Co-movements of International Term Structure Slopes and Affine Term Structure Models

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This present paper provides both theoretical and empirical analyses of multi-factor joint affine term structure models (JATSMs) in explaining the co-movements of international term structure slopes. We extend the single-country affine term structure models of Dai and Singleton (2000) to a two-country setup. Using the efficient method of moments and reprojection analysis, we find that a JATSM with two square-root factors and one Gaussian factor performs best in capturing the correlation between the US and the UK term structure slopes.

Keywords: Affine term structure models, International term structure models, Correlations among international term structure slopes, Efficient method of moments, Reprojection method

JEL Classification: C14, C15, F31, G12

I. Introduction

With the increased globalization of international financial markets, understanding the co-movements of cross-country term structures of interest rates is important for various market participants, such as policy

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makers and practitioners. It has been well established that the dynamics of term structures can be classified into changes in the levels, slopes, and curvatures.[1] Whereas most studies on international term structure focus on correlations between levels, the present paper investigates the co-movements of international term structure slopes.

Co-movements of cross-country term structure slopes are critically important for various reasons. First, joint dynamics among international term structure slopes are key to the management of international bond portfolios. Second, most large industrial firms regularly borrow from international financing markets. For these firms, knowledge of co-movements among international term structure slopes is important in deciding the optimal term structure of debt. Third, as suggested by Estrella and Mishkin (1997), term spreads between long- and short-term bonds reflect the markets' expectations on future monetary policy. The future paths of foreign countries' monetary policies play an important role in the determination of domestic monetary policy. Therefore, joint dynamics of term structure slopes across countries play an important role in the coordination of global monetary policies. Fourth, a long line of research has accumulated robust evidence that changes in term structure slopes can anticipate turning points in the business cycle.[2] For this reason, co-movements of international term structure slopes may be a good indicator for the global business cycle.

Existing literature on joint affine term structure models (JATSMs) focuses mainly on the implications of the determination of the exchange rate and the valuation of currency derivatives. Bansal (1997), Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002), Han and Hammond (2003), Benati (2006), and Brennan and Xia (2006) investigate the performance of their international term structure models in explaining the observed exchange rate dynamics. Recently, Ahn (2004), Inci and

[1] See Littermam and Scheinkman (1991) and Dai and Singleton (2000), among many others.

[2] Estrella and Hardouvelis (1991) and Estrella and Mishkin (1997) show that the term structure slope is a good predictor of both future economic activity and inflation for the US and EU countries. See also Harvey (1991) for Germany, Davis and Henry (1994) for the UK, and Hu (1993) for the G-7 countries. Recently, Jung (2001) and Seo and Kim (2007) provide theoretical explanations for the predictive contents of term structure slopes. Jung (2001) provides a sticky price model that can explain the relationship between term structure slopes and real economic activity. Seo and Kim (2007) demonstrate that monetary policy rules play an important role in the prediction performance of term structure slopes on future inflation.

Lu (2004), Mosburger and Schneider (MS) (2005), Inci (2007), Leippold and Wu (2007), and Egorov, Li, and Ng (ELN) (2008) investigate the joint behavior of cross-country term structures of interest rates.

Although these papers consider various maturities of international yield curves, little effort has been made in examining the correlations between international term structure slopes. The present paper fills this gap. By extending the canonical model for single-country affine term structure models (ATSMs) provided by Dai and Singleton (DS) (2000) to a two-country setup, we investigate the performance of JATSMs in capturing the correlations of the term structure slopes of two countries. To the best of our knowledge, the current paper is the first to investigate directly the adequacy of various JATSMs in capturing the correlation dynamics of international term structure slopes. Given the two-country term structure dynamics, we derive the implied stochastic process of the exchange rate using the technique developed by Backus, Foresi, and Telmer (2001) and Ahn (2004). Therefore, we simultaneously model the joint dynamics of term structure slopes of two countries and their exchange rates.

Traditionally, the estimation of JATSMs is challenging because they include latent state variables, and there is no analytical expression available for the discrete conditional density. Recent advances in econometric methods have enabled researchers to address this issue using simulated method of moments techniques. The current paper uses the efficient method of moments (EMM) developed by Gallant and Tauchen (1996) to estimate the parameters of our JATSMs. This methodology has been widely used in the estimation of various multi-factor term structure models.[3] As noted by Chernov, Gallant, Ghysels, and Tauchen (2003), the advantages of using the EMM are as follows: (1) it offers formal statistical tests of a model's fit; (2) it offers formal diagnostics of a model's inadequacies; and, most importantly, (3) non-nested specifications can be compared in a meaningful way because the EMM forces all models to confront the same set of moment conditions.

[3] Anderson and Lund (1997) estimate the parameters of various short-rate models. Relying on the EMM method, DS investigate the empirical performance of single-country ATSMs. Ahn, Dittmar, and Gallant (2002) and Ahn, Dittmar, Gallant, and Gao (2003) estimate their single-country quadratic term structure models and hybrid term structure models, respectively, using the EMM estimation procedure. Bansal and Zhou (2002) and Bansal, Tauchen, and Zhou (2004) adopt the EMM method for the estimation of their discrete-time regime-switching term structure models.

In particular, we complement the EMM specification tests using the reprojection analysis of Gallant and Tauchen (1998). The advantage of the reprojection method is that it enables us to compare directly the conditional density for the observed international term structure slopes implied by our JATSMs with a conditional density directly extracted from the data. Relying on this method enables us to investigate how well our JATSMs reproduce the co-movements of international term structure slopes compared with those implied by the data. As such, the present paper provides a comprehensive picture about the models' performance in capturing the actual correlation dynamics of international term structure slopes.

The current paper is organized as follows. In Section 2, we introduce our JATSMs. In Section 3, we discuss the data and provide a brief summary of the EMM estimation procedure. The empirical results of the EMM estimation complemented by the reprojection analysis are provided in Section 4. Section 5 concludes the paper.

II. Theoretical Models

A. JATSMs

In this section, we establish two-country JATSMs. At the outset, we assume that the world economy consists of two countries, that is, a domestic country d and a foreign country f, and is represented by a filtered probability space $(\Omega, \mathfrak{I}, F, P)$, where $F=\{\mathfrak{I}_t\}_{0\leq t<T}$ is the filtration of the information structure under the probability measure P. We also assume that the uncertainty in the world economy is driven by N independent Brownian motions adapted to F. We denote the time t price of a zero-coupon bond denominated in currency $k\in\{d, f\}$, with the unit face value maturing at time $T=t+\tau$ by $P_k(t, T)$. In the absence of arbitrage opportunities in the world economy, the prices of zero-coupon bonds are given as follows:

$$P_k(t,\tau) = E_t^P\left[\frac{M_k(T)}{M_k(t)}\right], \tag{1}$$

where $E_t^P[\cdot]$ denotes the expectation conditional on the information at time t, $\mathfrak{I}_t$ under the physical probability measure P. $M_k(t, T)$ is the global stochastic discount factor (SDF) expressed in currency k, which

discounts payoffs at time T into the time t value under the stochastic economy. Ahn (2004) demonstrates that if the world economy is complete, then the stochastic differential equations (SDEs) of the unique $M_k(t, T)$ result in

$$\frac{dM_k(t)}{M_k(t)} = -r_k(t)dt - \xi_k(t)'dw_N(t), \tag{2}$$

where $r_k(t)$ is the nominal instantaneous interest rate of country k, $\xi_k(t)$ is an N-dimensional vector-valued function for $k \in \{d, f\}$, and $w_N(t)$ is an N-dimensional vector of standard Brownian motions.

Following Ahn, Dittmar, and Gallant (ADG) (2002) and Dai and Singleton (2003), we directly explore the stochastic processes of the global SDFs by specifying the following three assumptions:[4]

(A1) the relationship between the interest rates, $r_d(t)$ and $r_f(t)$, and the underlying state variables, $X(t)$;
(A2) the SDEs of the state variables, $dX(t)$; and
(A3) the diffusion processes of the global SDFs, $\xi_d(t)$ and $\xi_f(t)$.

Extending a term structure model to a two-country setup requires an additional assumption on the factor structure of the world economy. We assume that the stochastic nature of the world economy is governed by N common factors, which can affect the bond prices of both countries.

As noted, we extend the single-country ATSMs of DS to a two-country setup. First, we assume that the interest rates of the two countries, $r_d(t)$ and $r_f(t)$, are affine functions of the state variables

$$r_d(t) = \delta_0^d + \delta_1^{d'}X(t) \text{ and } r_f(t) = \delta_0^f + \delta_1^{f'}X(t), \tag{3}$$

where δ_0^d and δ_0^f are scalars, and δ_1^d and δ_1^f are N-dimensional vectors of constants. $X(t)$ denotes an N-dimensional vector of common factors. Second, we assume that $X(t)$ follows an affine diffusion under the physical probability measure P

[4] ADG and Dai and Singleton (2003) verify that any single-country term structure model can be fully characterized by specifying these three assumptions.

$$dX(t) = \mathrm{K}\left(\Theta - X(t)\right)dt + \Sigma\sqrt{S(t)}\,d\omega(t), \tag{4}$$

where Θ is an N-dimensional vector of constants, K and Σ are N-dimensional square matrices of constants, and $S(t)$ is an N-dimensional diagonal matrix, with the ith elements on the main diagonal given by

$$\left[\, S(t) \,\right]_{ii} = \alpha_i + \beta_i' X(t),$$

where α_i is a scalar and β_i is an N-dimensional vector of constants. We impose both the identification and admissibility conditions provided by DS.[5] Third, we assume that the diffusions of the global SDFs are

$$\xi_d(t) = \sqrt{S(t)}\,\lambda^d \;\text{ and }\; \xi_f(t) = \sqrt{S(t)}\,\lambda^f, \tag{5}$$

where λ^d and λ^f are N-dimensional vectors of constants.

