Robust Synchronization to Frequency Selective Fading Channel in OFDM Systems

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주파수 선택적 페이딩 채널에서 강인한 OFDM 동기 기법

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Abstract

The synchronization is one of the most important issues in the OFDM systems. Although the cyclic prefix has widely been used for symbol synchronization, it may not provide performance robust to the multi-cell environment. Therefore, the training symbol method is usually employed in multi-cell environment. Although the use of matched filter can provide best performance, it may have a problem of a large amount of computational complexity. To reduce the complexity, Cox introduced a method that utilizes a training symbol that has two repetitive patterns. However, it may not properly work in frequency selective fading channel due to the delay spread. In this paper, we propose three-step synchronization scheme robust to frequency selective fading channel. Firstly, the receiver obtains frame timing and then the symbol timing in the next step. Finally, the delay is compensated to correct residual timing offset. To facilitate the three-step operation, we design a training symbol that has four repetitions within a symbol period. Numerical results show that the proposed scheme can provide robust performance with low complexity.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been considered as one of the most promising modulation techniques to support multimedia services in mobile radio communications [1, 2]. However, the OFDM system requires to accurately maintain symbol and frame timing synchronization.

There have been a number of researches on the synchronization method for the OFDM system. Beek proposed a symbol synchronization scheme by using correlation with the cyclic prefix (CP) [3]. A three-step cell search algorithm was proposed to find the symbol and frame timing based on the Beek's scheme [4-6]. However, these schemes may provide performance degradation in frequency selective fading channel because the CP is contaminated by multi-path delay spread. Moreover, it may not properly work in a multi-cell, multi-user environment. Since the output of correlation with the CP is directly proportional to the transmit power from the base station, the user can be synchronized to a wrong base station that has higher power than the base station nearby. As a result, the more users the base station has, the more likely new users are to be synchronized to that base station, causing severe unbalancing cellular network load.

On the other hand, the use of a special OFDM training symbol can avoid the problem involved in multi-cell environment since the transmit power of the signal can be maintained at the same level. While matched filter scheme can provide optimum performance, it is difficult to implement due to large complexity. To reduce the implementation complexity, Cox introduced a synchronization method that utilizes a training symbol that has two repetitive patterns [7]. However, since this scheme has very large uncertainty region and high ambiguity at the boundary of timing metric, it can be vulnerable to frequency selective fading channel [9].

In this paper, we propose a timing synchronization scheme that can provide robust performance and low computational complexity in frequency selective fading channel.

II. OFDM SYSTEM MODEL

Fig. 1 illustrates the discrete-time OFDM system model. The OFDM transmitter modulates X_n using N_c subcarriers by inverse discrete Fourier transform (IDFT). The last N_g samples are inserted as a CP to form the OFDM symbol s_k. The channel impulse response h_k affects the signal s_k. Thus, the received signal, r_k, is given by

$$r_k = h_k * s_k + n_k \qquad (1)$$

Here $*$ denotes convolution. The data Y_n are obtained by discarding the first N_g samples (CP) of r_k and demodulating the N_c remaining samples of each OFDM symbol by means of a

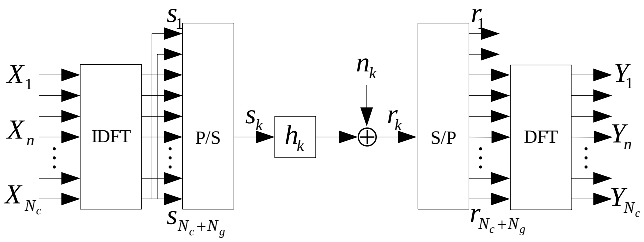

Fig. 1. OFDM system

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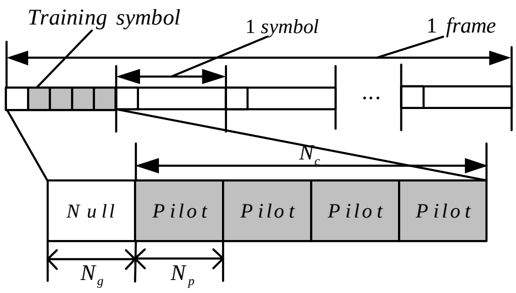

Fig. 2. The structure of OFDM frame

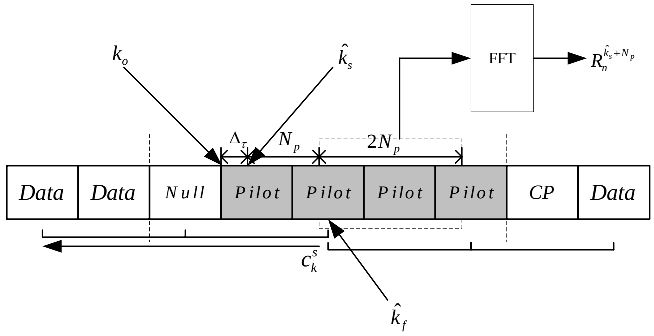

Fig. 3. Symbol timing synchronization and delay estimation

discrete Fourier transform (DFT).

