Effective SNR Based MIMO Switching in Mobile WiMAX Systems

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Abstract

The mobile WiMAX system considers the use of (2×2) *multiple-input multiple-output (MIMO) schemes for performance improvement. It is required for mobile stations to appropriately switch the MIMO mode; space–time block code (STBC) or vertical spatial multiplexing (VSM). In this paper we consider a novel switching scheme for the MIMO mode based on effective signal-to-noise ratio. Assuming the use of a minimum mean square error (MMSE) receiver for the VSM and a maximum ratio combining receiver for the STBC, the threshold for the mode switching is analytically derived in a closed form. Simulation results show that the proposed switching scheme outperforms conventional switching schemes in terms of the capacity and outage probability.*

1. Introduction

The mobile WiMAX system (m-WiMAX) based on the IEEE 802.16e-2005 specifications considers the use of (2×2) multiple-input multiple-output (MIMO) antenna technologies to improve the system performance [1]. The system profile for the m-WiMAX [2] includes two MIMO modes for the downlink. One is (2×2) vertical spatial multiplexing (VSM) [3] and the other is space–time block code (STBC) for transmit diversity [4].

The VSM mode can improve the system capacity by generating multiple spatial layers when the number of receive antennas is larger than or equal to that of the transmit antennas, enabling to remove interference signals from other layers [3]. To obtain a spatial multiplexing gain, however, the MIMO channel should be spatially independent. As the rank of the MIMO channel decreases, the performance of the VSM can abruptly deteriorate. Moreover, when the signal to noise (SNR) is low, the capacity of the system can significantly decrease due to split transmit power over multiple spatial layers. On the other hand, the STBC mode exploits spatial diversity by employing multiple antennas at the transmitter and possibly at the receiver. Since it utilizes the whole transmit power for the transmission of a single data stream, it can effectively be applied to low SNR environments. The spatial diversity gain enables the receiver to reliably decode the signal in fading channels, almost independent of the property of the channel rank. However, it cannot achieve substantial capacity gain due to the absence of spatial multiplexing gain [4]. It is desirable to maximize the system capacity of the m-WiMAX by transmitting data with the use of a proper MIMO mode according to the time-varying channel condition.

Previous studies consider the prediction and selection of a MIMO mode by using short-term channel state information (CSI) [5], [6] or long-term CSI of the channel [7]–[9]. The MIMO mode can be selected to maximize the minimum distance of the received signal constellation, increasing the spectral efficiency [5], [6]. However, this switching scheme was designed for uncoded transmission schemes and thus it may not work well in coded ones. The MIMO mode switching based on approximated performance may cause switching errors, resulting in performance degradation. Long-term CSI based switching schemes [7]–[9].consider the mode selection to maximize the channel capacity in an average sense, significantly reducing the system complexity and feedback signaling overhead due to slow time-varying nature of the long-term channel characteristics. However, the use of only long-term CSI may degrade the switching performance due to the fact that it may not properly track instantaneous channel condition which varies much faster than the long-term channel characteristics.

In this paper, we consider the use of instantaneous CSI for the MIMO mode switching associated with time-varying channels. We select the MIMO mode based on the effective SNR of the received signal which is directly related to the instantaneous capacity of the MIMO channel. By deriving the effective SNR of the VSM and STBC schemes in a simple closed form, the MIMO mode can easily be switched using a simple threshold. It is shown that the proposed scheme outperforms the previous schemes in both the capacity and the outage probability.

The rest of this paper is organized as follows. The system model in consideration is described in Section II. The effective SNR of the VSM and STBC is derived in a closed form and then it is applied to the switching of the MIMO mode in Section III. The performance of the

