Performance Analysis of Forward Link DS-CDMA Systems using Random and Orthogonal Spreading Sequences

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Abstract

The characteristics of spreading sequences significantly affect the performance of the forward link DS-CDMA system in fading channel. The bit error rate (BER) is a function of the signal-to-interference ratio (SIR) which has different characteristics depending upon the spreading sequence. In this paper, the SIR is analyzed when orthogonal codes and pseudo random codes are used as the spreading sequence. The BERs are obtained by Monte Carlo analysis when the convolutional code is used or not. The use of orthogonal codes can cancel out the interference signals having the same path delay, but the use of random codes can't, resulting in significant performance degradation. As a result, the BER performance of the orthogonal spreading system is much better than that of the random spreading system. Even in the case of many multipaths, the number of users achieving a fixed BER value in the orthogonal spreading system is twice larger than that in random spreading system. The analytical results are verified by computer simulation.

I. Introduction

In multipath fading channel, the performance of the DS-CDMA system is affected by the characteristics of the spreading sequences. The bit error rate (BER) of the CDMA system is a function of the signal-to-interference ratio (SIR) of the received signal. The use of orthogonal sequences and random sequences as the spreading sequence may result in significantly different SIR performance. However, few results have been reported on this issue. Recently, the BER performance was evaluated by Monte Carlo simulation when no channel coding is used [1]. It has been reported that the BER performance of the forward link DS-CDMA system using orthogonal spreading sequences is better than that using random spreading sequences. However, since the interference from other users is under-estimated in [1], the BER performance difference between the two spreading systems is not large. Besides, as the number of multipaths increases, the performance difference becomes negligible.

In this paper, we analyze the performance of the forward link DS-CDMA system when the two types of spreading sequences are employed with and without the use of convolutional channel coding. It is shown that the use of random sequences results in poorer performance than the use of orthogonal sequences due to significant increase of the interference. The capacity of the forward link DS-CDMA system with the use of orthogonal sequences is at least twice larger than that with the use of random sequences.

In the following, the transceiver and channel are desciribed in section 2. The BER performances of no channel coded and convolutionally coded DS-CDMA system are analyzed in section 3 and section 4. The analytical results are verified by computer simulation. Finally, conclusions are drawn in section 5.

II. System modeling

For ease of analysis, we consider a single-cell environment where users are connected with one base station. An equivalent baseband transmitter model of the base station is depicted in Fig. 1. For ease of description, we call a DS-CDMA system employing orthogonal sequence the orthogonal spreading method (OSM) and that using random (PN) sequence the random spreading method (RSM). The transmitter employs a convolutional encoder with constraint length *K* and an interleaver with depth *D* and width *W*. In the case of no channel coding, the convolutional encoder and the interleaver are excluded.

Assuming that the signals from each path are independent and their relative delays are spaced by a multiple of chip times, the impulse response of a time varying channel at time *t* can be represented as

$$h(t,\tau) = \sum_{l=0}^{L-1} \alpha_l (t - d_l T_c) e^{j\phi_l (t - d_l T_c)} \delta(\tau - d_l T_c) \qquad (1)$$

where $\alpha_l(t)$ and $\phi_l(t)$ are the channel gain and phase of the *l*-th path at time *t*, respectively, *L* is the number of channel propagation paths, $\delta(t)$ is *Dirac delta* function and d_l is a nonnegative integer.

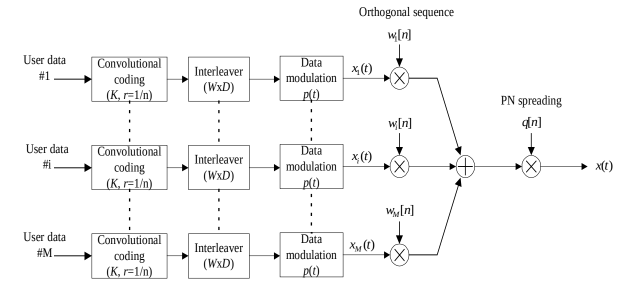
If we assume that the channel response is not varying within one symbol time interval and the channel is perfectly estimated, the rake receiver output of the *i*-th user and *k*-th symbol using the maximal ratio combining (MRC) method, $Y_i[k]$, can be represented as

$$\begin{aligned} Y_i[k] &= \sum_{l=0}^{L-1} s_{i,l}[k] + i_{i,l}[k] + m_{i,l}[k] + u_{i,l}[k] + n_{i,l}[k] \\ &= S_i[k] + I_i[k] + M_i[k] + U_i[k] + N_i[k] \end{aligned} \qquad (2)$$

