Mean-Variance Efficiency of Reserve Portfolios

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This paper analyzes the mean-variance efficiency of the reserve portfolios of central banks in an effort to shed light on the recent debate regarding the need for portfolio diversification. Using likelihood ratio test statistics, we examine the efficiency of the reserve portfolios of 18 countries from 2000 to 2009. The null hypothesis of efficiency is rejected for approximately half of the countries. However, overall inefficiency appears to have decreased over time, particularly in those countries that previously had inefficient portfolio diversification. Along with the continued dominance of the US dollar in reserve portfolios, our findings suggest that the status of the US dollar as an international reserve currency did not decline.

Keywords: Reserve portfolio, Mean-variance efficiency, Liquid portfolio, Hedging portfolio

JEL Classification: F31, F33, G11

I. Introduction

Following the onset of the global financial crisis, the role of the US dollar as the international reserve currency has become a focus of debate. Anxieties over substantial depreciation of the US dollar appear to have undermined confidence in the currency. For example, the governor of the People's Bank of China suggested that a new transnational currency should be created, and that in the meantime, the SDR should be afforded

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a more important role in an international monetary system. In reality, finding a viable alternative to the US dollar in terms of private sector accessibility, as well as in terms of benefits of economy of scale in international trade and investment, is difficult to accomplish (Cooper 2009; Eichengreen 2009). In this regard, the global financial crisis has surprisingly had little effect on the dollar's dominance over the euro or the yen.

When the value of the US dollar declines, central banks can respond in one of three ways: (i) do nothing, (ii) buy dollars in hopes of stabilizing the market, or (iii) sell dollars in the hope of profiting from the dollar's decline. Truman and Wong (2006) refer to these responses as passive diversification, stabilizing diversification, and active diversification, respectively. From the aggregate IMF-COFER data, Lim (2007) finds evidence for stabilizing diversification, suggesting that central banks are not moving away from the US dollar, but are instead buying more of it. Using individual country data, however, Truman and Wong (2006) and Wong (2007) claim that developed countries are engaged in passive and stabilizing diversification, whereas developing countries are engaged in active diversification. Wooldridge (2006) suggests that developing countries have indeed moved away from the US dollar in recent years.

Some authors have proposed that an optimal reserve portfolio is composed of two constituent portfolios, namely, the hedging portfolio and the mean-variance efficient portfolio. Putnam (2004) proposes segregating the reserve portfolio into two parts: the liquid portfolio and the "liquidity-challenged" portfolio. The liquid portfolio is constructed with a crisis in mind. The liquidity-challenged portfolio is constructed with the objective of achieving the highest returns. Beck and Rahbari (2008) and Beck and Weber (2010) work out the optimization problems. Their solution implies that optimal reserve portfolio weights are a weighted average of mean-variance efficient portfolio weights and hedging portfolio weights.

If reserve portfolios are indeed portfolios of hedging portfolios and mean-variance efficient portfolios, we can evaluate the mean-variance efficiency of reserve portfolios, and may determine which countries are more interested in the returns objective and which are more motivated by the hedging objective. In this paper, we attempt to determine whether reserve portfolios are moving toward mean-variance efficiency. The resolution of this issue is relevant to the recent debate on reserve portfolio diversification. Our analysis is intended to clarify whether the continued dominance of the US dollar holding exerts a negative effect on mean-variance efficiency. The answers will raise implications regarding the hedging ef-

fectiveness of the US dollar and, to some extent, the status of the US dollar as a hedging currency.

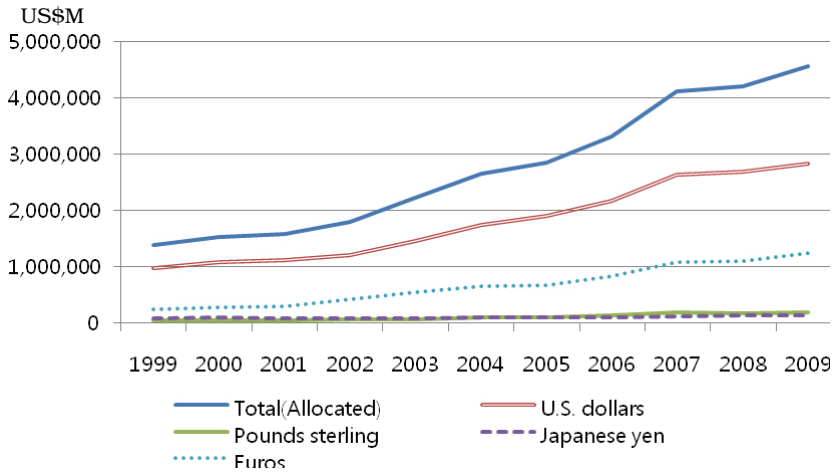
The rest of this article is organized as follows. Section II takes note of the salient changes in official foreign exchange reserves holding and the reserve currency composition of major countries. Section III discusses the manner by which the mean-variance efficiency of reserve portfolios is measured. Section IV analyzes the efficiency of the reserve portfolios of 18 countries from 2000 to 2009. Section V presents the concluding remarks. The Appendix presents another set of results under alternative assumptions regarding how central banks evaluate investment performance.