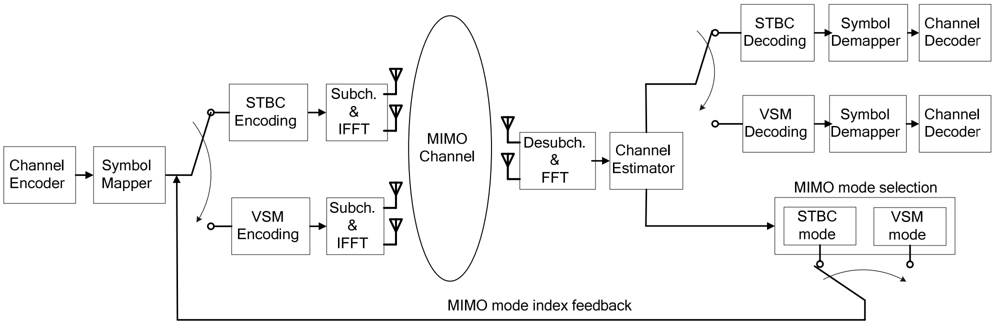


Fig. 1. M-WiMAX wave-2 system with adaptive MIMO switching.

proposed switching scheme is verified by computer simulation in Section IV. Finally, conclusions are summarized in Section V.

2. System model

Consider (2×2) MIMO (i.e., two transmit and two receive antennas) schemes suggested in the IEEE 802.16e-2005 specifications [1] and the m-WiMAX wave-2 profile, as illustrated in Fig. 1 [2]. The received signal can be represented as

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and $\mathbf{r} = \begin{bmatrix} r_1 & r_2 \end{bmatrix}^T$ denote the transmit and received signal vector, respectively, $\mathbf{H}$ is the (2×2) MIMO channel given by

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \end{bmatrix}, \tag{2}$$

and $\mathbf{n} = \begin{bmatrix} n_1 & n_2 \end{bmatrix}^T$ denotes zero-mean additive white Gaussian noise (AWGN) with covariance $N_0 \cdot \mathbf{I}_2$. Here, the superscript T denotes the matrix transpose, $\mathbf{I}_2$ denotes a (2×2) identity matrix, h_{ji} denotes the channel response from transmit antenna i to receive antenna j, and $\mathbf{h}_i$ denotes the channel vector originated from transmit antenna i. Letting $P_T = E[\mathbf{x}^H\mathbf{x}]$ be the transmission power, the average SNR can be defined as

$$\bar{\gamma} = \frac{P_T}{N_0}. \tag{3}$$

The transmit signal vector in the VSM mode can be represented as $\mathbf{x} = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T$, where s_1 and s_2 are data symbols transmitted by transmit antenna 1 and 2, respectively. The VSM mode can achieve a spatial multiplexing gain by transmitting independent data streams through two transmit antennas. In the STBC mode, data symbol s_1 and s_2 are transmitted during two-symbol duration: the transmit signal vector is $\mathbf{x}_1 = \begin{bmatrix} s_1 & s_2 \end{bmatrix}^T$ and $\mathbf{x}_2 = \begin{bmatrix} -s_2^* & s_1^* \end{bmatrix}^T$ during the first and second symbol time, respectively. The received signal in the STBC can be represented as [4]

$$\mathbf{r}_i = \mathbf{H}\mathbf{x}_i + \mathbf{n}_i \tag{4}$$

where $i = 1, 2$ is the symbol time index.

In the m-WiMAX system, the mobile station (MS) determines the MIMO mode and reports the index of the selected mode to the base station (BS) via a low-rate feedback channel in the uplink [1]. Letting Γ_{VSM} and Γ_{STBC} be a performance metric of the VSM and the STBC, respectively, the MIMO mode can be determined with the corresponding index π as

$$\pi = \begin{cases} 0, & \Gamma_{VSM} < \Gamma_{STBC} \\ 1, & \Gamma_{VSM} > \Gamma_{STBC} \end{cases}. \tag{5}$$

3. Proposed Adaptive MIMO Switching

It is desirable to define a proper performance metric for each MIMO mode to optimize the MIMO performance. In this paper, we consider an effective SNR as the performance metric for the MIMO mode selection.

