Fast Training of Fractionally-Spaced Modified Decision Feedback Equalizer in Slow Frequency Selective Fading Channels

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***Abstract*- To reduce the equalizer training time with robustness to the timing phase error in fixed wireless channels, we propose a fractionally-spaced modified DFE (FS-MDFE) scheme. The proposed FS-MDFE is an extension of the previously proposed symbol-spaced modified DFE (SS-MDFE) [1]. The performance of the FS-MDFE is analytically evaluated in terms of the mean squared error. The optimum MMSE solution is derived for initialization of the feedforward filter of the MDFE. We also propose a simplified initialization method to reduce the implementation complexity.**

I. INTRODUCTION

In fixed wireless communication systems, slow frequency selective fading often causes the channel in non-minimum phase condition. In this case, the use of a conventional adaptation method such as the least mean square (LMS) algorithm may not be feasible to train the equalizer within a given training time. To reduce the equalizer training time, there have been a number of propositions including the use of the cyclic training patterns [2], fast Fourier transform (FFT) [3], recursive least square (RLS) algorithm, fast RLS algorithm and lattice structure [4]. There also have been research works on the equalization problem in non-minimum phase channels, such as the bi-directional equalizer (BDE) [5] and allpass equalizer [6]. Most of them, however, may not be practical for real application mainly due to their high computational complexity and stability problem.

Another approach considers modification of a conventional decision feedback equalizer (DFE) structure comprising the feedforward and feedback filters. The feedforward filter (FFF) is followed by the feedback filter (FBF) in a conventional DFE (CDFE), whereas the FBF is performed prior to the FFF in the modified DFE (MDFE) [1][7]. Since the coefficient of the FBF of the MDFE can be initialized using an estimate of the channel impulse response, it is only required to initialize and train the FFF of the MDFE for the start-up. As a result, the MDFE can reach to the steady-state faster than the CDFE.

The design of the MDFE was considered for the symbol-spaced scheme [1]. It is well known that the symbol-spaced equalizer is vulnerable to the timing phase error, but the fractionally-spaced equalizer can provide performance robust to the timing phase error. The performance of the symbol-spaced equalizer can much more be sensitive to the timing phase error in fixed wireless channels, because the received signal from each multipath can have different timing phases. We propose a fractionally-spaced MDFE (FS-MDFE) structure that can provide performance robust to the timing phase error. Although the design of the MDFE scheme was considered in [1][7], no analytic design was reported based on the minimum mean squared error (MMSE) method. The proposed FS-MDFE is analytically designed, and its performance is analyzed in terms of the mean squared error (MSE).

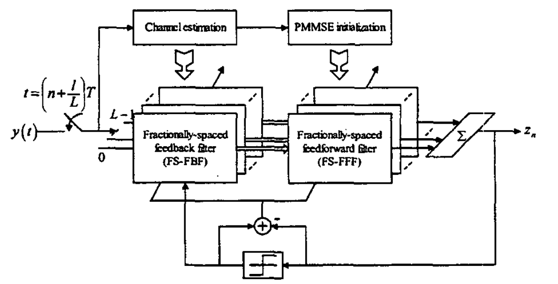


Fig. 1 Block diagram of the fractionally-spaced modified DFE.

To fast train the proposed FS-MDFE even in non-minimum phase channels, we propose a simplified MMSE initialization method called partial MMSE (PMMSE) initialization, that first initializes a few main coefficients of the FFF. Then all the FFF coefficients are trained to reach the steady-state using a conventional LMS adaptation method as shown in Fig. 1.

Following Introduction, Section II describes the structure of the FS-MDFE structure. Section III derives the MMSE solution of the FS-MDFE. Section IV presents the PMMSE initialization for fast training of the FFF. The performance of the proposed FS-MDFE is evaluated by computer simulation in fixed wireless channels in Section V. Finally conclusions are summarized in Section VI.

