# Effect of channel estimation error on packet-based multi-user OFDM systems

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## Abstract

In this paper, we consider the performance of a packet-based multi-user orthogonal frequency division multiplexing (OFDM) wireless system in the presence of channel estimation error. To investigate the effect of incorrect channel information on the system performance, we consider the use of two channel estimation schemes; one is a simple linear and the other is an optimum Wiener-type channel estimator. We analyze the effect of channel impulse response (CIR) estimation error on coherent reception and the effect of instantaneous signal to interference and noise ratio (SINR) estimation error on channel aware techniques associated with packet scheduling. It is shown that the performance of coherent reception is relatively less susceptible to the CIR error, but the performance of channel aware schemes can be very sensitive to the SINR estimation error, mainly due to incorrect scheduling. To alleviate this scheduling problem, we propose an improved scheduling method which can be employed even with the use of a simple channel estimator. Finally, we verify the performance of the proposed scheduling scheme by computer simulation.

## I. Introduction

In recent years, multi-user orthogonal frequency division multiplexing (OFDM) has attracted much attention as an effective transceiver technique for high-speed packet-based multi-user wireless systems that can provide high spectral efficiency by incorporating various advanced core technologies. It is known that the opportunistic scheduling technique with the use of adaptive modulation and coding (AMC) and hybrid automatic repeat request (HARQ) can significantly enhance the average spectral efficiency of OFDM systems [1]. Most of previous researches assume the use of perfect channel state information (CSI) in the transmitter [2]. However, it is impractical to obtain perfect CSI such as the channel impulse response (CIR) and signal to interference and noise power ratio (SINR).

Many researchers have investigated the effect of channel estimation error on the single-user link-adaptation scheme, AMC and HARQ and so on [2, 3]. However, to the authors' best knowledge, no result has been reported on the effect of channel estimation error on multi-user systems. In this paper, we consider the performance of a packet-based multi-user OFDM wireless system in the downlink when the channel estimation error exists, especially in view of packet scheduling and link adaptation. To this end, we consider the use of two estimation schemes; one is a simple linear and the other is an optimum Wiener-type CSI estimator [4, 5]. Then, we analyze the effect of CIR estimation error on coherent reception and the effect of instantaneous SINR estimation error on the channel aware schemes associated with the packet scheduling.

The rest of the paper is organized as follows. Section II and III describe the system framework and the effect of CSI error, respectively. Then, we propose an improved scheduling scheme in Section IV. Finally, Section V summarizes the conclusions.

## II. System framework

Consider an OFDM downlink system, where $X_m(n,k)$ represents the $m$ –th user signal at the $n$ –th symbol time and $k$ -th subcarrier, $m \in \{0,1,\cdots,M-1\}$ and $k \in \{0,1,\cdots,K-1\}$. The frequency domain symbol is converted into a time domain signal using the inverse fast Fourier transform (FFT). A cyclic prefix (CP) is inserted to preserve the orthogonality between the subcarriers and to eliminate the interference between the adjacent OFDM symbols. We assume that each data packet comprises $N_t$ OFDM symbols and $N_f$ subcarriers, and the pilot symbol is regularly inserted in a rectangular shape (i.e., apart by $d_t$ and $d_f$ symbols in the time and frequency grid, respectively).

Then, the received signal after FFT can be represented as

$$Y_m(n,k) = H_m(n,k)X(n,k) + Z_m(n,k) \qquad (1)$$

where $X(n,k)$ is the data signal for a selected user and $H_m(n,k)$ is the frequency response of fast fading channel from the transmitter to the *m*-th user, and $Z_m(n,k)$ is the background noise plus interference term which can be approximated as zero mean additive white Gaussian noise (AWGN) with variance $\sigma^2_{m,Z}$ . $H_m(n,k)$ is the discrete Fourier transform of the time domain CIR represented as

$$h_m(t,\tau) = \sum_{l=0}^{L-1} h_{m,l}(t)\delta(\tau - \tau_{m,l}) \qquad (2)$$

where $L$ is the number of multipaths, $\delta(\cdot)$ is Kronecker delta function, $\tau_{m,l}$ and $h_{m,l}(t)$ are the delay and complex-valued CIR at time *t* of the *l*-th path for user *m*, respectively.

In the receiver, all users estimate the CSI and report their estimated CSI to the transmitter through a feedback channel in the uplink. Then, the transmitter selects a user for the next packet time. Accordingly, the AMC parameters for the scheduled user are determined based on the instantaneous CSI. In what follows, we describe a procedure for the scheduling, link-adaptation and channel estimation.