Given the assumptions described in Equations (3)-(5), the prices of zero-coupon bonds are given by an exponential affine function of the state variables

$$P_d(t,\tau) = \exp[\,A_d(\tau) + B_d(\tau)' X(t)] \;\text{ and }\; P_f(t,\tau) = \exp[\,A_f(\tau) + B_f(\tau)' X(t)].$$

The yields of zero-coupon bonds are affine functions of the state variables:

$$yld_d(t,\tau) = A_d(\tau)/\tau + (B_d(\tau)'/\tau)X(t) \;\text{ and }\; yld_f(t,\tau) = A_f(\tau)/\tau + (B_f(\tau)'/\tau)X(t),$$

where $A_d(\tau)$ and $A_f(\tau)$ are scalar functions, and $B_d(\tau)$ and $B_f(\tau)$ are N-dimensional vector-valued functions. Then, as shown by Duffie and Kan (1996) and DS, $A_k(\tau)$ and $B_k(\tau)$ satisfy the ordinary differential equations

$$\frac{dA_k(\tau)}{d\tau} = -\Theta_k^{*\prime}\mathrm{K}_k^{*\prime} B_k(\tau) + \frac{1}{2}\sum_{i=1}^{N}\left[\,\Sigma' B_k(\tau)\,\right]_i^2 \alpha_i - \delta_o^k,$$

$$\frac{dB_k(\tau)}{d\tau} = -\mathrm{K}_k^{*\prime} B_k(\tau) - \frac{1}{2}\sum_{k=1}^{N}\left[\Sigma' B_k(\tau)\right]_k^2 \beta_k + \delta_1^k,$$

[5] See DS for the details of these conditions.

with the initial conditions $A_k(0)=0_{1\times 1}$, and $B_k(0)=0_{N\times 1}$. $K^*_k=K+\Sigma\Phi_k$ and $\Theta^*_k=(K^*_k)^{-1}[K\Theta-\Sigma\Psi_k]$ are the parameters of the SDEs of the state variables under the equivalent martingale measure defined for country k, where Φ_k is an $N\times N$ matrix with the ith row given by $\beta_i'[\lambda^k]_i$, and Ψ_k is an $N\times 1$ vector with the ith element given by $\alpha_i[\lambda^k]_i$.

Therefore, the term structure slopes defined as the difference between the long and short maturity zero-coupon bond yields are given as

$$\begin{aligned} &Slope_d(t,\tau_l,\tau_s)=yld_d(t,\tau_l)-yld_d(t,\tau_s)\\ &\text{and}\\ &Slope_f(t,\tau_l,\tau_s)=yld_f(t,\tau_l)-yld_f(t,\tau_s), \end{aligned} \tag{6}$$

where τ_l and τ_s denote long and short maturity, respectively, with $\tau_l>\tau_s$.

B. Three-factor JATSMs

This section specifies each of the models investigated and presents their implications for stochastic correlation between the domestic and foreign term structure slopes. We focus only on the three-factor models. The choice of the number of factors is related to our empirical investigation. Recently, MS evaluate the performance of their three-factor JATSMs in capturing the joint dynamics of the US and UK bond prices, whereas ELN explore the empirical performance of their four-factor models in explaining the joint behavior of the US and EU term structures.[6] Unlike these papers, we investigate the performance of JATSMs in capturing the joint dynamics of the international term structure slopes. As will be discussed later, our data consist of the US and UK term structure slopes, defined as the difference between the five-year and six-month yields for each country and the dollar−pound exchange rates. Therefore, our choice of three factors is not conservative. Following the notation of DS, let $JA_m(N)$ denote a JATSM with m common square-root factors and $N-m$ common Gaussian factors. With three factors (*i.e.*, $N=3$), there exist four non-nested subfamilies of JATSMs: $JA_0(3)$, $JA_1(3)$, $JA_2(3)$, and $JA_3(3)$. The current paper focuses only on JATSMs with $m>0$. The family of $JA_0(3)$ is incapable of generating the stochastic second moments of the term structure slopes, which is clearly counterfactual.

[6] Except for ELN, most papers consider only three-factor models. See, for example, Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002), Ahn (2004), and Brennan and Xia (2006), among many others.

a) $JA_1(3)$

The family of $JA_1(3)$ is characterized by the assumption that one of the state variables derives the stochastic volatility of all three state variables. The assumptions of $JA_1(3)$ are as follows. First, $r_d(t)$ and $r_f(t)$ are affine functions of the three common state variables:

$$r_d(t) = \delta_0^d + \sum_{i=1}^{3} \delta_{1i}^d X_i(t) \text{ and } r_f(t) = \delta_0^f + \sum_{i=1}^{3} \delta_{1i}^f X_i(t). \tag{7}$$

Equation (7) states that $r_d(t)$ and $r_f(t)$ can have different sensitivities to the same state variables. Second, the dynamics of $X(t)$ are given as

$$\begin{pmatrix} dX_1(t) \\ dX_2(t) \\ dX_3(t) \end{pmatrix} = \begin{pmatrix} \kappa_{11} & 0 & 0 \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 - X_1(t) \\ -X_2(t) \\ -X_3(t) \end{pmatrix} dt$$
$$+ \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{1+\beta_{21}X_1(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)} \end{pmatrix} \begin{pmatrix} d\omega_1(t) \\ d\omega_2(t) \\ d\omega_3(t) \end{pmatrix}. \tag{8}$$

Third, the market prices of factor risks are given as

$$\xi_d(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{1+\beta_{21}X_1(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^d \\ \lambda_2^d \\ \lambda_3^d \end{pmatrix},$$

$$\xi_f(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{1+\beta_{21}X_1(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^f \\ \lambda_2^f \\ \lambda_3^f \end{pmatrix}.$$

Applying Ito's lemma to the theoretical term structure slopes described in Equation (6) yields the following instantaneous second moments for the two term structure slopes:

$$Var_d(t, \tau_s, \tau_l) = \left(b_{d,1}(\tau_l) - b_{d,1}(\tau_s)\right)^2 X_1(t)$$
$$+ \left(b_{d,2}(\tau_l) - b_{d,2}(\tau_s)\right)^2 \left(1 + \beta_{21}X_1(t)\right) \tag{9}$$

$$+\left(b_{d,3}(\tau_l)-b_{d,3}(\tau_s)\right)^2\left(1+\beta_{31}X_1(t)\right),$$

$$\begin{aligned} Var_f(t,\tau_s,\tau_l) &= \left(b_{f,1}(\tau_l)-b_{f,1}(\tau_s)\right)^2 X_1(t) \\ &+\left(b_{f,2}(\tau_l)-b_{f,2}(\tau_s)\right)^2\left(1+\beta_{21}X_1(t)\right) \\ &+\left(b_{f,3}(\tau_l)-b_{f,3}(\tau_s)\right)^2\left(1+\beta_{31}X_1(t)\right), \end{aligned} \tag{10}$$

$$\begin{aligned} &Cov\left(t, slope_d(t,\tau_s,\tau_l),\ slope_f(t,\tau_s,\tau_l)\right) \\ &=\left(b_{d,1}(\tau_l)-b_{d,1}(\tau_s)\right)\left(b_{f,1}(\tau_l)-b_{f,1}(\tau_s)\right)X_1(t) \\ &+\left(b_{d,2}(\tau_l)-b_{d,2}(\tau_s)\right)\left(b_{f,2}(\tau_l)-b_{f,2}(\tau_s)\right)\left(1+\beta_{21}X_1(t)\right) \\ &+\left(b_{d,3}(\tau_l)-b_{d,3}(\tau_s)\right)\left(b_{f,3}(\tau_l)-b_{f,3}(\tau_s)\right)\left(1+\beta_{31}X_1(t)\right). \end{aligned} \tag{11}$$

Therefore, the second moments of the term structure slopes are represented as an affine function of the square-root factor $X_1(t)$. The other two state variables cannot contribute to generating the heteroskedastic second moments of the term structure slopes. Investigating the role of the negative correlations among the factors is of interest in explaining the dynamics of the term structure slopes.[7] Among the three JATSMs investigated in the current paper, $JA_1(3)$ is the most flexible in inducing the flexible correlation structure among the factors. In $JA_1(3)$, all the three factors can have both positive and negative correlations.[8]

The parametric restrictions of $JA_1(3)$ are given as

$$\kappa_{11}\theta_1 > 0,\ \kappa_{11} > 0,\ \theta_1 \geq 0,\ \beta_{21} \geq 0,\ \beta_{31} \geq 0,$$

$$\text{at least one of } (\delta_2^d, \delta_2^f) \text{ is non-negative,}$$

$$\text{at least one of } (\delta_3^d, \delta_3^f) \text{ is non-negative.}$$

[7] In their single-country affine setup, DS find that accommodating the negative correlations among the factors is important in matching the higher moments of the US bond yields.

[8] As demonstrated by ADG, the form of ATSMs requires a trade-off between the structure of bond price volatilities and admissible non-zero correlations of the factors. Admissibility of an ATSM requires non-negative correlations among the square-root factors. As such, an increase in the number of square-root factors limits the flexibility of the ATSM in specifying correlations while giving more flexibility in generating heteroskedastic volatility.

b) $JA_2(3)$

The family of $JA_2(3)$ is characterized by the assumption that two of the state variables derive the stochastic volatility of all three state variables. The assumptions of $JA_2(3)$ are as follows. First, $r_d(t)$ and $r_f(t)$ are affine functions of the three common state variables

$$r_d(t) = \delta_0^d + \sum_{i=1}^{3} \delta_{1i}^d X_i(t) \text{ and } r_f(t) = \delta_0^f + \sum_{i=1}^{3} \delta_{1i}^f X_i(t). \tag{12}$$

Second, the dynamics of $X(t)$ are given as

$$\begin{aligned}\begin{pmatrix} dX_1(t) \\ dX_2(t) \\ dX_3(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & \kappa_{12} & 0 \\ \kappa_{21} & \kappa_{22} & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 - X_1(t) \\ \theta_2 - X_2(t) \\ -X_3(t) \end{pmatrix} dt \\ &+ \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)+\beta_{32}X_2(t)} \end{pmatrix} \begin{pmatrix} d\omega_1(t) \\ d\omega_2(t) \\ d\omega_3(t) \end{pmatrix}.\end{aligned} \tag{13}$$

Third, the market prices of factor risks are given as

$$\xi_d(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)+\beta_{32}X_2(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^d \\ \lambda_2^d \\ \lambda_3^d \end{pmatrix},$$

$$\xi_f(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}X_1(t)+\beta_{32}X_2(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^f \\ \lambda_2^f \\ \lambda_3^f \end{pmatrix}.$$

The parametric restrictions of $JA_2(3)$ are given as

$$\begin{aligned}&\kappa_{11}\theta_1 + \kappa_{12}\theta_2 > 0,\ \kappa_{12} \le 0,\ \kappa_{21}\theta_1 + \kappa_{22}\theta_2 > 0,\ \kappa_{21} \le 0, \\ &\kappa_{33} > 0,\ \theta_1 \ge 0,\ \theta_2 \ge 0,\ \beta_{31} \ge 0,\ \beta_{32} \ge 0, \\ &\text{at least one of } (\delta_3^d,\ \delta_3^f) \text{ is non-negative.}\end{aligned}$$

