Using Mixed Research Methods to Study Students' Affective Change in an Inquiry-Oriented Mathematics Class*

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Abstract

The study presented in this paper was designed to evaluate an inquiry-based first course in differential equations by investigating the course's impact on students' mathematical competence, particularly focusing on the question of how students' views about mathematics changed through their participation in the class. For the evaluation, a mixed methods approach was employed. The primary purpose of this paper is to discuss the advantages and challenges of the mixed methods approach.

One advantage of a mixed methods approach is the development of a reliable evaluation through convergent

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validation. Moreover, each different method provides evidence of the change in the students' views about mathematics from a unique angle.
On the other hand, one of the serious challenges raised by the use of mixed methods approaches concerns data synthesis. While mixed methods approaches contribute to the development of elaborated findings by providing a chance for researchers to study data from various angles, findings from various sources can be divergent. It is recommended that researchers avoid thinking in either-or ways about mixing methods. Instead, a researcher should reflect on what each method tells about the phenomenon under inquiry. This understanding of multiple layers of reality via multiple methods will extend the horizon of our understanding and form a useful basis for future educational improvement.

Key words: mixed methodology, qualitative methodology, quantitative methodology, views about mathematics.

I Introduction

Based on the continuing evolution of research methodologies and the perceived legitimacy of both quantitative and qualitative research, there is an increasing tendency to adapt and use mixed methods approaches which employ strategies to collect and analyze both qualitative and quantitative data (Cresswell, 2003; Jick, 1979; Lawrenz & Huffman, 2002; Sieber, 1973; Waysman & Savaya, 1997). The purposes of this paper are to introduce a theoretical background for mixed methods approaches, to present the process in which a mixed methods approach was employed in our developmental research project, and finally, to discuss advantages and challenges of using mixed methods approaches.

Ⅱ Theoretical Background

In the domain of social and human sciences, there have been significant historical debates concerning the legitimacy of quantitative and qualitative paradigms to understand the social world. At a fundamental level, the debates are concerned with what knowledge claim is legitimate (Bredo & Feinberg, 1982; Cresswell, 2003; House, 1990; Smith & Falls, 1983). The quantitative paradigm is based on the positivism that historically originated with Comte (Bredo & Feinberg, 1982). The positivist philosophy supports the view that society has gone through an inevitable evolution from seeing the world using a theological perspective to a metaphysical perspective and eventually to the positivist perspective. Based on this historical evolution, the positivist assumes a hierarchy of individual sciences from mathematics at the top through experiential sciences such as physics down to sociology. In this perspective, the positivist considers that social scientists should borrow the methodology of the physical sciences to develop law-like knowledge about the social and human world. In contrast, the qualitative paradigm, which is opposed to positivism, argues that the scientific investigation of the social world should take a different approach from the physical sciences, in the sense that the physical sciences are concerned with inanimate objects outside humans, which is not the case for the social sciences. The qualitative paradigm is based on interpretivism, which emphasizes developing a holistic understanding of the lived in world from the viewpoint of participants.

Alongside this purist dispute over research paradigms, there have also been claims that a combination of methods may produce theory superior to those by any single-method approach (Cresswell, 2003; Jick, 1979; Lawrenz & Huffman, 2002; Sieber, 1973; Waysman & Savaya, 1997). While these claims partly reflect the acknowledgement of the legitimacy of each research method to make a unique contribution to theory construction, the claims are basically rooted in pragmatism. In the pragmatist perspective, knowledge claims arise out of action, situations, and consequences. Truth is what works at a certain time and space. In the pragmatist tradition, research is basically concerned with knowledge as applications and solutions to problems. This suggests that research is more concerned with "a problem" rather than "a method"itself. Thus, researchers liberally use pluralistic approaches to derive knowledge in order to best understand the phenomenon under inquiry. From this perspective, mixed methods approaches gradually have become common practice, especially in program evaluation, which is primarily concerned with providing the best answers to evaluation questions.