II. Recent Changes in Reserve Currency Composition

Interestingly enough, the role of the US dollar remains substantial as a reserve currency, even long after the collapse of the Bretton Woods system. The share of the US dollar in international reserves declined, but this decline was gradual and occurred against the background of the rapid accumulation of reserves by the world's central banks. The share, which was as much as 78.4% in 1969, declined to 50.6% in 1990. However, a reversal of this trend occurred after the East Asian financial crisis of 1997. With the rapid increases in foreign exchange holdings, the share of the US dollar substantially increased. The share of the dollar was recorded at more than 70% from 1999 to 2001, despite strong challenge from the newly introduced euro. Figure 1 shows the currency composition of the total foreign exchange reserves as reported by the IMF.

In spite of the risk of long-run depreciation, the global financial crisis of 2008 had little effect on the status of the dollar as the key international currency.[1] The balance of foreign exchange reserves (of which a significant portion was held in dollar-denominated assets) continued to rise except for a brief period between late 2008 and early 2009 when the central banks of several emerging market economies intervened in the foreign exchange market to stabilize exchange rates in the midst of substantial capital outflows. As signs of financial stability returned, the balance of foreign exchange reserves began to rise again from the second quarter of 2009. Although the share of the US dollar is on a declining

[1] After the global financial crisis, there has been ongoing debate on how to build a crisis-resilient macro-financial system. Capital mobility with or without capital control and flexibility of exchange rate are among the key issues (Lee *et al.* 2010).



Source: IMF, *Currency Composition of Official Foreign Exchange Reserves*, 2010.

FIGURE 1
OFFICIAL FOREIGN EXCHANGE RESERVE HOLDINGS (1999-2009)

trend, this may not be indicative of a permanent change in the currency's status. As of 2009, the US dollar's share in total identified official holdings of foreign exchange was around 62%, a level last seen before the 1997 Asian financial crisis.

The euro is the only plausible competitor to the US dollar. Given Europe's economic size and the volume of its intra-regional trade, it comes as no surprise that the euro's share in the global balance of international reserves picked up after its introduction in 1999. The share of the euro increased from 19.0% in 2001 to 24.8% in 2009 in developed countries, and increased from 19.7% in 2001 to 30.1% in 2009 in developing countries. By contrast, the share of the US dollar has decreased in developed countries from 70.6% to 65.4%, and from 73.8% to 58.5% in developing countries.

Meanwhile, information on the reserve currency-composition of individual countries is limited. We extend the reserve currency-composition data of Truman and Wong (2006) and Wong (2007) to the year 2009. Truman and Wong's data include the reserve currency-composition of 23 countries from years 2000-2005. We collect the reserve currency-composition of the same 23 countries for 2006-2009 from the same sources as those employed by Truman and Wong. For most countries,

TABLE 1
COMPOSITION OF OFFICIAL FOREIGN EXCHANGE RESERVES

Country	Year 2000			Year 2005			Year 2009		
	USD	EUR	JPY	USD	EUR	JPY	USD	EUR	JPY
Australia	40.0	30.0	30.0	45.0	45.0	10.0	45.0	45.0	10.0
Bulgaria	10.0	88.9	0.0	4.5	93.7	0.0	1.1	98.6	0.0
Canada	74.7	21.9	3.4	53.3	43.7	3.0	52.7	41.4	0.2
Colombia	80.4	15.3	4.2	85.0	12.0	3.0	85.0	12.0	3.0
Croatia	26.2	69.6	0.0	15.1	84.9	0.0	23.7	73.1	0.0
Finland	30.0	0.0	15.0	30.0	0.0	5.0	18.5	3.3	62.0
Germany	99.2	0.0	0.7	98.2	0.0	1.7	97.5	0.0	2.5
Iceland	--	--	--	40.0	40.0	5.0	20.0	32.0	15.0
Latvia	53.8	32.5	4.8	39.9	56.7	2.2	28.7	63.4	5.0
Lithuania	82.6	15.7	1.0	1.7	88.5	9.8	0.6	93.6	2.4
New Zealand	52.7	16.6	30.6	52.1	42.8	0.0	25.0	25.0	5.0
Norway	21.9	45.5	12.6	36.6	39.8	6.6	34.9	40.9	4.2
Philippines	91.7	1.2	5.4	82.0	8.8	4.3	82.0	11.0	5.0
Romania	73.0	24.3	0.0	35.9	59.3	0.0	29.4	62.9	0.0
Slovak Republic	22.4	74.7	2.9	22.2	77.8	0.0	--	--	--
Slovenia	21.1	72.2	0.0	14.6	75.9	0.0	32.1	57.2	0.0
Sweden	--	--	--	20.0	50.0	0.0	29.9	50.0	0.0
Switzerland	40.7	44.1	3.4	34.9	47.1	3.1	30.0	58.0	5.0
U.K.	35.8	38.1	26.2	30.6	59.1	10.3	10.5	79.3	10.2
US	0.0	47.3	52.7	0.0	57.4	42.6	0.0	54.6	45.4
Uruguay	--	--	--	80.7	14.5	2.5	93.8	2.9	3.3

Sources: Truman and Wong (2006). The values for 2009 were collected by the authors from the sources listed in Truman and Wong (2006).

the US dollar share, the euro share, and the yen share are available. However, only the dollar share is available for Peru and Hong Kong. Therefore, we exclude these two countries from further consideration.