3.1. Performance metric of the VSM mode

In the VSM mode, the transmit signal s_i, whose transmit power is $P_T/2$, can be decoded using an appropriate receiver weight vector $\mathbf{w}_i$. Assuming the use of a minimum mean-square error (MMSE) receiver, $\mathbf{w}_i$ can be determined by [11]

$$\mathbf{w}_i = \left(N_0\mathbf{I}_2 + \frac{P_T}{2}\sum_{k\neq i}\mathbf{h}_k\mathbf{h}_k^H \right)^{-1}\mathbf{h}_i. \tag{6}$$

Then, the received signal after the MMSE filtering can be represented as

$$\begin{aligned} \mathbf{w}_i^H\mathbf{r} &= \mathbf{w}_i^H\mathbf{h}_i s_i + \mathbf{w}_i^H\left(\sum_{k\neq i}\mathbf{h}_k s_k + \mathbf{n}\right) \\ &= \mathbf{w}_i^H\mathbf{h}_i s_i + \mathbf{w}_i^H\mathbf{z}_i \end{aligned} \tag{7}$$

where the first term is the desired signal and the second term is the interference signal from other layer plus AWGN. The post detection SNR of the received signal is [11]

$$\gamma_i = \frac{P_T}{2}\mathbf{w}_i^H \mathbf{h}_i \,. \qquad (8)$$

The corresponding system capacity of the VSM mode can be represented as

$$\begin{aligned} C_{VSM} &= \log_2(1+\gamma_1) + \log_2(1+\gamma_2) \\ &= \log_2\left(1+(\gamma_1+\gamma_2)+\gamma_1\gamma_2\right) \\ &= \log_2\left(1+\gamma_{VSM}\right) \end{aligned} \qquad (9)$$

where γ_{VSM} represents the effective SNR of the VSM mode given by

$$\gamma_{VSM} \triangleq (\gamma_1+\gamma_2)+\gamma_1\gamma_2 \,. \qquad (10)$$

To this end, first calculate the post detection SNR γ_1 associated with weight

$$\mathbf{w}_1 = \left(N_0\mathbf{I}_2 + \frac{P_T}{2}\mathbf{h}_2\mathbf{h}_2^H\right)^{-1}\mathbf{h}_1 \,. \qquad (11)$$

Using a matrix inversion formula [10]

$$\begin{aligned} &\left(N_0\mathbf{I}_2 + \frac{P_T}{2}\mathbf{h}_2\mathbf{h}_2^H\right)^{-1} \\ &= \frac{\bar{\gamma}^2}{P_T\left\{\left(\left|h_{12}\right|^2+\left|h_{22}\right|^2\right)\bar{\gamma}+2\right\}} \begin{bmatrix} \left|h_{22}\right|^2 + \frac{2}{\bar{\gamma}} & -h_{12}h_{22}^* \\ -h_{12}^*h_{22} & \left|h_{12}\right|^2 + \frac{2}{\bar{\gamma}} \end{bmatrix}, \end{aligned} \qquad (12)$$

it can be shown that

$$\gamma_1 = \frac{P_T}{2}\mathbf{w}_i^H\mathbf{h}_i = \frac{\bar{\gamma}^2\left(\det\left(\mathbf{H}\mathbf{H}^H\right)+\left(\frac{2\left\|\mathbf{h}_1\right\|_F^2}{\bar{\gamma}}\right)\right)}{2\left(\left\|\mathbf{h}_2\right\|_F^2\bar{\gamma}+2\right)}, \qquad (13)$$

where $\|\cdot\|_F$ denotes the Frobenius norm. Similarly, it can be shown that the post detection SNR γ_2 for the second layer is

$$\gamma_2 = \frac{\bar{\gamma}^2\left(\det\left(\mathbf{H}\mathbf{H}^H\right)+\left(\frac{2\left\|\mathbf{h}_2\right\|_F^2}{\bar{\gamma}}\right)\right)}{2\left(\left\|\mathbf{h}_1\right\|_F^2\bar{\gamma}+2\right)} \,. \qquad (14)$$

Finally, the effective SNR of the VSM mode can be represented as

$$\gamma_{VSM} = \frac{\bar{\gamma}^4\left(\det\left(\mathbf{H}\mathbf{H}^H\right)+\frac{2\left\|\mathbf{H}\right\|_F^2}{\bar{\gamma}}+\frac{4}{\bar{\gamma}^2}\right)^2}{4\left(\left\|\mathbf{h}_1\right\|_F^2\bar{\gamma}+2\right)\left(\left\|\mathbf{h}_2\right\|_F^2\bar{\gamma}+2\right)} - 1 \,. \qquad (15)$$

