Long-Term CSI Based Multi-User Scheduling for Collaborative Spatial Multiplexing in Mobile-WiMAX

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Abstract

In this paper, we propose a multi-user scheduling scheme to maximize the ergodic sum-rate of the uplink of a multi-user multiple-input multiple-output wireless system. The proposed scheduling scheme only utilizes the receive correlation information of users without instantaneous channel state information (CSI). It can be applied to the collaborative spatial multiplexing mode specified in the IEEE 802.16e mobile-WiMAX system, where the number of users in transmission is limited to the number of receive antennas of the BS at each time slot. An upper-bound analysis and numerical results show that the proposed scheduling scheme outperforms conventional random scheduling scheme in terms of the ergodic sum-rate without loss of opportunity fairness. Moreover, the proposed scheduling scheme can provide an ergodic sum-rate comparable to full-CSI based scheduling scheme in the presence of user mobility, while significantly reducing the feedback signaling overhead.

Index Terms—Collaborative spatial multiplexing (CSM), ergodic sum-rate, feedback signaling overhead, Mobile-WiMAX system, receive correlation, scheduling complexity.

I. INTRODUCTION

The sum-rate of the uplink of multi-user multiple-input multiple-output (MIMO) systems can be increased by using multiple receive antennas at the base station (BS) [1]–[4]. For example, the use of two receive antennas at the BS can almost double the sum-rate compared to the use of a single receive antenna, even when users use only a single transmit antenna [3], [4]. The sum-rate capacity of a multi-user uplink

MIMO system can be achieved by employing a minimum mean-squared error-successive interference cancellation (MMSE-SIC) receiver that decodes the receive signal from all users in a sequential manner [3], [4]. However, it may not be applicable to practical systems due to huge pilot signaling overhead for the channel estimation of all users [4], [5].

This problem can be alleviated by limiting the number of users in transmission to the number of receive antennas of the BS (i.e., the number of available degrees of freedom) at each time slot [4]–[7]. As a consequence, much effort has been devoted to multi-user scheduling methods to maximize the sum-rate. The BS can maximize the sum-rate by scheduling users with the use of instantaneous channel state information (CSI) of all users [8], [9]. However, these full-CSI based scheduling schemes may require a large amount of feedback or training overhead [10]–[12], and may also suffer from so-called channel mismatch problem in the presence of user mobility [11], [13], making it impractical to be employed. Therefore, the use of random scheduling is often considered at the expense of sum-rate reduction [6], [7].

The use of spatial correlation information has recently been considered for multi-user scheduling in the uplink of multi-user MIMO systems [14], since the spatial correlation characteristics often remain unchanged for a relatively long duration. In fact, the spatial correlation is little changed in an interval of about 100 ms even when the user is in a mobility of 1000 km/h [15]. By using this property, [14] can remarkably improve the sum-rate performance with low feedback signaling overhead. However, this work only considers the use of the transmit correlation (i.e., spatial correlation at the user) for multi-user scheduling assuming no receive correlation (i.e., spatial correlation at the BS). Although this assumption may be appropriate in indoor environments, but not in outdoor environments where few scatters exist near the BS and many scatters are near the users [14], [16]. Moreover, the receive correlation is more meaningful than the transmit correlation in the uplink since the use of multi-antenna configuration is more feasible at the BS than the user due to the terminal complexity issues [6], [7].

In this paper, we propose a new multi-user scheduling scheme that utilizes the receive correlation to maximize the ergodic sum-rate of the uplink in multi-user MIMO systems. We assume that the number of users in transmission is limited to the number of receive antennas of the BS at each time slot and that each user has a single transmit antenna. In fact, this is considered as one of key multi-user MIMO techniques in the uplink of the IEEE 802.16e Mobile-WiMAX system, so-called collaborative spatial multiplexing (CSM) [6]. To optimize the multi-user scheduling in the CSM mode, we first analyze the impact of the

receive correlation on the ergodic sum-rate in the CSM mode. Then, we find the condition to maximize the ergodic sum-rate assuming that the BS only knows the receive correlation information of all users. Finally, we propose a multi-user scheduling scheme that can maximize the ergodic sum-rate in the CSM mode. It is shown that the proposed scheduling scheme outperforms the random scheduling scheme in terms of the ergodic sum-rate without loss of opportunity fairness. Furthermore, it is shown that the proposed multi-user scheduling scheme provides an ergodic sum-rate comparable to full-CSI based multi-user scheduling scheme in the presence of user mobility, while noticeably reducing the feedback signaling overhead.

The remainder of this paper is organized as follows. Section II describes the system and channel model in consideration. The impact of the receive correlation on the ergodic sum-rate in the CSM mode is analyzed in Section III. Section IV describes the proposed receive correlation-based multi-user scheduling scheme that maximizes the ergodic sum-rate in the CSM mode. Section V verifies the performance of the proposed multi-user scheduling scheme in terms of the ergodic sum-rate, feedback signaling overhead, and scheduling complexity via analysis and computer simulation. Finally, conclusions are given in Section VI.

II. System and Channel Model

Consider the uplink of a multi-user MIMO wireless system, where the BS uses M antennas for the reception and each of N users employs a single antenna for the transmission. In the CSM mode [6], the BS receives the signal from M users at each time slot as illustrated in Fig. 1. Without loss of generality, it can be assumed that user 1, user 2, …, user M are scheduled to transmit the signal at any time slot. Let $\mathbf{x} = [x_1 \ \cdots \ x_M]^T$ be the transmitted signal vector from the M scheduled users and $\mathbf{h}_n = [h_{1n} \ \cdots \ h_{Mn}]^T$ be the channel vector from user n to the BS, where $n = 1, \ldots, M$, and the superscript T denotes transpose. The channel matrix from the M scheduled users to the BS can be represented as $\mathbf{H} = [\mathbf{h}_1 \ \cdots \ \mathbf{h}_M]$. Then, the received signal vector at the BS can be represented as [3]

$$\begin{aligned} \mathbf{y} &= \sum_{n=1}^{M} \mathbf{h}_n x_n + \mathbf{w} \\ &= \mathbf{H}\mathbf{x} + \mathbf{w} \end{aligned} \tag{1}$$

where $\mathbf{w} = [w_1 \ \ldots \ w_M]^T$ denotes the additive noise vector. Here, w_m are independent and identically

distributed (i.i.d.) zero-mean Gaussian random variables with the same variance N_0, where $m = 1, \ldots, M$.

