Utilization of Revoicing Based on Learners' Thinking in an Inquiry-Oriented Differential Equations Class*

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Abstract

Researchers of mathematics education are increasingly interested in a teacher's discursive moves, which refer to deliberate actions taken by a teacher to participate in or influence debate and discussion in the mathematics classroom. This study explored one teacher's discursive moves in an undergraduate inquiry-oriented mathematics class. The data for this study come from four class sessions in which students investigated initial value problems as represented by the phase portrait of a system of differential equations. Through the analysis and a review of the literature, we identified four categories of a teacher's discursive moves: revoicing, questioning/requesting, telling, and managing. This report focuses on the roles of revoicing as it relates to the development of mathematical ideas and student beliefs about themselves and mathematics. The results show that the teacher used revoicing in the following ways: revoicing as a binder, revoicing as a springboard, revoicing for ownership, revoicing as a means for socialization.

Key words: Classroom discourse, Inquiry-oriented instruction, Revoicing, Differential equations

Ⅰ. Introduction

In past decades, school mathematics reform recommendations suggest that mathematics instruction should resemble the practice of mathematicians (e.g., Forman, Larreamendy-Joerns, Stein, & Brown, 1998). In other words, mathematics is fundamentally a discipline of inquiry in which mathematicians conjecture, prove and communicate their results to their peers. From this perspective, the inquiry-oriented mathematics class, which is the subject of the present investigation, was designed to provide opportunities for students to learn mathematics through active participation into the authentic practice of mathematics. Providing opportunities for students to learn mathematics in ways that simulate authentic mathematical practice requires that teachers bring their instructional methods in line with recent educational reform recommendations. In particular, reform documents emphasize the teacher's discursive role to facilitate and orchestrate students' practice of mathematics in the classroom (e.g., NCTM, 1991). In this regard, the analysis of teacher's discourse in relation to the changed teacher's role in the inquiry-oriented mathematics classroom has become a significant and timely research topic.

While there are clear calls for inquiry in both science and mathematics classrooms (e.g., National Research Council, 1996; NCTM, 1991; Richards, 1991), what exactly characterizes an inquiry-oriented classroom is less clear. To clarify the nature of inquiry-oriented classrooms and to provide a more comprehensive perspective on the complexity of teaching and learning, Rasmussen and Kwon (2007) characterize inquiry in terms of both student activity and teacher activity. In particular, students learn new mathematics by inquiry, which involves solving novel problems, debating mathematical solutions, posing and following up on conjectures, and explaining and justifying one's thinking. The first function that student inquiry serves is to learn new mathematics by engaging in genuine argumentation. The second function that student inquiry serves is to empower learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity. On the other hand, teachers also engage in inquiry. Teacher inquiry centers on inquiring into their students' mathematical thinking and reasoning. Teacher inquiry into student thinking serves three functions. First, it enables teachers to interpret how their students build mathematical ideas. Second, it provides an opportunity for teachers to learn something new about particular mathematical ideas in light of student thinking. Third, it better positions teachers to follow up on students' thinking by posing new questions and tasks.

This paper focuses on the teacher's revoicing in an inquiry-oriented classroom, because it is one of the discursive strategies that often occurs in the teaching of mathematics, but which has received limited attention in mathematics education research at the undergraduate level. In work at the K-12 level, Forman et al. (1998) highlighted revoicing as a critical feature of a teacher's discourse by which s/he orchestrates students' discussion. They found that a teacher recruits students' attention to point out important aspects of students' argumentation through revoicing. O'Connor and Michaels (1993) characterized revoicing as affording the teacher tools to coordinate the elements of academic task structure and social participation structure, while simultaneously bringing students into the process of intellectual socialization.