II. STRUCTURE OF THE MODIFIED DFE

A. *Fractionally-Spaced System Model*

We consider signal transmission over an additive Gaussian noise channel, whose input and output can be represented as

$$y(t)=\sum_k a_k h(t-kT)+v(t) \tag{1}$$

where T is the symbol time, $\{a_k\}$ denotes a complex-valued data sequence of zero mean random variable, $h(t)$ is the overall impulse response of the channel including the transmit filter, physical channel and receive filter, and $v(t)$ denotes zero mean additive complex Gaussian noise statistically independent of the data sequence $\{a_k\}$.

Assume that the channel output $y(t)$ is over-sampled at a rate of L/T and that the channel impulse response has a finite span over the time interval $[-K_1T,\ K_2T]$. Then, the input-output relation for the discrete-time equivalent channel has a form of

$$\mathbf{y}_n=\sum_{k=-K_1}^{K_2} a_{n-k}\mathbf{h}_k+\mathbf{v}_n \tag{2}$$

where

$$\mathbf{y}_n \triangleq \begin{bmatrix} y\left((n+{}^{L-1}\!/_{L})T\right) \\ \vdots \\ y(nT) \end{bmatrix} = \begin{bmatrix} y_{n,L-1} \\ \vdots \\ y_{n,0} \end{bmatrix},$$

$$\mathbf{h}_n \triangleq \begin{bmatrix} h\left((n+{}^{L-1}\!/_{L})T\right) \\ \vdots \\ h(nT) \end{bmatrix} = \begin{bmatrix} h_{n,L-1} \\ \vdots \\ h_{n,0} \end{bmatrix},$$

$$\mathbf{v}_n \triangleq \begin{bmatrix} v\left((n+{}^{L-1}\!/_{L})T\right) \\ \vdots \\ v(nT) \end{bmatrix} = \begin{bmatrix} v_{n,L-1} \\ \vdots \\ v_{n,0} \end{bmatrix}.$$

For ease of analysis, we assume that the channel has the maximum amplitude $h_{0,0}$. As a consequence, the channel impulse response is not causal, but it can be made causal by introducing a delay of K_1T.

Assuming that the channel is time-invariant over a block of F symbols, (2) can be written in a matrix form as

$$\begin{bmatrix} \mathbf{y}_{n+F-1} \\ \mathbf{y}_{n+F-2} \\ \vdots \\ \mathbf{y}_n \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{-K_1} & \mathbf{h}_{-K_1+1} & \cdots & & \mathbf{h}_{K_2} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h}_{-K_1} & \mathbf{h}_{-K_1+1} & \cdots & & \mathbf{h}_{K_2} & & \vdots \\ \vdots & & \ddots & & & & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{-K_1} & \mathbf{h}_{-K_1+1} & \cdots & & \mathbf{h}_{K_2} \end{bmatrix} \cdot \begin{bmatrix} a_{n+K_1+F-1} \\ a_{n+K_1+F-2} \\ \vdots \\ a_{n-K_2} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{n+F-1} \\ \mathbf{v}_{n+F-2} \\ \vdots \\ \mathbf{v}_n \end{bmatrix} \quad (3)$$

or

$$\mathbf{y}_{n+F-1:n} = \mathbf{H}_{(K_1+K_2+F-1)\times(F-1)}\mathbf{a}_{n+K_1+F-1:n-K_2} + \mathbf{v}_{n+F-1:n}. \quad (4)$$

where the first and last components, separated by a colon, are given as a subscript to the vector to emphasize the components of the vector.

