Designing a Microworld for Mathematical Creativity and Gifted Education*

Han Hyuk Cho, Suh-Ryung Kim, Hyuk Han,
Manyoung Jin, Hwakyung Kim and Minho Song**

Department of Mathematics Education
College of Education, Seoul National University

Abstract

The purpose of this paper is to introduce a microworld designed for mathematical creativity and gifted education. We have developed a microworld named JavaMAL by combining LOGO and DGS microworlds. Using JavaMAL, students can make and manipulate semi-dynamic objects called "tiles" to explore mathematics. We have tested JavaMAL for creative mathematics program and gifted education, and we have confirmed that there are positive educational results from the teaching experiments.

Key words: tile, semi-dynamic, LOGO, JavaMAL,
microworld, DGS, creative environment,
gifted education

** About the authors
Address: Department of Mathematics Education, College of Education, Seoul National University, Shinlim-dong, Kwanak-gu Seoul 151-748, Korea;
E nail: hancho@snu.ac.kr, srkim@snu.ac.kr, hyuk0916@snu.ac.kr, jin4489@snu.ac.kr, indices2@snu.ac.kr, mino96@snu.ac.kr

Ⅰ　Introduction

In his Enrichment Triad Model Theory, Renzulli(1991) considers three different levels of learning for mathematically gifted students: (1) surveying and understanding the given problems, (2) developing their strategic skills in solving the problems, (3) producing the final solutions and products through relevant project activities. In this sense, it is important to investigate and design educational environments for the gifted learners to explore problems and make creative products. Accordingly, the purpose of this paper is to describe　the design of a microworld(Edwards, 1995), a playground for creative mathematical education. In brief, microworlds are primarily exploratory learning environments where the learners can manipulate or create objects and test their effects through discovering spaces and constraining simulation of real-world phenomena (Jonassen, 1996).

LOGO(Abelson & diSessa, 1980; Clements & Battista, 2001; Hoyles & Sutherland, 1999) a representative microworld, is an environment where students can create and produce mathematical figures using the most basic commands **fd** and **rt**. Fig. 1 shows the textual command and the figure drawn by the turtle executing the command. As a result of this textual command in the LOGO environment, the learners would be able to produce more visually and creatively oriented configurations. This is the most salient feature of the LOGO microworld. The basic commands **fd** and **rt** are easily understood, and by appropriate arrangement and operation of these two commands, the learners are able to make more creative and diverse commands to draw figures.
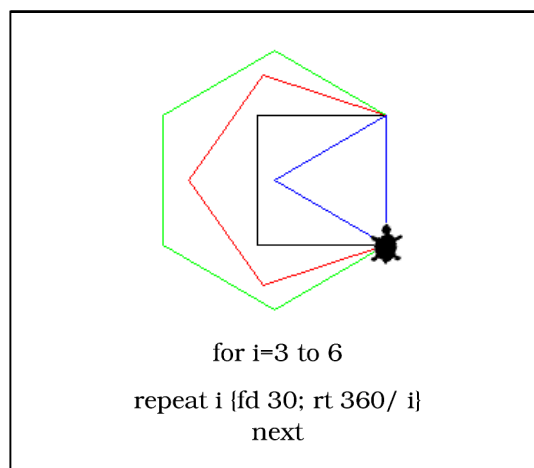
for i=3 to 6

repeat i {fd 30; rt 360/ i}
next

Figure 1. LOGO

Previous research on LOGO opened the gates of new possibilities in creative mathematics education. As new computer techniques and technology are being developed, however, the LOGO environment needs to be revised. Based on its fundamental philosophy, constructionism, we have improved the LOGO environment to be more dynamic and diverse for creative math education.

Meanwhile, research studies have also been conducted on the Dynamic Geometry System (DGS) along with Cabri(Laborde, 2000) and GSP and its application to overcome the static aspect of "usual" school geometry. The DGS is an experimental environment in which the learners hypothesize, justify, and confirm geometric and algebraic properties through mouse exploration, and the DGS helps students overcome the static features of the diagrams and mathematical figures in the textbook. In short, the DGS system plays an important role by providing an excellent environment for conjecturing, testing and confirming.