Influenced by the work of Forman et al. (1998) and O'Connor and Michaels (1993), we approach teacher's revoicing as a discursive move, which is defined as teacher's deliberate actions situated within the context of the mathematical communication (Krussel, Edwards, & Springer, 2004). Our broad goal is to contribute to the field's understanding of the complicated process of the co-construction of mathematics in an inquiry-oriented mathematics classroom. More specifically, we investigated how a teacher's revoicing can facilitate the co-construction of undergraduate mathematics in an inquiry-oriented differential equations (IODE) classroom. We take the perspective that revoicing can play an important role promoting both student and teacher inquiry.

Ⅱ. Theoretical Background

Since the 1970s, educational researchers have adapted sociolinguistic perspectives to examine a teacher's discursive moves in classroom settings. Early studies were interested in the sequential pattern of the interaction of a teacher and students. For example, Mehan (1979) suggested an Initiation - Reply - Evaluation (IRE) pattern as a basic elicitation sequence. Whereas Mehan's construct suggested that traditional teachers often fall into a pattern in which they funnel correct answers by evaluating students' short responses, Bowers and Nickerson (2001) observed a cyclical pattern in each phase of a concept-centered class. In the concept-centered class, when the teacher initiated a new activity, it was observed that the interaction pattern included teacher's elicitation, student's response, and teacher's elaboration (ERE pattern). In addition, they observed another type of communicative routine in which the teacher or a student would make a proposition, and others would discuss it (PD pattern).

While previous studies approached teacher's discourse as a communicative routine in a certain sequential order, the present study adapts the notion of discursive move to analyze a teacher's discourse as an "action" that a teacher deliberately takes in the context of communication (Krussel, Edwards, & Springer, 2004). This notion of discursive move emphasizes the mutual relation between a teacher and students in classroom discourse. That is, when considering teacher's discourse as action, it emphasizes the teacher's intention to participate in the on-going classroom communication and to influence the flow of the communication as one of the participants. In studies about teachers' discursive moves in mathematics classes, researchers have identified diverse verbal forms such as telling, questioning, and revoicing, and have discussed their significance in the teaching and learning of mathematics. For example, Lobato, Clarke, and Ellis (2005) analyzed the role of telling as a way of stimulating students' mathematical thoughts via the introduction of new ideas into a classroom conversation. Clegg (1987) characterized teacher questioning as strategies to review, check on learning, probe thought processes, pose problems, seek out different or alternative solutions, and challenge students to reflect on critical issues or values they had not previously considered. Boaler and Humphreys (2005) posit that questioning helps students develops critical mathematical concepts in student-centered learning environment.

In addition to telling and questioning, revoicing is another discursive move that teachers use to facilitate students' learning. Revoicing involves the reuttering of another person's speech through repetition, rephrasing, expansion, and reporting (Forman et al., 1998). O'Connor and Michaels (1996) focused on the notion of revoicing to illustrate that the instructional process depends upon the skillful orchestration of classroom discussion by the teacher. They claim that revoicing by the teacher may change the way students see themselves and each other as legitimate participants in the activity of making, analyzing, and evaluating claims, hypotheses, and predictions. Forman et al. (1998) emphasized that the teacher is able to orchestrate discussion through revoicing by recruiting attention and participation from students in the class, aligning learners with argumentative positions through reported speech, highlighting positions through repetition, and pointing out important aspects of their arguments through expansion. Also, Forman and Ansell (2002) found that the teacher legitimized student contributions to the discussion by revoicing their arguments.

Researchers have shown that revoicing is one significant form of a teacher's discursive moves in reform-oriented classrooms. The analysis in this article contributes to this emerging body of research by examining how a teacher's revoicing can support the co-construction of mathematics in an inquiry-oriented mathematics classroom. In the analysis, we approached teacher's revoicing as situated within the context of the classroom practice of mathematics; in other words, we consider revoicing as a teacher's action allowing her/him to participate in the collective construction of mathematics with students. Thus, instead of singling out teacher's revoicing for the analysis, teacher's revoicing is considered as a type of discursive move that is integrated with students' discourse, which in turn contributes to the collective building of mathematical ideas and dispositions.