### A. Packet scheduling

Recently, it was known that the spectral efficiency can significantly be improved by employing an intelligent packet scheduling scheme taking account of wireless channel condition, so-called opportunistic packet scheduling [7-10]. Maximum SINR scheduling and proportional fair (PF) scheduling are the examples of opportunistic scheduling.

The maximum SINR scheduling selects a user whose instantaneous SINR is the largest among all the users as

$$m = \arg\max_{m\in\{0,\cdots M-1\}} \left[\gamma_m(n,k)\right] \quad (3)$$

where, $\gamma_m(n,k)$ denotes the instantaneous SINR of user $m$. If we assume that $M$ users are uniformly allocated in each subcarrier, the subcarrier index $k$ in (3) can be omitted without loss of generality. This implies that the multi-carrier system with an opportunistic scheduler can be treated as a simple parallel extension of a single-carrier time division multiplexing system [8]. Therefore, we will omit the index $k$ in what follows. The maximum SINR scheduling maximizes the spectral efficiency by achieving the multi-user diversity (MUD) gain, but it cannot guarantee the fairness if the difference between the average SINR $\bar{\gamma}_m$ of each user is large. This fairness problem can be alleviated by employing a PF scheduling scheme [9].

Letting $R_m(n)$ be the possible transmission data rate at symbol time $n$ and $\bar{R}_i(n)$ be the average data rate up to symbol time $n$, the PF scheduler selects the user according to

$$m = \arg\max_{m\in\{0,\cdots M-1\}} \left[\frac{R_m(n)}{\bar{R}_m(n)}\right]. \quad (4)$$

Then, (4) can be described as [10]

$$m = \arg\max_{m\in\{0,\cdots M-1\}} \left[\frac{\gamma_m(n)}{\bar{\gamma}_m}\right] = \arg\max_{m\in\{0,\cdots M-1\}} \left[|H_m(n)|^2\right]. \quad (5)$$

In this paper, we consider the use of PF scheduling considering the trade-off issue between the spectral efficiency and fairness.

### B. AMC and HARQ

The AMC adjusts the modulation level and code rate according to the channel status, requiring accurate CSI such as the instantaneous SINR. However, the performance of AMC can significantly be degraded in the presence of channel estimation error. This problem can be alleviated with combined use of HARQ that supports the rate adaptation by retransmission. The HARQ combines the channel coding and ARQ to make the retransmissions more reliable [11]. It often employs a stop-and-wait method to save the radio resource. The HARQ with chase combining (CC) repeatedly sends a copy of originally transmitted packet when a decoding failure occurs. The receiver softly combines the received packet and previously received packets in a dynamic manner. The HARQ with incremental redundancy (IR) transmits an additional redundant data when the transmission fails. Thus, the HARQ-IR needs rate-compatible codes supported by a simple puncturing operation. It is known that the HARQ-IR can provide better performance than the HARQ-CC due to an additional coding gain at the cost of increased decoding complexity. The effective code rate is adjusted according to the channel status with acknowledgement signaling (i.e., ACK or NACK). Since the HARQ does not require the channel information at the transmitter, it can provide performance robust to the channel measurement error. In this paper, we employ an AMC scheme with fixed power constraint using square QAM and Zig-zag coding [12]. In addition, we employ an HARQ-IR scheme to balance inaccurate AMC

### C. Channel estimation

Accurate CSI is indispensable for the implementation of channel aware techniques. CSI can be estimated using the received pilot symbols. The CIR corresponding to the pilot symbol for user $m$ can be estimated as

$$\tilde{H}_m(n_p,k_p) = Y_m(n_p,k_p)/X(n_p,k_p) = H_m(n_p,k_p) + \tilde{Z}_m(n_p,k_p) \quad (6)$$

where $n_p$ and $k_p$ denote the symbol and subcarrier index of the pilot symbol, respectively, and $\tilde{Z}_i(n_p,k_p)$ denotes the noise term. The CIR at the data symbol is finally estimated by interpolating the CIR (6) at the pilot symbol. For the purpose filtering, we consider the use of two interpolation filters, Wiener and linear interpolation filter as the channel estimation filter (CEF) [4, 5]. The Wiener CEF minimizes the mean square error (MSE) of the estimated CIR at the cost of large implementation complexity, while the liner interpolation CEF can be implemented with ease at the sacrifice of performance degradation.