Sherin(2002) proposed yet another up-graded DGS by adding textual commands to the system. We consider Sherin's

attempt educationally desirable and have tried to combine the LOGO and DGS microworlds, keeping the features of both microworlds but adding new educational functions. In this environment, named JavaMAL microworld, "making" is emphasized and the "making" comes first followed by experimenting and confirming. For this attempt, we introduce a textual command system using the keyboard as well as the mouse-clicking sequence command system in addition to the already existing pull-down menu system. In sum, this paper proposes the designing of new functions that can connect the LOGO with the DGS using newly invented command systems, and we will demonstrate some examples of creative mathematical education programs that are possible in this new environment.

II Tile: Semi-Dynamic Object

In the previous section, we surveyed the strengths of the LOGO and the dynamic function of the DGS. In our effort to connect the two, we have introduced a dynamic feature to the LOGO on one hand and have created a function that makes the DGS more static on the other hand.

Such complementary functions are needed in mathematics education for the following reasons: Let's observe the following procedure of making creative products in the LOGO environment. The left hand side of Fig. 2 shows a command ⓐ for drawing basic figure using the primitive commands **fd** and **rt**. The command ⓑ in the middle of Fig. 2 shows that the previous process has been "internalized" and "condensed", and the right side of Fig. 2 shows a function command ⓒ that can be used to draw more complicated figures when one encapsulates the process of drawing as a basic object.
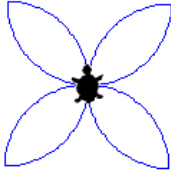

ⓐ
  fd 1; rt 2;
  fd 1; rt 2;
  fd 1; rt 2;
  fd 1; rt 2;
  fd 1; rt 2;
     ...
	ⓑ
	repeat 45 {fd 1; rt 2}
	rt 90;
	repeat 45 {fd 1; rt 2}
	rt 90;
		ⓒ
		function leaf {
		repeat 45 {fd 1; rt 2;}; rt 90;
		repeat 45 {fd 1; rt 2;}; rt 90;
		}
		repeat 4 {leaf; rt 90}

Figure 2. Making in LOGO

The three processes in Fig. 2 are similar to those of Sfard's(1991) concept formation scheme: interiorization, condensation and reification. In fact, Fig. 2 can be regarded as a prime representaion in the LOGO environment that shows the necessary steps of concept formation development. Following the Circulation Theory in cognitive development, we design a "tile" command so that a semi-dynamic object "tile" is created by the commands **fd** and **rt**, and the object can be manipulated again by the commands **fd** and **rt**. This is the basic idea of the tile command.

The following Fig. 3 shows a sequence of making tiles. That is, a student draws a basic figure using the Logo commands **fd** and **rt**, and makes tiles using a tile command. Then the student makes another tile by copying and reducing the tile, and the student moves and rotates the tiles, again using **fd** and **rt** tile commands. Hence, the student can control not only the turtle's movements of the turtle, but also those of the tiles constructed by the turtle using the "**fd** tile" and the "**rt** tile" commands. Again, as shown in Fig. 3, a tile is an object made by **fd** and **rt**

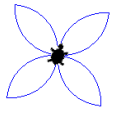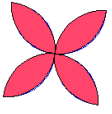commands, and it is a basic object moved freely by mouse dragging or by **fd** and **rt** commands.
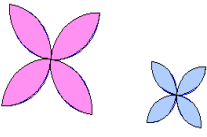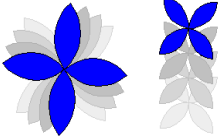
drawing a figure (fd, rt commands)	making a tile (tile command)	making another tile (copy, reduction)	rotating and forwarding