Mixed methods approaches are based on the notion of method triangulation as a means for seeking convergence across qualitative and quantitative methods (Tashakkori & Teddlie, 1998). The concept of triangulation, broadly defined as “the combination of methodologies in the study of the same phenomenon” (Denzin, 1978: 291), is based on the assumption that any bias inherent in a particular data source, investigator, or method would be neutralized when used in conjunction with other data sources, investigators, and methods (Jick, 1979). Combining various methods, triangulation is primarily concerned with the notion of convergent validation; that is, the convergence between various methods enhances the validity of the research findings. In addition to convergent validation, triangulation with mixed methods can also capture a more complete, holistic, and contextual portrayal of the units under study than either could provide separately (Lawrenz & Huffman, 2002; Russek & Weinberg, 1993). In general, the mixed methods approach is useful to capture the best of both quantitative and qualitative approaches and expected to create more reliable and deeper explanation.

Because of these methodical advantages, researchers in mathematics education are increasingly adapting mixed methods approaches. However, the process of the employment of a chosen approach is not straightforward. In particular, triangulation efforts do not always lead to convergent findings. Various methods often produce inconsistent or contradictory results. In those cases, the lack of guidance for data synthesis has been pointed out as one of the most serious challenges (Jick, 1979). In order to resolve the challenge, several researchers have developed methods for synthesizing findings such as weighing different types of data (e.g., Caracelli & Greene, 1993). However, our experience with a mixed methods approach tells us that data synthesis is not merely an issue of weighing different types of data. According to our experience in a mixed methods approach, inconsistency or contradiction among what various methods reveal sometimes opens up a new horizon of understanding. From this perspective, this paper presents our experience with a mixed methods approach to evaluate an instructional design of a university level mathematics class. In particular, our discussion will focus on what the process of the mixed methods approach was like, what challenges we encountered, how we dealt with the challenges, and what we have learned through the process.

Ⅲ Research Design

As part of a developmental research program, this study was conducted primarily to provide evaluation for an inquiry oriented differential equations (IODE) course at a Korean university. Developmental research for the IODE class originated with the critical reflection of traditional differential equations courses that have emphasized the acquisition of a set of computational skills (Artigue & Gautheron, 1983; Rasmussen, 2001). In traditional differential equations classes, instructors present algorithms for solving specific types of differential equations and students practice how to use them. Students are rarely given opportunities to participate in the construction of mathematical knowledge in the class. Although modeling has played a critical role in the development of differential equations, modeling and application are most often neglected in teaching and learning differential equations.

Alternatively, based on the philosophy of Realistic Mathematics Education (RME), the instructional design of the IODE course promotes students' participation in the active construction of mathematics through progressive mathematization (Treffers, 1987). Thus, in the class under study, context problems created by the research team formed the grounds for progressive mathematization. The context problems were designed to reflect mathematical phenomena which are realistic to the students experientially and mathematically. In the research class, there were 19 female students at the end of the semester. Most of the students were freshmen from the department of mathematics education with their major mathematical background being calculus. However, several students came from outside departments such as educational engineering and special education; some of these students were pursuing a transfer to the department of mathematics education for teaching credentials in their future. All of the 19 students were taking a linear algebra course during the same semester.

In the IODE class, there was little direct instruction by the instructor. The class began with the students' mathematical investigation of context problems. The students formed small groups of three or four to work on the context problems. While the students worked in a small group setting, the instructor interacted with the students. After the small group discussion, the students joined together for the purpose of a whole class discussion. Several students came to the front to share their results and the students in the audience asked questions or gave comments on their presentation. The students' discussion was not merely concerned with seeking specific solutions for given tasks but negotiating their understanding of the mathematical concepts and principles behind the tasks, and ultimately led to the gradual emergence of a taken-as-shared meaning.