B. Fractionally-Spaced Modified DFE

In the FS-MDFE, the channel output $\mathbf{y}_{n+F-1:n}$ will be fed into the feedback section of the MDFE to precancel the postcursor ISI

$$\begin{bmatrix} \mathbf{x}_{n+F-1} \\ \mathbf{x}_{n+F-2} \\ \vdots \\ \mathbf{x}_n \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{n+F-1} \\ \mathbf{y}_{n+F-2} \\ \vdots \\ \mathbf{y}_n \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+F} & \cdots & \hat{\mathbf{h}}_{K_2} & \mathbf{0} & \cdots & \mathbf{0} \\ \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+F-1} & \cdots & & \hat{\mathbf{h}}_{K_2} & & \vdots \\ \vdots & & & & \ddots & \mathbf{0} \\ \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+1} & & \cdots & \cdots & & \hat{\mathbf{h}}_{K_2} \end{bmatrix} \begin{bmatrix} \hat{a}_{n+(K_1+F-1)-\Delta-1} \\ \hat{a}_{n+(K_1+F-1)-\Delta-2} \\ \vdots \\ \hat{a}_{n-K_2} \end{bmatrix} \quad (5)$$

or

$$\begin{aligned} \mathbf{x}_{n+F-1:n} &= \mathbf{y}_{n+F-1:n} - \mathbf{H}_{post}\hat{a}_{n+(K_1+F-1)-\Delta-1:n-K_2} \\ &= \mathbf{y}_{n+F-1:n} - \mathbf{V}_{F-1:0}(n) \end{aligned} \quad (6)$$

where $\mathbf{x}_{n+F-1:n}$ denotes the output of the FBF (*i.e.*, the input of the FFF of the FS-MDFE), $\hat{\mathbf{h}}_n$ is the FBF tap coefficient obtained from the channel estimate, $\hat{a}_n$ is the detected data symbol, Δ indicates the decision delay of the FS-MDFE due to the channel and the FFF, $\mathbf{H}_{post}$ is the FBF section,

$$\mathbf{H}_{post} \triangleq \begin{bmatrix} \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+F} & \cdots & \hat{\mathbf{h}}_{K_2} & \mathbf{0} & \cdots & \mathbf{0} \\ \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+F-1} & \cdots & & \hat{\mathbf{h}}_{K_2} & & \vdots \\ \vdots & & & & \ddots & \mathbf{0} \\ \hat{\mathbf{h}}_{\Delta-(K_1+F-1)+1} & & \cdots & \cdots & & \hat{\mathbf{h}}_{K_2} \end{bmatrix}, \quad (7)$$

and $\mathbf{V}_q(n)$ is the postcursor ISI term related to each feedforward tap coefficient $\mathbf{w}^*_{-q}$

$$\mathbf{V}_q(n) \triangleq \sum_{m=1}^{B} \hat{a}_{n+(K_1+F-1)-\Delta-m}\hat{\mathbf{h}}_{\Delta-(K_1+F-1)+m+q}. \quad (8)$$

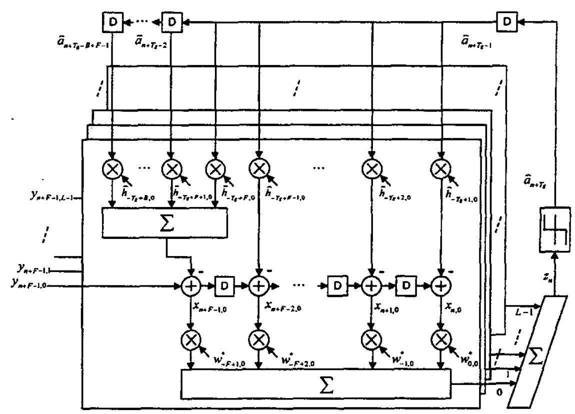


Fig. 2 Structure of the fractionally-spaced modified DFE.

Note that the FBF tap coefficient $\hat{\mathbf{h}}_n$ is represented as a vector unlike in the SS-MDFE. As a result, the FS-MDFE has a fractionally-spaced FBF, requiring the computational complexity approximately L times higher than that of the fractionally-spaced CDFE (FS-CDFE).