Figure 3. Tile in LOGO

Laborde(2000) showed that the DGS contains a rich learning context for proof since the "dragging" sequence in the invariant geometric properties stimulate students' thinking. The DGS helps learners to experiment with moving figures on a screen, which is an impossible task for the generation whose tools consisted of pen-and-paper. Be that as it may, even though it is possible for students to make conjectures about certain geometric properties by dragging his or her mouse in the DGS, these properties need to be justified. In the following Fig. 4, we can see that the tile ACE is made by mouse-clicking sequences, and is rotated to confirm the conjecture $\triangle$ ACE $\equiv$ $\triangle$ DCB.
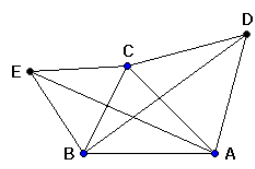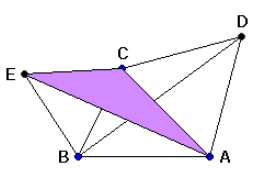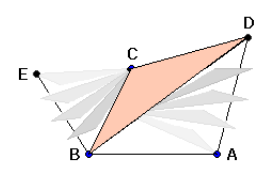
Confirming $\triangle$ ACE $\equiv$ $\triangle$ DCB		
constructing a figure	making a tile ACE	rotating a tile

Figure 4. Tile in DGS

The purpose of the DGS environment is to explore the invariant geometric properties by mouse dragging when the situation is changed. The DGS is good for providing an appropriate environment for exploration and experimentation of various properties. The reasoning of properties begins, however, with a fixed instance of dynamic situations. That is, mouse dragging dynamic representations assures the invariability of a figure, but deductive reasoning takes a step-by-step process between two static situations. For this purpose, we introduce a "tile command" to take snapshots of the dynamic movements, and then the tiles are manipulated to check the figure's properties such as congruence and similarity properties. In part, the tile is expected to be a medium for deductive reasoning. In Fig. 4 one is able to assume that the triangles are congruent and check one's assumption by manipulating the tile. Fig. 5 shows the tile's geometric operation to check congruency and similarity of figures. By allowing the blue triangle to be semi-dynamic, that is movable but keeping the same shape and size, and by using the tile commands, one can transform it in many ways (translation, reflection, rotation, and dilation).

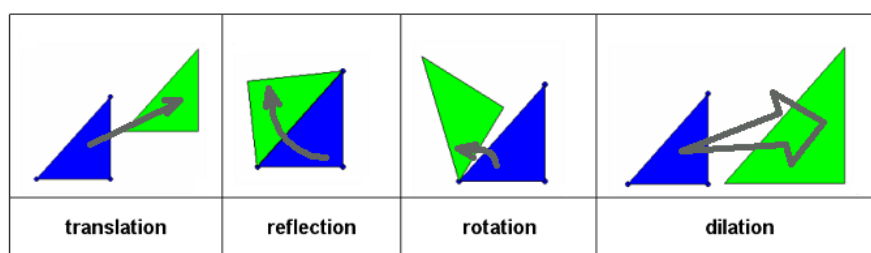| translation | reflection | rotation | dilation |

Figure 5. Tile transformation

Students can move a tile with the LOGO commands on a keyboard or manipulate it with the mouse. Students can drag a tile, just as he drags a point. Unlike a point,

however, he can reflect and rotate the tile, which is semi-dynamic with a shape and size. There are two ways to move a tile: using textual commands on the keyboard and the mouse-clicking sequences to connect the LOGO and the DGS.