The Wiener CEF estimate the CIR at the $n$-th symbol time and the $k$-th subcarrier by

$$\hat{H}(n,k) = \sum_{p=0}^{N_w-1} w_p(n,k)\tilde{H}(n_p,k_p) \quad (7)$$

where $N_w$ denotes the number of filter taps and $w_p(n,k)$ denotes the $p$-th coefficient of the Wiener CEF. The optimum coefficient of the Wiener CEF can be determined as [5]

$$\mathbf{w}^T(n,k) = \boldsymbol{\theta}^T(n,k)\boldsymbol{\Phi}^{-1} \quad (8)$$

where the superscript $T$ denote the matrix transpose, $\boldsymbol{\Phi}$ is the $(N_w \times N_w)$ auto-covariance matrix of the received pilot symbol and $\boldsymbol{\theta}$ is the $(N_w \times 1)$ cross-covariance vector between the desired signal and received pilot symbol.

A linear interpolation CEF estimates the CIR at the data symbol as [4]

$$\begin{aligned}\hat{H}(n_p,k_p+k) &= \tilde{H}(n_p,k_p) + \frac{k}{d_f}\left(\tilde{H}(n_p,k_p+d_f) - \tilde{H}(n_p,k_p)\right),\ 0<k<d_f\\ \hat{H}(n_p+n,k) &= \tilde{H}(n_p,k) + \frac{n}{d_t}\left(\tilde{H}(n_p+d_t,k) - \tilde{H}(n_p,k)\right),\ 0<n<d_t.\end{aligned} \quad (9)$$

The channel SINR can be estimated using the estimated CIR. The average SINR can be estimated by a long term average with relatively high accuracy [6]. Therefore, we assume that the average SINR $\bar{\gamma}_m$ can be estimated very accurately. Then, the instantaneous SINR can be estimated as

$$\hat{\gamma}_m(n,k) = |\hat{H}_m(n,k)|^2\,\bar{\gamma}_m \quad (10)$$

## III. Effect of channel estimation error

The CSI cannot be estimated perfectly due to imperfect condition, such as the interference, background noise and the property of fading channel. Inaccurate CSI degrades the system performance in two aspects. The CIR estimation error affects coherent reception, and the instantaneous SINR estimation error affects the user scheduling and the choice of AMC parameters.

We first examine the effect of the CIR estimation error on coherent reception. **Fig. 1** depicts the average spectral efficiency with the use of perfect scheduling and AMC selection, assuming that all users experience the same Doppler frequency and delay spread, and that the users are uniformly distributed in a cell. The legend 'LI' and 'ISINR' denote the linear interpolation and instantaneous SINR, respectively. For reference, the performance with ideal channel estimation is also depicted. The simulation condition is summarized in Table 1. It can be seen that the average spectral efficiency increases as the number of user increases by an order of $\log(\log N)$ due to the MUD gain [8]. It can also be seen that the coherent detection is not significantly affected by the CIR estimation error in a packet-based multi-user OFDM system.

Next, we investigate the effect of inaccurate SINR on the scheduling and AMC. **Fig. 2** depicts the average spectral efficiency of the PF scheduling and AMC assuming perfect coherent detection. It can be seen that the system performance is significantly affected by the SINR estimation error. When the instantaneous SINR is used with simple linear interpolation, the MUD gain is severely degraded. This is mainly due to the fact that the scheduler selects an inappropriate user and the AMC level is also incorrectly determined because of inaccurate SINR information. As a result, actual transmission rate can be much lower than the anticipated one with the use of a simple estimator. In this figure, we also depict the result with the use of erroneous scheduling and perfect AMC selection. It can be seen that the performance degradation due to inaccurate AMC selection is significantly mitigated by the HARQ-IR. Therefore, it can be said that the performance degradation is mainly due to incorrect scheduling. Erroneous CSI estimation by users in poor channel condition can make such users be selected as the best user, preventing users in good condition from transmitting the data at a high rate. Therefore, the PF scheduler cannot guarantee the proportional fairness. Note that the performance degradation is getting worse as the number of users increases, especially when a simple estimator is employed.

## IV. Improved PF scheduling robust to incorrect channel information

Although the Wiener-type CEF can provide acceptable performance, it may not be easily employed due to the implementation complexity [5]. Instead, it may be desirable to employ a scheduling scheme robust to inaccurate CSI. The proposed scheme separates the users into several groups based on the accuracy of estimated CSI.

The proposed scheduler monitors the average CSI of all users. Then, the transmitter classifies the users into several groups according to the condition of channel estimation accuracy. Since we assume that all users experience the same Doppler shift and delay spread, the channel estimation accuracy is mainly affected by the average SINR. If we classify the user into $I$ user groups according to the susceptibility of channel estimation accuracy, each group comprises as

$$U_{i+1} \in m \ \ (\breve{\gamma}_i \le \overline{\gamma}_m < \breve{\gamma}_{i+1}),\ i = 0,1,\cdots,I-1 \qquad (11)$$

where $U_i$ denote the $i$ -th user group and $\breve{\gamma}_0 = -\infty$ , $\breve{\gamma}_1 < \breve{\gamma}_2 \cdots < \breve{\gamma}_{I-1}$ , $\breve{\gamma}_I = \infty$ denote the threshold for the classification. Then, different scheduling algorithms are employed to each user group to alleviate the incorrect user selection problem. Therefore, if we assume the all the users are uniformly distributed in the cell, the proposed PF scheduler can provide asymptotically proportional fairness even in the presence of large channel estimation error.