The instantaneous second moments for the two term structure slopes are then given as

$$\begin{aligned} Var_d(t,\, \tau_s,\, \tau_l) = &\left(b_{d,1}(\tau_l) - b_{d,1}(\tau_s)\right)^2 X_1(t) \\ &+\left(b_{d,2}(\tau_l) - b_{d,2}(\tau_s)\right)^2 X_2(t) \\ &+\left(b_{d,3}(\tau_l) - b_{d,3}(\tau_s)\right)^2 \left(1 + \beta_{31}X_1(t) + \beta_{32}X_2(t)\right), \end{aligned} \tag{14}$$

$$\begin{aligned} Var_f(t,\, \tau_s,\, \tau_l) = &\left(b_{f,1}(\tau_l) - b_{f,1}(\tau_s)\right)^2 X_1(t) \\ &+\left(b_{f,2}(\tau_l) - b_{f,2}(\tau_s)\right)^2 X_2(t) \\ &+\left(b_{f,3}(\tau_l) - b_{f,3}(\tau_s)\right)^2 \left(1 + \beta_{31}X_1(t) + \beta_{32}X_2(t)\right), \end{aligned} \tag{15}$$

$$\begin{aligned} &Cov\left(t,\, slope_d(t,\, \tau_s,\, \tau_l),\; slope_f(t,\, \tau_s,\, \tau_l)\right) \\ &\quad +\left(b_{d,1}(\tau_l) - b_{d,1}(\tau_s)\right)\left(b_{f,1}(\tau_l) - b_{f,1}(\tau_s)\right) X_1(t) \\ &\quad +\left(b_{d,2}(\tau_l) - b_{d,2}(\tau_s)\right)\left(b_{f,2}(\tau_l) - b_{f,2}(\tau_s)\right) X_2(t) \\ &\quad +\left(b_{d,3}(\tau_l) - b_{d,3}(\tau_s)\right)\left(b_{f,3}(\tau_l) - b_{f,3}(\tau_s)\right)\left(1 + \beta_{31}X_1(t) + \beta_{32}X_2(t)\right). \end{aligned} \tag{16}$$

In Equations (14)-(16), $X_1(t)$ and $X_2(t)$ can generate the stochastic second moments of the term structure slopes. Therefore, $JA_2(3)$ is more flexible than $JA_1(3)$ in generating the stochastic second moments of the term structure slopes. However, $JA_2(3)$ is less flexible than $JA_1(3)$ in generating the negative correlations among the factors. As shown by DS, the admissibility conditions require that the two square-root factors, $X_1(t)$ and $X_2(t)$, cannot be negatively correlated. In our specification, these restrictions require that neither κ_{12} nor κ_{21} can have a positive value. On the contrary, the correlations between the square-root factor $X_1(t)$ or $X_2(t)$ and the Gaussian factor $X_3(t)$ can have negative signs.

c) $JA_3(3)$

The family of $JA_3(3)$ is characterized by the assumption that three of the state variables derive the stochastic volatility of all three state variables. The assumptions of $JA_3(3)$ are as follows. First, $r_d(t)$ and $r_f(t)$ are affine functions of the three common state variables

$$r_d(t) = \delta_0^d + \sum_{i=1}^{3} \delta_{1i}^d X_i(t) \text{ and } r_f(t) = \delta_0^f + \sum_{i=1}^{3} \delta_{1i}^f X_i(t). \tag{17}$$

Second, the dynamics of $X(t)$ are given as

$$\begin{pmatrix} dX_1(t) \\ dX_2(t) \\ dX_3(t) \end{pmatrix} = \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 - X_1(t) \\ \theta_2 - X_2(t) \\ \theta_3 - X_3(t) \end{pmatrix} dt + \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{X_3(t)} \end{pmatrix} \begin{pmatrix} d\omega_1(t) \\ d\omega_2(t) \\ d\omega_3(t) \end{pmatrix}. \tag{18}$$

Third, the market prices of factor risks are given as

$$\xi_d(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{X_3(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^d \\ \lambda_2^d \\ \lambda_3^d \end{pmatrix},$$

$$\xi_f(t) = \begin{pmatrix} \sqrt{X_1(t)} & 0 & 0 \\ 0 & \sqrt{X_2(t)} & 0 \\ 0 & 0 & \sqrt{X_3(t)} \end{pmatrix} \begin{pmatrix} \lambda_1^f \\ \lambda_2^f \\ \lambda_3^f \end{pmatrix}.$$

The parametric restrictions of $JA_3(3)$ are given as

$$\begin{aligned} &\textstyle\sum_{j=1}^{3} \kappa_{ij}\theta_i > 0, \text{ for } i = 1, 2, 3, \\ &\kappa_{ij} \le 0, \text{ for all } i \ne j,\ i, j = 1, 2, 3, \\ &\kappa_{ii} > 0,\ \theta_i \ge 0 \text{ for } i = 1, 2, 3. \end{aligned}$$

Then, the instantaneous second moments for the two term structure slopes are

$$\begin{aligned} Var_d(t, \tau_s, \tau_l) = &\left(b_{d,1}(\tau_l) - b_{d,1}(\tau_s)\right)^2 X_1(t) \\ &+ \left(b_{d,2}(\tau_l) - b_{d,2}(\tau_s)\right)^2 X_2(t) \\ &+ \left(b_{d,3}(\tau_l) - b_{d,3}(\tau_s)\right)^2 X_3(t), \end{aligned} \tag{19}$$

$$\begin{aligned} Var_f(t, \tau_s, \tau_l) = &\left(b_{f,1}(\tau_l) - b_{f,1}(\tau_s)\right)^2 X_1(t) \\ &+ \left(b_{f,2}(\tau_l) - b_{f,2}(\tau_s)\right)^2 X_2(t) \end{aligned} \tag{20}$$

$$+\left(b_{f,3}(\tau_l)-b_{f,3}(\tau_s)\right)^2 X_3(t),$$

$$\begin{aligned} &Corr\left(t,\ slope_d(t,\ \tau_s,\ \tau_l),\ slope_f(t,\ \tau_s,\ \tau_l)\right) \\ &\quad = \left(b_{d,1}(\tau_l)-b_{d,1}(\tau_s)\right)\left(b_{f,1}(\tau_l)-b_{f,1}(\tau_s)\right)X_1(t) \\ &\quad +\left(b_{d,2}(\tau_l)-b_{d,2}(\tau_s)\right)\left(b_{f,2}(\tau_l)-b_{f,2}(\tau_s)\right)X_2(t) \\ &\quad +\left(b_{d,3}(\tau_l)-b_{d,3}(\tau_s)\right)\left(b_{f,3}(\tau_l)-b_{f,3}(\tau_s)\right)X_3(t). \end{aligned} \tag{21}$$

As described in Equations (19)-(21), all factors can generate the stochastic second moments of the term structure slopes. Therefore, $JA_3(3)$ is the most flexible in inducing heteroskedastic volatility. Furthermore, $JA_3(3)$ is the only model that guarantees the positivity of the nominal interest rates $r_d(t)$ and $r_f(t)$. However, $JA_3(3)$ is incapable of generating the negative correlations among the factors. As discussed, admissibility conditions require non-negative correlations among the square-root factors. Therefore, JATSMs cannot simultaneously allow for negative correlations among the factors and guarantee the positivity of the nominal interest rates.

C. Dynamics of the Exchange Rate

To characterize completely the risk exposure of an international bond portfolio, we also need to model the dynamics of the exchange rate. If the world economy is complete and permits no arbitrage trading opportunity, then there exists a unique exchange rate, which is defined as the ratio of the global SDFs, as shown by Bansal (1997), Backus, Foresi, and Telmer (2001), and Ahn (2004).

$$\frac{Y(T)}{Y(t)} = \frac{M_f(t,T)}{M_d(t,T)}, \tag{22}$$

where $Y(t)$ is the exchange rate defined as the number of units of domestic currency per one unit of foreign currency. Applying Ito's lemma to Equation (22), given the dynamics of the global SDFs described in Equation (2), results in the following dynamics of the exchange rate:

$$d\ln Y(t) = \left[\left(r_d(t)-r_f(t)\right) + \frac{1}{2}\sum_{i=1}^{3}\left(\lambda_i^{d^2}-\lambda_i^{f^2}\right)\left(\alpha_i+\beta_i' X(t)\right)\right]dt \tag{23}$$

$$+\sum_{i=1}^{3}\left(\lambda_i^d - \lambda_i^f\right)\sqrt{\alpha_i + \beta_i' X(t)}\, dw(t)\,.$$

By Girsanov theorem, the dynamics of the exchange rate under the equivalent martingale measure Q are

$$d\ln Y(t) = \left(r_d(t) - r_f(t)\right)dt + \sum_{i=1}^{3}\left(\lambda_i^d - \lambda_i^f\right)\sqrt{\alpha_i + \beta_i' X(t)}\, dw^Q(t), \tag{24}$$

where $w^Q(t) = w(t) + \int_0^t \Sigma\sqrt{S(\tau)}\, d\tau$. As pointed out by Ahn (2004), Equation (24) states that the uncovered interest rate parity holds under the probability measure Q. However, it does not hold under the physical probability measure P. Equation (23) indicates that the exchange rate compensates for the difference not only between the interest rates but also between the market prices of factor risks required in the two countries. Therefore, our two-country term structure model extends the uncovered interest rate parity to the physical probability measure.

III. Data and Estimation Method

A. Data

Our EMM estimation analysis is based on bi-weekly (Thursday-to-Thursday) observations for the US and UK term structure slopes and the dollar-pound exchange rate return from April 16, 1987 to June 28, 2007 (528 observations). We retrieve the LIBOR rates of 6- and 12-month maturities and the swap rates for maturities of 2-5 years for the US and UK. We then use these rates to bootstrap zero-coupon LIBOR and swap yields according to Piazzesi (2001). Term structure slopes are defined as $yld(t, \tau_l) - yld(t, \tau_s)$, where $yld(t, \tau_l)$ is the 5-year zero-coupon yield and $yld(t, \tau_s)$ is the 6-month zero-coupon yield for each country. The bi-weekly return of the dollar-pound exchange rate is defined as $100 \cdot (\log Y(t) - \log Y(t-1))$, where $Y(t)$ is the dollar-pound exchange rate obtained from Morgan Stanley Capital International. Both interest rates and exchange rate data are provided by Datastream.