Assuming perfect channel estimation and no decision error, (5) can be rewritten as

$$\begin{bmatrix} \mathbf{x}_{n+F-1} \\ \mathbf{x}_{n+F-2} \\ \vdots \\ \mathbf{x}_n \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{-K_1} & \mathbf{h}_{-K_1+1} & & \cdots & & \mathbf{h}_{\Delta-(K_1+F-1)+F-1} \\ \mathbf{0} & \mathbf{h}_{-K_1} & & \cdots & & \mathbf{h}_{\Delta-(K_1+F-1)+F-2} \\ \vdots & & \ddots & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{-K_1} & \cdots & \mathbf{h}_{\Delta-(K_1+F-1)} \end{bmatrix} \cdot \begin{bmatrix} a_{n+K_1+F-1} \\ a_{n+K_1+F-2} \\ \vdots \\ a_{n+K_1+F-1-\Delta} \end{bmatrix} + \begin{bmatrix} \mathbf{v}_{n+F-1} \\ \mathbf{v}_{n+F-2} \\ \vdots \\ \mathbf{v}_n \end{bmatrix} \quad (9)$$

or

$$\mathbf{x}_{n+F-1:n} = \mathbf{H}_{pre}a_{n+(K_1+F-1):n+(K_1+F-1)-\Delta} + \mathbf{v}_{n+F-1:n} \quad (10)$$

where

$$\mathbf{H}_{pre} \triangleq \begin{bmatrix} \mathbf{h}_{-K_1} & \mathbf{h}_{-K_1+1} & & \cdots & & \mathbf{h}_{\Delta-(K_1+F-1)+F-1} \\ \mathbf{0} & \mathbf{h}_{-K_1} & & \cdots & & \mathbf{h}_{\Delta-(K_1+F-1)+F-2} \\ \vdots & & \ddots & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}_{-K_1} & \cdots & \mathbf{h}_{\Delta-(K_1+F-1)} \end{bmatrix}. \quad (11)$$

Define the FFF tap coefficient by

$$\mathbf{w}^* \triangleq \begin{bmatrix} \mathbf{w}^*_{-(F-1)} & \cdots & \mathbf{w}^*_{-1} & \mathbf{w}^*_0 \end{bmatrix} \quad (12)$$

where $\mathbf{w}^*_{-q} \triangleq \begin{bmatrix} w^*_{-q,L-1} & \cdots & w^*_{-q,1} & w^*_{-q,0} \end{bmatrix}$ and the superscript $*$ denotes the complex conjugate transpose of a matrix or vector and the complex conjugate of a scalar. Then, the output of the FS-MDFE z_n is represented as

$$z_n = \mathbf{w}^*\mathbf{x}_{n+F-1:n}, \quad (13)$$

and the error sequence is given by

$$e_n = a_{n+(K_1+F-1)-\Delta} - z_n = a_{n+(K_1+F-1)-\Delta} - \mathbf{w}^*\mathbf{x}_{n+F-1:n}. \quad (14)$$

Concerning the decision delay Δ of the FS-MDFE, we define a new variable T_g by

$$T_g \triangleq (K_1+F-1) - \Delta \quad (15)$$

which indicates the position of the main tap in the FFF. For example, if $T_g = 0$, the position of the main tap will be on $w^*_{0,0}$ of the FFF, if $T_g = 1$, $w^*_{-1,0}$, and so on. As T_g increases, the position of the main tap will move from $w^*_{0,0}$ to $w^*_{-(F-1),0}$.