For example, in the case of two group classification, the threshold $\breve{\gamma}_1$ can be determined as the median value considering the distribution of user's average SINRs. **Fig. 3** depicts the performance of the proposed scheduler, assuming that the average SINR is distributed from $-10$ to 30 dB for each user, where $\breve{\gamma}_1 = 10$dB. The users in good condition are scheduled using a PF scheduler, while the user in poor condition are scheduled using a non-opportunistic scheduler (e.g., round-robin type). It can be seen that the proposed scheduler significantly improves the performance in the presence of large channel estimation error. Thus, the proposed scheme is quite useful with the use of a simple channel estimator, substantially reducing the implementation complexity for channel estimation. Note that similar approaches can also be applied to the case when the users experience different Doppler shifts and delay spreads.

## V. Conclusions

In this paper, we have investigated the effect of channel estimation error in a packet-based multi-user OFDM system. It was suggested that the CIR estimation error is not critical for the performance of coherent reception, but instantaneous SINR estimation error is very affective to adaptive transmission techniques (e.g., packet scheduling). We have proposed an improved scheduling scheme to mitigate the effect of channel estimation error. The proposed scheme classifies the users into several groups considering the average CSI as an indication of the accuracy of CSI. The simulation results show that the proposed scheme works well even with the use of a simple channel estimator, making it quite practical

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**Table 1. System parameters**

| Index | Value | Index | Value |
|---|---|---|---|
| Carrier freq. | 5.8 GHz | Number of sub-carriers | 2048 |
| Cell radius | 2.5 km | Packet size ($N_t$, $N_f$) | (64,8) |
| Guard interval | 5 $\mu$ sec | Pilot spacing ($d_t$, $d_f$) | (64,4) |
| Symbol duration ($T_s$) | 20.48 $\mu$ sec | Pilot pattern | Rectangular |
| Bandwidth | 100 MHz | Doppler effect | Jakes model max. 16 Hz |
| Duplex | FDD | Power delay profile ($L$=18) | rms delay 167 $n$ sec |

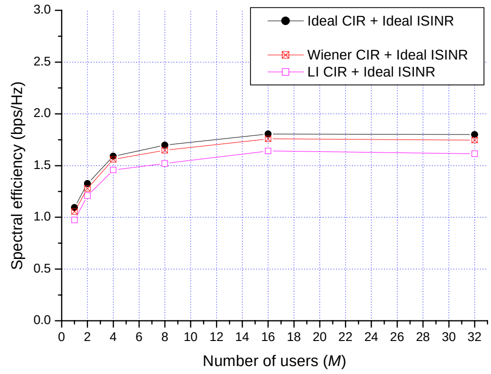


**Fig. 1. Average spectral efficiency with imperfect coherent detection**

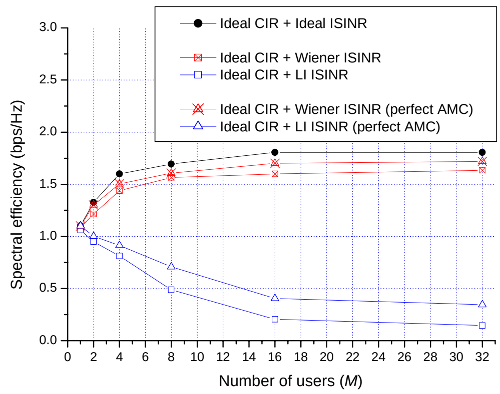


**Fig. 2. Average spectral efficiency with imperfect instantaneous SINR**

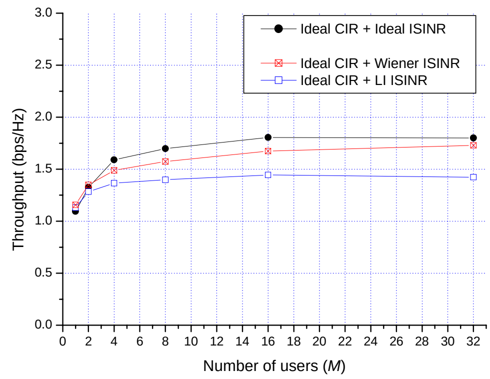


**Fig. 3. Performance of the proposed scheduling method**