Generally, the IODE course emphasized the students' active participation in the construction of mathematical meaning based on the theory of RME. Our evaluation of the project investigated the impact of the IODE course on the students' mathematical competence. Since we employed a comprehensive notion of mathematical competence for the evaluation, we assessed diverse aspects of students' learning of mathematics such as conceptual reasoning, modeling ability, retention, and so on. In this study, our evaluation specifically focused on the students' affective aspect, in line with the recent emphasis on the affective aspect of mathematics learning (e.g., NCTM, 1989, 2000). We collected both quantitative and qualitative data, specifically surveys, video-recordings of class, worksheets, and journals in order to investigate the change in the students' views about mathematics through their course participation. The VAMS (Views About Mathematics Survey), a quantitative survey designed to assess a respondent's view about knowing and learning mathematics (Carlson, 1999), was administered to the students in the IODE class at the beginning and at the end of the semester for comparison. Two responses were excluded from the t-test because two students did not take the pre-survey and post-survey of VAMS. We also observed all the sessions as participant observers and collected video recordings of all the class sessions through the semester. All the video recordings were transcribed for discourse analysis. We also collected all documents authored by students such as reflective journals and worksheets. The VAMS and the transcripts formed the main body of data for analysis. The students' document data complemented the results of the analysis based on the survey and class observation.

Ⅳ Convergence and Divergence in Mixed Methods Approach

A. Quantitative Analysis

As described above, the research team was deeply engaged with the IODE class. Through this involvement, we noticed remarkable change in the students' attitudes toward mathematics, especially in how they viewed and studied mathematics. In order to check our hypothesis about the positive impact of the IODE on the students' views about mathematics, formal analysis began with the t-test of the VAMS, a quantitative survey designed to assess a respondent's view about knowing and learning mathematics. For the assessment, VAMS items used a design based on the Contrasting Alternative Design (Carlson, 1999). Each item presented a question to which a respondent selected either an exclusive choice of belief (corresponding to options 1 or 5) or a weighted combination of the two alternatives (corresponding to options from 2 to 4). One of the alternative views corresponded to the view most commonly held among expert mathematicians. The opposite view represented the naï e view which can be attributed to the lay community and mathematics students with little or no mathematics background. Thus, for each VAMS item, a respondent's response can be roughly classified as expert (a response between 4 and 5), mixed or transitional (a response between 3 and 4), or naï e (a response of 2 or below). Figure 1 shows the response choice diagram of a typical item from VAMS.

When studying mathematics in a textbook or in course materials:

(a) I memorize it the way it is presented.

(b) I make sense of the material so that I can understand it.

①	②	③	④	⑤	○
(a) ◀				▶ (b)	
only (a) never(b)	more (a) than (b)	equally (a) and (b)	more (a) than (b)	only (a) never (b)	Neither (a) nor (b)

Figure 1. Samples items from VAMS

The VAMS items were grouped into two dimensions: philosophical and pedagogical. The philosophical dimension included the categories of questions concerning the structure of mathematical knowledge, the methods of mathematics, and the validity of mathematical knowledge. The pedagogical dimension included the categories of questions concerning the learnability of mathematics, the role of reflective thinking, and personal relevance of mathematics. Individual VAMS items were devised to assess the view of a respondent corresponding to one of these categories.

The result of the quantitative analysis shows that the overall means of pre- and post-surveys are 3.55 and 3.65, respectively (see Table 1). The initial mean value suggests that the students' views about mathematics belong to the upper-transitional state; in other words, that the students already possessed expert-like views about mathematics in the beginning of the semester. However, the comparison of the overall means of the pre- and the post-tests supports our hypothesis of the positive impact of the IODE class on the students' views about mathematics since the mean rose from 3.55 to 3.67 (p=0.024).