III. PROPOSED SYNCHRONIZATION SCHEME

Fig. 2 depicts the frame structure of an OFDM signal. Each frame contains n_s symbols including a training symbol at the head of the frame. We consider the use of training symbol for the timing synchronization in the OFDM system. As illustrated in Fig. 2, the training symbol has four repetitive patterns within a symbol time and a guard interval portion with zero power. We define a time index set by

$$\Phi_1 \triangleq \{k : k = \theta + 2N_p \cdot i, i = 0,1,\cdots,I_\theta\} \;,0 \le \theta \le N_f - 1 \qquad (2)$$

where $N_p = N_c/4$, $N_f = (N_g + N_c)n_s$ and $I_\theta = \lfloor N_f/(2N_p) - 1 \rfloor$. Here $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. The set Φ_1 contains the correlation time instants, i.e., from time θ for each $2N_p$

To obtain the framing timing, the receiver performs a block-wise correlation defined as

$$c_k^f = \sum_{i=0}^{N_p-1} r_{k+i} r_{k+i+N_p}^* \;,\;\; k \in \Phi_1 \qquad (3)$$

where the superscript * denotes complex conjugate. In the timing region of the training symbol, c_k^f has a peak correlation value due to the nature of the training symbol. To obtain a reliable detection, the correlation is averaged over L_f frames

$$\bar{c}_k^f = \frac{1}{L_f}\sum_{u=1}^{L_f} c_{k+(u-1)N_f}^f \;. \qquad (4)$$

The frame timing can be found by selecting the timing index $\hat{k}_f$ by

$$\hat{k}_f = \arg\max_{k\in\Phi_1} \bar{c}_k^f \;. \qquad (5)$$

After the first step, the timing ambiguity is reduced within a symbol interval. As the next step, we search for the symbol timing, as illustrated in Fig. 3. The receiver performs correlation c_k^s for N_c samples

$$c_k^s = \sum_{i=0}^{2N_p-1} r_{k+i} r_{k+i+2N_p}^* \;,\;\; k \in \Phi_2 \qquad (6)$$

where $\Phi_2 \triangleq \{\hat{k}_f - N_c + 1,\; \hat{k}_f - N_c + 2,\; \cdots, \hat{k}_f\}$. If the received signal does not experience multi-path propagation, c_k^s has a peak correlation value at the exact symbol time k_o. Similar to the first step, c_k^s is averaged over L_s frames as

$$\bar{c}_k^s = \frac{1}{L_s}\sum_{u=1}^{L_s} c_{k+(u-1)N_f}^s \;. \qquad (7)$$

The symbol time can be found by choosing the time index $\hat{k}_s$ by

$$\hat{k}_s = \arg\max_{k\in\Phi_2} \bar{c}_k^s \;. \qquad (8)$$

Practically, the received signal experiences the multi-path propagation. So the symbol time $\hat{k}_s$ estimated in second step can be shifted by Δ_τ from the exact symbol time k_o as illustrated in Fig. 3.

$$\hat{k}_s = k_o + \Delta_\tau \;. \qquad (9)$$

Therefore it is required to compensate the delay Δ_τ as the third step. When $N_p \ge N_g$, the samples apart by N_p samples are independent from the delay spread. The amount of delay Δ_τ can be estimated in the frequency domain by $2N_p$-point DFT using the samples apart by N_p samples from $r_{\hat{k}_s}$. The $2N_p$-point DFT apart by $\hat{k}_s + N_p$ is given by

$$\begin{aligned} R_n^{\hat{k}_s+N_p} &= \sum_{m=0}^{2N_p} r_{m+k_o+\Delta_\tau+N_p} e^{-j2\pi mn/(2N_p)} \\ &= \sum_{m=0}^{2N_p} r_{m+k_o+\Delta_\tau} e^{-j2\pi mn/(2N_p)} \\ &= R_n^{\hat{k}_s} \end{aligned} \qquad (10)$$