The structure of the FS-MDFE is illustrated in Fig. 2. The FS-CDFE has a fractionally-spaced FFF and a symbol-spaced FBF, but the FS-MDFE has a fractionally-spaced FFF and FBF. Although the FS-CDFE and FS-MDFE have different external appearance, their internal structures and analytical performances are the same [8]. Note that, if $B \geq K_2 + (K_1 + F - 1) - \Delta$, the FS-MDFE can be implemented using (16) depicted in Fig. 2 instead of using (8).

$$\mathbf{V}_{q-1}(n) = \begin{cases} \mathbf{V}_q(n-1) + a_{n+(K_1+F-1)-\Delta-1}\mathbf{h}_{q+\Delta-(K_1+F-1)}, & q = 1,2,\cdots,F-1 \\ \sum_{m=1}^{B} a_{n+(K_1+F-1)-\Delta-m}\mathbf{h}_{m+\Delta-(K_1+F-1)+F-1}, & q = F \end{cases} \tag{16}$$

III. PERFORMANCE ANALYSIS

It can easily be shown that the optimum coefficient of the FBF is equal to the postcursor term of the channel impulse response from (8). Defining the coefficient of the FBF by

$$\mathbf{c}_{B:1} \triangleq [\mathbf{c}_B \quad \cdots \quad \mathbf{c}_1]^T \tag{17}$$

where $\mathbf{c}_m \triangleq [c_{m,L-1}, \cdots, c_{m,0}]^T$ and the superscript T denotes the transpose of a matrix. The optimum coefficient is given by

$$\mathbf{c}_{B:1,opt} = \mathbf{h}_{\Delta-(K_1+F-1)+B:\Delta-(K_1+F-1)+1}. \tag{18}$$

The optimum coefficient of the FFF can be found in the MMSE sense using the *orthogonality principle* that the optimal error is uncorrelated with the observed data, *i.e.*,

$$E\{e_n \mathbf{x}^*_{n+F-1:n}\} = \mathbf{0}. \tag{19}$$

Combining (10), (14) and (19) and assuming that the data and noise are white and uncorrelated with each other, the optimum coefficient of the FFF is given by

$$\mathbf{w}^*_{opt} = \mathbf{e}^T_\Delta \mathbf{H}^*_{pre}\left(\mathbf{H}_{pre}\mathbf{H}^*_{pre} + \frac{1}{\gamma}\mathbf{I}\right)^{-1} \tag{20}$$

where $\gamma = S_a/S_v$, $S_a = E\{|a_k|^2\}$, $S_v = E\{|v_k|^2\}$ and $\mathbf{e}_i$ denotes the ith unit vector with the unit magnitude only in the ith position and zeros elsewhere, $i = 0, 1, 2, \cdots$.

The MMSE $J_{\min}$ of the FS-MDFE is given by

$$\begin{aligned} J_{\min} &= E\{|e_n|^2\}\Big|_{\mathbf{w}^* = \mathbf{w}^*_{opt}} \\ &= S_a\left[1 - \mathbf{e}^T_\Delta \mathbf{H}^*_{pre}\left(\mathbf{H}_{pre}\mathbf{H}^*_{pre} + \frac{1}{\gamma}\mathbf{I}\right)^{-1} \mathbf{H}_{pre}\mathbf{e}_\Delta\right]. \end{aligned} \tag{21}$$

Note that the optimum FS-MDFE minimizes the MSE by canceling all the postcursor ISI (assuming no decision error) and then minimizing the precursor ISI and filtered noise variance.

IV. FAST INITIALIZATION

Once the impulse response $\mathbf{h}_n$ of the channel is estimated using an appropriate preamble, the coefficient of the FFF and FBF can be initialized using the MMSE solution of the FS-MDFE from (18) and (20). However, this MMSE initialization may require large implementation complexity as the span F of the FFF increases. We consider an initialization method for the FFF of the FS-MDFE to reduce the implementation complexity.

As a simple method, only the main tap coefficient of the FFF can be first initialized as

$$w^*_{\Delta-(K_1+F-1),0} = \frac{1}{h_{0,0}} = \frac{h^*_{0,0}}{|h_{0,0}|^2}. \tag{22}$$

The one-tap initialization is feasible when the precursor ISI term is small and short. This corresponds to the most of minimum phase condition. However, when the channel is in non-minimum phase condition, the one-tap initialization may not accomplish fast start-up. To avoid this problem, we propose a partial MMSE (PMMSE) initialization method.