The term $\det\left(\mathbf{H}\mathbf{H}^H\right)$ highly depends on the eigenvalues of $\mathbf{H}\mathbf{H}^H$ and it can be represented by the product of two eigenvalues of $\mathbf{H}\mathbf{H}^H$. When the MIMO channel is highly correlated, one of the eigenvalues of $\mathbf{H}\mathbf{H}^H$ is likely very small, resulting in a small $\det\left(\mathbf{H}\mathbf{H}^H\right)$. This implies that the VSM mode may not be applicable in spatially correlated channel environments.

3.2. Performance metric of the STBC mode

Assuming that the channel characteristics are not noticeably varying during two-symbol time duration, the received signal vectors in the STBC mode at symbol time 1 and 2 can respectively be represented as

$$\mathbf{r}_1 = \begin{bmatrix} r_{1,1} \\ r_{2,1} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_{1,1} \\ n_{2,1} \end{bmatrix}, \qquad (16)$$

$$\mathbf{r}_2 = \begin{bmatrix} r_{1,2} \\ r_{2,2} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}\begin{bmatrix} -s_2^* \\ s_1^* \end{bmatrix} + \begin{bmatrix} n_{1,2} \\ n_{2,2} \end{bmatrix}. \qquad (17)$$

From (16) and (17), letting $\mathbf{y} = \begin{bmatrix} r_{1,1} & r_{2,1} & r_{1,2}^* & r_{2,2}^* \end{bmatrix}^T$ and $\mathbf{n} = \begin{bmatrix} n_{1,1} & n_{2,1} & n_{1,2}^* & n_{2,2}^* \end{bmatrix}^T$, the received signal vectors can be rewritten as

$$\begin{aligned} \mathbf{y} &= \sqrt{\frac{P_T}{2}}\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathbf{n} \\ &= \sqrt{\frac{P_T}{2}}\mathbf{H}_{eff}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \mathbf{n}. \end{aligned} \qquad (18)$$

The combined signal after the MRC can be represented as

$$\mathbf{H}_{eff}^H\mathbf{y} = \sqrt{\frac{P_T}{2}}\,\|\mathbf{H}\|_F^2\;\mathbf{I}_2\mathbf{s} + \mathbf{n}', \qquad (19)$$

where $\mathbf{n}' = \mathbf{H}_{eff}^H\mathbf{n}$ denotes an equivalent noise vector. Then the effective SNR of the STBC scheme is given by

$$\gamma_{STBC} = \frac{\left\|\mathbf{H}\right\|_F^2}{2}\bar{\gamma} \,. \qquad (20)$$

3.3 Criterion for the MIMO mode switching

The MIMO mode can be selected based on the effective SNR as

$$\gamma_{VSM} \underset{\pi=0}{\overset{\pi=1}{\gtrless}} \gamma_{STBC} \qquad (21)$$

where π denotes the selected MIMO mode index. Since (21) can be rewritten as

$$\frac{\left(\det\left(\mathbf{HH}^H\right)+\frac{2\|\mathbf{H}\|_F^2}{\bar{\gamma}}+\frac{4}{\bar{\gamma}^2}\right)^2}{4\left(\|\mathbf{h}_1\|_F^2\,\bar{\gamma}+2\right)\left(\|\mathbf{h}_2\|_F^2\,\bar{\gamma}+2\right)}\bar{\gamma}^4-1 \underset{\pi=0}{\overset{\pi=1}{\gtrless}} \frac{\|\mathbf{H}\|_F^2}{2}\bar{\gamma}\,. \quad (22)$$

The switching criterion can further be simplified to

$$\frac{\det\left(\mathbf{HH}^H\right)^2}{2\left(\|\mathbf{H}\|_F^2+\frac{2}{\bar{\gamma}}\right)}\bar{\gamma}+2\det\left(\mathbf{HH}^H\right) \underset{\pi=0}{\overset{\pi=1}{\gtrless}} \|\mathbf{h}_1\|_F^2\,\|\mathbf{h}_2\|_F^2\,. \quad (23)$$