Table 1 presents the summary statistics of the data. First, the US slope is steeper than that of the UK on average. As presented in Figure 1, which depicts the time series of our data, the UK term structure has negative values during the late 1980s, early 1990s, and late 1990s, whereas the US slope is positive most of the time. The average of cur-

TABLE 1

SUMMARY STATISTICS

Panel A: Summary statistics

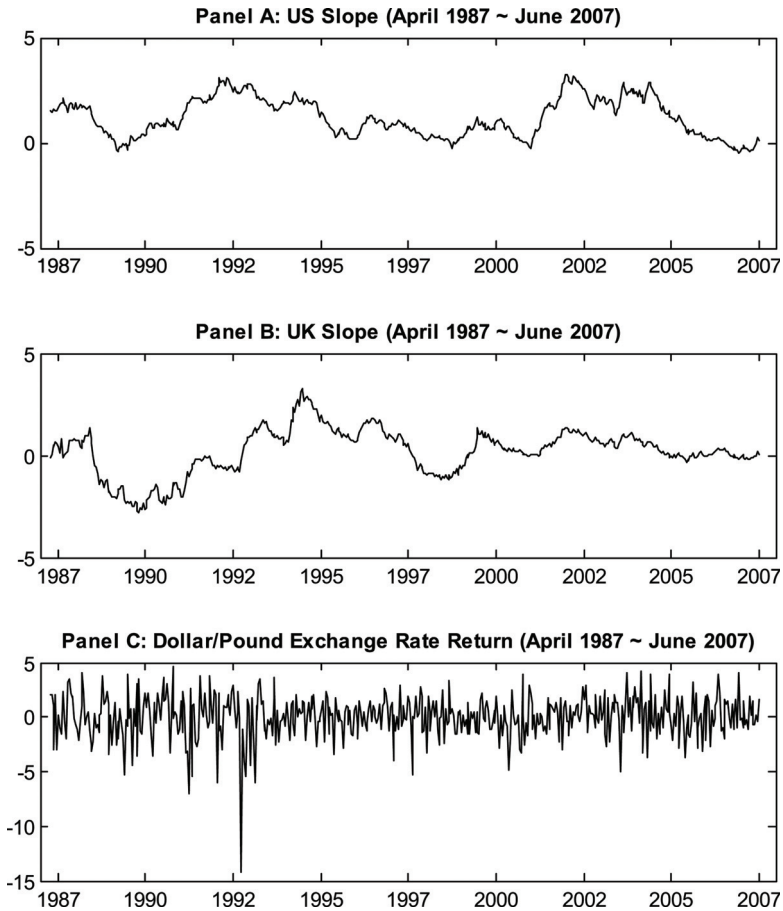
	Mean	Std. Dev.	Skewness	Kurtosis	Auto. Corr.	Min	Max	JB statistics
U.S. slope	1.1964	0.9213	0.2193	1.9603	0.9854	−0.4901	3.2342	28.3324
U.K. slope	0.1811	1.1215	−0.4173	3.2788	0.9862	−2.8053	3.2646	16.7956
FX return	0.0433	1.9170	−1.1439	8.8942	0.0177	−14.234	4.6602	870.0952

Panel B: Cross-correlations

	US slope	UK slope	FX return
U.S. slope	1	0.4197	0.0047
U.K. slope	0.4197	1	0.0160
FX return	0.0047	0.0160	1

This table presents the summary statistics of the bi-weekly data of the US and UK term structure slopes and the dollar-pound exchange rate return from April 16, 1987 to June 28, 2007 (528 observations). The term structure slopes are defined as $yld(t, \tau_l) - yld(t, \tau_s)$, where $yld(t, \tau_l)$ is the 5-year yield and $yld(t, \tau_s)$ is the 6-month yield for each country. The dollar-pound exchange rate return is defined as $100 \cdot (\log Y(t) - \log Y(t-1))$, where $Y(t)$ is the dollar-pound exchange rate. The JB statistics refer to the Jarque-Bera test statistics. All figures are expressed in percentages.

rency depreciation rates (bi-weekly log returns on the dollar price of the pound) is positive, which means that the dollar depreciates over the pound during our sample period. Second, the two term structure slopes are highly persistent, whereas the exchange rate return is relatively stationary. Third, the Jarque-Bera (JB) statistics in panel A of Table 1 clearly suggests that none of the tri-variate series are normally distributed, but they have different distributional characteristics. Both the UK slope and the exchange rate return are negatively skewed and leptokurtic, whereas the US slope is positively skewed and platykurtic. Many of these important dynamics of the data are recovered by the semi-nonparametric (SNP) density, which is presented in the following subsection.



The plots present the US and UK term structure slopes and the dollar-pound exchange rate return from April 16, 1987 to June 28, 2007 (528 observations). The term structure slopes are defined as $yld(t, \tau_l) - yld(t, \tau_s)$, where $yld(t, \tau_l)$ is the 5-year yield and $yld(t, \tau_s)$ is the 6-month yield for each country. The dollar-pound exchange rate return is defined as $100 \cdot (\log Y(t) - \log Y(t-1))$, where $Y(t)$ is the dollar-pound exchange rate. The data are sampled bi-weekly.

FIGURE 1
DATA

B. EMM

The EMM method is briefly described here.[9] As noted by Gallant and Tauchen (1996), the EMM method consists of a two-step process. The

first step is fitting a consistent estimator of the conditional density of the observed data. Let $f(y_t|x_{t-1}, \Theta)$ denote this approximation to the density, where y_t denotes the current observations, x_{t-1} denotes the lagged observations, and Θ denotes a parameter vector of the density approximation. In the current paper, y_t is a vector of the US slope, the UK slope, and the dollar-pound exchange rate return. We approximate this density using the SNP procedure of Gallant and Nychka (1987) and Gallant and Tauchen (1989). The SNP density used in this paper is a new version provided by Gallant and Tauchen (2007a). In this SNP density, a Gaussian vector autoregression (VAR) process captures the conditional first moments of the data, and a BEKK-GARCH of Engle and Kroner (1995) describes the conditional second moment dynamics of the data. As in previous versions of the SNP density, a Hermite polynomial expansion captures the deviations from conditional normality. Denoting a demeaned transformation of y_t as $z_t = R^{-1}_{x_{t-1}}(y_t - \mu_{x_{t-1}})$, where the conditional mean function $\mu_{x_{t-1}}$ is a VAR on L_u lags,

$$\mu_{x_{t-1}} = b_0 + B\,x_{t-1},$$

and the conditional variance function $\Sigma_{x_{t-1}} = R^2_{x_{t-1}}$ is a BEKK-GARCH on (L_g, L_r) lags:

$$\Sigma_{x_{t-1}} = R_0 R_0' + \sum_{i=1}^{L_g} Q_i \Sigma_{x_{t-i}} Q_i' + \sum_{i=1}^{L_r} P_i (y_{t-i} - \mu_{x_{t-1-i}})(y_{t-i} - \mu_{x_{t-1-i}})' P_i',$$

where R_0 is an upper triangular matrix, and the matrices P_i and Q_i can be scalar, diagonal, or full matrices. The SNP density of z_t is given by

$$f_K(z_t \mid x_{t-1}, \Theta) = \frac{[\,P(z_t, x_{t-1})\,]^2\,\phi(z_t)}{\int [\,P(u, x_{t-1})\,]^2\,\phi(u)\,du},$$

$$P(z_t, x_{t-1}) = \sum\nolimits_{|\alpha|=0}^{K_z} \left(\sum\nolimits_{|\beta|=0}^{K_x} a_{\beta\alpha}\, x_{t-1}^{\beta} \right) z_t^{\alpha},$$

where $P(z_t, x_{t-1})$ is a polynomial in (z_t, x_{t-1}) of degree (K_z, K_x), and $\phi(z_t)$ denotes the standard normal density function.

The second step in the EMM process involves estimating a parameter

[9] For details on the EMM, see Gallant and Tauchen (1996, 2001, 2007b).

vector for our JATSMs. The procedure takes a set of initial starting values for the model and simulates a long set of data. In our case, we simulate 10,000 series of the US slope, the UK slope, and the log exchange rates by the standard Euler scheme. The SNP model is fit to the simulated data and the scores of the fitted model with respect to the SNP parameters are estimated. Designate the parameters of the structure model (*i.e.*, JATSMs) as ρ and the estimated parameters of the SNP density as $\tilde{\Theta}$. The scores of the fitted SNP model are used as moment conditions, $m'(\rho, \tilde{\Theta})$, and the quadratic form

$$m'(\rho,\tilde{\Theta})\ \tilde{I}^{-1}\, m(\rho,\tilde{\Theta})$$

is estimated, where $\tilde{I}^{-1}$ denotes the quasi-information matrix from the quasi-maximum likelihood estimation of Θ. If a structural model is correctly specified, then the statistic

$$nm'(\rho,\tilde{\Theta})\,\tilde{I}^{-1}\, m(\rho,\tilde{\Theta})$$

is asymptotically chi-squared on $l_\Theta - l_\rho$ degrees of freedom, where l_Θ and l_ρ are the lengths of parameter vectors Θ and ρ, respectively.

IV. Estimation Results

This section discusses the EMM estimation results for the three JATSMs investigated in the current paper. The first subsection reports the estimation of the SNP score generator. The following subsections present the various diagnostics that enable us to understand the strengths and weaknesses of the different model specifications. We focus on the specification testing based on the quantitative quasi t-ratios obtained from the EMM estimation procedure in the second subsection. We then analyze the models further by examining their ability to match specific conditional moments of the data through the reprojection method.

A. Estimation of the SNP Density

We fit an SNP model to the US slope, the UK slope, and the dollar-pound exchange rate return using the procedure outlined by Gallant and Tauchen (2007a). The authors suggest an upward fitting strategy, in which the parameters of parts of the SNP model are tuned to min-

imize the Schwartz (1978) criterion [*i.e.*, Bayes Information Criterion (BIC)] and are then used as starting points for the fitting of the next part of the model. In the present paper, we adopt a more general strategy for the determination of the variance function. As previously discussed, the current version of the SNP model provided by Gallant and Tauchen (2007a) enables the matrices P_i and Q_i to be scalar, diagonal, or full matrices. We examine the BIC values of all combinations of P_i and Q_i given the VAR(1) mean dynamics. Except for the estimation of the variance function, we follow the upward fitting strategy. Our BIC-preferred SNP density is described by $\{L_u, L_g, L_r, K_z, I_z, K_x\}=\{1, 1s, 1f, 4, 0, 0\}$.

$L_u=1$ implies that one lag of the data is sufficient to describe the mean dynamics in the VAR. $\{L_g, L_r\}=\{1s, 1f\}$ suggests that the BEKK-GARCH(1, 1) model, where P_1 is a three-dimensional matrix and Q_1 is a scalar, describes the conditional second moments of the data. We need a fourth-order Hermite polynomial in standardized innovation to capture the shape deviations from conditional normality. $I_z=0$ implies that the interaction terms in the orders of the polynomials are suppressed. Finally, $K_x=0$ suggests that incorporating the lags of the process is not necessary in modeling the coefficients of the Hermite polynomials.

