Qualitative Approach to Graphs of Functions in a Microworld

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Abstract.

This paper deals with a teaching experiment of how 6th grade students understand the rate of change with covariational reasoning in a LOGO-based microworld JavaMAL. We introduce the function graphs using the traces of a turtle swimming in the JavaMAL microworld in which textual commands are used not only for controlling the turtle's motion but also for modifications and reflections on students' thinking. We found that visual and symbolic representations for the rate of change can work together for a meaningful qualitative approach to the graphs of functions.

Key words : LOGO, JavaMAL, Microworld, Graph, Vector

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1. Introduction.

Along with the development in computer technology, the pedagogical studies of teaching variables and functions in new environments have also advanced. The research studies can be viewed as branching out into two broad fields: one type of study focuses on learning variables and functions by using symbolic processing or programming and the other type of study deals with it using the visuality of computers. The qualitative calculus (Stroup, 2002) approaches the understanding of function not as correspondence, which is its definition in modern mathematics, but as covariation which is more intuitive. The covariation of function can be seen as the change of a motion with the visual and active environment. The Computer-based Ranger (Berry et al., 2003), Motion Detector (Nemirovskyet al., 1998) and Math Worlds (Kaput, 1998) are the tools for a qualitative approach to function and its graph. These research studies deal with the relation between the speed vs. time graph and position vs. time graph using the visual and active tools. In particular, measuring the rate of change using a tool is the main topic in this research.

Our study begins by examining the possibility of introducing the concept of a speed vs. time graph or a position vs. time graph to younger students by means of intuitively and visually clear actions. Having introduced the graph through intuitively clear real situation to students who lack comprehension of such concepts, our study will deal with the function graph within such context and the rate of change by using a qualitative approach. For this purpose, we introduce function graphs in a more intuitive manner by using the trace of a turtle swimming in a river. For these implications, we design a LOGO-based JavaMAL microworld[1] in which the various mouse and textual commands including the following command are available to control the speed of the river and the speed of the turtle simultaneously:

move a, b
moves turtle horizontally **a** steps and vertically **b** steps

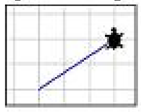


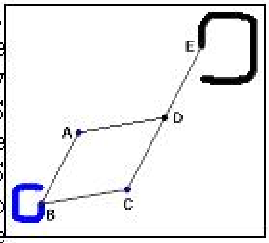
Diagram 1: move command and move 3, 2

Covariation focuses on the relationship of change between independent variables and dependent variables. The amount of change within graph expression can be represented by (x, y). Students can express their thoughts through the turtle's movements from such covariational viewpoints by using "move x, y" as the medial representation. In other words, it acts as an intermediary within the process of attaining mathematical expression from formal system. By employing a teaching method which uses an environment that illustrates the turtle's movements, we attempt to find out how students comprehend the rate of change when given new commands within the tool and how such a tool ought to be designed. We are trying to examine the viewpoints of both the students and the researcher in terms of the employment of the command provided above. Our study deals with JavaMAL microworld with the addition of a new command, "move".

[1] JavaMAL microworld is an internet based java applet which allows not only LOGO but also DGS environment through on-line interaction using textual commands that can be saved on the internet based board (http://edunet4u.snu.ac.kr).

2. Representations and JavaMAL microworld.

Carlson et al. (2002) and Laborde et al. (2001) state that the covariation point of view clarifies the notion of change[2], which is an intuitive characteristic of function, and assert that the rate of change should be introduced not as numerical but as visual motion. Compared to the period of paper-and-pencil which lacked the representation system that can show change visually, the development of computer technology has provided ways of learning functions such as the change of motion. Namely, the development of visual representation systems (Kaput, 1998) has enabled ways of introducing the graph of function and the rate of change to younger learners intuitively and visually. Nemirovsky et al. (1998) and Berry et al. (2003) each formulated an environment by using a motion detector and computer-based ranger in order to allow students to approach the graph intuitively and visually. Such development of new representation system emphasizes



[2] Goldberg et al. (1992) and Laborde et al. (2001) discuss the acquisition of the concept of function in a dynamic geometry system DGS. We support the view that DGS is a new context which shows the change of function, and the JavaMAL DGS microworld is designed so that it is easy to endow dependency among points. From the right side figure, points A, B, C are free points, point D is a dependent point which turns the quadrilateral ABCD into a parallelogram, and E is a symmetrical point of C and D. This problem of making prediction of the movement of E when B is moved is related to the concept of function composition, and is desirable in context which deals with the change of functions (geometric and algebraic relations).

more on the "knowing with" point of view rather than the "know how" or "know what".