When the channel is spatially correlated, the channel matrix $\mathbf{H}$ can be generated using a complex white Gaussian random matrix $\mathbf{H}^w = [\mathbf{h}_1^w \ \cdots \ \mathbf{h}_M^w]$ whose entries are i.i.d. zero-mean Gaussian random variables with unit-variance, i.e., [17]

$$\mathrm{vec}(\mathbf{H}) = \mathbf{R}^{1/2}\mathrm{vec}(\mathbf{H}^w) \tag{2}$$

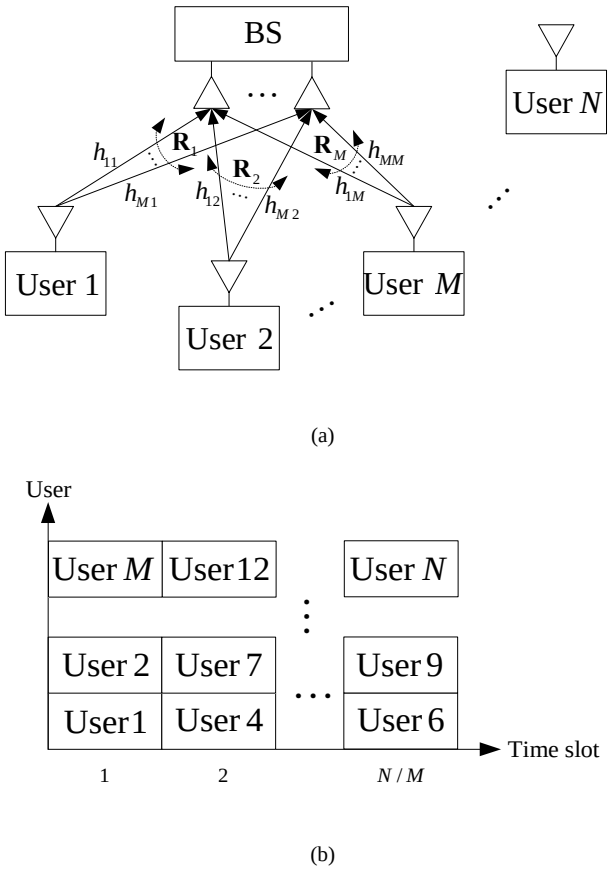

Fig. 1. Model of the CSM mode. (a) Signal transmission at any time slot. (b) An example of multi-user scheduling.

where $\mathrm{vec}(\mathbf{H})$ denotes an operator that stacks matrix $\mathbf{H}$ into a vector columnwise (i.e., $\mathrm{vec}(\mathbf{H}) = [\mathbf{h}_1^T \ \cdots \ \mathbf{h}_M^T]^T$) and $\mathbf{R}^{1/2}$ denotes the Hermitian positive definite square root of the channel correlation matrix (i.e., $\mathbf{R} = \mathbf{R}^{1/2}\mathbf{R}^{*1/2}$). Here, the superscript $*$ denotes conjugate transpose. Since users are not physically co-located in real environments, it can be assumed that there is no transmit correlation between the users [18]–[21]. In other words, the columns of $\mathbf{H}$ are uncorrelated as

$$E\{\mathbf{h}_{n_1}\mathbf{h}_{n_2}^*\} = \mathbf{O}, \quad \text{for } n_1 \neq n_2, \tag{3}$$

where $\mathbf{O}$ denotes a zero matrix whose all entries are zero. Thus, the CSM mode has a channel correlation matrix $\mathbf{R}$ represented as [20]

$$\begin{aligned}\mathbf{R} &= E\{\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^*\} \\ &= \begin{bmatrix} E\{\mathbf{h}_1\mathbf{h}_1^*\} & \cdots & E\{\mathbf{h}_1\mathbf{h}_M^*\} \\ \vdots & \ddots & \vdots \\ E\{\mathbf{h}_M\mathbf{h}_1^*\} & \cdots & E\{\mathbf{h}_M\mathbf{h}_M^*\} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R}_1 & \cdots & \mathbf{O} \\ \vdots & \ddots & \vdots \\ \mathbf{O} & \cdots & \mathbf{R}_M \end{bmatrix}\end{aligned} \tag{4}$$

where $\mathbf{R}_n$ denotes the receive correlation matrix of user n defined as

$$\begin{aligned}\mathbf{R}_n &= E\{\mathbf{h}_n\mathbf{h}_n^*\} \\ &= \begin{bmatrix} E\{h_{1n}h_{1n}^*\} & \cdots & E\{h_{1n}h_{Mn}^*\} \\ \vdots & \ddots & \vdots \\ E\{h_{Mn}h_{1n}^*\} & \cdots & E\{h_{Mn}h_{Mn}^*\} \end{bmatrix}.\end{aligned} \tag{5}$$

III. Ergodic Sum-Rate in The CSM Mode

To analyze the impact of the receive correlation on the ergodic sum-rate in the CSM mode, we assume that user 1, user 2, …, user M are scheduled at time slot t without loss of generality and that the BS perfectly estimates the channel of the M scheduled users (i.e., user 1, user 2, …, user M) [6]. Then, the BS can get an ergodic sum-rate by means of an MMSE-SIC process, given by [3], [4]

$$\begin{aligned}C(t) &= E\left\{\log_2 \det\left(\mathbf{I} + \sum_{n=1}^{M} \frac{\gamma_n \mathbf{h}_n \mathbf{h}_n^*}{M}\right)\right\} \\ &= E\left\{\log_2 \det\left(\mathbf{I} + \frac{\mathbf{H}\boldsymbol{\Gamma}\mathbf{H}^*}{M}\right)\right\}\end{aligned} \tag{6}$$

where $\mathbf{I}$ is an identity matrix, $\boldsymbol{\Gamma}$ is a diagonal matrix whose n-th diagonal element γ_n represents the average signal-to-noise ratio (SNR) of user n, which equals P_n / N_0, and P_n denotes the average transmit power of user n for $n = 1, \ldots, M$. From the concave property of the logarithmic function and after some mathematical handling, it can be shown that the ergodic sum-rate (6) is upper bounded by