As shown in Table 1, developed and developing countries exhibit somewhat different patterns. In addition, the location and stage of economic development matter. In the case of Latin American countries such as Columbia and Uruguay, the US dollar accounts for a substantial proportion of foreign exchange. Likewise, the share of the euro is dominant in European countries, particularly for East European countries such as Bulgaria, Lithuania, and Romania.

Comparing the composition patterns of the US and Germany is an interesting endeavor. For the US, the shares of the euro and Japanese

TABLE 2

SIZE OF OFFICIAL FOREIGN EXCHANGE RESERVES AND EXCHANGE RATE REGIMES

Country	Reserves (US$M)			Reserves/GDP (%)			Exchange Rate Regime*		
	2000	2005	2009	2000	2005	2009	2000	2005	2009
Australia	16782	40970	33002	4.2	5.6	3.4	Flexible	Flexible	Flexible
Bulgaria	3028	7992	16117	23.5	27.7	33.2	C-Board	C-Board	C-Board
Canada	29019	30662	42602	4.0	2.7	3.2	Flexible	Flexible	Flexible
Colombia	8409	14205	23158	8.4	9.7	9.8	Flexible	Managed	Managed
Croatia	3377	8799	14419	15.8	19.8	22.7	Managed	Managed	Fixed
Finland	7341	10075	7403	1.0	0.9	0.5	Euro	Euro	Euro
Germany	49667	39763	36929	1.3	0.7	0.6	Euro	Euro	Euro
Iceland	365	1009	3639	4.2	6.2	30.1	Flexible	Flexible	Managed
Latvia	851	2232	6445	10.9	13.9	24.9	Fixed	Fixed	Fixed
Lithuania	1310	3720	6238	11.5	14.3	16.9	C-Board	C-Board	C-Board
New Zealand	3619	8693	13982	6.8	7.7	12.0	Flexible	Flexible	Flexible
Norway	26707	46375	45719	15.9	15.4	12.3	Flexible	Flexible	Flexible
Philippines	12975	15799	37504	17.1	16.0	23.3	Flexible	Flexible	Managed
Romania	2469	19871	39345	6.6	20.0	24.1	Crawling	Managed	Managed
Slovak Rep.	4022	14899	50	20.3	32.0	0.1	Managed	Fixed	Euro
Slovenia	3110	8013	590	15.6	22.4	1.3	Crawling	Fixed	Euro
Sweden	13757	21381	38543	5.6	5.8	9.5	Flexible	Flexible	Flexible
Switzerland	30854	35419	91615	12.3	9.5	18.6	Flexible	Flexible	Managed
U.K.	34163	35852	38026	2.3	1.6	1.7	Flexible	Flexible	Flexible
US	31238	37836	50520	0.3	0.3	0.4	Flexible	Flexible	Flexible
Uruguay	2432	3068	7644	12.1	17.7	24.4	Flexible	Managed	Managed
Argentina	24414	22742	42923	8.6	12.4	13.9	Managed	Managed	Managed
Brazil	32433	53217	231889	5.0	6.0	14.6	Flexible	Flexible	Managed
Chile	14686	16690	23849	19.5	14.1	14.8	Flexible	Flexible	Flexible
China	165573	818889	2399161	13.9	35.9	48.1	Fixed	Fixed	Fixed
Hong Kong	107541	124247	255770	63.6	69.9	122.2	C-Board	C-Board	C-Board
Japan	347210	828831	996962	7.4	18.2	19.8	Flexible	Flexible	Flexible
Korea	95854	209973	265205	18.0	24.9	31.8	Flexible	Flexible	Managed
Malaysia	27432	69378	92866	29.2	50.3	48.1	Fixed	Managed	Managed
Mexico	35142	73016	94103	6.1	8.6	10.7	Flexible	Flexible	Managed
Russia	24262	175694	405827	9.3	22.9	32.5	Managed	Managed	Managed
Singapore	79723	115715	186006	84.5	92.3	101.5	Managed	Managed	Managed
Thailand	31933	50503	133600	26.0	28.6	50.7	Managed	Managed	Managed

Source: IMF, *Annual Report and Exchange Rate Arrangements and Exchange Restrictions*, various issues.

Note: C-board denotes currency board. Pegged exchange rate regime within horizontal bands is classified as the fixed. The managed floating regime is also classified as fixed to distinguish it from a free-floating regime.

yen were 54.6% and 45.4% in 2009, implying diversification of foreign exchange reserves. On the other hand, Germany holds almost 100% of the foreign exchange reserves in the form of US dollar denominated assets. Our task is to examine to what extent this kind of diversity in reserve portfolios can be explained from the perspective of mean-variance efficiency.