Table 1. T-test of VAMS for the IODE Class

	n	Mean	SD	t	Sig.
Pre-Test	17	3.55	.169	2.49	.024
Post-Test	17	3.67	.211		

This result of the quantitative data analysis largely matched our initial hypothesis about the positive impact of IODE on the students' views about mathematics based on our class observation. However, there were several points that our experience of participatory observation did not support. First of all, it did not seem fitting to say that the students already held views about mathematics similar to those held by expert mathematicians. For instance, for the

VAMS item asking "When I experience a difficulty while studying mathematics: (a) I immediately ask for help, or give up trying (b) I try hard to figure it out on my own or with others", the means of the pretest and the posttest are 3.59 and 3.53, respectively. In addition, for the VAMS item asking about the teacher's role in mathematics class: "The role of a mathematics teacher is to (a) show me how to work specific problems (b) guide me in learning to solve problems," the means of the pretest and the posttest are 3.94 and 3.88, respectively. Thus, the quantitative data suggest that the students were more active learners at the beginning of the semester.

However, our class observation in the beginning of the semester appeared to contradict these results. It was observed that the students were often frustrated when they encountered ill-structured mathematical contexts. When the students were invited to justify their solutions, they tended to rely on knowledge from exterior sources. They often expected the instructor to tell them "the right answer." They regarded mathematics as law-like knowledge given by somebody with authority, such as well-informed people and textbooks. This kind of view about mathematics contrasts with the views about mathematics held by expert mathematicians who consider mathematics as "an endless and playful search for knowledge "(Burton, 2004; Ju, 2001; Pickering & Stephanides, 1997).

Based on this observation, it seemed that students' participation in the IODE class resulted in a conflict with the students' views about mathematics developed through their experience in previous mathematics classes. This discrepancy between the students' expectations and the classroom reality led to confusion and resistance. Moreover, the instructor explicitly stated the course expectation of the students' own construction of mathematics through active participation. The instructor took a proactive role to encourage the students to talk about what they thought in their own words. In general, by helping the students' reflect on their own mathematical practice, the IODE class appeared to create a ground for negotiation of students' notions of what mathematics is, how to do mathematics, and in general, their views about mathematics. Thus, our research team anticipated much more dramatic and fundamental changes in the students' views about mathematics than those measured by the VAMS.

B. Qualitative Analysis

As discussed, although the quantitative analysis matched our hypothesis on the positive impact of the IODE on the students' views about mathematics, there were several points of discrepancy from our class observation. This motivated systematic examination of the hypothesis based on qualitative data from the class. For this purpose, we took an interpretive approach to the students' classroom discourse under the assumption that language is the microcosm of a worldview carried by a speaker (Whorf, 1956). In the discourse analysis, each author looked for patterns of language use by the students to reflect the students' views about mathematics. We tested the validity of the discourse patterns by reviewing the transcripts and analyzed the patterns to confirm the cases most related to the students' views about mathematics. Through the process, we noticed that the students most often revealed their views about mathematics by their ways of positioning themselves in their mathematical arguments. The most salient pattern of language use related to the change in students' views about mathematics is the switch from "the third-person perspective" mode to "the first-person perspective" mode of speaking. In the analysis, we connected the dimensions developed in the VAMS such as methodology, validity, critical thinking, and so on with the students' perspective mode used in the discourse. For instance, in the VAMS taxonomy, "methodology" is one of the dimensions of philosophical aspects of views about mathematics. "Methodology" is concerned with notions about how to do mathematics, for instance, understanding the nature of mathematical methods, mathematical modeling, the role of technology in mathematical problem solving, etc. the following is an example of the third-person perspective discourse,

(4th session)
1. Student A: It should be e to the 0.2t.
2. Student B: How did we get the solution?
3. Student C: That's what the formula says⋯) write with e⋯
4. Student B: Ah⋯

This is a conversation between two students in a small group during the fourth session of the course. In this transcript, the students compare an exact solution with a numerical solution for a given differential equation $dP/dt = 0.2P$. Student A remembered that another student in class had presented a formula with the exact solution of an autonomous differential equation during the previous session. Student A recollected the formula and applied it to get the exact solution. When Student B asks for justification for the solution in line 2, Student C supported Student A by referring to the formula. However, her voice revealed some hesitation which showed that she did not feel confident with the formula. Student B, with some hesitation, accepted the solution produced by "the formula."