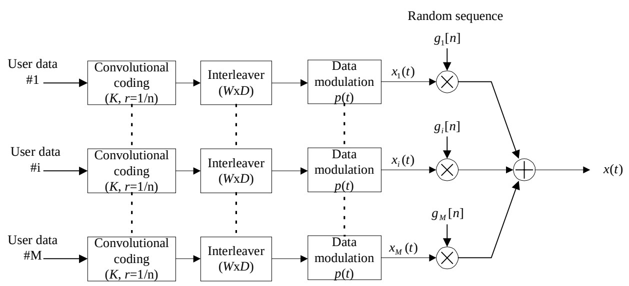
where $s_{i,l}[k]$ is the *l*-th path desired signal of the user *i*, $i_{i,l}[k]$ is the inter-path interference of the user *i*, $m_{i,l}[k]$ is the other users' interference of the *l*-th path, $u_{i,l}[k]$ is the other users' interference of all the paths except the *l*-th path and $n_{i,l}[k]$ is the interference due to background noise. Note that the performance difference between the two spreading systems is created by the term $m_{i,l}[k]$

$$m_{i,l}[k] = \begin{cases} 0\,; & \text{OSM} \\ \alpha_l^2[k]\sqrt{E_c T_c} \displaystyle\sum_{q=1,\, q\neq i}^{M} x_{q,k} \sum_{z=0}^{N-1} \mathrm{Re}\left\{ e^{j(\psi_q[kN+z]-\psi_i[kN+z])} \right\}; & \text{RSM} \end{cases} \qquad (3)$$

where *M* is the number of users, E_c is the transmitted energy per chip, $x_{q,k}$ is the *q*-th user's message signal having a value of 1 or –1 at the *k*-th symbol time. T_c is the chip duration, *N* is the processing gain and $\psi_q[n]$ is the phase of the transmitted signal of the *q*-th user at chip time



(a) Orthogonal spreading (OSM)



(b) Random spreading (RSM)

Fig. 1 Baseband transmitter model of DS-CDMA base station

n. In the RSM, all the terms except $\alpha_{i,l}^{\,2}[k]$ in $m_{i,l}[k]$ have the same sign irrespective of *l*. Since the signs of all the rake receiver outputs due to this interference are equal, interference effect will be added in the same direction, significantly increasing the interference power on the contrary in [1].

If there exists a large number of users, we can approximate each interference term as a Gaussian random variable by the central limit theorem. All the statistical characteristics of the interference can be described by the mean and variance of each term. The means of each terms are zero and variances can be obtained as

$$V\{I_i[k]\} = \frac{1}{2} E_c T_c N \sum_{l=0}^{L-1} \sum_{s=0, s\neq l}^{L-1} \alpha_l^2[k] \alpha_s^2[k]$$

$$V\{M_i[k]\} = \begin{cases} 0\,; & \text{OSM} \\ \dfrac{1}{2} E_c T_c N (M-1) \left\{ \displaystyle\sum_{l=0}^{L-1} \alpha_l^2[k] \right\}^2 ; & \text{RSM} \end{cases}$$

$$V\{U_i[k]\} = \frac{1}{2} E_c T_c N (M-1) \sum_{l=0}^{L-1} \sum_{s=0, s\neq l}^{L-1} \alpha_l^{\,2}[k] \alpha_s^{\,2}[k] \qquad (4)$$

$$V\{N_i[k]\} = \frac{N_o}{2} T_c N \sum_{l=0}^{L-1} \alpha_l^{\,2}[k]$$

where N_o is the one side power spectrum of zero mean AWGN.

III. Performance of uncoded systems

If there are many users in the channel, the interference due to background noise can be ignored without the loss of generality. If the channel gain $\boldsymbol{a} = \{\alpha_0, \alpha_1, \ldots\ldots, \alpha_{L-1}\}$ is given, the conditional SIR $\lambda(\boldsymbol{a})$ can be represented by

$$\lambda(\boldsymbol{a}) = \frac{2N\left(\displaystyle\sum_{l=0}^{L-1} \alpha_l^{\,2}\right)^2}{\chi(M-1)\left(\displaystyle\sum_{l=0}^{L-1} \alpha_l^{\,2}\right)^2 + M \displaystyle\sum_{l=0}^{L-1} \sum_{p=0, p\neq l}^{L-1} \alpha_l^{\,2} \alpha_p^{\,2}} \qquad (5)$$

where $\chi = 0$ for the OSM and $\chi = 1$ for the RSM. Therefore, the bit error probability of coherent BPSK can be given as

$$P_b = \int_0^\infty p_b(\boldsymbol{a}) f(\boldsymbol{a}) d\boldsymbol{a}, \quad p_b(\boldsymbol{a}) = Q\left(\sqrt{\lambda(\boldsymbol{a})}\right) \qquad (6)$$

where $f(\boldsymbol{a})$ is the probability density function of $\boldsymbol{a}$

determined by the fading characteristics of the channel and $p_b(\boldsymbol{a})$ is the conditional bit error probability given $\boldsymbol{a}$. Although P_b can not be expressed by a closed form, it can be calculated by Monte Carlo method.

