alpha_1}{1+\alpha_2}\right)\left(\frac{d_1}{d_2}\right)^{-\tau}} p_0 \tag{19}$$

$$p_{2,\text{Eig.BF}}^{\text{opt}} = \frac{1}{1+\left(\frac{1+\alpha_2}{1+\alpha_1}\right)\left(\frac{d_2}{d_1}\right)^{-\tau}} p_0 \tag{20}$$

(19) and (20) show that the optimum power allocation rule $p_{\text{Eig.BF}}^{\text{opt}}$ depends on the transmit correlation amplitude (i.e., α_1 and α_2) as well as the average channel gain. This is because the eigen beamforming can partially achieve the beamforming gain by only using the average CSI. Therefore, the larger the transmit correlation amplitude of the first hop is, the smaller the allocated power $p_{1,\text{Eig.BF}}^{\text{opt}}$ at the source is, and vice versa.

Ⅳ. Performance evaluation

We compare the rate achieved by the proposed power allocation strategy with the rate achieved by the equal power allocation strategy. In the equal power allocation, the power of hops is shared without regard to the channel conditions. We first define the parameter ΔC for the performance measurement.

$$\Delta C = C\left(p^{\text{opt}}\right) - C\left(p^{\text{eq}}\right) \tag{21}$$

where $C\left(p^{\text{opt}}\right)$ and $C\left(p^{\text{eq}}\right)$ denote the rates achieved by the optimum power allocation and equal power allocation, respectively. The common simulation parameters are summarized in Table I.

Fig. 2 depicts the parameter ΔC according to average SNR when $d_1 = 0.7$ km, $d_2 = 0.3$, $\alpha_1 = 0.9$ and $\alpha_2 = 0.5$. It can be seen that the achievable rate of the proposed power allocation strategy is larger than that of the equal power allocation strategy through all SNR region. The reason is mainly because the source and the relay adaptively allocate their power according to the channel conditions. As the average SNR γ_0 increases, the parameter ΔC approaches to

0.47 bps/Hz. This means that the gain of the proposed power allocation strategy is limited as high SNR.

Fig. 3 describes the proposed power allocation p^{opt} with the eigen beamforming according to the difference of amplitude of correlation coefficients $\Delta\alpha$ when $\gamma_0 = 10$ dB, $\alpha_1 = 0$ and $d_1 = d_2 = 0.5$ km. It can be seen that more power is allocated at the source that the relay. The reason is because the beamforming gain of the second hop is larger than that of the first hop, and vice versa.

V. Conclusion

We have proposed the optimum power allocation strategy in terms of spectral efficiency of the regenerative relay channels with the transmit beamforming. The analytic and simulation results show that the optimum power allocation at the source and the relay highly depends on the average channel gain and the achievable beamforming gain between the first hop and the second hop.

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Table 1. Simulation parameters

PARAMETERS	Values
Considering system	Two-hop regenerative relay system
Cell radius (r)	1 Km
Frequency reuse factor	1
# of source node's tx. antennas (M_1)	2
# of relay node's rx., tx. antennas (N_1, M_2)	1, 2
# of destination node's rx. antennas (N_2)	1
Available bandwidth (w_0)	1
Sum power (p_0)	1
Fading channel	Rayleigh fading
Link adaptation	Ideal (i.e., using the Shannon's capacity formula)

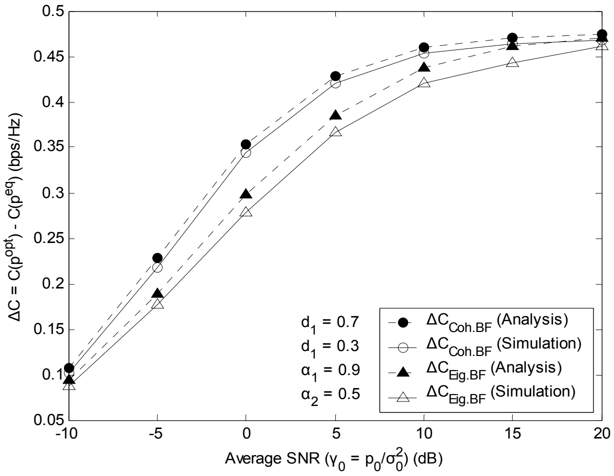

Fig 2. Performance of the proposed scheme according to average SNR.

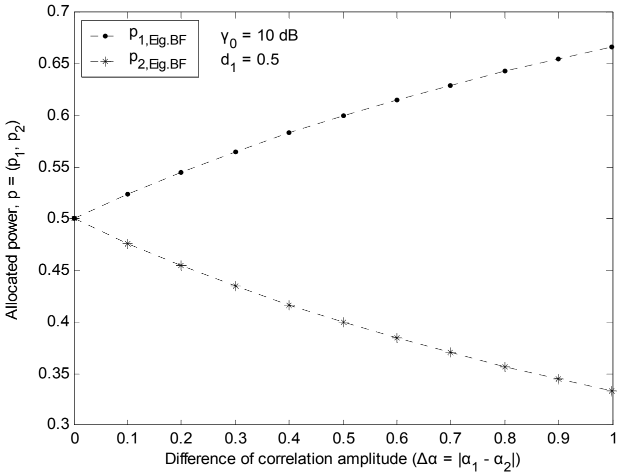

Fig 3. Allocated power of the proposed scheme according to difference of correlation amplitude.