The PMMSE initialization is an approximate version of the MMSE initialization (20). A small number of coefficients of the FFF including the main tap are initialized as

$$\tilde{\mathbf{w}}^*_{opt} = \mathbf{e}^T_\Delta \tilde{\mathbf{H}}^*_{pre}\left(\tilde{\mathbf{H}}_{pre}\tilde{\mathbf{H}}^*_{pre} + \frac{1}{\gamma}\mathbf{I}\right)^{-1} \tag{23}$$

while the rest of tap coefficients of the FFF are set to zero, where

$$\tilde{\mathbf{H}}_{pre} \triangleq \begin{cases} \mathbf{H}_{pre}\langle(\Delta-K_1+1)L : (\Delta-K_1+1)L+r-1\rangle, \\ \qquad r \leq \text{FL-}(\Delta-K_1+1)L+1 \\ \mathbf{H}_{pre}\langle FL-r+1 : FL\rangle, \\ \qquad r > \text{FL-}(\Delta-K_1+1)L+1 \end{cases} \tag{24}$$

$$\tilde{\mathbf{w}}^* \triangleq \begin{cases} \mathbf{w}^*\langle(\Delta-K_1+1)L : (\Delta-K_1+1)L+r-1\rangle, \\ \qquad r \leq \text{FL-}(\Delta-K_1+1)L+1 \\ \mathbf{w}^*\langle FL-r+1 : FL\rangle, \\ \qquad r > \text{FL-}(\Delta-K_1+1)L+1 \end{cases} \tag{25}$$

$\mathbf{H}_{pre}\langle x:y\rangle$ denotes a submatrix of $\mathbf{H}_{pre}$ comprising the elements from the x-th to the y-th row of $\mathbf{H}_{pre}$ and $\mathbf{w}^*\langle x:y\rangle$ denotes a subvector of $\mathbf{w}^*$ comprising the elements from the x-th to the y-th element of $\mathbf{w}^*$. That is, the r number of rows from the main tap row (*i.e.*, the $(\Delta-K_1+1)L$-th row) of $\mathbf{H}_{pre}$ are selected for $\tilde{\mathbf{H}}_{pre}$, and the r number of elements from the main tap element of $\mathbf{w}^*$ are selected for $\tilde{\mathbf{w}}^*$. The value of r should be larger than the delay spread of the main path enough to eliminate the main ISI term from each multipath. For example, when the delay spread of the channel is about $2T$ long, r should be larger than $2L$. Note that the matrix inversion of size $(FL \times FL)$ is reduced to the one of size $(r \times r)$, significantly reducing the computational complexity.

Once the major coefficients of the FFF are initialized, all the coefficients of the FS-MDFE are trained using an LMS adaptation method,

$$\mathbf{w}^*_{-q}(n+1) = \mathbf{w}^*_{-q}(n) + 2\alpha e_n \mathbf{x}^*_{n+q}, \qquad q = 0,1,\cdots,F-1 \tag{26}$$

$$\mathbf{c}_m(n+1) = \mathbf{c}_m(n) - 2\beta e_n \sum_{q=0}^{\min(m-1,F-1)} \mathbf{w}_{-q}(n) a^*_{n+(K_1+F-1)-\Delta-m+q}, \quad m = 1,2,\cdots B \tag{27}$$

Note that the LMS adaptation algorithm for the FFF is similar to that of the FS-CDFE, whereas the algorithm for the FBF is quite different, requiring increased computational complexity.

V. PERFORMANCE EVALUATION

The performance of the proposed FS-MDFE is evaluated by computer simulation when high-level QAM signals are transmitted in a burst mode over fixed wireless channels with frequency selective fading. We consider transmission of the QAM signal using the time division multiple access with frequency division duplex (TDMA/ FDD), where the frame has a duration of 6 ms, comprising 8 time slots. The QAM signal is modulated at a symbol rate of 1.6 Mbauds with a modulation level of up to 256. Each time slot contains 1200 symbols including the 120-symbol preamble. Both the transmit and receive filters use a square root raised cosine filter with roll-off factor 0.25. We assume that the sampling rate is $2/T$, $F=8$, $B=8$ and $T_g=3$.