From Fig. 6, we can see that the Pythagoras trees are given not only by the LOGO's turtle command, but by the DGS construction command. We can also see that the tiles are created by a turtle command and manipulated by mouse dragging.
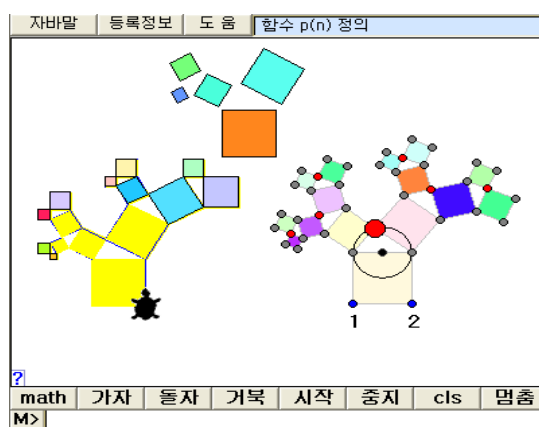

Figure 6. Pythagoras tree

The JavaMAL microworld is designed not only to combine the LOGO and the DGS, but also to introduce bridges between the two such as "tiles". From our teaching experiments, we find that the tiles in JavaMAL can provide a rich environment such that students can make their own creative mathematical products and explore mathematical properties from the given mathematical premises. Since JavaMAL is written in JAVA language, it can be used on the internet (see: http://edunet4u.snu.ac.kr).

Communications between students and teachers in cyber

space are also available on the internet, the unified environment of democratic access(Kaput, 1999) and internet board. It is accessible to anyone, anytime and anyplace. Now some new examples of creativity education will be given in the next two sections that the JavaMAL microworld can provide.

III Example: Group Activity by Making Modules

In creativity education, small group as well as an individual projects are encouraged. Social constructivism emphasizes the social construction of knowledge. Group activities can stimulate the group members so that they can produce more creative results after they share ideas through communication. Group activities in cyber spaces as well as in classrooms should be emphasized, and the JavaMAL microworld is designed for this purpose. Since the JavaMAL microworld is available on the internet, the group activity can be realized through the internet board using its textual command system. Fig. 7 shows an example of how small group members work together not only in the classroom but also on the web. The group members draw a sketch of "The Garden," after exchanging their ideas in the classroom. The garden is divided into the 'sun', 'tree' and 'butterfly,' and each module is made by each member for movements, which, in turn, come together to make the whole project complete. Feedback from the group members for an individual module can be exchanged for further discussion not only in the classroom but also in the cyber space.
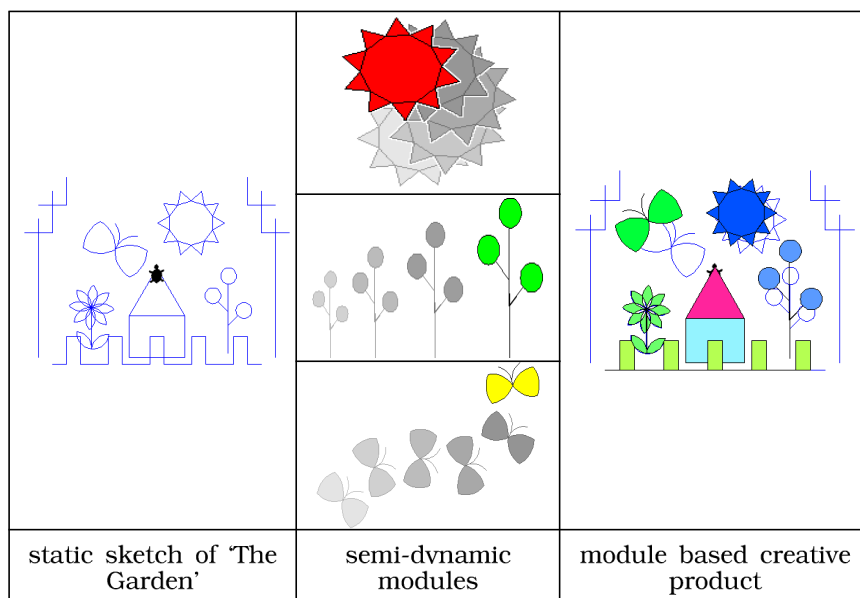
Figure 7. Module based group activity in the JavaMAL

The function of the tiles here is, therefore, to add semi-dynamic features for the modules in the sketch and prompt them to move. It is the tile that makes the 'sun' blazing, the 'tree' growing, and the 'butterfly' flying. Also, each tile comes together to construct a creative product as a whole. This kind of module based group activity is a good example of creativity education in a microworld environment since it fosters both individual and small group projects.