(refer to Appendix I)

$$C(t) \le \log_2 \left[\begin{array}{l} (-1)^{2M}\left[(0!)^M\{(1+\gamma_1 \mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_M \mathrm{tr}(\mathbf{R}_M)/M)\}\right] \\ +(-1)^{2M-1}\left[\frac{(1!)^1(0!)^{M-2}}{M^2}\left\{\begin{array}{l}\gamma_1\gamma_2\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2)(1+\gamma_3\mathrm{tr}(\mathbf{R}_3)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-2}\mathrm{tr}(\mathbf{R}_{M-2})/M)\end{array}\right\}\right] \\ +(-1)^{2M-2}\left[\begin{array}{l}\frac{(2!)^1(0!)^{M-3}}{M^3}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3)(1+\gamma_4\mathrm{tr}(\mathbf{R}_4)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-2}\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-3}\mathrm{tr}(\mathbf{R}_{M-3})/M)\end{array}\right\} \\ +\frac{(1!)^2(0!)^{M-4}}{M^4}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\gamma_4\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2)\mathrm{tr}(\mathbf{R}_3\mathbf{R}_4)(1+\gamma_5\mathrm{tr}(\mathbf{R}_5)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-3}\gamma_{M-2}\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-3}\mathbf{R}_M)\mathrm{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1})(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-4}\mathrm{tr}(\mathbf{R}_{M-4})/M)\end{array}\right\}\end{array}\right] \\ +\cdots \\ +(-1)^{M+1}\left[\frac{((M-1)!)^1}{M^M}\gamma_1\gamma_2\times\cdots\times\gamma_M\{\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\cdots\mathbf{R}_M)\}\right] \end{array}\right] \quad (7)$$

$$\triangleq \widehat{C}(t).$$

Consider a simple case where $M = 2$ for ease of mathematical tractability. Then, (7) can be simplified to

$$C(t) \le \log_2((1+\gamma_1 \mathrm{tr}(\mathbf{R}_1)/2)(1+\gamma_2 \mathrm{tr}(\mathbf{R}_2)/2) - \gamma_1\gamma_2 \mathrm{tr}(\mathbf{R}_1\mathbf{R}_2)/4)\,. \quad (8)$$

where

$$\mathbf{R}_n = \begin{bmatrix} 1 & \rho_n \\ \rho_n^* & 1 \end{bmatrix}. \quad (9)$$

for $n = 1, 2$. Here, $\rho_n = \alpha_n e^{j\theta_n}$ denotes the receive correlation coefficient of user n, where α_n ($0 \le \alpha_n < 1$) and θ_n ($0 < \theta_n \le 2\pi$) denote the amplitude and the phase of the receive correlation coefficient, respectively. Since $\mathrm{tr}(\mathbf{R}_1) = \mathrm{tr}(\mathbf{R}_2) = 2$ and $\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2) = 2 + \rho_1\rho_2^* + \rho_1^*\rho_2$, (8) can be rewritten as

$$C(t) \le \log_2\left(1+\gamma_1+\gamma_2+\frac{\gamma_1\gamma_2}{4}(2-\rho_1\rho_2^*-\rho_1^*\rho_2)\right). \quad (10)$$

Noting that $\rho_n = \alpha_n e^{j\theta_n}$ for $n = 1, 2$, we have

$$C(t) \le \log_2\left(1+\gamma_1+\gamma_2+\frac{\gamma_1\gamma_2}{2}(1-\alpha_1\alpha_2\cos(\theta_1-\theta_2))\right). \quad (11)$$

Fig. 2 depicts the ergodic sum-rate in the CSM mode with the upper-bound (11) for an average SNR of 10 dB (i.e., $\gamma_1 = \gamma_2 = 10$ dB). It can be seen that the ergodic sum-rate in the CSM mode highly depends on the receive correlation coefficient of the two scheduled users (i.e., user 1 and user 2). Notice that the ergodic sum-rate increases as the phase difference approaches to π (i.e., $\theta_1 - \theta_2 \to \pi$), and the product

of the two amplitudes approaches to one (i.e., $\alpha_1 \alpha_2 \to 1$). It can also be seen that although the simulation results and the analytic upper-bound (11) have some discrepancy due to Jensen's inequality loss (i.e., the ergodic sum-rate difference between (33) and (34) in Appendix II), they have a tendency very similar to each other. Thus, the analytic upper-bound can be applied to the multi-user scheduling to maximize the ergodic sum-rate in the CSM mode. In other words, the BS can maximize the ergodic sum-rate by finding a pair of users whose receive correlation coefficients have a phase difference close to π and an amplitude product close to one.

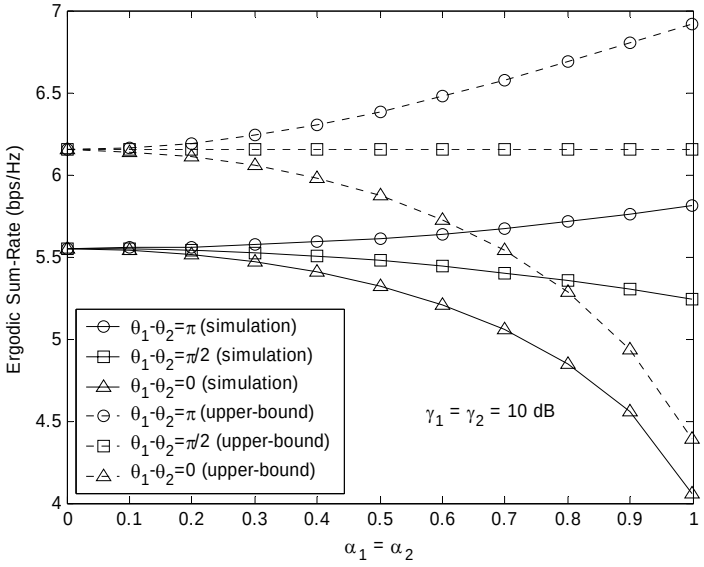

Fig. 2. Ergodic sum-rate according to receive correlation coefficient when $M = 2$.

IV. Proposed Multi-User Scheduling in The CSM Mode

Consider multi-user scheduling that utilizes only the receive correlation information without the use of instantaneous CSI of all users in the CSM mode. To provide fair scheduling opportunity to all N users, it is assumed that each of N users is scheduled once every N/M time slots (i.e., M times every N time slots) in an average sense. Under the above assumption, there can be ${}_N C_M$ scheduling cases at time slot 1, where

$$ {}_x C_y \triangleq \frac{x!}{(x-y)!\,y!} \tag{12} $$

and $x! \triangleq x \cdot (x-1) \cdots 3 \cdot 2 \cdot 1$. At time slot 2, there can be ${}_{N-M} C_M$ scheduling cases since the M

scheduled users at time slot 1 are excluded for the scheduling. In this manner, there can be ${}_{N-Mt+M}C_M$ scheduling cases at time slot t. Thus, there can be a total S number of scheduling cases in N/M time slots, where

$$S = \prod_{t=1}^{N/M} \frac{{}_{N-Mt+M}C_M}{(N/M)!}. \tag{13}$$

Since $\prod_{t=1}^{N/M} {}_{N-Mt+M}C_M = \prod_{t=1}^{N/M} {}_{Mt}C_M$, it can be shown that

$$S = \frac{N!}{(M!)^{N/M}\left(N/M\right)!}. \tag{14}$$

Let $\widehat{C}_s(t)$ be the analytic upper-bound (7) corresponding to the s-th scheduling case at time slot t, where $s = 1, \ldots, S$, and $t = 1, \ldots, N/M$. Then, the optimum scheduling case can be found by

$$s_{\text{opt}} = \arg\max_{s\in\{1,\ldots,S\}} \frac{M}{N}\sum_{t=1}^{N/M} \widehat{C}_s(t). \tag{15}$$