The computer simulation is performed under fixed wireless channel conditions that have the line of sight (LOS) and multipaths. The fading model is approximated as a product of the two Ricean random gains of the fast and slow fading gains [9]. The slow fading is modeled as a Ricean random variable and fast fading is modeled as a Ricean or Rayleigh. The resulting 3-ray channel model has the form of a tapped delay line (TDL) with gain and delay summarized in Table1, where a1, a2 and a3 are the average energy of each path with a Doppler frequency of 0.4 Hz.

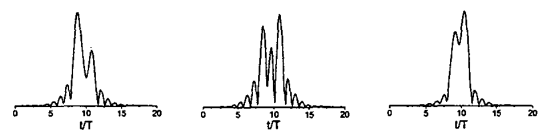
We consider the three channels to represent the minimum phase, nearly non-minimum phase and non-minimum phase condition. Fig. 3 depicts the absolute value of the overall impulse response, including the transmit filter, physical channel and receive filter. We assume that the channel is "quasi-stationary," *i.e.*, the channel characteristics are unchanged during a slot interval.

In a fixed wireless channel, the symbol-spaced equalizer may not properly work since each multipath is not separated regularly by an integer multiple of the symbol period T. If the timing phase of each path (*i.e.* timing phase when each path gain has the maximum value) is different, a small timing phase error can cause significant performance degradation in the symbol-spaced equalizer. We define the *output SNR* by the ratio of the signal to decision error power, and the *input SNR* by the ratio of the signal to noise power after the receive filter. Fig. 4 depicts the performance of the FS-MDFE and SS-MDFE due to the symbol timing phase error, when the input SNR is 40 dB, and ideal symbol timing phase is based on the first path. It can be seen that the FS-MDFE provides performance robust to the symbol timing phase error and outperforms the SS-MDFE. Note that the performance degradation is not proportional to the symbol timing phase error.

In order to evaluate the performance of the initialization methods, Fig. 5 depicts the MSE when the input SNR is 40 dB, where the one-tap initialization is denoted by *ONETAP*, the MMSE initialization by *MMSE*, and the PMMSE initialization by *PARTIAL*, that first initializes only 4 tap coefficients $(r=4)$ among 16 tap coefficients $(F=8)$ of the FFF. It can be seen that the one-tap initialization shows poor convergence behavior, but the PMMSE initialization is applicable to all the cases. Note that the PMMSE initialization can provide fast convergence and high SNR enough to receive 256-QAM signal.

Table 1 The characteristics of the channel.

Slow fading	Fast fading	a1	Delay (ns)	a2	Delay (ns)	a3	Delay (ns)
9.25 dB	7.3 dB	0.76	0	0.3	468.75	0.57	1015.625



(a) Channel A (b) Channel B (c) Channel C

Fig. 3 Overall response of the sampled channels.

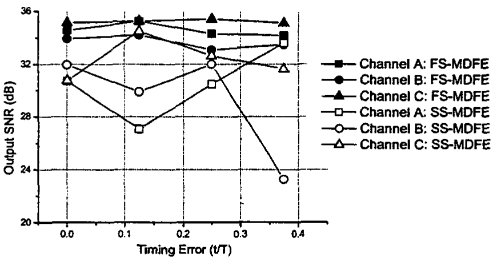
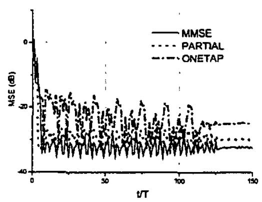


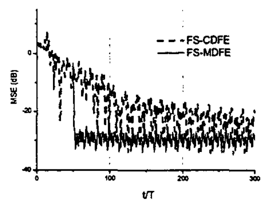
Fig. 4 The output SNR due to the symbol timing phase error.