4. Performance evaluation

The performance of the proposed switching scheme is verified by computer simulation, where the system parameters are summarized in Table 1. For comparison, the performance of a previous switching scheme in [5] is considered, where the mode is selected based on the minimum Euclidean distance as

$$\frac{\lambda_{\min}^2}{N_T} d_{\min,vsm}^2 \underset{\pi=0}{\overset{\pi=1}{\gtrless}} \frac{1}{N_T}\|\mathbf{H}\|_F^2\, d_{\min,stbc}^2\,. \quad (24)$$

Here, $\lambda_{\min}$ is the minimum eigenvalue of $\mathbf{HH}^H$, and $d_{\min,vsm}$ and $d_{\min,stbc}$ are the minimum distance of the received signal constellation in the VSM and the STBC mode, respectively. For reference, the performance of non-adaptive MIMO schemes that use a fixed MIMO mode (i.e., VSM or STBC) is also considered. The performance of the proposed scheme is evaluated in terms of the capacity in the presence of transmit antennas correlation, where the spatial correlation is defined by

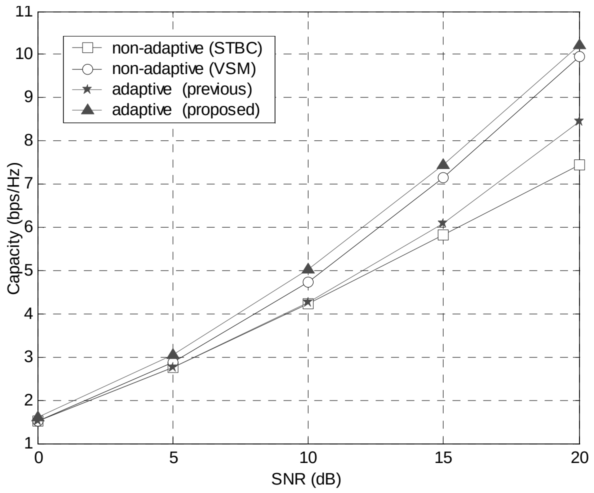
$$\rho = \frac{E\left\{\mathbf{h}_1^H\mathbf{h}_2\right\}}{2}\,. \quad (25)$$

The performance is evaluated in Rayleigh flat fading channel as in the previous scheme [5].

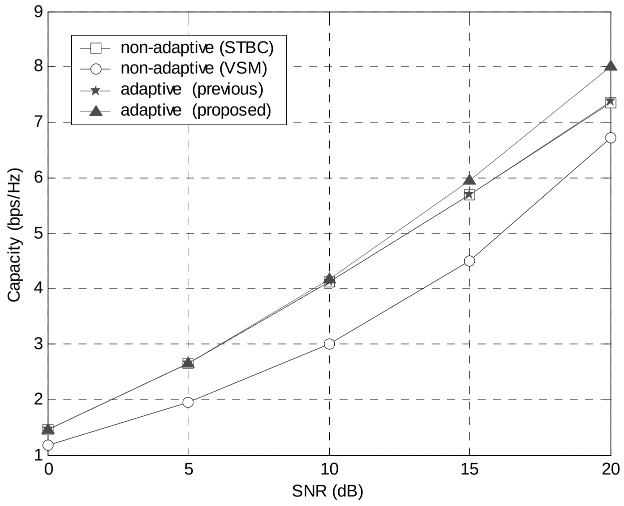
Fig. 2 depicts the capacity performance when the transmit channel is weakly (e.g., $|\rho| = 0.1$) and highly (e.g., $|\rho| = 0.9$) correlated. It can be seen that the proposed scheme outperforms the other schemes by properly switching the MIMO mode according to the channel condition. Especially, it can also be seen that the previous switching scheme has severe performance degradation in $|\rho| = 0.1$ and high SNR region where the VSM mode is preferred to the STBC mode. This is mainly due to the fact that the previous scheme is designed to favor the STBC mode, which leads to capacity loss at the cost of reliable transmission. The proposed scheme provides a capacity gain of about 0.8 bps/Hz over the previous scheme when $|\rho| = 0.1$ and 0.1 bps/Hz when $|\rho| = 0.9$ at SNR 10 dB.