As a simple example to clarify the above procedure, consider the application of the proposed multi-user scheduling to a CSM mode with $M = 2$, where 4 users have the same average SNR (i.e., $\gamma_n = \gamma$), and the receive correlation coefficients whose amplitudes are the same (i.e., $\alpha_n = \alpha$) and whose phases are equally scattered on $(0, 2\pi]$ (e.g., $\theta_n = 2n\pi/N$ for $n = 1, \ldots, 4$). Then, there can be 3 possible cases for the scheduling of 4 users in $N/M (= 2)$ time slots as illustrated in Fig. 3 (i.e., $\{(1, 2), (3, 4)\}$, $\{(1, 3), (2, 4)\}$, $\{(1, 4), (2, 3)\}$, where the numbers denote the user index). The upper-bound of the ergodic sum-rate corresponding to these three scheduling cases can be represented as

$$\frac{1}{2}\sum_{t=1}^{2}\widehat{C}_1(t) = \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}\right), \tag{16}$$

$$\frac{1}{2}\sum_{t=1}^{2}\widehat{C}_2(t) = \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}\left(1+\alpha^2\right)\right), \tag{17}$$

$$\frac{1}{2}\sum_{t=1}^{2}\widehat{C}_3(t) = \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}\right). \tag{18}$$

Thus, it can easily be found that the second scheduling case (i.e., $\{(1, 3), (2, 4)\}$ in Fig. 3 (b)) is the optimum case (i.e., $s_{\text{opt}} = 2$) that maximizes the ergodic sum-rate in this CSM mode.

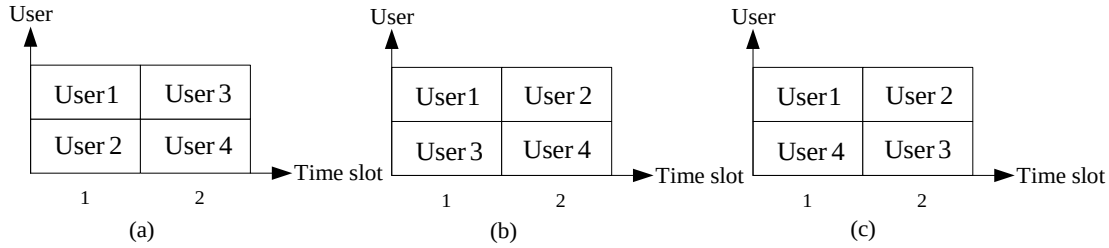

Fig. 3. Three scheduling cases for $M = 2$ and $N = 4$. (a) $s = 1$. (b) $s = 2$. (c) $s = 3$.

The proposed multi-user scheduling scheme can maximize the ergodic sum-rate by only utilizing the receive correlation information without loss of opportunity fairness. However, it may involve scheduling complexity. This complexity problem can be alleviated by sub-optimizing the scheduling by means of user shedding-based exhaustive search: At time slot 1, the BS schedules M users that maximize the ergodic sum-rate based on the analytic upper-bound (7) among N users. At time slot 2, the BS selects M users that maximize the analytic upper-bound (7) among $(N - M)$ users, shedding the M scheduled users at time slot 1. In this manner, all N users are scheduled in N / M time slots. This sub-optimum method can significantly reduce the scheduling complexity without noticeable sum-rate performance degradation which will be verified in Section V.

V. Performance Evaluation

In this section, we verify the performance of the proposed multi-user scheduling scheme in terms of the ergodic sum-rate, feedback signaling overhead, and scheduling complexity. For reference, the performance of conventional multi-user scheduling schemes (i.e., full-CSI based and random multi-user scheduling scheme) is also evaluated. To make the performance comparison tractable, we consider a simple case where the BS has two receive antennas (i.e., $M = 2$). We assume that all users have the same average SNR (i.e., $\gamma_n = \gamma$), the same speed of v, and the receive correlation coefficients whose amplitudes are the same (i.e., $\alpha_n = \alpha$) and whose phases θ_n are uniformly distributed on $(0, 2\pi]$.

A. Ergodic Sum-Rate Gain

We analyze the ergodic sum-rate gain of the proposed scheduling scheme over the random scheduling scheme by using the analytic upper-bound (11). Based on the above assumption, the analytic upper-bound

(11) can be simplified to

$$C(t) \le \hat{C}(t) = \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}(1 - \alpha^2 \cos(\theta_{n_1} - \theta_{n_2}))\right) \tag{19}$$

where θ_{n_1} and θ_{n_2} are the phase of the receive correlation coefficients of the two scheduled users, user n_1 and n_2, at time slot t.

The random multi-user scheduling scheme randomly picks up a pair of users at each time slot. The corresponding ergodic sum-rate can be represented as

$$\begin{aligned} E\{C_r\} &\le \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} \hat{C}(t) d\theta_{n_1} d\theta_{n_2} \\ &= \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}(1 - \alpha^2 \cos(\theta_{n_1} - \theta_{n_2}))\right) d\theta_{n_1} d\theta_{n_2} \\ &\triangleq E\{\hat{C}_r\}. \end{aligned} \tag{20}$$

Note that (20) corresponds to the ensemble average of the ergodic sum-rate of the random multi-user scheduling scheme. It can be shown from the definition of the ergodic sum-rate [22] that (20) can be approximated in terms of the time averaged sum-rate as

$$E\{\hat{C}_r\} \approx \frac{1}{S}\frac{2}{N}\sum_{s=1}^{S}\sum_{t=1}^{N/2} \hat{C}_s(t). \tag{21}$$

Unlike the random scheduling scheme, the proposed scheduling scheme assigns users to maximize the ergodic sum-rate among S number of scheduling cases. The corresponding ergodic sum-rate of the proposed scheduling scheme can be represented as

$$\begin{aligned} E\{C_p\} &\le \frac{2}{N}\sum_{t=1}^{N/2} \hat{C}_{s_{\text{opt}}}(t) \\ &\triangleq E\{\hat{C}_p\}. \end{aligned} \tag{22}$$