Ⅲ. Method

Our research team has been engaged in conducting teaching experiments in undergraduate differential equations for the past eight years. The resulting IODE course materials were inspired by the instructional design theory of Realistic Mathematics

Education (RME) (Gravemeijer, 1994). In particular, the materials recruit situations (real world situations and mathematical situations) that are experientially real for learners. Instructional tasks are organized into a sequence of questions designed to promote student mathematization. The materials have been revised through teacher reflection and detailed analysis of student thinking over the course of teaching experiments at different sites (e.g., see Rasmussen & Keynes, 2003; Rasmussen, Stephan, & Allen, 2004).

An important consequence of RME design principles for teaching practice is the necessarily proactive role by the teacher in supporting students' reinvention of mathematical ideas and methods for solving problems (Rasmussen, & Marrongelle, 2006). In this regard, the IODE approach builds on what Richards (1991) refers to as an "inquiry-oriented" instructional model, in which important mathematical ideas and methods emerge from students' problem-solving activities and discussions about their mathematical thinking.

The data for this analysis came from a fifteen-week IODE course taught in a large state university in the United States in 2005. The course was taught by one of the authors of this paper. Eight of these fifteen weeks were video-recorded with two cameras. In this article, we focus on the video recordings that were captured during four consecutive class sessions. In those sessions, students investigated a system of differential equations to learn how to draw solution curves using straight line solutions. All utterances of both the teacher and the students were transcribed. Each of the four class session transcripts was uploaded into an Excel spreadsheet. Teacher and student utterances were placed on individual rows of the spreadsheet. An example of a spreadsheet is shown in Table 1. The teacher's utterances are on lines 188, 190, and 192. The student's utterances are on lines 189 and 191. Each utterance was given a Main Lesson Code of either whole class discussion or small group work, to denote the setting within the class in which the utterance took place. Each utterance was assigned a Discourse Move Code (described in more detail below). A space for recording coding notes and observations was also included for each utterance.

Line Number	Main Lesson Code	Discursive Move Code	Discourse	Comments
188			Okay, Brian, why don't you say a little bit. Do you want to come up here to the board. So, Brian and Jeff had a way to think about this and they were using this form of the x(t) and y(t) equations, so come and show us your arguments.	
189			For this one?	
190			Yeah, initial condition, (-4,6).	
191			Why it curves which way?	
192			Yeah, why it curves which way.	

Table 1: Sample of Excel Spreadsheet Coding Template

Through analysis of related literature, we identified four broad categories of teacher's discursive move: (1) revoicing; (2) questioning/requesting (3) telling; and (4) managing. Revoicing is broadly defined as reuttering or saying again (could be verbal, symbolic, or gestural) of someone else's utterances (symbolizing or gesturing). Questioning is a discursive move in which a teacher checks for understanding, requests to explain thinking, requests to justify thinking and so on. Telling is defined narrowly as stating information or demonstrating procedures in the more traditional sense (Smith, 1996) in order to clearly distinguish this form of discursive move from others. Managing is consisted of arranging, directing, motivating, and checking.

Beginning with our review of the literature, we made preliminary observations of video and simultaneously highlighted the teacher's discursive moves in the transcripts. A coding scheme was then developed. We refined and revised our coding scheme based on further review of the literature (e.g., Forman et al., 1998; Krussel, Edwards, & Springer, 2004; Lobato, Clarke, & Ellis, 2005; Mehan, 1979; Smith, 1996) and multiple passes through our data. The collaborative coding procedure by multiple members of our research team minimized biases by each individual researcher and eliminated interpretations not grounded in the data. When a more stable coding scheme emerged, we applied it to the whole set of data to check whether the coding scheme could cover all the cases from the classroom discourse. The coding scheme developed into a more comprehensive set of codes through this process.