where $R_n^{\hat{k}_s}$ is $2N_p$-point DFT apart by $\hat{k}_s$ on the assumption that it does not contaminated by delay spread. And $2N_p$-point DFT at k_o is given by

$$R_n^{k_o} = \sum_{m=0}^{2N_p} r_{m+k_o} e^{-j2\pi mn/(2N_p)} \;. \qquad (11)$$

The time delay can be represented by phase rotation in the frequency domain [8]. So $R_n^{\hat{k}_s}$ is the phase-rotated version of $R_n^{k_o}$

$$R_n^{\hat{k}_s} = R_n^{k_o} e^{j2\pi\Delta_\tau n/(2N_p)} . \quad (12)$$

The averaged phase rotation can be obtained by

$$\hat{\theta} = \frac{1}{N_p - 1} \sum_{i=1}^{N_p - 1} \arg\left(\frac{R_{2i+1}^{\hat{k}_s} R_{2i+1}^{k_o *}}{R_{2i-1}^{\hat{k}_s} R_{2i-1}^{k_o *}} \right) . \quad (13)$$
$$= 2\pi\left(\hat{\Delta}_\tau / N_p\right)$$

Thus, the amount of delay can be estimated as

$$\hat{\Delta}_\tau = \left\lfloor \frac{N_c \hat{\theta}}{8\pi} \right\rfloor . \quad (14)$$

Finally, the symbol timing index is estimated as

$$\hat{k}_o = \hat{k}_s - \hat{\Delta}_\tau . \quad (15)$$

IV. PERFORMANCE EVALUATION

The performance of the proposed scheme is evaluated in respect to uncertainty, computational complexity and detection probability.

A. Uncertainty

As illustrated in Fig. 5, peak value of correlation for the use of matched filter is high and shape of timing metric is very sharp. Thus, this scheme has outstanding performance in terms of uncertainty [9, 10]. On the other hand, Cox's method has high ambiguity at the boundary of timing metric due to the flat synchronization timing region as depicted in Fig. 5. However, since proposed scheme has no flat region at the correlation peak, it can reduce the uncertainty compared to Cox's scheme. Moreover, it can obtain timing synchronization faster than other schemes since uncertainty of the initial timing offset can be reduced with three-step approach.

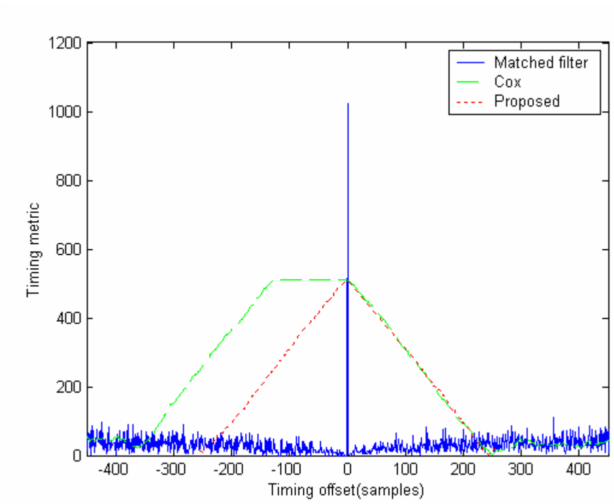

Fig. 5. Comparison of window timing metric

TABLE I. COMPARISON OF COMPUTATIONAL COMPLEXITY

Scheme	Complex multiplication
Matched filter	$N_c N_f$
Cox	$N_c/2 + N_f - 1$
Proposed	$N_f/2 + N_c/2 + N_c - 1$

TABLE II. SIMULATION CONDITION

N_c	512
N_g	125
Number of FFT and IFFT	512
n_s	16
Training symbol	
- Matched filter	512 extended PN sequence
- Cox	2 × 256 extended PN sequence
- Proposed	4 × 128 extended PN sequence

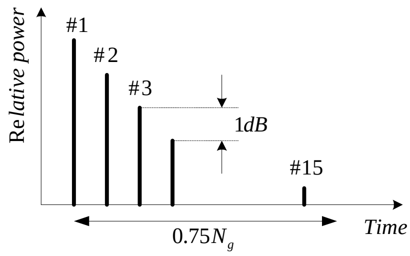

Fig. 4. Channel model

B. Computational complexity

Table I compares the computational complexity of the three schemes. The scheme using matched filter need too much computational complexity to implement. Cox reduced the computational complexity by implementing the correlation using the iterative formula [7]. Proposed scheme has low computational complexity by the block-wise correlation in the frame synchronization step. Since $N_f \gg N_c$, we can see that Cox's scheme has approximately N_f complex multiplication while proposed scheme has $N_f/2$ complex multiplication. Therefore, proposed scheme has about a half of computational complexity compared to Cox's method.