In order for the two spreading systems to have the same BER, the denominator in (5) must be equal. Using the fact that

$$\Gamma = \sum_{l=0}^{L-1} \alpha_l^{\,4} - \frac{1}{L-1}\sum_{l=0}^{L-1}\sum_{p=0,p\neq l}^{L-1} \alpha_l^{\,2}\alpha_p^{\,2} = \frac{1}{L-1}\sum_{l=0}^{L-1}\sum_{p=l+1}^{L-1}(\alpha_l^{\,2}-\alpha_p^{\,2})^2 \ \geq 0 \qquad (7)$$

it can be seen that

$$M_O \geq (2+\frac{1}{L-1})M_R - \frac{L}{L-1} \qquad (8)$$

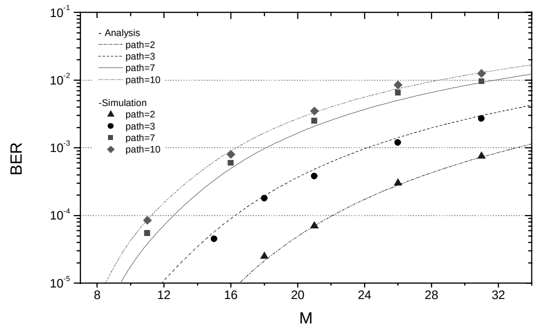
where M_R is the number of users with the RSM and M_O is the number of users with the OSM. Therefore, the number of users satisfying a specific BER with the OSM is always twice more than that with the RSM. Besides, as the number of multipaths decreases, Γ in (7) increases and the capacity difference between the two systems becomes further larger.

To verify the performance analysis, computer simulation is carried out under the same simulation conditions as in the IS-95 system when the channel is Rayleigh faded and each path has the same gain in average [2]. We assume that the receiver is operating with perfect carrier and timing recovery, and ideal channel estimation. The BER performance of the OSM and the RSM is depicted in Fig. 2 as a function of the number users and the channel paths, where the analytical result (6) is also shown as "Analysis".

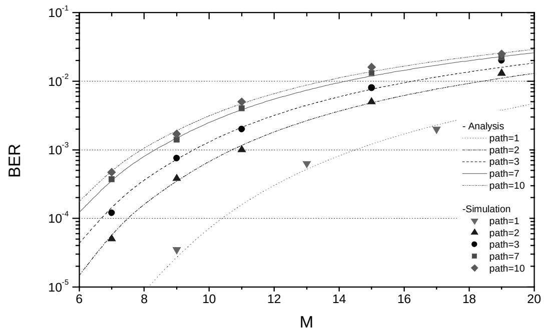
It can be seen that each system requires significantly different E_b/I_0 to have the same BER. This performance difference is mainly due to the interference from other users. In the case of the forward link, as the desired signal suffers from the same fading as other user signals, the SIR becomes a constant independent of fading if we ignore the background noise. Therefore, the diversity effect due to increase of the channel paths can not be obtained. Instead, as the number of the channel paths increases, the inter-path interference increases, resulting in poorer BER performa-nce. At a BER of 10^{-3}, it can be seen that the OSM can support the users about two or three times more than the RSM depending on the number of paths.

IV. Performance of convolutionally coded systems

When a convolutional code is employed with perfect



(a) Orthogonal spreading (OSM)



(b) Random spreading (RSM)

Fig. 2 The BER performance of uncoded DS-CDMA

interleaving, the BER corresponding to the Hamming distance d can be represented by

$$P_d = \int_0^\infty p_d(\boldsymbol{\alpha}) f_d(\boldsymbol{\alpha})d\boldsymbol{\alpha} \qquad (9)$$

where

$$p_d(\boldsymbol{\alpha}) = Q(\sqrt{\lambda_d(\boldsymbol{\alpha})}) = Q\left(\sqrt{\frac{2N\left(\sum_{k=1}^{d}\sum_{l=0}^{L-1}\alpha_l^{\,2}[k]\right)^2}{\sum_{k=1}^{d}\left\{\chi(M-1)\left(\sum_{l=0}^{L-1}\alpha_l^{\,2}[k]\right)^2 + M\left(\sum_{l=0}^{L-1}\sum_{p=0,p\neq l}^{L-1}\alpha_l^{\,2}[k]\alpha_p^{\,2}[k]\right)\right\}}}\right) \qquad (10)$$