The size of the optimal holding of official foreign exchange reserve is another matter. Table 2 provides insights into this issue. Among the countries reported in Table 1, the ratio of foreign exchange reserve with respect to the GDP exceeds 20% in 2009 for Bulgaria, Croatia, Iceland, Latvia, Philippines, Romania, and Uruguay. These countries are mostly developing countries with fixed rate or managed floating rate regimes. The ratio substantially increased in these countries after the global financial crisis. By contrast, the foreign reserve holdings of Slovak Republic and Slovenia dramatically decreased after accession to the euro area in 2007 and 2009, respectively.

Regrettably, we have no official information on the currency composition patterns of East Asian countries, which are often criticized for accumulating too much foreign reserves.[2] As shown in Table 2, the ratio of reserves with respected to the GDP was about 50% for China, Malaysia, and Thailand, but more than 100% for Hong Kong and Singapore in 2009. All these countries also adopt either fixed rate (currency board) or managed floating rate regimes.[3] By contrast, Latin American countries with similar exchange rate regimes show much lower ratios. For example, the ratio amounted to 14.6% (14.8%) for Brazil (Chile), and 10.7% for Mexico in 2009.

III. Measurement of Mean-Variance Efficiency

Our principal objective is to evaluate the mean-variance efficiency of the reserve portfolios of central banks. To this end, we evaluate official

[2] According to Kim *et al.* (2009), the share of the US dollar in Bank of Korea's foreign reserves equals to 63.1%, whereas those of the euro and the yen are presumed to be 20.0% and 15.5%, respectively.

[3] Stiglitz (2010, p. 338) provides an illuminating answer: "Another reason for high savings is a result of the 1997-1998 East Asia crisis. Countries do not want to expose themselves to that kind of risk. So countries started accumulating hundreds of billions of dollars of reserves, globally. That increased their security, but presented globally what is known as the paradox of thrift — an increase in savings may lead to a weaker economy."

reserve portfolios as we would evaluate any other investor's portfolio.

The evaluation of mean-variance efficiency of a portfolio can be carried out in two steps. In the first step, we identify the investment possibility set, *i.e.*, the set of mean-variance combinations that can be achieved from the available assets. The frontier of such a set is called the mean-variance efficient frontier. Drawing the mean-variance efficient frontier in the mean-standard deviation space rather than in the mean-variance space is customary practice (Figure 2). In the second step, we assess whether the given portfolio is far from the efficient frontier. A conventional measure of the efficiency of a portfolio is the Sharpe ratio, which is defined as the ratio of the mean to the standard deviation of the portfolio excess returns (*i.e.*, the portfolio returns in excess of the risk-free rate). We can compare the Sharpe ratio of the given portfolio with the maximum Sharpe ratio that can be obtained from the efficient frontier. The portfolio that has the maximum Sharpe ratio is called the tangent portfolio because the line passing the risk-free rate is tangent to the efficient frontier at the tangent portfolio (t in Figure 2).

Ross (cited by Kandel 1984) proposed a statistic based on this idea — comparing the Sharpe ratio of a given portfolio to the maximum Sharpe ratio. The statistic is called the likelihood ratio test (LRT) statistic given that it can be interpreted as a ratio of two likelihoods. It is defined as follows:

$$Q = T \log\left(\frac{1+x}{1+y}\right) \tag{1}$$

where x is the squared Sharpe ratio of the tangent portfolio, and y is the squared Sharpe ratio of the test portfolio. T is the number of return observations used in the estimation of the mean vector and the variance-covariance matrix. If the test portfolio is mean-variance efficient, then the distribution of Q is the Chi-squared distribution with degrees of freedom equal to the number of risky assets.

Figure 2 illustrates the LRT statistic of Ross. The risk-free asset is denoted by r, the test portfolio is denoted by α, and the tangent portfolio is denoted by t. $\sqrt{x}$ is the Sharpe ratio of the tangent portfolio, while $\sqrt{y}$ represents the Sharpe ratio of the test portfolio. The likelihood ratio test statistic of Ross is given by $T\log[(1+x)/(1+y)]$. If the test portfolio is efficient, point α would be identical to point t. In such cases, $\sqrt{x}$ (the slope of the line connecting t and r) would be identical to $\sqrt{y}$

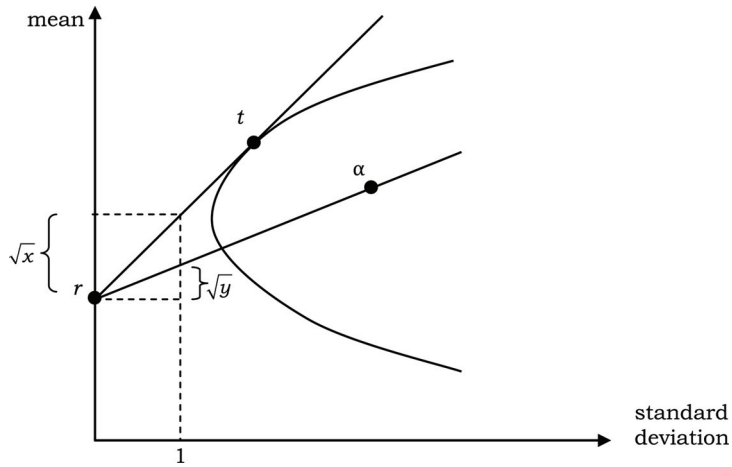


FIGURE 2
LIKELIHOOD RATIO TEST STATISTIC OF ROSS

(slope of the line connecting α and r). The larger the difference between $\sqrt{x}$ and $\sqrt{y}$, the more inefficient the test portfolio.