The ergodic sum-rate gain of the proposed scheduling scheme over the random scheduling scheme can be represented as

$$E\{\hat{C}_p\} - E\{\hat{C}_r\} = \frac{2}{N}\sum_{t=1}^{N/2} \hat{C}_{s_{\text{opt}}}(t) - \frac{1}{S}\frac{2}{N}\sum_{s=1}^{S}\sum_{t=1}^{N/2} \hat{C}_s(t). \tag{23}$$

Since (23) cannot be represented in a closed form, we consider an asymptotic ergodic sum-rate gain as the

number of users increases to infinity (i.e., $N \to \infty$). In the proposed scheme with this assumption, it is possible to choose a pair of users whose receive correlation coefficient has a phase difference close to π (i.e., $\theta_{n_1} - \theta_{n_2} \approx \pi$) at each time slot. The corresponding asymptotic ergodic sum-rate of the proposed scheduling scheme can be represented as

$$\lim_{N\to\infty} E\{\hat{C}_p\} \approx \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}(1+\alpha^2)\right). \tag{24}$$

It can be shown that (refer to Appendix II)

$$E\{\hat{C}_r\} = \log_2\left(1 + 2\gamma + \frac{\gamma^2}{2}\right) - \frac{1}{\sqrt{2\pi}\ln 2}\sum_{k=1}^{\infty}\left(\frac{\alpha^2\gamma^2}{2+4\gamma+\gamma^2}\right)^k \frac{(k-1)!!}{k\cdot k!!} \tag{25}$$

where "!!" denotes the double factorial defined by [23]

$$x!! \triangleq \begin{cases} x\cdot(x-2)\ldots 5\cdot 3\cdot 1 & x>0 \text{ odd} \\ x\cdot(x-2)\ldots 6\cdot 4\cdot 2 & x>0 \text{ even} \\ 1 & x=-1,\ 0. \end{cases} \tag{26}$$

Thus, the asymptotic gain by the proposed multi-user scheduling scheme can be represented as

$$\lim_{N\to\infty}\left[E\{\hat{C}_p\} - E\{\hat{C}_r\}\right] \approx \log_2\left(1 + \frac{\alpha^2\gamma^2}{2+4\gamma+\gamma^2}\right) + \frac{1}{\sqrt{2\pi}\ln 2}\sum_{k=1}^{\infty}\left(\frac{\alpha^2\gamma^2}{2+4\gamma+\gamma^2}\right)^k \frac{(k-1)!!}{k\cdot k!!}. \tag{27}$$

It can be seen that the asymptotic ergodic sum-rate gain by the proposed scheduling scheme increases as γ and α increase. It can be seen that the proposed scheduling scheme has an asymptotic ergodic sum-rate gain of up to about 2.45 bps/Hz as $N \to \infty$, $\gamma \to \infty$, and $\alpha \to 1$, i.e.,

$$\begin{aligned}\lim_{\alpha\to 1}\lim_{\gamma\to\infty}\lim_{N\to\infty}\left[E\{\hat{C}_p\} - E\{\hat{C}_r\}\right] &= 1 + \frac{1}{\sqrt{2\pi}\ln 2}\sum_{k=1}^{\infty}\frac{(k-1)!!}{k\cdot k!!} \\ &\approx 2.45 \text{ bps/Hz}\end{aligned} \tag{28}$$

since $\sum_{k=1}^{\infty}(k-1)!!/(k\cdot k!!) \approx 2.52$.

B. Feedback Signaling Overhead

We analyze the feedback signaling overhead of the proposed, full-CSI based and random multi-user scheduling schemes according to N. The full-CSI based scheduling scheme requires N feedback signals at each time slot since all users should report their instantaneous CSI at each time slot. Thus, the

amount of feedback signaling overhead in the full-CSI based scheduling scheme increases in linear proportion to N [10]–[12]. On the other hand, the proposed scheduling scheme only requires N feedback signals in N/M time slots (i.e., M feedback signals at each time slot) as illustrated in Section IV. Thus, the amount of feedback signaling overhead for the proposed scheduling scheme is fixed to M regardless of N, remarkably reducing the feedback signaling overhead over the full-CSI based scheme. The random scheduling scheme can work without feedback signal, but it yields the worst sum-rate performance as will be shown in Section V-D.

C. Scheduling Complexity

We compare the scheduling complexity of the proposed, full-CSI based, and random multi-user scheduling schemes. In the full-CSI based scheduling scheme, the BS needs to calculate the sum-rate for ${}_N C_M$ scheduling cases at each time slot to find out the optimum scheduling case. On the other hand, in the proposed scheduling scheme, the BS needs to calculate the ergodic sum-rate for $N!/\{(M!)^{N/M}(N/M)!\}$ scheduling cases in N/M time slots (i.e., $(N-1)!M/\{(M!)^{N/M}(N/M)!\}$ at each time slot). This complexity can be reduced by employing a shedding-based exhaustive search scheme, requiring the calculation for $\sum_{t=1}^{N/M} {}_{N-Mt+M}C_M/(N/M)$ scheduling cases at each time slot. For example, when $M=2$, the scheduling complexity of the proposed scheme is much larger than that of the full-CSI based scheme when N is larger than 8 (e.g., 19305 vs. 91 at each time slot when $N=14$). However, the scheduling complexity of the proposed scheduling with shedding-based exhaustive search is always lower than that of the full-CSI based multi-user scheduling scheme since ${}_{N-Mt+M}C_M \le {}_N C_M$ (e.g., 36 vs. 91 at each time slot when $N=14$).

D. Simulation Results

We verify the analytic design and the sum-rate performance of the proposed scheduling scheme by computer simulation. The system parameters for the simulation are taken from those of the uplink of IEEE 802.16e Mobile-WiMAX system, where the carrier frequency $f_c = 2.3$ GHz, the frame length $T = 5$ ms, the number of BS receive antennas is two (i.e., $M=2$) and users employ a single transmit antenna [6].