Along with LOGO, JavaMAL microworld's distinctive feature is that it itself is a form of programming language. JavaMAL programming allows teachers to construct an environment in which mathematics can be materialized and interaction with computers can be made possible. In addition, JavaMAL microworld allows students to express their thinking through symbolic language so that the students' expressions can be used for modifications and reflection. Our researchers have constructed an environment, illustrated in figure 1, which uses JavaMAL command to describe the movement of the turtle in accordance with the position of the speed control bar. Let us examine how the students understood the rate of change by using this environment.

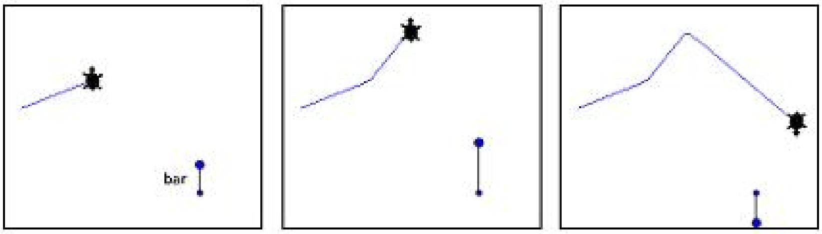


Figure 1: swimming turtle controlled by the speed control bar

3. Episodes.

We now provide two episodes concerning the rate of change. We analyzed the recorded videotapes of the 6th grade students while they were working out the problems in a classroom with computers. The problem is to write out the construction order of the positions of the speed control bar for the given swimming trace as in figure 2. Two types of answer given by two 6th grade students (12-13 years of age) are provided in figure 2.

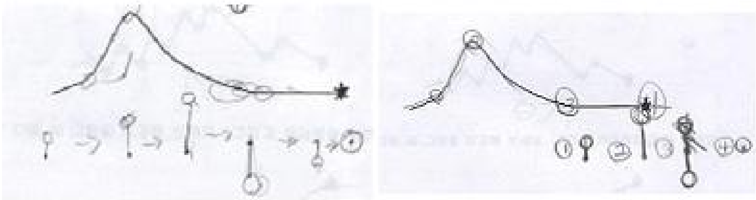


Figure 2: representing the rate of change using the speed control bar

Initially, we paid attention to the fact that the position of the bar enumerates in accordance with time. The forms of the speed control bar are shown in order following its distinctive position. When we connect the moving points of the bar at the bottom of the students' answer, it becomes the speed vs. time graph. The answer on the left is a continuous form, but the one on the right reveals the bar for the specified moments. Although there are slight differences, both answers form the qualitative approach to the rate of change. Subsequently, by using the experience of the construction through the given tool as background, we requested the students to try to formulate commands to draw their designed pictures by introducing "move a, b", as shown in figure 3. Figure 1 illustrates an environment that can provide a tool perspective for the rate of change by action on speed control bar, and figure 3 illustrates the students' process of making the swimming trace by using the textual command on their own.

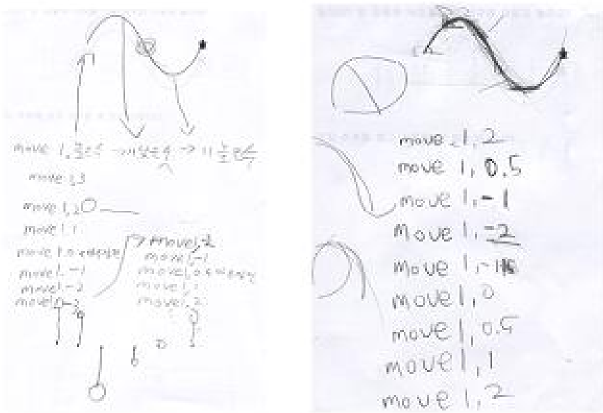


Figure 3: representing the rate of change using move commands

At this point, it is important to focus on the fact that one emphasizes the action while the other stresses the symbolic representations for the process of making. In particular, "move 1, a" becomes the medial language which allows the students to express their thoughts of the turtle's movement which is swimming in a river, where a is the speed of the turtle and the river is flowing from left to right in the constant speed 1.