Kandel (1984) proposed the use of the LRT statistic in cases in which no risk-free asset exists. Kandel's test, however, treats a portfolio as efficient even if the portfolio is on the part of the frontier below the global minimum variance portfolio. Needless to say, this "lower part" of the frontier is dominated by the "upper part" of the frontier and cannot be efficient, regardless of whether a risk-free asset exists. As our test portfolios often have expected returns below the expected return of the global minimum variance portfolio and thus closer to the "lower part" of the frontier, Kandel's statistic is deemed inappropriate for the current work.

Other statistics have been proposed. Roll (1977) shows that the test of the efficiency of the market portfolio is equivalent to the test of the linear relationship between expected returns and betas. This idea can be generalized to any portfolio that is not the market portfolio. That is, if a linear relationship is found between expected returns of individual stocks and their betas with respect to the given portfolio, we may conclude that the given portfolio is efficient. Shanken (1985) examines what he calls the cross-sectional regression test and a Lagrange multiplier test. Gibbons, Ross, and Shanken (1989) discuss an F-test. The likelihood ratio test of Ross and the F-test of Gibbons, Ross, and Shanken (1989) are asymptotically equivalent. We employ the test of Ross because it has a simple geometric interpretation.

IV. Empirical Analysis

In analyzing the mean-variance efficiency of reserve portfolios, we consider only the shares of three major reserve currencies: the US dollar, the euro, and the Japanese yen. We disregard all other currencies in the reserve portfolios and consider only the relative weights among these three currencies. The number of countries considered in the analysis is 18, less than the 21 countries shown in Table 1. We exclude two eurozone countries — Germany and Finland — as well as the US from our efficiency analysis.[4] Given that these countries do not hold euro or US dollar assets in reserve, the analysis of the efficiency becomes complicated.

We assume that the entire US dollar share is invested in US government bonds, the entire euro share is invested in German government bonds, and the entire yen share is invested in Japanese government bonds. We collect the monthly returns of the US, German, and Japanese government bonds with a five-year maturity for the period spanning the beginning of 1990 to the end of 2009. We then calculate the monthly returns of these three assets in terms of the US dollar, the euro, and the SDR, as well as in local currency terms for each of the 18 countries in our database.

To calculate the LRT statistics, we must decide (i) how to define returns, *i.e.*, in local currency term, in common currency terms, or in a certain combination, and (ii) how to calculate the risk-free rate.

The definition of returns was guided by the following consideration: To the extent that central banks care about the investment performance of reserve portfolios, in what currency terms would they think of investment performance? For example, would the central bank of the Philippines assess the investment performance in Philippine peso terms or in US dollar terms? Our answer to this question is that it critically depends on the exchange regime of the Philippines. If the Philippine peso is freely floating, then the central bank would probably ponder on investment performance in peso terms; no obvious alternative exists. However, if the Philippines manages its currency with the goal of maintaining the exchange rate against the US dollar, then the central bank of the Philippines is likely to ponder on investment performance in US dollar terms because the Philippine peso is unstable, and investment performance calculated

[4] We also exclude Slovenia and Slovak Republic from the 2009 analysis because both have become members of the eurozone by 2009.

in peso terms is regarded as unreliable.[5]

We determine the base currency for return calculation on the basis of the exchange rate regime, as listed in Table 3. When the exchange regime is flexible, we use local currency as the base currency. If not, we use the US dollar or the euro. The euro is used as the base currency for Bulgaria, Croatia, Iceland (only in 2009), Latvia, Lithuania, Romania, Slovak Republics (in 2000 and 2005), Slovenia (in 2000 and 2005), and Switzerland (only in 2009). The US dollar is used as the base currency for Colombia (in 2005 and 2009), Philippines (only in 2009), and Uruguay (in 2005 and 2009).[6]

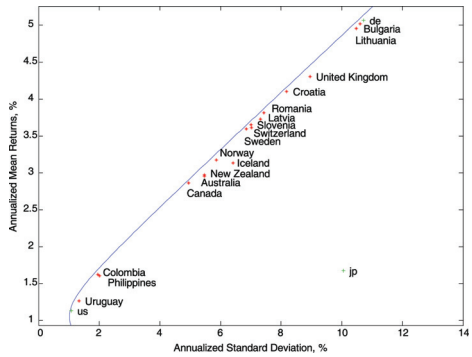
The risk-free rate is set to zero. For many investors, short-term government bonds can be regarded as risk-free assets because they present very low levels of risk. For central banks, however, even this low level of risk cannot be disregarded. Central banks make investment choices among low-risk low-return alternatives.