The performance of the FS-CDFE and FS-MDFE is compared in terms of the training time. Fig. 6 depicts the MSE of the FS-CDFE and FS-MDFE, where the average signal power is 3 dB. The coefficients of the FFF and FBF of the FS-CDFE are trained using the LMS algorithm with all zero coefficients. Once the channel is estimated using the 48 symbols of the preamble, the coefficients of the FS-MDFE are initialized by the PMMSE initialization $(r=4)$ before being trained by the LMS adaptation method. It can be seen that the FS-MDFE begins the coefficient training from the 49-th symbol time due to the channel estimation, but it outperforms the FS-CDFE in view of the convergence speed and steady-state SNR. Note that the FS-CDFE cannot provide high SNR enough for reception of 256-QAM signal even at high input SNR (=40 dB) when the channel has nearly non-minimum phase or non-minimum phase condition.

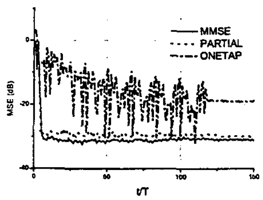
Fig. 7 depicts the achievable SNR of the FS-CDFE and FS-MDFE with the analytic SNR bound of (21). As the input SNR increases, the output SNR of the FS-CDFE becomes saturated much



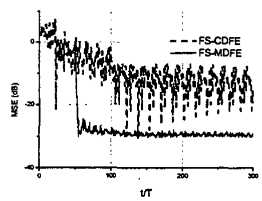
(a) Channel A



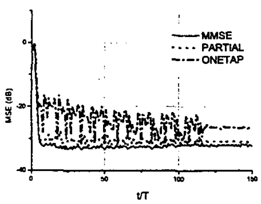
(a) Channel A



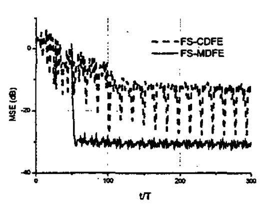
(b) Channel B



(b) Channel B



(c) Channel C



(c) Channel C

Fig. 5 Learning curve with three initialization methods.

Fig. 6 Learning curve of the FS-CDFE and FS-MDFE.

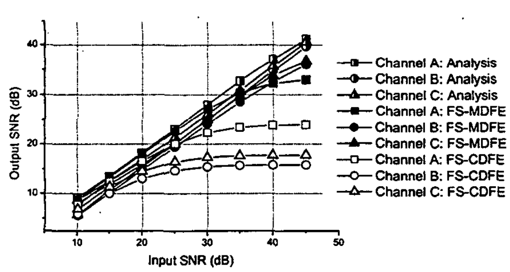


Fig. 7 Achievable output SNR of the FS-CDFE and FS-MDFE.

earlier than the FS-MDFE. This implies that the FS-CDFE may not be appropriate for reception of high-level QAM signals in fixed wireless channels.

VI. Conclusion

In this paper, we have proposed a fractionally-spaced scheme as a generalization of the SS-MDFE. The MMSE solution of the proposed FS-MDFE has been analytically evaluated. Since the MMSE solution of the FFF of the FS-MDFE includes a matrix inversion requiring large computational complexity, a two-step training method has been proposed to reduce the complexity. A small number of main coefficients of the FFF are first initialized using a partial MMSE (PMMSE) initialization. Then, all the coefficients are trained using a conventional LMS adaptation method. The FBF of the FS-MDFE needs more computation than that of the FS-CDFE, because the FS-MDFE has a fractionally-spaced FBF and its postcursor-precancelling structure requires the use of more complicated LMS tracking algorithm. At a small expense of additional computational complexity, the FS-MDFE can provide fast convergence and high SNR enough for reception of 256-QAM signal in fixed wireless channels. The FS-MDFE can provide performance robust under non-minimum channel condition due to the frequency selective fading.

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