B. EMM Specification Tests

Estimation results for the three JATSMs are presented in Table 2, which presents the parameter estimates and specification tests for each of the model. The bottom rows of Table 2 present the χ^2 statistics for the model fit and the z-statistic for the goodness of fit that is asymptotically standard normal and adjusted for degrees of freedom.[10]

Table 2 shows that all models are rejected, suggesting that our JATSMs are incapable of capturing the joint dynamics of the US slope, the UK slope, and the exchange rate return. Although all the models are sharply rejected, $JA_2(3)$ shows the best performance, followed by $JA_1(3)$. Interestingly, $JA_3(3)$, which is the best popular model in the international term structure modeling literature, shows the worst performance. The worst performance of $JA_3(3)$ clearly indicates that allowing the negative correlations among the factors plays a critical role in our data. As discussed, $JA_3(3)$ is the only model that cannot accommodate the negative correlation structure among the factors. The estimated

[10] The z-statistic is calculated as $(\chi^2 - df)/\sqrt{2df}$ and represents a degrees of freedom normalization of the χ^2 statistic.

TABLE 2
PARAMETER ESTIMATES AND SPECIFICATION TESTS

$JA_1(3)$		$JA_2(3)$		$JA_3(3)$	
δ_0^d	0.0143	δ_0^d	0.0362	δ_0^d	0.0288
	(0.0044)		(0.0005)		(0.0011)
δ_0^f	0.0217	δ_0^f	−0.0050	δ_0^f	−0.0003
	(0.0030)		(0.0010)		(0.0019)
δ_1^d	0.0078	δ_1^d	0.0033	δ_1^d	0.0051
	(0.0002)		(0.0008)		(0.0003)
δ_2^d	0.0088	δ_2^d	0.0018	δ_2^d	−0.0068
	(0.0004)		(0.002)		(0.0001)
δ_3^d	0.0016	δ_3^d	0.0015	δ_3^d	0.0032
	(0.0002)		(0.0008)		(0.0002)
δ_1^f	0.0060	δ_1^f	0.0075	δ_1^f	0.0096
	(0.0006)		(0.0006)		(0.0004)
δ_2^f	0.0009	δ_2^f	0.0049	δ_2^f	0.0061
	(0.0007)		(0.0002)		(0.0005)
δ_3^f	-0.0059	δ_3^f	−0.0001	δ_3^f	0.0016
	(0.0003)		(1.9e-05)		(0.0006)
κ_{11}	0.7159	κ_{11}	6.8961	κ_{11}	1.6578
	(0.1294)		(0.0410)		(0.0327)
κ_{21}	2.4700	κ_{21}	−1.8768	κ_{21}	−1.9661
	(0.1432)		(0.0406)		(0.1165)
κ_{31}	4.8469	κ_{31}	9.1254	κ_{31}	−9.1370
	(0.6268)		(0.0787)		(0.5192)
κ_{22}	6.2050	κ_{12}	−1.0133	κ_{12}	−0.8311
	(0.2257)		(0.0063)		(0.0451)
κ_{32}	−1.4908	κ_{22}	0.7566	κ_{22}	2.8267
	(1.2925)		(0.0162)		(0.1006)
κ_{32}	−1.1843	κ_{32}	−1.0639	κ_{32}	−0.1582
	(0.3148)		(0.0102)		(0.1646)
κ_{33}	5.4570	κ_{33}	0.5086	κ_{13}	−0.0124
	(0.1662)		(0.0469)		(0.0085)
θ_1	2.4819	θ_1	2.9806	κ_{23}	−0.1318
	(0.0236)		(0.0102)		(0.0235)
β_{21}	1.8873	θ_2	5.8042	κ_{33}	2.4937
	(0.3325)		(0.1302)		(0.0586)
β_{31}	0.4274	β_{31}	2.9806	θ_1	1.6918
	(0.0152)		(0.2542)		(0.0596)
λ_1^d	−0.0100	β_{32}	5.2763	θ_2	1.6090
	(0.0002)		(0.2535)		(0.0524)

(Table 2 Continued)

TABLE 2
(CONTINUED)

$JA_1(3)$		$JA_2(3)$		$JA_3(3)$	
λ_2^d	0.0155	λ_1^d	0.0422	θ_3	7.5373
	(0.0002)		(0.0001)		(0.0568)
λ_3^d	−0.0264	λ_2^d	−0.0159	λ_1^d	−0.0331
	(0.0006)		(5.9e-05)		(0.0002)
λ_1^f	0.0047	λ_3^d	0.0050	λ_2^d	0.0104
	(0.0007)		(0.0002)		(0.0008)
λ_2^f	−0.0066	λ_1^f	0.0012	λ_3^d	0.0263
	(0.0005)		(0.0005)		(0.0009)
λ_3^f	0.0139	λ_2^f	−0.0006	λ_1^f	−0.0058
	(0.0003)		(0.0007)		(0.0008)
		λ_3^f	−0.0001	λ_2^f	−0.0030
			(7.1e-05)		(0.0005)
				λ_3^f	−0.0009
					(8.5e-05)
χ^2	52.93		48.23		54.74
df	16		15		14
p-value (percent)	0.0008		0.0023		0.0001
z-value	6.53		6.07		7.70

This table presents the parameter estimates and goodness-of-fit tests for the JATSMs. Standard errors are given in parentheses. The last four rows report the χ^2 statistics for the goodness of fit of the models, the degrees of freedom, p-values, and corresponding z-values.

parameters for k_{21} and k_{31} of $JA_1(3)$ are 2.4700 and 4.8469, respectively, and they are highly significant. These results indicate that the two Gaussian factors, $X_2(t)$ and $X_3(t)$, are negatively correlated with the square-root factor $X_1(t)$. Similarly, the estimated value for k_{31} of $JA_2(3)$ is 9.1254 and is highly significant.

The superior performance of $JA_2(3)$ over $JA_1(3)$ reveals that the stochastic volatility factors also play an important role in fitting the observed joint dynamics of the data. In $JA_2(3)$, the two square-root factors, $X_1(t)$ and $X_2(t)$, contribute to generating the stochastic second moments of the data, whereas only the first factor, $X_1(t)$, can induce the stochastic volatility in $JA_1(3)$. Therefore, our finding suggests that the theoretical trade-off of JATSMs in generating the negative correlations among the factors and the stochastic second moments of the data hampers

their empirical performance. Although MS and ELN commonly find similar results for international term structure level data, our results are new because these papers do not directly investigate the performance of their JATSMs in explaining international term structure slopes.

Investigating the sensitivities of the interest rates to the three common factors is of interest. For $JA_1(3)$, the estimated sensitivity of the UK slope to the second factor is close to zero (0.0009) and is not significant, whereas the sensitivity of the US slope to the second factor is 0.0088 and is highly significant. For $JA_2(3)$, the sensitivity of the UK slope to the third factor is far less than that of the US slope. Therefore, there seem to be clear differences in the magnitudes of the factor sensitivities of the interest rates.

Additional insight into the performances of the models can be derived from analyzing the scores of the best model fits with respect to the SNP parameter vector. Table 3 reports the quasi t-ratios for the 40 moment conditions for the models. For a reasonable model specification, these 40 scores should be close to zero. Gallant and Long (1997) and Tauchen (1998) show that a quasi t-ratio above 2.0 in magnitude indicates that the model fails to explain the corresponding score.

Table 3 suggests that all the models perform fairly well in capturing the mean dynamics of the VAR part of the fitted SNP density. None of the models have t-ratios greater than 2.0 in magnitude. However, note that our evidence does not indicate that the term structure factors alone can explain the mean dynamics of the exchange rate because we estimate the models relying on both the exchange rate data and the structure slopes data.[11]

The scores with respect to the BEKK-GARCH terms reveal interesting patterns. $JA_2(3)$ has no t-ratio greater than 2.0 and shows the best performance in capturing the second moment dynamics of the data. $JA_1(3)$ has two t-ratios greater than 2.0. $JA_1(3)$ has some difficulty in matching the second moment dynamics of the data, suggesting that one square-root factor is insufficient to capture correctly the variance dynamics of the data. Interestingly, $JA_3(3)$ has one t-ratio greater than 2.0. This result shows the importance of the negative correlations among the factors in fitting the conditional second moments of the data.

The scores with respect to the Hermite terms of the two term struc-

[11] Brandt and Santa-Clara (2002) and Inci and Lu (2004) provide evidence for the existence of the exchange rate factors independent of the term structure factors.

TABLE 3

DIAGNOSTIC T-RATIOS

	$JA_1(3)$	$JA_2(3)$	$JA_3(3)$
$b_0(1)$	−1.8529	−1.5209	−1.4656
$b_0(2)$	1.3085	0.7853	1.5486
$b_0(3)$	−1.2212	−1.1312	−0.4935
$B(1, 1)$	0.7850	0.4072	0.5874
$B(2, 1)$	−0.8233	0.1297	−1.0707
$B(3, 1)$	1.2208	0.1253	1.0113
$B(1, 2)$	−0.8635	−0.4909	0.4488
$B(2, 2)$	0.9776	0.7779	0.3958
$B(3, 2)$	1.0707	0.5307	0.4250
$B(1, 3)$	0.3009	0.8221	−0.1920
$B(2, 3)$	0.9583	0.6106	0.2447
$B(3, 3)$	1.1961	1.1785	0.5834
$R_0(1, 1)$	0.7175	−0.6524	−0.0957
$R_0(1, 2)$	1.0789	0.7622	1.4234
$R_0(2, 2)$	0.4358	1.2543	1.4343
$R_0(1, 3)$	−0.6658	1.7817	−0.2272
$R_0(2, 3)$	1.1248	0.1101	0.1030
$R_0(3, 3)$	**−2.4415**	−0.2609	−1.1442
$P_1(1, 1)$	0.1791	−0.8282	−0.6052
$P_1(2, 1)$	−0.1015	0.5850	0.9130
$P_1(3, 1)$	**−2.1794**	0.3066	−0.1835
$P_1(1, 2)$	−1.3645	−0.2583	−0.8006
$P_1(2, 2)$	1.8760	0.8738	**2.3578**
$P_1(3, 2)$	−0.7125	0.4089	0.2752
$P_1(1, 3)$	−0.8166	1.0253	0.7196
$P_1(2, 3)$	1.2552	0.8030	0.6889
$P_1(3, 3)$	−1.5801	0.3052	−0.3518
Q_1	−0.2039	−0.4496	−0.9072
$\alpha(0, 0, 1)$	−0.3727	−0.4554	−0.1643
$\alpha(0, 0, 2)$	**−2.7764**	−1.1339	−1.5168
$\alpha(0, 0, 3)$	**2.5538**	**2.1458**	1.7850
$\alpha(0, 0, 4)$	0.3541	−1.5917	−0.8327
$\alpha(0, 1, 0)$	−0.6430	−1.0837	0.2793
$\alpha(0, 2, 0)$	1.6541	−0.3301	**2.6150**
$\alpha(0, 3, 0)$	**−2.1009**	−0.6093	**−2.1172**
$\alpha(0, 4, 0)$	−0.7047	−0.2133	−1.6836
$\alpha(1, 0, 0)$	−0.4516	**−2.0295**	−1.2015
$\alpha(2, 0, 0)$	0.0788	−1.9965	−0.5268
$\alpha(3, 0, 0)$	**−3.3193**	**−2.9556**	**−3.2596**
$\alpha(4, 0, 0)$	−0.0146	−0.0532	−0.5964

This table presents the t-ratio diagnostics for the EMM scores. The t-ratios are the test statistics of the null hypothesis that the scores, with respect to the parameters of the SNP density, are equal to zero. $\alpha(i, j, k)$ refers to the parameter before the polynomial term with the ith degree of power on the US slope, the jth degree of power on the UK slope, and the kth degree of power on the exchange rate return.