In this episode, Student C talked about mathematics in the third-person perspective. This kind of third-person perspective reflected the students' assumption that the agency of justification is "the formula," that is, mathematics. In other words, mathematics justifies itself. Thus, this third-person perspective discourse showed that the students perceived mathematics as a self-contained system of knowledge independent of how humans think. Mathematics has its own predetermined order which can be justified only by itself. In the third-person perspective narrative, the students regard themselves as passive agents to accept the given order imposed by mathematics and tend to rely on some authoritative sources such as formula, textbooks, and knowledgeable people. Such a tendency was reflected in other students' use of third-person perspective narratives:

(2nd session)
Teacher: What is the meaning of this equation?
Student: I don't know because I did not preview the textbook.
(2nd session)
Student: My friend taught me how to integrate this kind of differential equation.
(5th session)
"Uh... before... when we learned about this equation⋯ Professor derived an equation of P from dP/dt."

However, the IODE class emphasized the active role of the students in the construction of mathematics. As a result, the students began to see themselves as active practitioners of mathematics and this change is indicated by their use of the first person perspective discourse:

> (14th session)
> We computed the average values for the data from the table. We connected the points on the plane to find that the rate of change was decreasing. The rate of change became almost zero near 21. So we thought that the solution space has an equilibrium solution at 21 and the curves decrease near the equilibrium solution.

In this transcript, the student described the method used to mathematically model the situation of cooling coffee. After the brief description, the student described the solution in terms of its equilibrium solution and presented the conclusion. In the previous example of the third-person perspective discourse, students sought legitimacy of their mathematical practice from authoritative sources outside themselves. In contrast, in this episode, the student applied Euler's method creatively to model the situation of the temperature change. In this presentation, she did not attempt to seek an answer legitimized by outside authority. The student confidently presented the method and the conclusion of her mathematical activity. Furthermore, she positioned herself as an active agent producing mathematics and this way of positioning is revealed by use of the first person perspective discourse.

So far, we have discussed two different types of perspective mode used in the students' discourse in the IODE course. The discourse analysis shows that the students' speech patterns became transformed from the third-person perspective mode to the first-person perspective mode through their participation in the IODE class. This overall pattern of switching between the two types of perspective mode in mathematical narratives is parallel to the transformation of the students' views about mathematics, in particular, how they view themselves as agents of mathematical practice. In the beginning of the semester, the students often spoke in the third-person perspective mode, which suggests that mathematics is a set of truths independent of what the students think. In this regard, the students positioned themselves as passive recipients or consumers of mathematics in the practice of mathematics. Through the semester, the students gradually came to speak in the first-person perspective mode in which they talk about mathematics using their own voices. The students represented themselves as active agents of producing mathematics.

This transformation in the speech pattern confirms findings from the studies of scientific discourse. It is often believed that scientific discourse is decontextualized and the subjective involvement of a practitioner is minimal. However, this belief is criticized by some who claim that the stances of detachment in scientific discourse are culturally constructed and historically situated within the tradition of Western science (Burton, 2004; Latour & Woolgar, 1979; Latour, 1987; Lynch & Woolgar, 1990; Pickering, 1992). Research has analyzed professional scientists' everyday discourse to show that scientists use a number of referential practices for grammatically encoding their subjective involvement in scientific practice. Ochs and her colleagues analyzed the discourse of physicists working at a laboratory (Ochs, et als., 1994; 1996). One of the patterns that they identified is "physics-centered discourse" characterized by using agentless passive structures or syntactically active structures. In physics-centered discourse, scientists foreground inanimate physical entities as the thematic focus. Another pattern is the "physicist-centered" account of scientific phenomena, in which the scientists refer to themselves as the thematic agents and experiencers of the physical phenomena under inquiry. In the physicist-centered utterances, scientists encode themselves as thematic foci and position themselves as active participants in the making of science and scientific discovery. Ochs and her colleagues compared these two discourse patterns to argue that although much has been made of the different rhetorical effects created by the discourse practice depending on the kind of thematic focus, both patterns presuppose that scientist and objects of inquiry are separate and distinct entities. In this context, the analysis was extended to the cases in which physicists and physics are treated as blended thematic foci. Ochs and her colleagues interpreted these cases of indeterminacy as discursive practice which blurs the distinction between scientists and physics, that is, an extreme case of subjective involvement with science, by arguing:

> [R]eferential ambiguity is a necessary poetics of mundane scientific problem solving in that by using indeterminate constructions as a linguistic heuristic, scientists constitute an empathy with entities they are struggling to understand. Such a referential poetics allows interlocutors to symbolically participate in events from the perspectives of entities in worlds no physicists could otherwise experience. (Ochs, et als., 1996, p.348)

Studies of the practice of professional scientists reveal that science including mathematics is a cultural product and emerges through daily human practice. In this perspective, mathematics exists in the form of a formal system as an historical product. Mathematics as a system is dialectically reconstituted by the practice of mathematicians who take "interpretive journeys" (Ochs, et als, 1994) between the two intertwined realms of the system of mathematics and mathematical imagination (Burton, 2004; Ju, 2001; Pickering & Stephanides, 1992). Thus, in the practice of mathematics, professional mathematicians position themselves as active agents in the production of mathematics and mathematics flourishes through the human agents' "endless and playful" engagement.

Earlier, we discussed that the students' alternation between the different types of perspective mode is parallel to the different ways of positioning oneself in the practice of mathematics. In particular, these students' discourse in the first-person perspective mode resembled the professional physicists' "physician-centered" discourse in the sense that in the first-person perspective mode narrative, the students foreground themselves as active participants in the construction of mathematics. As the students gradually switched from the third-person perspective mode speech to the first-person perspective mode speech, the students interwove their mathematical ideas with previously shared mathematical meanings into their practice of mathematics. Through the process, they came to view mathematics as a system of knowledge endlessly growing through their engagement, and themselves as active producers of mathematics. Thus, the first-person perspective speech reveals the students' views about mathematics as lively human practice, which is close to the views held by the professional mathematicians.

V Conclusion: The Multilayered Reality and the Mixed Methods Approach

So far, we have focused on the description of the motivation and the process during our experience with a mixed methods approach. Next, the discussion will center on the issue of advantages and challenges of the mixed methods approach. One of the advantages of the mixed methods approach is the development of reliable evaluation tools. In our case, the analysis of the VAMS and of classroom discourse largely confirms the positive impact of the IODE course on the students' views about mathematics. This convergent validation increases the reliability of the analysis by cross-checking findings from diverse data sources.

Secondly, since quantitative data are embedded in a set of prescribed criteria, quantitative data may provide a tentative guide for qualitative data collection and analysis. In our case, when we began discourse analysis, we employed the dimensions presented by VAMS such as methodology, validity and critical thinking to organize how the students talked about mathematics. In this way, the quantitative VAMS analysis organized our qualitative discourse analysis.

The third advantage of our mixed methods approach is the fact that each method provided evidence of the change in the students' views about mathematics from a unique angle. In this regard, the convergence of findings does not simply mean the reproduction of identical results. In our research, even though the VAMS and the discourse analysis confirmed the change in the students' views about mathematics, each method revealed the change in different ways. This difference is coming from the unique convention that each research method adapts. Quantitative analysis determines whether there is a significant difference between two sets of data according to a predetermined scale. For instance, in the VAMS survey, a respondent's view about mathematics was transformed into a set of numerical values according to theoretical criteria embedded in the design of the survey. The numerical values indicate the extent and the category according to the scale employed. The VAMS analysis proved a change in views about mathematics by examining whether the difference between numerical values from pre- and post-tests was significant at a given level of statistic significance. The numerical values enabled us to deduce overall characteristics of the students' views about mathematics. However, it does not provide a detailed description of the students' views about mathematics, which is more often achieved by a qualitative analysis.