As a method of summarizing data, Figure 2 plots the mean-variance frontier for 2009 using the US dollar, the euro, and the SDR as base currencies. Dots represent reserve portfolios, while plus signs indicate investment assets, *i.e.*, US, German, and Japanese government bonds. In plot (a), the frontiers are calculated out of returns in US dollar terms; in plot (b), returns are in euro terms; and in plot (c), returns are in SDR terms. Both the mean and standard deviation are annualized. The estimates of the mean vector and variance-covariance matrix are based on monthly returns from January 2000 to December 2009. Not surprisingly, the relative positions of countries in the plot are affected significantly by the selection of the base currency.

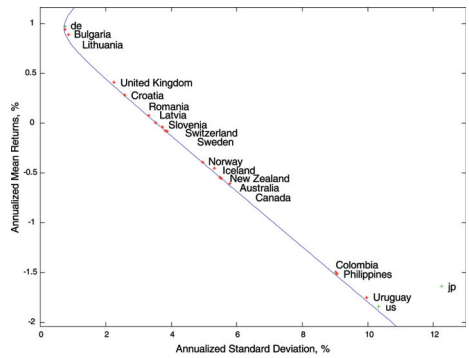
The estimates of the mean vector and variance-covariance matrix are

[5] If the Philippine peso is pegged to the US dollar, then whether we ponder on investment performance in terms of US dollars or in terms of Philippine pesos is irrelevant.

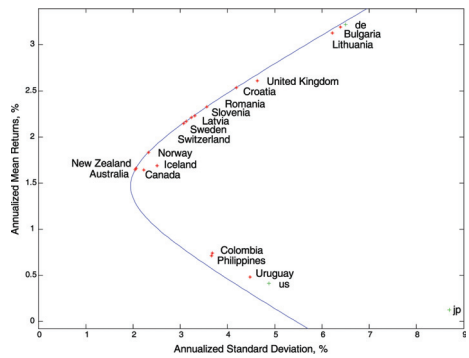
[6] Using the US dollar or the euro instead of the local currency as the base currency poses some implications on the calculated efficiency. The asset denominated in the base currency has no exchange risk, making the asset appear more attractive. Thus, when the base currency is the US dollar, the dollar-denominated asset looks more attractive. Increasing its weight makes the portfolio look more efficient. On the other hand, the entire efficient frontier moves to the left as a result of having a no-exchange-risk asset. This tends to make our test more effective in discriminating inefficient portfolio. In the Appendix, we present the results of the analysis using the local currency as the base currency, regardless of the exchange regime. The difference between the results reported in the main text and those reported in the Appendix can be partly explained by the effect described above.



(a) USD Based



(b) EUR Based



(c) SDR Based

FIGURE 3

MEAN-VARIANCE EFFICIENT FRONTIERS (2009)

TABLE 3
LRT STATISTICS (2000-2009)

(a) Year 2000

Country	SR Reserve	SR Tangent	LRT Ross	P Value
Norway	0.223	0.224	0.034	0.998
United Kingdom	0.130	0.154	0.796	0.850
Switzerland	0.115	0.148	1.001	0.801
Philippines	0.203	0.228	1.254	0.740
New Zealand	0.171	0.205	1.478	0.687
Australia	0.138	0.205	2.653	0.448
Colombia	0.488	0.525	3.590	0.309
Canada	0.190	0.283	5.009	0.171
Bulgaria	0.415	0.598	17.599	0.001
Slovak Republic	0.275	0.598	27.969	0.000
Slovenia	0.274	0.598	27.988	0.000
Croatia	0.248	0.598	29.535	0.000
Latvia	0.177	0.598	32.977	0.000
Romania	0.159	0.598	33.712	0.000
Lithuania	0.154	0.598	33.880	0.000

(b) Year 2005

Country	SR Reserve	SR Tangent	LRT Ross	P Value
Switzerland	0.098	0.101	0.082	0.994
Sweden	0.107	0.112	0.124	0.989
Philippines	0.236	0.239	0.165	0.983
Norway	0.051	0.070	0.265	0.966
Canada	0.039	0.052	0.323	0.956
Australia	0.028	0.060	0.341	0.952
New Zealand	0.015	0.056	0.355	0.949
Iceland	0.011	0.057	0.369	0.947
United Kingdom	0.036	0.057	0.383	0.944
Bulgaria	0.412	0.426	1.185	0.757
Colombia	0.144	0.314	8.849	0.031
Uruguay	0.127	0.314	9.404	0.024
Croatia	0.256	0.426	12.380	0.006
Slovenia	0.245	0.426	12.970	0.005
Lithuania	0.219	0.426	14.361	0.002
Slovak Republic	0.200	0.426	15.293	0.002
Romania	0.144	0.426	17.531	0.001
Latvia	0.134	0.426	17.848	0.000