Of the four different discursive moves, revoicing, questioning/requesting, telling, and managing, we found that revoicing accounted for over 22% of the teacher utterances, and hence represented a significant portion of the teacher's discursive moves. Teacher revoicing may be a direct restatement or it may involve an adaptation of the original utterance. It may or may not include a short follow up question to determine if the revoicing was consistent with what the student said. Consistent with Forman et al. (1998), we distinguished four different types of revoicing: repetition, rephrasing, expansion, and reporting. Repetition occurs when a teacher repeats a student's utterance using the same words or a portion thereof. Rephrasing is when a teacher states a student's utterance in a new or different way, but without adding significantly new or different information. Expansion is similar to rephrasing in that a teacher restates a student's utterance in a new or different way, but also adds something significantly new or different. Reporting occurs when a teacher explicitly attributes an idea, claim, and argument to a specific student. This explicit attribution of reporting will then be in the form of repetition, rephrasing, or expansion, and therefore all reportings were double coded.

In our view, these different forms of revoicing are strongly related to the development of mathematical ideas and students' beliefs about themselves and the nature of mathematics. In particular, repeating, rephrasing, and expanding enable students to learn new mathematics through their engagement in genuine argumentation. Reporting empowers learners to see themselves as capable of reinventing mathematics and to see mathematics itself as a human activity. The following section details more thoroughly the role of revoicing in inquiry-oriented classrooms and hence furthers our understanding of the different functions of revoicing that facilitate student inquiry.

Ⅳ. Results and Discussions

Our analysis indicates that the teacher's revoicing carries out critical functions in the process of collective construction of mathematics in the class. In particular, our analysis addresses the questions: How did the teacher's revoicing facilitate the co-construction of mathematics in the IODE? What happened when the teacher participated in the mathematical communication by revoicing? Through our analysis, we identified four functions of revoicing, outlined below, and addressed in the episodes that follow.

A. Revoicing as a binder

O'Connor and Michaels (1993, 1996) argued that a teacher's revoicing works to signal that a mathematical position has been identified and that a speaker is aligned with a certain position. In our analysis, the teacher's revoicing created a context for students to bring up and align with diverse mathematical positions, which supported the negotiation of mathematical meaning. In the context of negotiation, the teacher continued to recast upcoming students' positions to highlight the trajectory of the students' practice of mathematics and to reveal the mathematical connection behind the students' claims. In this way, a teacher's revoicing enables students to attend to critical ideas in order to generate more comprehensive mathematics by connecting diverse perspectives. We show an example of revoicing as a binder in Episode 1.

B. Revoicing as a springboard

We found that a teacher's revoicing recruits students' attention to a specific claim and prompts the speaker to clarify and elaborate her/his own claim. Thus, a teacher's revoicing provides scaffolding for students to clarify, to elaborate, and to extend their mathematical positions through reflection. Moreover, the concepts highlighted by a teacher through revoicing subsequently come up in the small group discussion and shape students' follow-up inquiry. This suggests that revoicing plays the role of springboard in the inquiry of students. We show an example of revoicing as a springboard in Episode 1.

C. Revoicing for ownership

Teacher's revoicing makes reference to whom the mathematical position belongs to and helps every classroom participant make sense of it. Also when the mathematical concepts or contents that the teacher wants students to discuss do not appear fully, revoicing enables a teacher to reveal available mathematical resources rising in the voices of students. As a consequence, mathematics is represented as being collectively constructed by the course participants themselves instead of being given by the teacher. In this regard, revoicing creates a sense of the classroom as a community of practice and a sense of mathematics as their own practice. We do not show an example of revoicing as ownership in this paper, but have discussed examples elsewhere (e.g., Kwon, et al., 2008).

D. Revoicing as a means for socialization

In revoicing, a teacher can demonstrate the cultural way of doing mathematics to support students' transformation as practitioners of mathematics. In this regard, teacher's revoicing contributes to transform students' practice of mathematics and ultimately to support their socialization into the cultural organization of mathematics community. We show an example of revoicing as a means for socialization in Episode 2.

The following episodes illustrate the teacher's revoicing and its roles in the collective construction of mathematics in the class.