While quantitative analysis proves a difference in a prescriptive way, the conventional way that a qualitative analysis proves a difference is descriptive. For instance, the qualitative discourse analysis in our research developed a pattern based on the observation of the students' authentic language use in the class. Under the assumption that language use reveals the worldview of a speaker (Whorf, 1956), the discourse analysis identified and compared the discourse pattern emerging through the students' course participation in order to explain how the students' views about mathematics have changed in terms of the change in their language use. Moreover, a qualitative analysis tends to produce an interpretation from an insider participant's perspective. That is, our analysis of the participant's ways of speaking basically reflected how the students thought of doing mathematics and how they positioned themselves in the practice of mathematics. As a consequence, the qualitative analysis not only showed whether the students' views about mathematics have been changed, it also described what the change is like and what the meaning of the change is from the perspective of the participants.

In general, the mixed methods approach provides a chance for a researcher to elaborate his/her findings from various angles. However, various methods do not always lead to convergent findings and this is where a challenge arises in mixed methods approach reality. The reality that we live in is not simple and clear in its organization but rather complicated and multilayered. What we learn depends on the angle from which we approach. Thus, it is possible that the mixed methods approach produces inconsistent or contradictory findings. In this context, data synthesis is one of the most serious challenges in mixed methods approaches (Jick, 1979; Lawrenz & Huffman, 2002). However, in data synthesis, a researcher is recommended to avoid thinking in either-or ways about mixing methods (Lawrenz & Huffman, 2002). Our experience tells us that when data synthesis is done in complex, integrative ways, the challenge of inconsistent findings often turns out to be constructive.

For instance, in our case, the quantitative and the qualitative analysis led to contrasting results about the initial status of the students' views about mathematics. In the mixed methods approach, such inconsistency created a situation in which a researcher needed to determine on which data s/he would put a priority. Some researchers tend to put more priority on qualitative data because qualitative data are more likely to illuminate elements of a context than quantitative data. However, in our case, supplementary data such as documents made by the students were used for cross-checking. It turns out that the analysis of this supplementary data matched the result of the discourse analysis. However, instead of throwing away the result of the quantitative analysis, we began pondering the inconsistency that happened and what we could learn from the inconsistent results.

Surveys are usually conducted in a semi-laboratory context which triggers a specific type of thinking for a respondent. While a view about mathematics is an underlying and guiding principle of how to do mathematics in the authentic context of mathematical practice, it becomes the topic of investigation in a perceived laboratory setting during the survey. In this regard, a survey may reveal singled-out views decontextualized out of the mathematical context. In the beginning of the semester, one student said to her peers, "This (making a hypothesis and checking numerically) is how to do science." In this language, the student represented her views about how professionals out there-not herself-are doing mathematics. However, her advanced view about mathematics was not yet integrated into her practice of mathematics. This discrepancy suggests that the students' views about mathematics are like a mosaic. In some areas they developed advanced views about mathematics. Even though the advanced views are not yet fully integrated, they are part of the students' views about mathematics which may emerge someday. In that regard, their mosaic of views laid a potential learning trajectory which can be actualized through mathematical interaction in class.

Divergent findings are hurdles for researchers. However, they also set up a starting point for further investigation in order to unfold the multilayered organization of the reality and eventually to expand the horizon of our understanding. In our research, a mixed methods approach contributed to the reliable evaluation of the project through convergent validation. At the same time, a mixed method approach revealed the multiple layers of the students' views about mathematics by providing divergent findings. This understanding of the multilayered organization will extend the horizon of our understanding of mathematics teaching and learning and form a useful basis to plan instructional support in future teaching to actualize students' mathematical potentials.

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