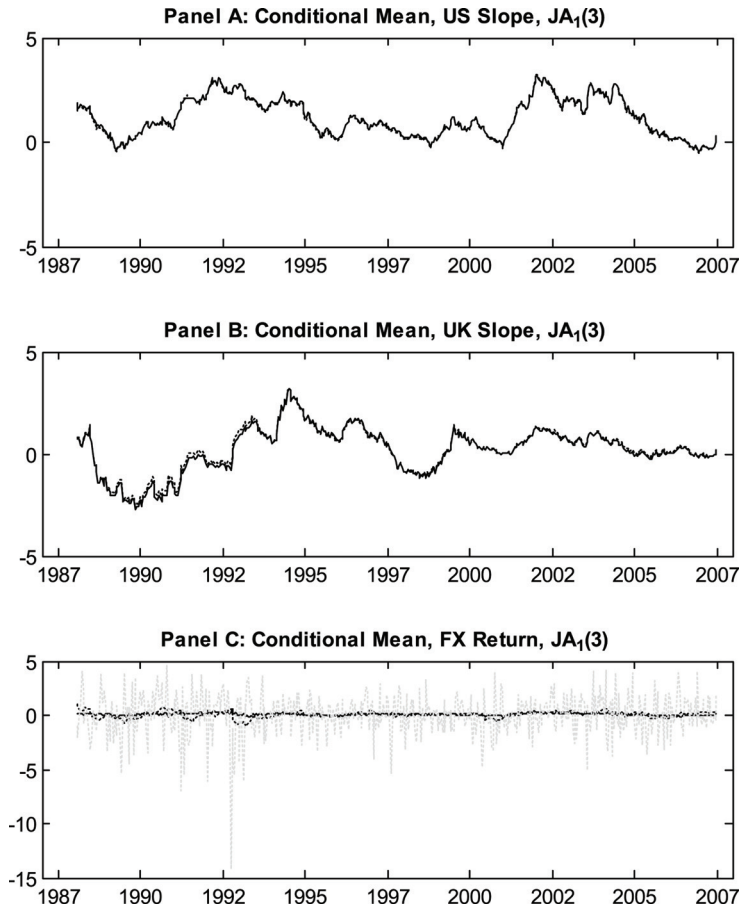
ture slopes reveal some important differences across the three JATSMs. First, $JA_3(3)$ performs worst in fitting the shape characteristic of the US and UK term structure slopes. Of the eight scores describing the conditional non-normality of the US and UK slopes, $JA_3(3)$ has three t-ratios greater than 2.0. However, both $JA_1(3)$ and $JA_2(3)$ have two t-ratios greater than 2.0, indicating that these models perform better than $JA_3(3)$ in capturing the Hermite terms. Overall, our results show the importance of the negative correlations among the factors in explaining the higher moment dynamics of the two term structure slopes. However, our results also indicate that none of the three models are able to match the higher moments successfully.

The t-ratios with respect to the Hermite terms of the exchange rate clearly show that $JA_3(3)$ performs best in explaining the shape characteristic of the conditional density for the exchange rate return data. $JA_3(3)$ has no t-ratio greater than 2.0, indicating that this model is capable of capturing the higher moments of the exchange rate return data. Interestingly, $JA_1(3)$ shows the worst performance. Among the four Hermite scores for the exchange rate return data, two are significant for $JA_1(3)$, and one is significant for $JA_2(3)$. This finding suggests that the negative correlations among the factors may not be a critical ingredient of the models in explaining the conditional skewness or the conditional kurtosis of the exchange rate data. On the contrary, the square-root factors seem to play an important role.

C. Reprojection

We briefly summarize the reprojection method here. A completed discussion is provided by Gallant and Tauchen (1998). The reprojection method provides additional diagnostics for the adequacy of the JATSMs. The idea behind the reprojection method is to characterize the dynamics of a given vector of observed variables conditional on its lags. In models where there are latent state variables, the reprojected conditional density provides a way to characterize the conditional density strictly in terms of observables. The reprojected density can be estimated by relying on simulated data for the data from a given estimated structural model. In our context, the reprojected density is the tri-variate conditional density for the two term structure slopes and the exchange rate return.

Let $p(y_t|x_{t-1})$ denote the conditional density for the data implied by the candidate JATSMs, where y_t denotes the contemporaneous data, and x_{t-1} denotes the lagged data. As no analytical expression of the

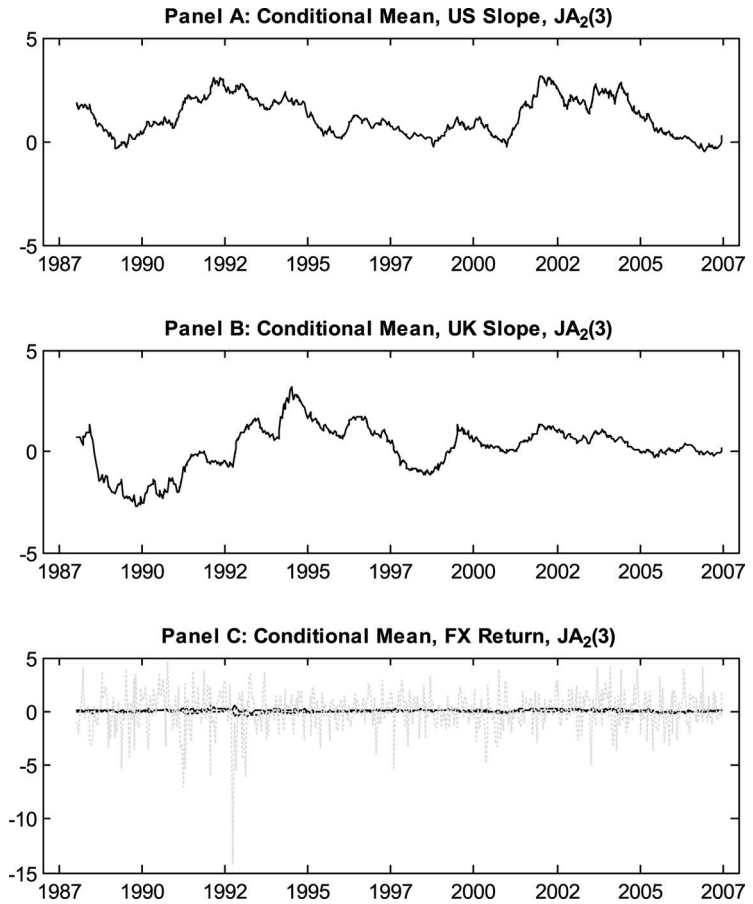


The plots present the reprojected conditional mean for $JA_1(3)$ against the projected conditional mean. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line. In the last plot, the dotted line represents the actual dollar-pound return.

FIGURE 2

PROJECTED AND REPROJECTED CONDITIONAL MEAN: $JA_1(3)$

conditional density implied by $JA_1(3)$, $JA_2(3)$, or $JA_3(3)$ model is known, we cannot estimate it by $\hat{p}(y_t|x_{t-1}) = p(y_t|x_{t-1}, \hat{\rho}_n)$, where $\hat{\rho}_n$ denotes the estimated model parameters presented in Table 2. Gallant and Tauchen (1998) suggest using $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ as an approximation of $\hat{p}(y_t|x_{t-1})$, where $\{\hat{y}_t, \hat{x}_{t-1}\}_{t=1}^{N}$ are simulated data generated by $\hat{\rho}_n$, and $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$

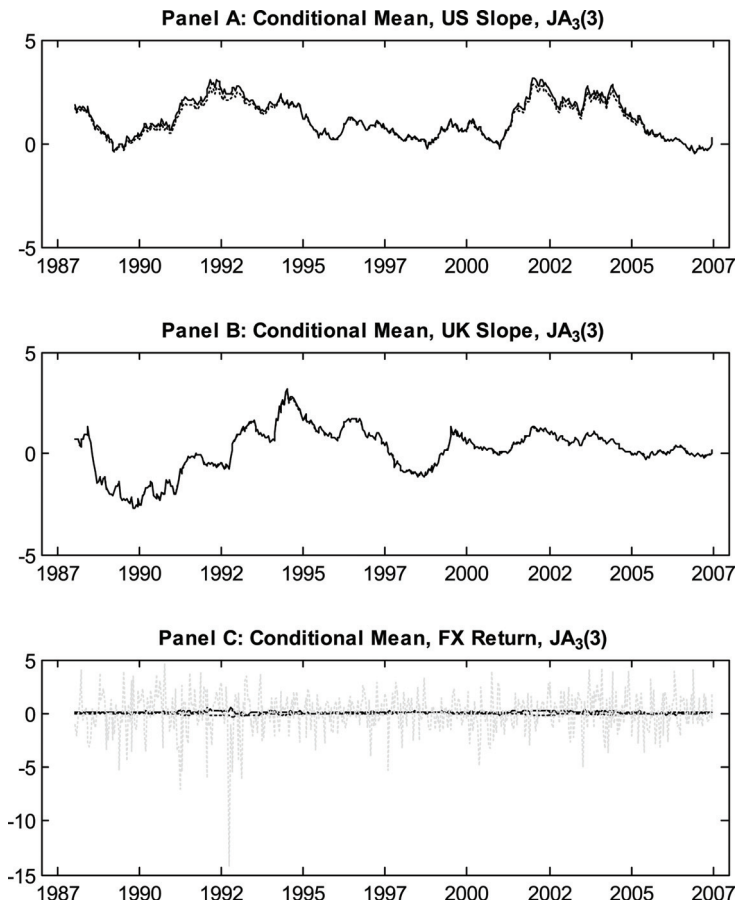


These plots present the reprojected conditional mean for $JA_2(3)$ against the projected conditional mean. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line. In the last plot, the dotted line represents the actual dollar-pound return.