Although P_d can not be expressed by a closed form, it can be obtained by Monte Carlo simulation as in the uncoded case. The BER when K=9 and r=1/2, can be expressed by [3]

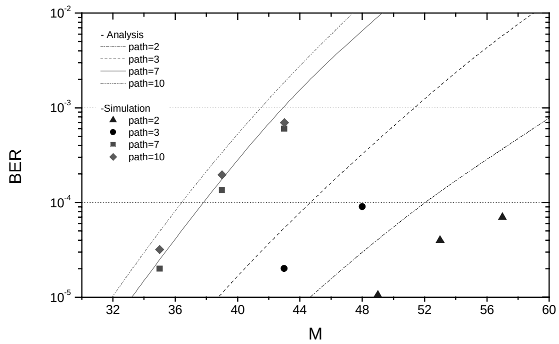
$$P_b \le \sum_{d=12}^{\infty} c_d P_d = 33P_{12} + 281P_{14} + 2179P_{16} + \ldots\ldots \quad (11)$$

Note that (8) is still valid with the use of convolutional codes.

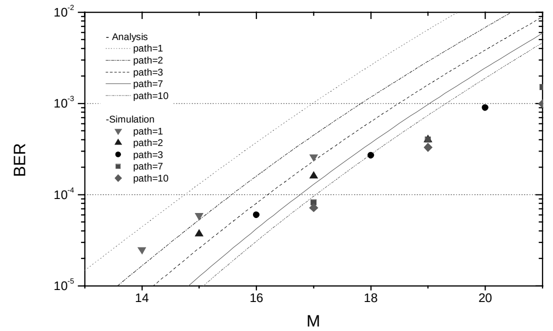
When the transmitted powers of all the users are equal, the number of acceptable users will be larger than the maximum number of Walsh codes at a BER of 10^{-3}. To be able to compare capacity of the OSM and the RSM, we keep the power of other users twice than that of the desired user. For sufficient interleaving, we use W=500 and D=300 assuming that the maximum Doppler frequency is 300Hz. The depth of the Viterbi decoding is set to 100. Other simulation conditions are the same as in the uncoded case. It can be seen from Fig. 3 that the OSM requires E_b/I_0 much smaller than the RSM does. It also can be seen that the analysis agrees well with the simulation results as the BER decreases or the number of channel paths increases. This is due to that the bit errors with minimum distance dominate the total bit errors and the central limit theorem works well. If the background noise effect is ignored, the desired and interference signals suffer from identical fading regardless of propagation characteristics such as the number of channel paths. Thus, the SIR becomes constant during each symbol time interval. However, the Viterbi decoding procedure is executed for several symbols interval, the total SIR λ_d varies as (10) and the diversity effect occurs. Moreover, the OSM and the RSM result in different BER performance behavior as the number of channel paths varies. Since $\chi = 0$ in the OSM, the orthogonality is heavily corrupted and the interference power increases as the number of channel propagation paths increases. Since $\chi = 1$ in the RSM, the interference increases due to orthogonality corruption and the diversity effect occurs simultaneously as the number of channel paths increases. Since the BER is more affected by the diversity effect than the interference, it decreases as the number of the channel propagation paths increases. It can be seen that the number of users in the OSM is 2.1(10 paths) ~ 3.3(2 paths) times larger than that in the RSM at a BER of 10^{-4}. Therefore, it is highly desirable to use orthogonal codes for the forward link DS-CDMA system.

V. Conclusions

In a multipath fading channel, the performance of the forward link DS-CDMA system highly depends on spreading sequences assigned to each user. In this paper, we have evaluated the BER performance of a DS-CDMA system with and without the use of convolutional coding when random sequences and orthogonal sequences are used as the spreading sequences. The both CDMA systems experience the same interference power due to signals having different propagation delay. However, other users' interference having the same path delay differently affects the receiver performance. The use of orthogonal spreading sequence can effectively reduce the interference from other users' signal. On the other hand, the use of random spreading sequence rather increases the interference power since the output signals of the rake receiver due to that interference have equal sign regardless of path delay. The use of orthogonal spreading sequences can provide higher system capacity at least twice than the use of random spreading sequence.



(a) Orthogonal spreading (OSM)



(b) Random spreading (RSM)

Fig. 3 The BER performance of convolutionally coded DS-CDMA

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