Ⅳ **Example: Tessellation and Tangram by Making Tiles**

So far we have talked about the functions of tiles as a snapshot of the dynamic DGS. The tile provides certain functions such as testing congruence and similarity (of

figures), and confirming and verifying conjectures. The tile is expected to contribute to creativity education in many ways. For example, imagine a space filled with many pieces of tiles, or a semi-dynamic tile which can play a role as teaching aids, such as tessellation and tangram. It is common to use ready made tiles in most activities such as LEGO activities and tangram puzzles. Meanwhile, in the JavaMAL environment the learners are able to "make" tiles by themselves, and design and solve the puzzle with tiles of their own making. Fig. 8 shows tessellation and tangram in which a student can 'make' a basic piece through a sequence of mouse-clicks, then by using the tile piece, he or she copies it and fills the space.
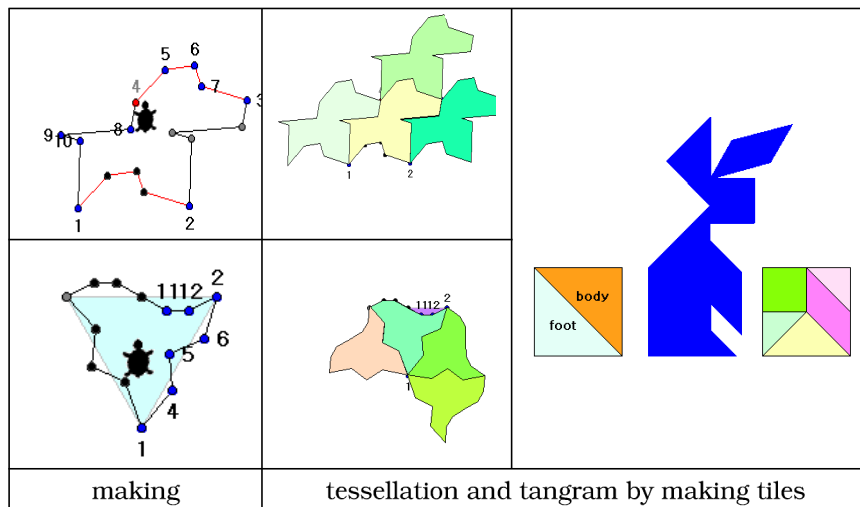

Figure 8. Tessellation and tangram by making tiles

While tessellation and tangram focus on puzzle-solving, JavaMAL also focuses on making a piece itself. In JavaMAL environment, making is emphasized prior to manipulation, and making is possible thanks to tiles.

V Conclusions and Future Work

In this paper we have surveyed the JavaMAL microworld as a playground for creativity education. In order to connect the static LOGO with the dynamic DGS, we introduced a new function, "tile". We presented some examples to apply what we studied to the notion of creativity education. From our pilot experiments, we confirmed that the JavaMAL microworld is well-designed for mathematical creativity and education for the gifted. This year we plan to do well-structured teaching experiments using well-organized curriculums and instructional designs based on Renzulli's Enrichment Triad Model Theory. We will modify the JavaMAL microworld by monitoring reactions from students, and we will try to obtain desirable educational outcomes by designing not only the JavaMal microworld but instructional strategies to provide an enriched learning environment based on the constructivism philosophy.