We consider the signal transmission over a spatially-correlated channel represented as [24]

$$\mathbf{h}_n(t) = \varepsilon \hat{\mathbf{h}}_n(t-T) + \sqrt{1-|\varepsilon|^2}\,\mathbf{R}_n^{1/2}\mathbf{h}^w \tag{29}$$

where $\mathbf{h}_n(t)$ denotes the channel vector from user n to the BS at time slot t, $\hat{\mathbf{h}}_n(t-T)$ denotes the estimated CSI of user n at the previous frame, $\mathbf{h}^w$ is a complex white Gaussian random vector whose entries are i.i.d. zero-mean Gaussian random variables with unit-variance, and ε is the time correlation coefficient associated with the user mobility v. The time correlation coefficient ε in spatially-correlated channel environments can be represented as [25]

$$\varepsilon = J_0\left(\frac{2\pi T f_c v}{c}\right) + J_2\left(\frac{2\pi T f_c v}{c}\right) \tag{30}$$

where c is the speed of light ($=3\times10^8$ m/s) and $J_l(x)$ is the l-th order Bessel function of the first kind defined by [26]

$$J_l(x) \triangleq \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{l+2k}}{k!\,\Gamma(l+k+1)}, \tag{31}$$

and $\Gamma(x)$ is the gamma function defined by [26]

$$\Gamma(x) \triangleq \int_0^{\infty} z^{x-1} e^{-z} dz. \tag{32}$$

In the full-CSI based scheduling scheme, the BS first selects a pair of users maximizing the sum-rate based on the CSI reported from N users at the previous frame time (i.e., at ($t-T$)) and then it informs them the scheduling information. Finally, the two scheduled users transmit the data signal to the BS at time slot t.

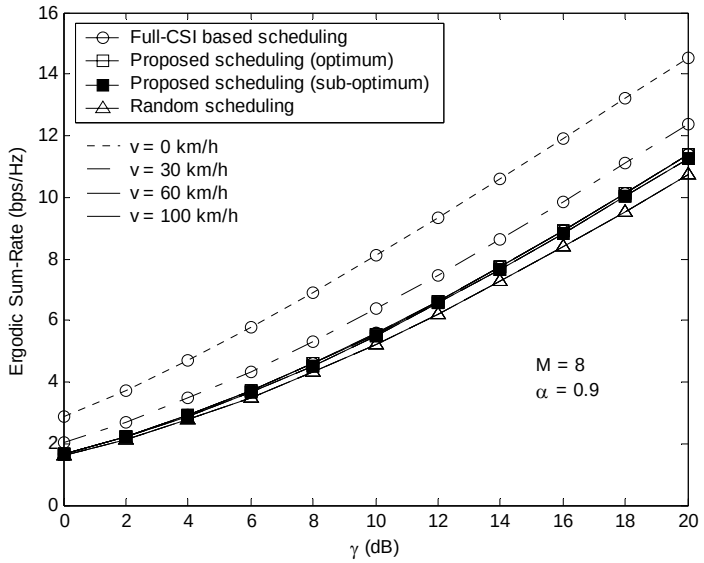

Fig. 4. Ergodic sum-rate according to γ when $N = 8$ and $\alpha = 0.9$.

Fig. 4 depicts the ergodic sum-rate in the CSM mode according to the average SNR γ with the use of the three multi-user scheduling schemes for $N = 8$ and $\alpha = 0.9$. It can be seen that the proposed sub-optimum scheme provides performance comparable to the proposed optimum scheme, while reducing the scheduling complexity by 13.75 at each time slot. It can also be seen that the full-CSI based scheduling scheme outperforms the proposed scheduling scheme, especially in low user mobility. However, the proposed scheme provides almost the same performance as the full-CSI based scheme when the user mobility is higher than 60 km/h. This is mainly due to the channel mismatch problem in the full-CSI based scheduling scheme [11], [13]. The proposed scheme outperforms the random scheduling scheme without loss of opportunity fairness. Moreover, the ergodic sum-rate gap between the two scheduling schemes increases as γ increases, which means that the proposed scheme is effective in high SNR regime.

Fig. 5 depicts the ergodic sum-rate of the three schemes according to the number of users when $\gamma = 10$ dB and $\alpha = 0.9$. It can be seen that as N increases, the full-CSI based scheme outperforms the other schemes in low mobility environments by exploiting so-called multi-user diversity gain [27], [28], but it suffers from the channel mismatch problem. It can also be seen that the proposed scheme works almost indifferently from user mobility and its ergodic sum-rate gain increases as the number of users increases. It can also be seen that both the proposed optimum and sub-optimum schemes provide similar performance. Thus, it is quite practical to employ the proposed sub-optimum scheme as N increases.

For example, when $N = 14$, the scheduling complexity of the two schemes is 19305 and 36 at each time slot, respectively. Notice that the random scheduling scheme cannot exploit any multi-user diversity gain.

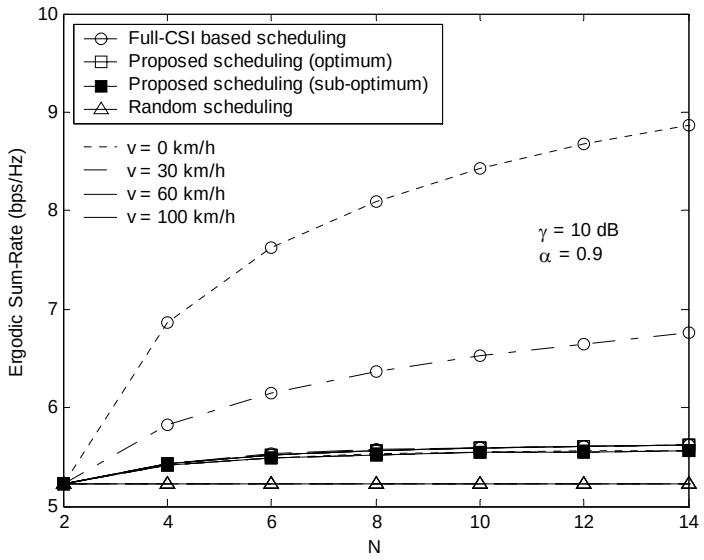

Fig. 5. Ergodic sum-rate according to N when $\gamma = 10$ dB and $\alpha = 0.9$.

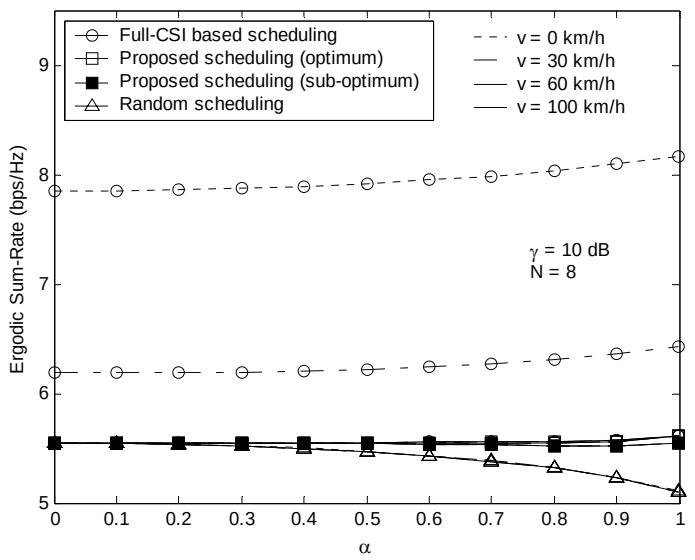

Fig. 6. Ergodic sum-rate according to α when $\gamma = 10$ dB and $N = 8$.