Mathematical Episode 1

The four class sessions used in the analysis encompass a teaching sequence in which students reinvent a method for identifying lines of eigenvectors (heretofore referred to as straight line solutions) and using the eigenvectors to find solutions to systems of linear differential equations. Prior to this first episode, students concluded that straight line solutions lie on the lines y = −x and y = −2x for the system of differential equations dx/dt = y, dy/dt = −2x−3y. The next task in the sequence involves students finding the solution equations for initial conditions that lie on either straight line solution. Figure 1 shows a problem prompting students to reason about the long-term behavior, or trajectory in the phase plane, of the solutions with initial conditions (−2, 4) and (−3, 6).

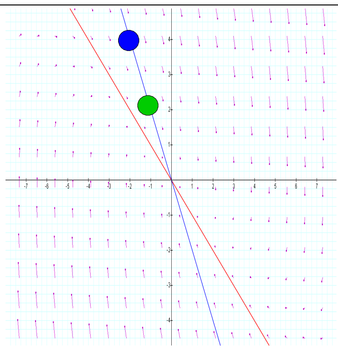
System of differential equations	Straight line solution	
$\frac{dx}{dt}=y$ $\frac{dy}{dt}=-2x-3y$	$y=-x$ $y=-2x$	

Figure 1. The problem for mathematical episode 1

This first episode is taken from a whole class discussion concerning how the solution curves for initial conditions (−2, 4) and (-3, 6) behave in the phase plane. Students took up questions such as: Do the two solution curves move in the same direction or different directions? Do the solutions move closer together, further apart, or does the distance between them remain the same? In this case, the teacher began the whole class discussion by inviting students to share their ideas about the behavior of the solutions with initial conditions (-2, 4) and (-3, 6). Harry was the first to present his group's thinking:

Teacher: Tell us what you are trying to think about as you're moving those.

Harry: **Keeping the same distance** and move along the straight line.

Teacher: So, you think ***the same distance?***

Students: No.

Teacher: What did you mean by that then? Do it there for us because you did ***keep the same distance***, right?

Harry: No.

Teacher: I mean ***the distance between the two points.***

Harry: I guess this one would go towards zero as this one moves closer to that one. Wouldn't it?

Teacher: Robert?

Robert: I don't agree. I don't think they should keep the same interval all the way towards zero. I think the top one, you got it right the first time actually go to faster.

Teacher: Do you want to come up and show us what you think?

Robert: It'll go like **one will move faster than the other. Not necessarily meet at the same time, but meet not at the same distance** [inaudible]

Teacher: So, you're saying ***they start here and this one starts to catch up.***

(italicized and bold faced for emphasis)

In this episode, Harry claimed that the curves move along the straight line toward the origin and keep the same distance. The teacher repeated "the same distance" from Harry's claim to ask clarification and Harry elaborated his claim. Then, instead of evaluating Harry's claim, the teacher called on Robert, who challenged Harry's claim. After Robert's presentation, the teacher summarized Robert's claim by rephrasing, "they start here and this one starts to catch up."

In this episode, one of the teacher's major discursive moves is revoicing. The teacher's revoicing fulfills several functions to facilitate and orchestrate students' communication in this episode. First, the teacher located Harry's position by repeating and rephrasing his claim. Also by rephrasing Robert's claim, the teacher aligned him with another position. This means that the teacher repeated or rephrased a student's claim to signal that a mathematical position has been identified and to align a speaker with a certain position. Second, the teacher's revoicing recruited students' attention to a given claim and prompted the speaker to clarify and elaborate the mathematical meaning of the claim. With these functions, instead of directly instructing or evaluating, the teacher's revoicing ultimately led the students to raise diverse mathematical positions for the negotiation of mathematical meaning. In this episode, the teacher's revoicing highlighted diverse mathematical positions raised by students and promoted negotiation of these positions. We interpret this as meaning that the teacher's revoicing connects diverse students' perspectives like a binder.