For a better understanding of covariational reasoning, we modified the tool environment so that the speed of the turtle was fixed while changes in the flow of the river could be manipulated instead. Could the students use the medial language, "move a, b" without confusion? We observed how the students expressed this context which corresponded to inverse function with their own languages, and the tool was designed in the same manner and the speed control bar in the given tool moved horizontally to control the speed of the river. Subsequently, the opportunity for students to employ commands was provided. One student's answer is shown in figure 4.

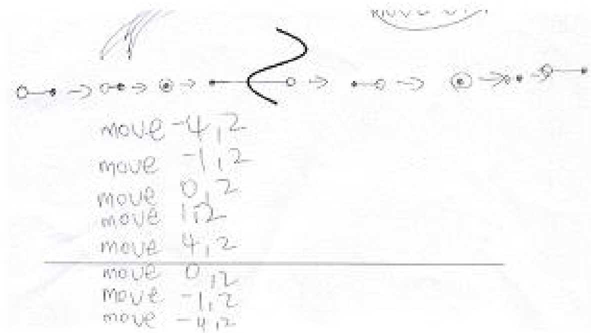


Figure 4: the reversed problem of representing the rate of change

Both situations point out that the students considered the speed control bar they had experienced before. Namely, the students tried to construct the commands that were converted to numerical value from their thoughts of where and how the length of the bar changed. "Move a, b" is a command that represents moment to moment movements. Although this does not differ completely from the point of view of the rate of change by limit, we think that it can act as a medial language within the process. In particular, by using "move", it is possible to explain the change of position as results of increase and decrease, or convex and concave.

The episodes discussed above illustrate the possibility of applying a qualitative approach to characteristics of function such as increase and decrease, and rate of change for younger students. The command is utilized and given in accordance with the speed control bar's movement. This is a step towards interiorization which then has to be condensed and go through the process of reification (Sfard, 1991).

4. Implications.

The above pedagogical study concerning a qualitative approach to graphs for elementary school students clarifies the possibility of applying a qualitative approach to the rate of change if appropriate visual and symbolic representations are available to the students. Thus, we designed a microworld so that it can provide flexibility in designing such a microworld situation to allow the students and teachers to interact freely. This paper suggests two things: First, prior to introducing function as correspondence to students, it is possible and advisable to introduce the graph intuitively and visually. It is desirable to use the formal symbol system during the process in which students continuously have experienced functional thinking and have gone through reification and transferred their actions to words. If formal definition is given when the students have not fully experienced the context, it can deprive them of the opportunity to understand its true nature. This opportunity can be provided through their usage of medial language.

The second thing is that computers should be actively applied from a "knowing with" point of view as in Early Algebra. In this regard, as the teachers adjust to the reactions of the students, the learning environment should be designed with room for readjustment by the teacher or the researcher, and should provide ways in which the computer and each student can communicate through medial language. The researcher (teacher) can design a tool by using microworld which allows a qualitative approach to functional context. Through microworld, the students interact with the tool by means of the learning environment which provides a possibility of constructing function on their own. In addition, the students express their thoughts in microworld using such experience as their background, and the teacher readjusts the tool environment in accordance with their reactions. At this point, by understanding the concept of rate of change and its corresponding function through a situational qualitative approach, the students are able not only to think and reflect using medial language but also to construct and realize their thoughts within a microworld context.

This paper only introduces a qualitative approach to function and its graph. Subsequent study should examine the teaching environment in which vertical mathematization[3] is reached through interiorization, condensation and reification.

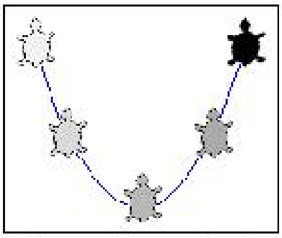


Figure 5: quadratic

The command "move a, b" can be used not only for learning the graphs of functions but also for teaching vectors. Figure 5 is the graph of quadratic function constructed by the turtle movement represented by the command "for i =-30 to 30 ; move 1, i ; next".

"Move a, b" also has the meaning of a vector, v=(a, b), and through this command the operation of the vector can be shown visually. Teaching suggestions concerning such subject should be examined in subsequent research.

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[3] **Freudenthal(1991) stated that "horizontal mathematisation leads from the world of life to the world of symbols, while vertical mathematisation means moving within the world of symbols."**

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