Fig. 6 depicts the ergodic sum-rate of the three scheduling schemes according to the variation of α when applied to the CSM mode with $N = 8$ at $\gamma = 10$ dB. It can again be seen the full-CSI based scheme works well in low user mobility and its performance somewhat increases as α increases as does the proposed scheme. It can also be seen that the performance of the proposed and random scheduling schemes is the same when $\alpha = 0$, but the performance of the random scheme decreases as α increases. This is mainly due to the fact that the random scheduling scheme has high probability of choosing pairs of users whose receive correlation coefficients have a phase difference close to 0, and the corresponding ergodic sum-rate rapidly decreases as α increases as shown in Fig. 2. As a consequence, the proposed scheduling scheme is quite effective compared to the random scheduling scheme in highly correlated channel environments.

VI. Conclusion

We have considered multi-user scheduling in the CSM mode specified in the IEEE 802.16e Mobile-WiMAX system. To minimize the feedback signaling overhead and improve the sum-rate performance in mobile environments, the proposed multi-user scheduling scheme maximizes the ergodic sum-rate by only utilizing the receive correlation information of users. The performance of the proposed scheduling scheme has been analyzed and optimized in terms of the ergodic sum-rate in the CSM mode by using an upper bound. The scheduling complexity of the proposed scheme can be reduced by employing a user shedding-based exhaustive search method without noticeable sum-rate performance degradation. The simulation results show that the proposed multi-user scheduling scheme is quite effective in the presence of receive channel correlation and user mobility. This work can be extended to transmit correlation-based user scheduling in the downlink of multi-user MIMO systems. This may require the use of a multi-beamforming technique based on the transmit correlation information of users.

Appendix I

Derivation of Upper-Bound (7)

It can be shown from $\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA})$ [1] that (6) can be rewritten as

$$C(t) = E\left\{\log_2 \det\left(\mathbf{I} + \frac{\mathbf{\Gamma H}^* \mathbf{H}}{M}\right)\right\}. \tag{33}$$

Applying Jensen's inequality, it can be shown that (33) is upper-bounded by

$$C(t) \le \log_2 E\left\{\det\left(\mathbf{I} + \frac{\mathbf{\Gamma H}^*\mathbf{H}}{M}\right)\right\}. \tag{34}$$

Or, (34) can explicitly be represented as

$$C(t) \le \log_2 E\left\{\det\begin{bmatrix} 1+\gamma_1 \mathbf{h}_1^*\mathbf{h}_1 / M & \cdots & \gamma_1 \mathbf{h}_1^*\mathbf{h}_M / M \\ \vdots & \ddots & \vdots \\ \gamma_M \mathbf{h}_M^*\mathbf{h}_1 / M & \cdots & 1+\gamma_M \mathbf{h}_M^*\mathbf{h}_M / M \end{bmatrix}\right\}. \tag{35}$$

Since $\mathbf{h}_n = \mathbf{R}_n^{1/2}\mathbf{h}_n^w$ from (2), (35) can be rewritten as

$$C(t) \le \log_2 E\left\{\det\begin{bmatrix} 1+\gamma_1 \mathbf{h}_1^{w*}\mathbf{R}_1\mathbf{h}_1^w / M & \cdots & \gamma_1 \mathbf{h}_1^{w*}\mathbf{R}_1^{*1/2}\mathbf{R}_M^{1/2}\mathbf{h}_M^w / M \\ \vdots & \ddots & \vdots \\ \gamma_M \mathbf{h}_M^{w*}\mathbf{R}_M^{*1/2}\mathbf{R}_1^{1/2}\mathbf{h}_1^w / M & \cdots & 1+\gamma_M \mathbf{h}_M^{w*}\mathbf{R}_M\mathbf{h}_M^w / M \end{bmatrix}\right\}. \tag{36}$$

Since $E\{\mathbf{h}_n^{w*}\mathbf{A}\mathbf{h}_n^w\} = \mathrm{tr}(\mathbf{A})$, $E\{\mathbf{H}_{n_1}^{w*}\mathbf{A}_1\mathbf{H}_{n_2}^w\mathbf{H}_{n_2}^{w*}\mathbf{A}_2\mathbf{H}_{n_1}^w\} = \mathrm{tr}(\mathbf{A}_1\mathbf{A}_2)$, $\cdots$, $E\{\mathbf{H}_{n_1}^{w*}\mathbf{A}_1\mathbf{H}_{n_2}^w\mathbf{H}_{n_2}^{w*}\mathbf{A}_2\mathbf{H}_{n_3}^w\mathbf{H}_{n_3}^{w*}\cdots\mathbf{H}_{n_M}^w\mathbf{H}_{n_M}^{w*}\mathbf{A}_M\mathbf{H}_{n_1}^w\} = \mathrm{tr}(\mathbf{A}_1\mathbf{A}_2\cdots\mathbf{A}_M)$

, we can have

$$C(t) \le \log_2\left[\begin{array}{l} (-1)^{2M}\left[(0!)^M\{(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)\}\right] \\ +(-1)^{2M-1}\left[\frac{(1!)^1(0!)^{M-2}}{M^2}\left\{\begin{array}{l}\gamma_1\gamma_2\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2)(1+\gamma_3\mathrm{tr}(\mathbf{R}_3)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-2}\mathrm{tr}(\mathbf{R}_{M-2})/M)\end{array}\right\}\right] \\ +(-1)^{2M-2}\left[\begin{array}{l}\frac{(2!)^1(0!)^{M-3}}{M^3}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3)(1+\gamma_4\mathrm{tr}(\mathbf{R}_4)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-2}\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-3}\mathrm{tr}(\mathbf{R}_{M-3})/M)\end{array}\right\} \\ +\frac{(1!)^2(0!)^{M-4}}{M^4}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\gamma_4\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2)\mathrm{tr}(\mathbf{R}_3\mathbf{R}_4)(1+\gamma_5\mathrm{tr}(\mathbf{R}_5)/M)\times\cdots\times(1+\gamma_M\mathrm{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-3}\gamma_{M-2}\gamma_{M-1}\gamma_M\mathrm{tr}(\mathbf{R}_{M-3}\mathbf{R}_M)\mathrm{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1})(1+\gamma_1\mathrm{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-4}\mathrm{tr}(\mathbf{R}_{M-4})/M)\end{array}\right\}\end{array}\right] \\ +\cdots \\ +(-1)^{M+1}\left[\frac{((M-1)!)^1}{M^M}\gamma_1\gamma_2\times\cdots\times\gamma_M\{\mathrm{tr}(\mathbf{R}_1\mathbf{R}_2\times\cdots\times\mathbf{R}_M)\}\right] \end{array}\right]. \tag{37}$$