(c) Year 2009

Country	SR Reserve	SR Tangent	LRT Ross	P Value
United Kingdom	0.147	0.159	0.409	0.938
Switzerland	0.046	0.084	0.848	0.838
New Zealand	0.005	0.092	1.004	0.800
Norway	0.002	0.093	1.043	0.791
Australia	0.007	0.095	1.064	0.786
Canada	0.002	0.096	1.105	0.776
Philippines	0.128	0.171	1.479	0.687
Bulgaria	0.363	0.390	2.131	0.546
Sweden	0.075	0.155	2.190	0.534
Uruguay	0.276	0.319	2.820	0.420
Colombia	0.239	0.319	4.947	0.176
Lithuania	0.301	0.390	6.580	0.087
Croatia	0.032	0.390	16.892	0.001
Romania	0.007	0.390	17.009	0.001
Latvia	0.001	0.390	17.014	0.001
Iceland	0.024	0.390	17.014	0.001

based on monthly returns of the 10-year period prior to the portfolio formation. We did not want to use longer periods given that these parameters may not be constant over a long time period. Additionally, we wanted to use information available to central bankers at the time they determined the reserve portfolios. For the portfolios in 2000, the estimates are based on the returns from January 1991 to December 2000; for those in 2005, the estimates are based on the returns from January 1996 to December 2005; for 2009 portfolios, the estimates are based on the returns from January 2000 to December 2009.

Table 3 shows the LRT statistic together with its two components: the Sharpe ratio of the reserve portfolio and the Sharpe ratio of the tangent portfolio. Under the null hypothesis that the reserve portfolio is efficient, the distribution of the LRT statistic is $\chi(3)$. The last column of Table 2 shows p-values for the null hypothesis. A low p-value indicates that the portfolio is inefficient. At the conventional significance level, the null hypothesis of efficiency is rejected for approximately half of the countries. For 2000, we cannot reject the null of efficiency for 8 out of 15 countries. For 2005, 10 of 18 countries are efficient. For 2009, 11 of 16 countries are efficient.

That some countries are efficient does not necessarily mean that these countries exhibit high Sharpe ratios. When we tested the hypothesis that the Sharpe ratios of all countries are identical using the z statistic

TABLE 4
JOBSON-KORKIE TEST OF DIFFERENCES IN SHARPE RATIOS

Hypothesis	Z Stat	P Value
All Sharpe ratios are equal in 2000	0.432	0.333
All Sharpe ratios are equal in 2005	0.902	0.184
All Sharpe ratios are equal in 2009	0.954	0.170

Note: The *z* statistic has the standard normal distribution under the null hypothesis.

TABLE 5
TEST OF DIFFERENCES IN EFFICIENCY OVER TIME

Hypothesis	T Stat	P Value
The average of all LRT stats is the same for 2000 and 2005.	0.941	0.173
The average of all LRT stats is the same for 2005 and 2009.	-0.113	0.455
The average of all LRT stats is the same for 2000 and 2009.	0.757	0.224
The average of significant LRT stats is the same for 2000 and 2005.	7.592	0.000
The average of significant LRT stats is the same for 2005 and 2009.	0.557	0.289
The average of significant LRT stats is the same for 2000 and 2009.	3.130	0.001

Note: P-values are based on asymptotic distribution, *i.e.*, standard normal distribution.

of Jobson and Korkie (1981),[7] we were unable to reject the hypothesis (see Table 4). Efficiency critically depends on the investment possibility set, not only on the Sharpe ratio of the portfolio. That is, even though the Sharpe ratio is low or even negative, the portfolio can still be efficient if the investment possibility set is small.

Some countries become efficient over time. For example, Bulgaria was inefficient in 2000, but became efficient in 2005. Two more countries — Uruguay and Colombia — became efficient in 2009. The values of the LRT statistics decline over time as well. Table 5 shows the statistical significance of the change. If we include all countries in our test, the change is only marginally significant. If we include only those countries that are initially inefficient, however, the change is more significant. That is, the average LRT statistics of inefficient countries become smaller over

[7] We use the correction by Cadsby (1986). The correction by Memmel (2003) is irrelevant to our multivariate test.

time. Considering inefficient countries only is a practical approach because efficient countries, by definition, have close-to-zero LRT statistics.[8]

V. Conclusion

The focus of the recent discussions of reform for the international reserve system stems from a crisis of confidence in the US dollar. If the long-term depreciation of the dollar is inevitable, the international monetary order centered on the US will face challenge from other currencies, such as the euro.

Using the likelihood ratio test statistics, we analyzed the efficiency of the reserve portfolios of 18 countries during the period spanning 2000-2009. The null hypothesis of efficiency is rejected for approximately half of the countries. However, overall inefficiency appears to have decreased over time, particularly in those countries that were previously inefficient in their portfolio diversification.

Therefore, we can conclude that the reserve portfolios of central banks are moving toward the mean-variance efficient frontier. This result implies that the continued dominance of the US dollar in reserve holdings may be justified from the perspective of efficiency. Moreover, the status of the US dollar as hedging currency appears to have decreased over the recent decade. This finding indicates that the role of the US dollar as the international reserve currency has not declined substantially, despite the current turmoil in the international monetary system. We are optimistic that our study will shed some light on the issue of reserve portfolio diversification.