In the future, we plan to up-grade the JavaMAL environment so that it can be used to represent three-dimensional geometric objects. As we were able to fill the space with the basic pieces, we'll attempt to design the environment where the pieces will make a polyhedron. We are going to add some new functions, that is, drawing the folding net of a polyhedron using the LOGO commands, folding it to make a polyhedron, and manipulating it. Here the tile acts as a medium that can make the folding net move. The folding net of a polyhedron is drawn by the basic LOGO command, **fd** and **rt**. The folding net is, in turn, folded and becomes three-dimentional, and a couple of polyhedrons merge together to make some creative product. Fig. 9 shows the process of the students' activities to make decahedron. First, they figure out what the folding net might

look like. Second, they draw the folding net using **fd** and **rt**. Then, they make the folding net to move and to fold using the functions of tile. Finally, they can translate or rotate the complicated polyhedron. Furthermore, the complicated polyhedrons can be changed and connected to each other, to make creative products such as a doll and a train.
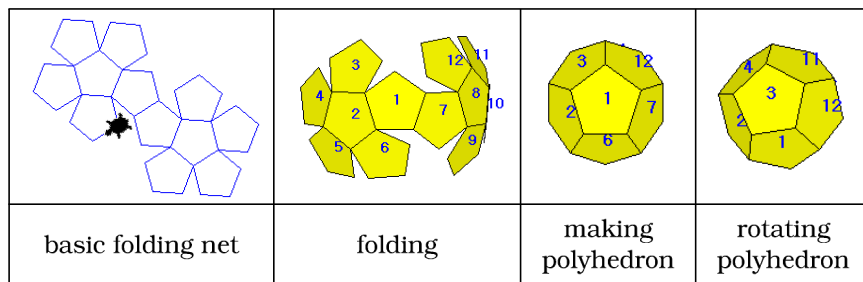

basic folding net	folding	making polyhedron	rotating polyhedron

Figure 9. Polyhedron in JavaMAL

This paper is primarily based on constructionism that emphasizes the active construction of the external artifact. We examined approaches to developing creativity in math education using different microworlds, and considered its educational values for creativity and education for the gifted. We propose the necessity of the tile functions and some programs using the tiles for developing creativity in math education. The designing of a microworld for creative math education as well as its application in a given situation will be an important contribution for the gifted and creativity education.

References

Abelson, H., & diSessa, A. A. (1980). *Turtle geometry: Computation as a medium for exploring mathematics.* Cambridge, MA: MIT Press.

Clements, D. H., & Battista M. T. (2001). *Logo and geometry.* Reston, VA: NCTM.

diSessa, A. A. (2000). *Changing minds.* Cambridge, MA: MIT Press.

Edwards, L. D. (1995). Microworlds as representation. In A. A. diSessa, C. Hoyles, R. Noss, & L. Edwards (Eds.), *Computers and exploratory learning.* Berlin: Springer.

Hoyles, C., & Sutherland, R. (1999). *Logo mathematics in the classroom.* London and New York: Routledge.

Kafai, Y., & Resnick, M. (Eds.) (1996). *Constructionism in practice: Designing, thinking, and learning in a digital world.* NJ: Erlbaum.

Kaput, J. (1999). The mathematics of change and variation from a millennial perspective. In C. Hoyles, C. Morgan, & G. Woodhouse (Eds.), *Rethinking the mathematics curriculum.* Falmer Press, 155~170.

Jonassen, H. D. (1996). *Computers as mindtools for schools*, Prentice Hall.

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics, 44,* 151~161.

Renzulli, J. S., & Reis, S. M. (1991). The schoolwide enrichment model: A comprehensive plan for the development of creative productivity. In N. Colangelo, & G. A. Davis (Eds.), *Handbook of gifted education*, Needham Heights, MA: Allyn and Bacon, 111~141.

Sfard, A. (1991). On the dual nature of mathematical conceptions: reflections on processes and objects as

different sides of the same coin. *Educational Studies in Mathematics, 22,* 1~36.

Sherin, B. (2002). Representing geometric constructions as programs: A brief exploration. *International Journal of Computers for Mathematical Learning, 7(1),* 101~115.

Wilensky, U. J. (1993). *Connected mathematics   building concrete relationship with mathematical knowledge.* Doctoral dissertation, MIT.