Table 1. Simulation parameters.

Parameters	Values
Channel model	Rayleigh fading
Channel estimation	Perfect
# of TX antenna	2
# of RX antenna	2
Receiver algorithm	MMSE (VSM) / Linear MRC (STBC)



(a) When $|\rho| = 0.1$



(b) When $|\rho| = 0.9$

Fig. 2. Capacity performance of the proposed scheme.

Fig. 3 depicts the capacity performance according to the spatial correlation at SNR 10dB. When the channel is heavily correlated, it is desirable to employ the STBC mode. On the other hand, when the channel is uncorrelated, the VSM mode is preferred to the STBC mode in an average sense. However, there are cases when the STBC mode works better than the VSM mode even in

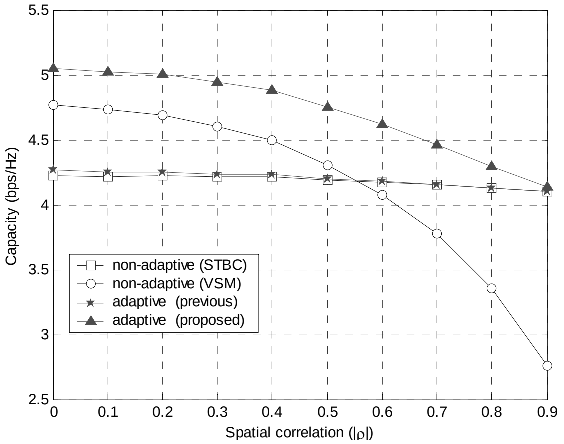


Fig. 3. Capacity performance according to $|\rho|$ at SNR 10dB.

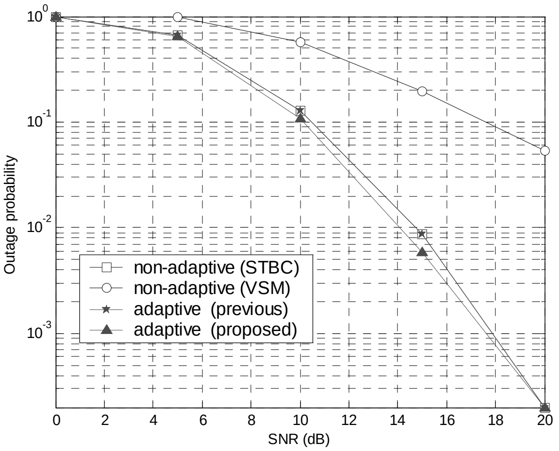


Fig. 4. Outage performance of the proposed switching scheme when $|\rho| = 0.9$.

low spatial correlation channel, and vice versa. As a consequence, it can be seen that the proposed switching scheme provides the best performance irrespective of spatial correlation condition. Since the previous switching almost selects the STBC scheme at SNR 10dB, it provides capacity performance very similar to the STBC scheme irrespective of the spatial correlation.

Fig. 4 depicts the outage probability at an outage capacity of 3 bps/Hz when the channel is highly correlated. It can be seen that the STBC mode is superior to the VSM mode and the difference increases as the SNR increases, which is mainly due to larger diversity gain of the STBC scheme. It can be seen that, although the proposed scheme shows less advantages in this condition, it still has a noticeable performance gain over the other schemes. In fact, the proposed scheme has a gain over the previous switching scheme about 0.5 dB when $|\rho| = 0.9$ in terms of the outage probability.

5. Conclusions

We have proposed a new switching scheme for the selection of the MIMO mode in the m-WiMAX system. Assuming the use of a MMSE receiver in the VSM mode and a MRC receiver in the STBC mode, the MIMO mode is simply switched based on a threshold which is analytically determined in terms of the effective SNR. The proposed scheme can outperform previous MIMO switching schemes in terms of the capacity and outage probability by exploiting the diversity and multiplexing gain. The simulation results show that the proposed switching scheme works well irrespective of the spatial correlation and SNR condition.

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