C. Detection probability

The detection probability of proposed scheme and conventional schemes are shown by computer simulation in AWGN channel and frequency selective channel. The simulation parameters are summarized in Table II. In the scheme using matched filter, when the frame timing is obtained within $\pm W/2$ sample from received timing of the first path, we define that the synchronization timing is detected. Here, W is a timing window size. In the Cox's scheme, when the frame timing is obtained between $-W$ and 0, we assume that the synchronization timing is detected. In the proposed scheme,

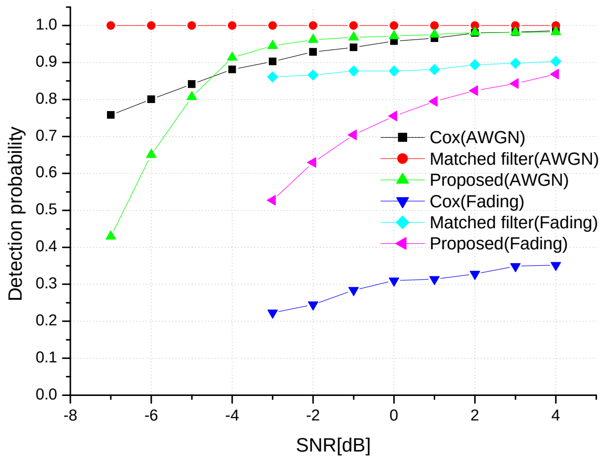

Fig. 6. Comparison of detection probability

when estimated symbol timing index $\hat{k}_o$ is obtained within $\pm W/2$ sample from received timing of the first path, we assume that the synchronization timing is detected. In AWGN channel, W is N_g. In frequency selective channel, the propagation model used in the simulation has 15 equally spaced multi-paths, each of which is subject to independent Rayleigh fading with different average power as depicted in . In this case, W is $0.25N_g$ since maximum delay spread is assumed to be $0.75N_g$.

The detection probability of three schemes is shown in Fig. 6. For fair comparison, all schemes use 3 frames for detection. In the AWGN channel, the scheme using matched filter has outstanding performance compared to other schemes. This is mainly due to the fact that the scheme using matched filter has high peak value of correlation and very sharp shaped timing metric. The Cox's scheme has detection probability above 0.9 for SNR ≥ -3dB in AWGN channel. The detection probability of this scheme slowly decreases according to reduction of SNR. This reduction is mainly due to the increase of uncertainty at the boundary of flat peak region of timing metric according to reduction of SNR. The detection performance of proposed scheme is better than that of Cox's scheme for SNR ≥ –4dB in AWGN channel. This is mainly due to the fact that shape of timing metric of proposed scheme is sharper than that of Cox's scheme. But the detection performance of proposed scheme is poorer than that of Cox's scheme for SNR ≤ -4dB in AWGN channel. This is mainly due to the fact that the peak value of correlation in the frame synchronization step is a half of Cox's. However, this is not practical SNR region

In the frequency selective fading channel, the scheme using matched filter has also better performance compared to other schemes. On the other hand, Cox's scheme has severely performance degradation in the frequency selective channel compared with AWGN channel. This is mainly due to the fact that each path has power variation by Rayleigh fading and the shape of timing metric is flat. In other words, since the first arrived path can be smaller than other paths frequently, the synchronization timing may be falsely detected behind flat region of timing metric. However, the proposed scheme has detection performance much better than Cox's scheme by the delay compensation step.

V. CONCLUSION

In this paper we have proposed a timing synchronization scheme that can provide performance robust to frequency selective fading channel and low computation complexity for the OFDM systems. To obtain the robustness to delay spread in frequency selective fading channel, we proposed three-step scheme, consisting of frame timing, symbol timing and delay compensation. Training symbol appropriate for three-step operation is also proposed. Numerical results show that the proposed detection scheme is practically more robust to frequency selective fading channel and less complex than conventional schemes .

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