$$
C(t) \le \log_2 \left[\begin{array}{l}
(-1)^{2M}\left[(0!)^M\{(1+\gamma_1 \operatorname{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_M \operatorname{tr}(\mathbf{R}_M)/M)\}\right] \\
+(-1)^{2M-1}\left[\dfrac{(1!)^1(0!)^{M-2}}{M^2}\left\{\begin{array}{l}\gamma_1\gamma_2\operatorname{tr}(\mathbf{R}_1\mathbf{R}_2)(1+\gamma_3\operatorname{tr}(\mathbf{R}_3)/M)\times\cdots\times(1+\gamma_M\operatorname{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-1}\gamma_M\operatorname{tr}(\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\operatorname{tr}(\mathbf{R}_1)/M)\times\cdots\times(1+\gamma_{M-2}\operatorname{tr}(\mathbf{R}_{M-2})/M)\end{array}\right\}\right] \\
+(-1)^{2M-2}\left[\begin{array}{l}\dfrac{(2!)^1(0!)^{M-3}}{M^3}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\operatorname{tr}(\mathbf{R}_1\mathbf{R}_2\mathbf{R}_3)(1+\gamma_4\operatorname{tr}(\mathbf{R}_4)/M)\times\cdots\times(1+\gamma_M\operatorname{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-2}\gamma_{M-1}\gamma_M\operatorname{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1}\mathbf{R}_M)(1+\gamma_1\operatorname{tr}(\mathbf{R}_1)/M)\times\cdots \\ \times(1+\gamma_{M-3}\operatorname{tr}(\mathbf{R}_{M-3})/M)\end{array}\right\} \\ +\dfrac{(1!)^2(0!)^{M-4}}{M^4}\left\{\begin{array}{l}\gamma_1\gamma_2\gamma_3\gamma_4\operatorname{tr}(\mathbf{R}_1\mathbf{R}_2)\operatorname{tr}(\mathbf{R}_3\mathbf{R}_4)(1+\gamma_5\operatorname{tr}(\mathbf{R}_5)/M)\times\cdots \\ \times(1+\gamma_M\operatorname{tr}(\mathbf{R}_M)/M)+\cdots \\ +\gamma_{M-3}\gamma_{M-2}\gamma_{M-1}\gamma_M\operatorname{tr}(\mathbf{R}_{M-3}\mathbf{R}_M)\operatorname{tr}(\mathbf{R}_{M-2}\mathbf{R}_{M-1})(1+\gamma_1\operatorname{tr}(\mathbf{R}_1)/M)\times\cdots \\ \times(1+\gamma_{M-4}\operatorname{tr}(\mathbf{R}_{M-4})/M)\end{array}\right\}\end{array}\right] \\
+\cdots \\
+(-1)^{M+1}\left[\dfrac{((M-1)!)^1}{M^M}\gamma_1\gamma_2\times\cdots\times\gamma_M\{\operatorname{tr}(\mathbf{R}_1\mathbf{R}_2\times\cdots\times\mathbf{R}_M)\}\right]
\end{array}\right].
$$

APPENDIX II

DERIVATION OF (25)

Letting $A = 1 + 2\gamma + \dfrac{\gamma^2}{2}$ and $B = \dfrac{\alpha^2\gamma^2}{2}$, (20) can be rewritten as

$$
\begin{aligned}
E\{\widehat{C}_r\} &= \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\log_2(A - B\cos(\theta_{n_1} - \theta_{n_2}))d\theta_{n_1}d\theta_{n_2} \\
&= \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\log_2 B + \frac{1}{\ln 2}\ln\left(\frac{A}{B} - \cos(\theta_{n_1} - \theta_{n_2})\right)d\theta_{n_1}d\theta_{n_2}.
\end{aligned} \tag{38}
$$

Since $\ln(a - x) = \ln a - \sum_{k=1}^{\infty}\dfrac{x^k}{ka^k}$, (38) can be rewritten as

$$
\begin{aligned}
E\{\widehat{C}_r\} &= \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\log_2 B + \frac{1}{\ln 2}\left(\ln\frac{A}{B} - \sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{B}{A}\right)^k\cos^k(\theta_{n_1} - \theta_{n_2})\right)d\theta_{n_1}d\theta_{n_2} \\
&= \frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\log_2 A - \frac{1}{\ln 2}\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{B}{A}\right)^k\cos^k(\theta_{n_1} - \theta_{n_2})d\theta_{n_1}d\theta_{n_2} \\
&= \log_2 A - \frac{1}{4\pi^2}\frac{1}{\ln 2}\sum_{k=1}^{\infty}\int_0^{2\pi}\int_0^{2\pi}\frac{1}{k}\left(\frac{B}{A}\right)^k\cos^k(\theta_{n_1} - \theta_{n_2})d\theta_{n_1}d\theta_{n_2} \\
&= \log_2 A - \frac{1}{4\pi^2}\frac{1}{\ln 2}\sum_{k=1}^{\infty}\frac{2\pi^{3/2}\left(\frac{B}{A}\right)^k\Gamma\left(\frac{k+1}{2}\right)}{k\Gamma\left(\frac{k+2}{2}\right)}.
\end{aligned} \tag{39}
$$

From [29], we have $\Gamma\left(\dfrac{k}{2}\right) = \dfrac{\sqrt{\pi}(k-2)!!}{2^{(k-1)/2}}$. Therefore, (39) can be rewritten as

$$
\begin{aligned}
E\{\widehat{C}_r\} &= \log_2 A - \frac{1}{\sqrt{2\pi}\ln 2}\sum_{k=1}^{\infty}\left(\frac{B}{A}\right)^k \frac{(k-1)!!}{k\cdot k!!} \\
&= \log_2\left(1+2\gamma+\frac{\gamma^2}{2}\right) - \frac{1}{\sqrt{2\pi}\ln 2}\sum_{k=1}^{\infty}\left(\frac{\alpha^2\gamma^2}{2+4\gamma+\gamma^2}\right)^k \frac{(k-1)!!}{k\cdot k!!}.
\end{aligned}
\tag{40}
$$

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