The whole class discussion of the behavior of the solutions with initial conditions (-2, 4) and (-3, 6) continued for some time. The following three excerpts from the whole class discussion occurred some time after the previous discussion with Harry and Robert. Due to space constraints, we cannot include all of the entire class discussion. Hence the following three excerpts represent relevant pieces of the whole class discussion. The ellipses represent omissions in the transcript.

Mike: I'm thinking that these points, they curve downward towards zero, **but never really touching zero.**

Teacher: He said that in kind of a question. So, do you agree or disagree with what he just said?

Karine: [inaudible]

Teacher: So you agree that ***they approach zero, but don't touch zero.***

(……)

Student: The graphs on the x(t) and y(t) plane are both negative **exponential.**

Teacher: Ok, ***exponential.*** All right, that is a good justification.

(……)

Emilian: So it would just be a **multiple** of itself, I guess.

Teacher: And the other point you were making to relate is that these are the same graphs, one has just shifted the other in 3 space. You just have ***multiple shifts.*** Just like in our case for autonomous differential equation for a single DE, 3-D graphs shifts along each other along the t-axis.

Again, we see the teacher repeating and rephrasing students ideas during this whole class discussion. Next, the teacher asked the class, in their small groups, to provide arguments for or against the student ideas about the behavior of the solutions with initial conditions (-2, 4) and (-3, 6) presented in the whole class discussion. In the following excerpt of one small group discussion we emphasize some of the students' discourse with bold letters. It is possible that these students' utterances reflect the teacher's revoicing in the earlier whole class discussion.

John: **Never touches zero.**

Diane: Okay, never touches zero because it's an **exponential.**

John: It's a **shift** on the t-axis. Same solution.

Diane: Because it's in terms of x and y.

John: It's always a **multiple** of itself, so t would give a different.

Sam: I think it's more like uniqueness, but oh well.

Diane: Right.

John: What would you do for that?

Sam: I don't know. I don't really have a strong opinion.

John: I'm interested in this thought. So, dx/dt = y. Right. And then take the partial derivative of that? NO, no, because we can say dx/dt = -2x and the partial derivative of that would be the partial of x with respect to -2?

Sam: Kay.

Aden: Can you explain to me why you wanted to take the partials?

Sam: Partials, because that's one of the things that was described by the uniqueness theorem. That was like one of the rules. So, I'm assuming we use that.

John: Well, I guess to build on that

Aden: How would you want to use it?

Diane: We're trying to figure out **whether or not it touches zero.**

We argue that this episode illustrates how a teacher's revoicing can highlight critical concepts and ideas under discussion so that the students might adapt those concepts and ideas into the follow-up inquiry in their small groups. The discussion in this small group eventually led to the uniqueness theorem of the second order differential equations followed by student's attention of the teacher's revoicing in the whole class discussion. One interpretation is that the teacher's revoicing ultimately worked as a springboard for students' construction of mathematics.

Mathematical Episode 2

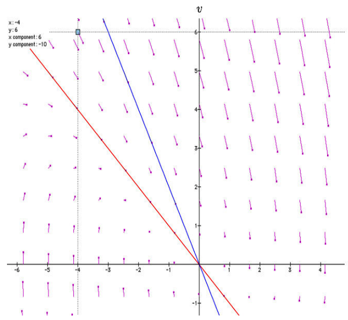
System of differential equations	Straight line solution	
$\frac{dx}{dt} = y$ $\frac{dy}{dt} = -2x - 3y$	$y = -x$ $y = -2x$	

Figure 2. The problem for mathematical episode 2

So far, we have illustrated a case in which students made claims without justification. Now we present an example where students were asked to provide justification for their mathematical claims. The task was to sketch the solution graph in the phase plane, as illustrated in Figure 3. In the whole class discussion, several students claimed that the solution graph was not a straight line and the teacher asked them to provide justification. After Harry's justification, the teacher used revoicing to expand it by introducing useful mathematical concepts.

Teacher: Another reason. Anyone have a different reason. Harry?