FIGURE 3

PROJECTED AND REPROJECTED CONDITIONAL MEAN: $JA_2(3)$

is an SNP density with the K-dimensional parameter vector $\hat{\Theta}$. Gallant and Long (1997) show that $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ converges to $\hat{p}(y_t|x_{t-1})$ as K goes to infinity. We estimate $f_K(\hat{y}_t|\hat{x}_{t-1}, \hat{\Theta})$ by re-estimating the parameters of the SNP density using the same specification used to characterize the observed tri-variate density for the US and UK slopes and the



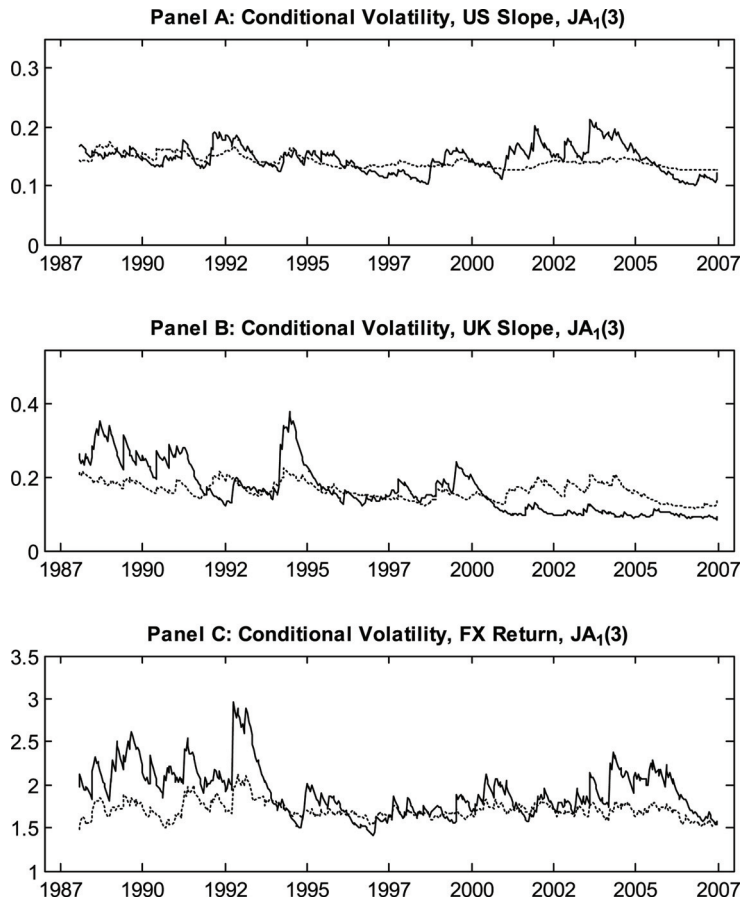
These plots present the reprojected conditional mean for $JA_3(3)$ against the projected conditional mean. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line. In the last plot, the dotted line represents the actual dollar-pound return.

FIGURE 4

PROJECTED AND REPROJECTED CONDITIONAL MEAN: $JA_3(3)$

exchange rate return.

Once the reprojected conditional density is estimated, specific moments, such as the conditional means, variances, and correlations implied by the model specification, can be computed. These conditional moments are simply continuous functions of the conditioning information (*i.e.*,

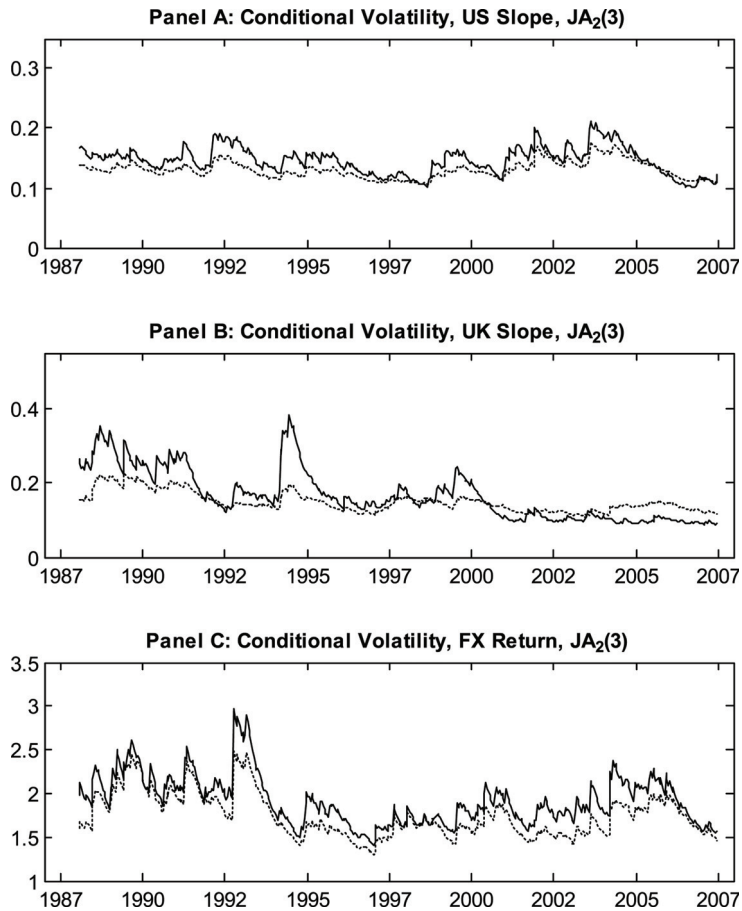


These plots present the reprojected conditional volatility for $JA_1(3)$ against the projected conditional volatility. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 5

PROJECTED AND REPROJECTED CONDITIONAL VOLATILITY: $JA_1(3)$

lagged data) used to estimate the reprojected density. Given the conditioning information, the implications of a given JATSM for any conditional moment of interest can be tracked down in the data and compared with the conditional moment implied by the unrestricted SNP density (*i.e.*, the fitted SNP density for the observed data). Therefore, the reprojected conditional density can be used to evaluate the perform-



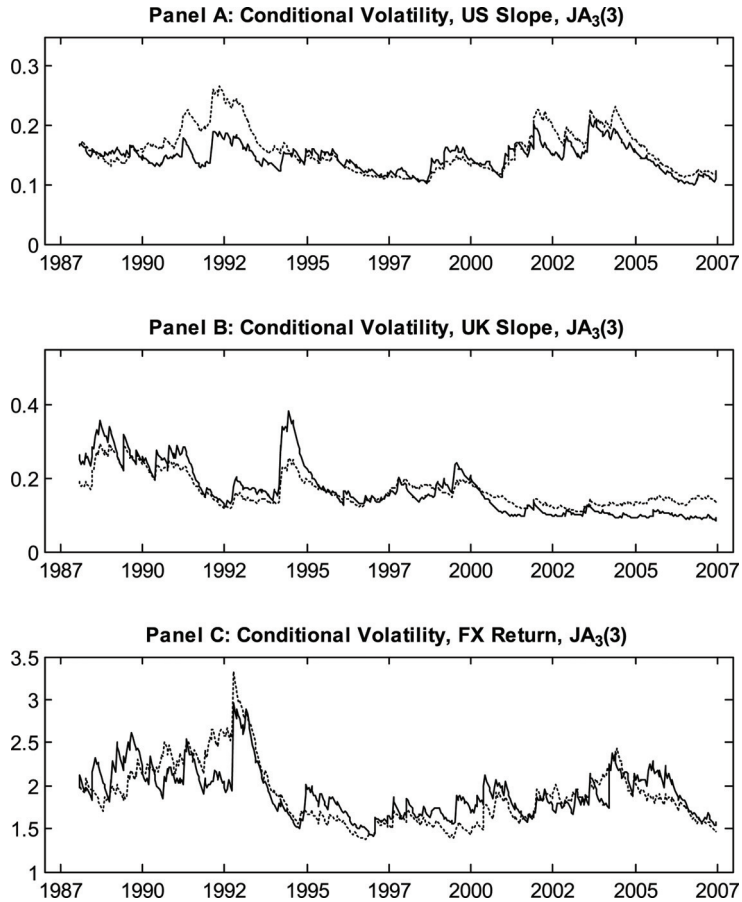
These plots present the reprojected conditional volatility for $JA_2(3)$ against the projected conditional volatility. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 6

PROJECTED AND REPROJECTED CONDITIONAL VOLATILITY: $JA_2(3)$

ance of a JATSM in reproducing the particular moments implied by the data. In the current paper, we compare the one-step-ahead conditional means, volatilities, and correlations implied by the three JATSMs with those implied by the data.

Figures 2-4 plot the conditional means implied by the fitted SNP model for the observed data and the conditional means implied by the



These plots present the reprojected conditional volatility for $JA_3(3)$ against the projected conditional volatility. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 7

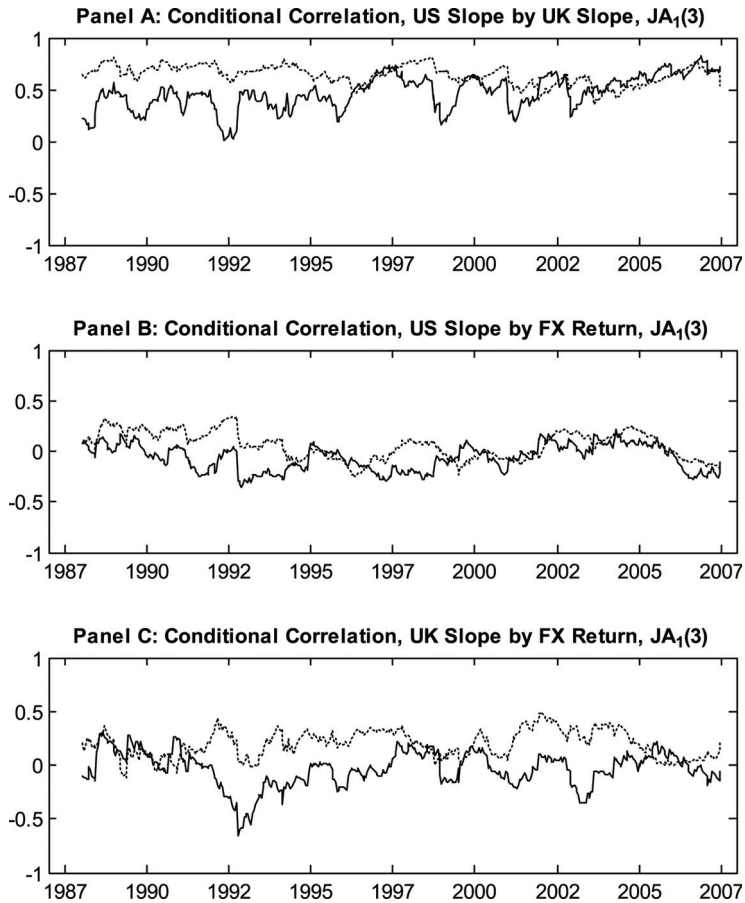
PROJECTED AND REPROJECTED CONDITIONAL VOLATILITY: $JA_3(3)$

three JATSMs. Consistent with the results of the EMM diagnostic t-ratios, all JATSMs are able to reproduce the VAR conditional mean dynamics of the data. In particular, $JA_2(3)$ can almost completely duplicate the mean dynamics implied by the data. $JA_1(3)$ slightly overestimates the conditional mean of the UK slope for the early 1990s. Similarly, $JA_3(3)$ slightly underestimates the conditional mean of the US slope for both

the early 1990s and early 2000s of the sample period. However, the plots suggest that both $JA_1(3)$ and $JA_2(3)$ can also adequately track the mean dynamics.