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[8] Given the low degree of freedom, these tests are only indicative.

Appendix

We present an alternative set of results using the local currency as the base currency for return calculation regardless of the exchange regime. When the regime is fixed, we continue to use the US dollar or the euro as the base currency. That is, the results reported in the appendix are based on the idea that central banks assess investment performance in the local currency terms even if it is in the managed exchange regime to maintain the exchange rate against a particular reference currency. This is somewhat a contradictory idea as indicated earlier. A managed exchange regime is adopted when the local currency is unstable, and in such a situation, the investment performance measured in local currency is likely to be unreliable. We report the results nonetheless to indicate the sensitivity of our analysis to the choice of base currency. Although the magnitude of statistics changes, the overall patterns are comparable to the ones reported in the main text.

TABLE A.1
LRT STATISTICS (2000-2009)

(a) Year 2000

Country	SR Reserve	SR Tangent	LRT Ross	P Value
Norway	0.161	0.175	0.426	0.935
Croatia	0.329	0.340	0.646	0.886
Romania	0.691	0.700	0.811	0.847
New Zealand	0.120	0.155	0.902	0.825
Philippines	0.268	0.287	0.951	0.813
United Kingdom	0.025	0.116	1.214	0.750
Australia	0.102	0.167	1.617	0.655
Switzerland	0.048	0.142	1.678	0.642
Slovenia	0.595	0.618	1.924	0.588
Colombia	0.405	0.434	1.925	0.588
Slovak Republic	0.270	0.316	2.384	0.497
Canada	0.158	0.246	3.222	0.359
Bulgaria	0.385	0.503	8.276	0.041
Latvia	0.178	0.503	18.423	0.000
Lithuania	0.157	0.503	19.082	0.000

(b) Year 2005

Country	SR Reserve	SR Tangent	LRT Ross	P Value
Switzerland	0.098	0.101	0.082	0.994
Sweden	0.107	0.112	0.124	0.989
Romania	0.418	0.419	0.163	0.983
Philippines	0.236	0.239	0.165	0.983
Norway	0.051	0.070	0.265	0.966
Canada	0.039	0.052	0.323	0.956
Australia	0.028	0.060	0.341	0.952
Croatia	0.091	0.106	0.354	0.949
New Zealand	0.015	0.056	0.355	0.949
Iceland	0.011	0.057	0.369	0.947
United Kingdom	0.036	0.057	0.383	0.944
Uruguay	0.281	0.292	0.686	0.877
Colombia	0.300	0.313	0.904	0.825
Bulgaria	0.412	0.426	1.185	0.757
Slovenia	0.245	0.426	12.970	0.005
Lithuania	0.219	0.426	14.361	0.002
Slovak Republic	0.200	0.426	15.293	0.002
Latvia	0.134	0.426	17.848	0.000

(c) Year 2009

Country	SR Reserve	SR Tangent	LRT Ross	P Value
United Kingdom	0.147	0.159	0.409	0.938
Switzerland	0.046	0.084	0.848	0.838
New Zealand	0.005	0.092	1.004	0.800
Norway	0.002	0.093	1.043	0.791
Australia	0.007	0.095	1.064	0.786
Canada	0.002	0.096	1.105	0.776
Romania	0.289	0.309	1.329	0.722
Uruguay	0.139	0.178	1.468	0.690
Philippines	0.128	0.171	1.479	0.687
Colombia	0.075	0.137	1.566	0.667
Bulgaria	0.363	0.390	2.131	0.546
Iceland	0.169	0.218	2.152	0.542
Sweden	0.075	0.155	2.190	0.534
Lithuania	0.301	0.390	6.580	0.087
Croatia	0.032	0.390	16.892	0.001
Latvia	0.001	0.390	17.014	0.001

TABLE A.2
JOBSON-KORKIE TEST OF DIFFERENCES IN SHARPE RATIOS

Hypothesis	Z Stat	P Value
All Sharpe ratios are equal in 2000	1.370	0.085
All Sharpe ratios are equal in 2005	1.828	0.034
All Sharpe ratios are equal in 2009	1.361	0.087

Note: The *z* statistic has the standard normal distribution under the null hypothesis.

TABLE A.3
TEST OF DIFFERENCES IN EFFICIENCY OVER TIME

Hypothesis	T Stat	P Value
The average of all LRT stats is the same for 2000 and 2005.	-0.019	0.492
The average of all LRT stats is the same for 2005 and 2009.	-0.274	0.392
The average of all LRT stats is the same for 2000 and 2009.	0.083	0.467
The average of significant LRT stats is the same for 2000 and 2005.	1.252	0.105
The average of significant LRT stats is the same for 2005 and 2009.	0.877	0.190
The average of significant LRT stats is the same for 2000 and 2009.	1.201	0.115

Note: P-values are based on asymptotic distribution, *i.e.*, standard normal distribution.

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