Harry: Well, first of all we assumed that t**here was a straight line solution and then we derived it through the dy or the um finding *x(t)* and *y(t)* and they did not come out to have the same powers in the huh exponents.** So, we had a contradiction. We concluded that there was no straight line.

Teacher: I see a couple of frowns. Like, huh? Um, let me write something on the board and tell me whether I just misconstrued. So, Harry said, ***suppose it were a straight line, then if you were to calculate the dx/dt and dy/dt components,***

> ***the ratio of the components, the dy/dt and dx/dt ought to be the exact same ratio as the y to the x. I mean that's how you get a straight line is that you have so, 1. If it were on a straight line, then we have to have dy/dt/dx/dt = dy/dx = y/x (ratio of y/x).That would have to be the case to be on a straight line. Your resultant vector, the dy/dx, would have to be exactly the same components of dy/dt, dx/dt as y to x. You have to have that proportionality going on.*** Well, let's see if we do have it. All right, well, if we're at the point, um, we're at the initial condition here.

In this case, the teacher's revoicing provided the mathematical foundation for the validity of Harry's justification. In other words, the teacher expanded the student's mathematical arguments for elaboration by bringing up the related formal concepts. In this way, the teacher's revoicing functioned as a bridge between a student's mathematical reasoning and the formal structure of mathematics. In other words, revoicing is a way that the teacher can demonstrate how to speak in the formal language of mathematics and demonstrates the cultural way of reasoning and speaking about mathematics that is shared in the community of mathematics. Since mathematics is communal practice, there is a set of norms that confers legitimacy to a practitioner's practice of mathematics. In addition to the system of mathematical facts and skills, the norm of how a teacher does mathematics in the classroom is an essential aspect of mathematics that students need to learn, but not readily teachable through direct instruction. This episode shows that a teacher's revoicing is a way to demonstrate the cultural way of doing mathematics in order to scaffold students' mathematical practice for their social transformation as practitioners of mathematics.

Ⅴ. Conclusions and Implications

Our analysis shows that a teacher's revoicing can constitute a major repertoire of his or her discursive moves and carries out critical functions in the context of mathematics practice in class. From that perspective, we have illustrated the roles of revoicing, in particular focusing how a teacher's revoicing facilitates the co-construction of mathematics through mathematization in the an IODE classroom. Specifically, in the collective construction of mathematics, our analysis shows that teacher's revoicing carries out the following three functions: Revoicing as a binder, Revoicing as a springboard, and Revoicing as a means for socilaization. Elsewhere we have argued that revoicing also has a fourth function: to assign ownership of an idea to a particular student (Kwon, et al., 2008).

Historically, differential equations have been invented as a language to express certain laws of nature. However, the conventional teaching and learning practice of differential equations heavily relies on drill and practice. It can hardly be said that students learn the historical spirit of differential equations. The development of the IODE approach has been initiated by the reflection on how to reform teaching differential equations in order for students to learn differential equation as a language for talking about their world.

It has been shown that the IODE approach positively contributes to students' conceptual understanding, problem solving, retention, justification, and attitudes toward mathematics (Cho, 2003; Ju, & Kwon, 2004, 2007; Kim, 2006; Kwon, Cho, Ju, & Shin, 2004; Kwon, Park, Kim, Ju, & Shin, 2004; Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006; Yackel, & Rasmussen, 2002). However, we still have to resolve the notorious dilemma of an inquiry-oriented mathematics classroom for teachers, that is, "how to teach without teaching?" In this paper, we have struggled with this dilemma by looking deeply into how the discourse move of revoicing can be a valuable resource for a teacher to guide students in the reinvention of mathematics. In this regard, this article provides an understanding of how a teacher can invite students into the classroom practice of mathematics and engage with students in the collective construction of mathematics. This study of revoicing can be extended by investigating the function of revoicing in conjunction with other verbal forms such as questioning in order to provide useful guidance for teachers how to effectively fulfill their role in an inquiry-oriented mathematics class.

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