Figures 5-7 depict the conditional volatilities implied by the fitted SNP model for the observed data and the conditional volatilities implied by the three JATSMs. Panel A of each figure reports the conditional volatility of the US slope. First, panel A of Figure 5 indicates that $JA_1(3)$ is able to reproduce neither the level nor the shape of the volatility of the US slope. Overall, the conditional volatility of the US slope implied by $JA_1(3)$ is too smooth. Furthermore, $JA_1(3)$ cannot generate the high level of volatility observed in the 2000s. Second, panel A of Figure 6 suggests that $JA_2(3)$ can reproduce the conditional volatility of the US slope. Third, panel A of Figure 7 suggests that $JA_3(3)$ shows intermediate performance. Although $JA_3(3)$ performs better than $JA_1(3)$ in matching both the level and shape of the conditional volatility of the US slope, $JA_3(3)$ largely overestimates the volatility in the early part of our sample period. Panel B of Figures 5-7 reports the conditional volatility of the UK slope. The plots suggest that both $JA_1(3)$ and $JA_2(3)$ fail to reproduce the volatility dynamics of the data. Panel B of Figure 5 suggests that $JA_1(3)$ is able to reproduce neither the level nor the shape of the conditional volatility of the UK slope. As presented in panel B of Figure 6, although $JA_2(3)$ is able to track the shape of volatility, it severely underestimates the level of volatility in the early part of the sample period. Panel B of Figure 7 clearly shows that $JA_3(3)$ is good at reproducing the volatility of the UK slope. Although $JA_3(3)$ slightly overestimates the level of volatility observed in the 2000s, it performs best among the three models. Panel C of Figures 5-7 reports the conditional volatility of the exchange rate return. Similar to the results of the US and UK slopes, $JA_1(3)$ fails to reproduce the conditional volatility of the exchange rate return. $JA_3(3)$ is able to track the volatility of the exchange rate return. However, $JA_3(3)$ has a problem in matching the volatility of the early part of the sample period. Panel C of Figure 6 indicates that $JA_2(3)$ is the best at capturing the conditional volatility of the exchange rate return. Although $JA_2(3)$ slightly underestimates the level of volatility for the entire sample, it can track the volatility path well.

In summary, our reprojection results clearly suggest that $JA_1(3)$ is incapable of explaining the volatility dynamics for any of the tri-variate data. $JA_2(3)$ and $JA_3(3)$ perform similarly. Although not complete, $JA_2(3)$ and $JA_3(3)$ can adequately reproduce the conditional volatility of the



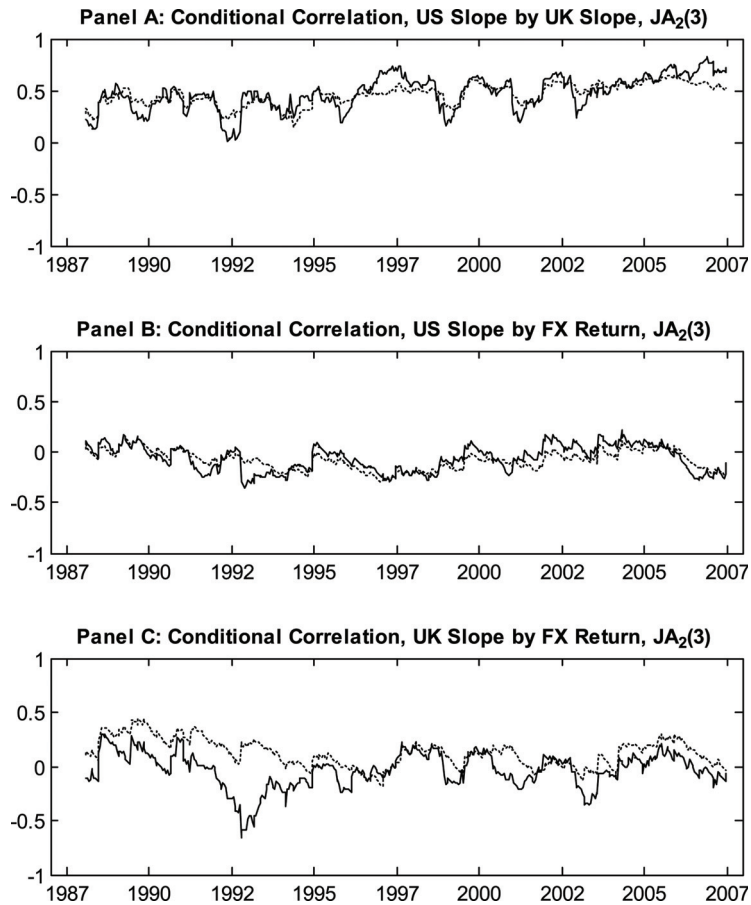
These plots present the reprojected conditional correlation for $JA_1(3)$ against the projected conditional correlation. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 8

PROJECTED AND REPROJECTED CONDITIONAL CORRELATION: $JA_1(3)$

data.

Panel A of Figures 8-10 compares the conditional correlation between the UK and US term structure slopes implied by the JATSMs with those implied by the data, which is the main focus of this paper. Our reprojection results clearly indicate that the three models show remarkably different capabilities in reproducing the conditional correlation of

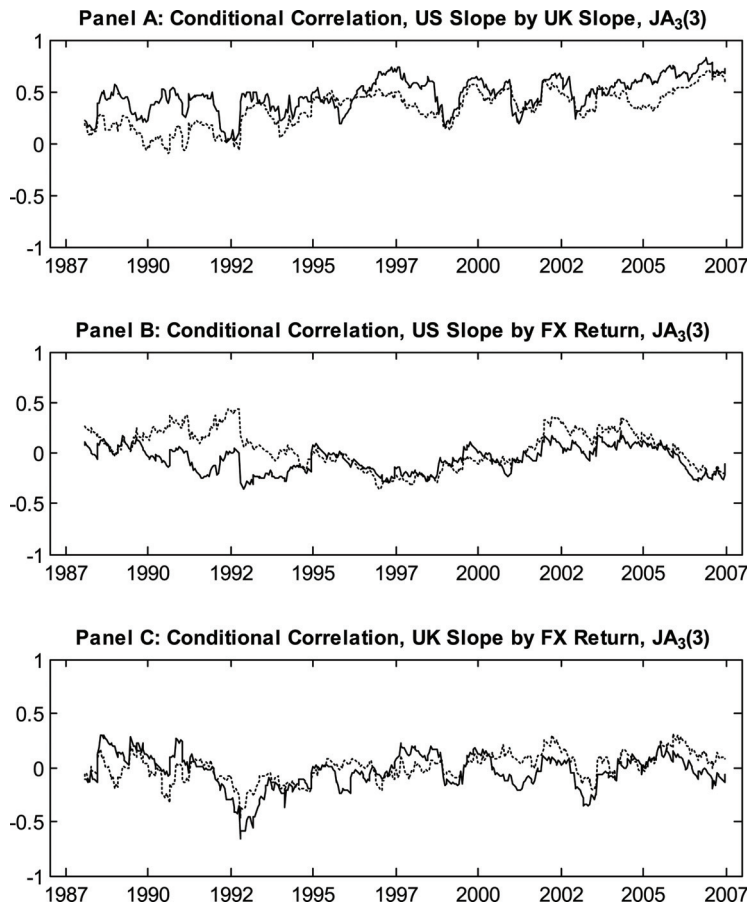


These plots present the reprojected conditional correlation for $JA_2(3)$ against the projected conditional correlation. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 9

PROJECTED AND REPROJECTED CONDITIONAL CORRELATION: $JA_2(3)$

the two term structure slopes. First, $JA_1(3)$ is not able to fit the correlation dynamics. $JA_1(3)$ has severe difficulty in matching the general level of correlation. Furthermore, conditional correlation reproduced by $JA_1(3)$ is too smooth. Therefore, $JA_1(3)$ fails to capture the shape of the correlation path. Second, $JA_3(3)$ performs slightly better than $JA_1(3)$. However, $JA_3(3)$ largely underestimates the level of correlation and



These plots present the reprojected conditional correlation for $JA_3(3)$ against the projected conditional correlation. The reprojected data are represented by the dashed line, and the projected data are represented by the solid line.

FIGURE 10

PROJECTED AND REPROJECTED CONDITIONAL CORRELATION: $JA_3(3)$

cannot reproduce the shape of correlation dynamics in the early 1990s. In addition, $JA_3(3)$ fails to reproduce the tendency of increasing the correlation in the late 2000s. Third, $JA_2(3)$ performs best in reproducing the conditional correlation of the data. Our result in panel A of Figure 9 indicates that $JA_2(3)$ does well in tracking the correlation dynamics. Although $JA_2(3)$ slightly underestimates the level of correlation in 1997

and 2007, it is capable of capturing both the level and shape of the conditional correlation between the US and UK term structure slopes.

In summary, the results of the reprojection analysis conform largely to the results of the EMM specification tests. All the models are successful in capturing the mean dynamics of our tri-variate data. However, $JA_1(3)$ fails to reproduce the second moment dynamics of the data. Our result suggests that tracking the second moment dynamics implied by the data with only one stochastic volatility factor within an affine framework is insufficient. $JA_2(3)$ and $JA_3(3)$ show similar performance in fitting the volatility dynamics. However, $JA_2(3)$ performs best in reproducing the correlation dynamics between the US and UK term structure slopes.

V. Conclusion

In the present paper, we develop two-country JATSMs by extending the single-country ATSMs of DS to a two-country setup. Relying on the EMM estimation process complemented by the reprojection analysis, we find that $JA_2(3)$ performs best in explaining the correlation dynamics between the US and UK term structure slopes. Our reprojection analysis reveals that $JA_2(3)$ is able to track reasonably the correlation dynamics of the data. Both $JA_1(3)$ and $JA_3(3)$ have some difficulty in capturing the correlation dynamics captured by our preferred SNP density. The poor performance of $JA_1(3)$ suggests that there should be at least two common square-root factors to track the correlation dynamics reasonably. The inferior performance of $JA_3(3)$ compared with that of $JA_2(3)$ suggests that flexible correlation structures among the factors also play an important role in capturing the observed correlation between the US and UK term structure slopes.

Although we focus on the completely affine models of DS, we can easily extend our analysis to the essentially affine models of Duffee (2002) and the more flexible risk premium specifications of Cheridito, Filipovic, and Kimmel (2006). This task is reserved for future research.

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