This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2005 proceedings.

Computationally Efficient Cancellation of Partially-overlapped Crosstalk in Digital Subscriber Lines

Minho Cheong, Yong-Hwan Lee* and Hyeong Jun Park
ETRI, Seoul National University*
161 Gajeong-dong, Yuseong-gu, Daejeon 305-350, Korea
Tel: 82-42-860-5635, E-mail: minho@etri.re.kr

Abstract- **We consider the crosstalk cancellation problem when the transmit and receive signal spectrum are partially overlapped with each other in the frequency domain. In this case, the use of conventional crosstalk cancellers may not be practical due to the computational complexity. When the overlapped bandwidth of the crosstalk signal is less than the bandwidth of the transmit signal, the crosstalk cancellation can be realized efficiently by processing in the baseband. The use of interpolation and decimation processing in the baseband makes it possible to realize the crosstalk canceller with low computational complexity. The proposed scheme can provide crosstalk cancellation performance comparable to the conventional one, while requiring the computational complexity less than one half the conventional one.**

I. INTRODUCTION

Explosive demand for multimedia services requires higher data transmission speed, which can be achievable by increasing the spectral efficiency or signal bandwidth [1]. Although the spectral efficiency has been dramatically improved in the past three decades, it is unlikely that the demand for high speed transmission can be solved by simply increasing the spectral efficiency. As a result, it is also required to increase the symbol rate or the signal bandwidth. However, the increase of the symbol rate may not always be feasible in practice due to the limited available bandwidth and implementation complexity. For example, in DSL environment, different frequency bands are used for the transmitter (Tx) and receiver (Rx) for duplex operation. The increase of the symbol rate can be achieved without the increase of overall bandwidth by making the Tx and Rx spectrum overlapped with each other in the frequency domain [1]. In this case, it is indispensable to suppress the crosstalk signal due to the use of imperfect wire shielding.

There have been extensive studies on the cancellation of crosstalk signals [3-4]. It is known that the inband data-driven crosstalk canceller scheme (IDXC) provides reliable performance, while being robust to the timing and frequency offset [3]. Conventional IDXC processes the crosstalk signal at a Nyquist rate, which can be implemented using a polyphase structure [4]. When the overlapped bandwidth of the crosstalk signal is smaller than the bandwidth of the Tx signal, the use of a conventional Nyquist rate IDXC uses the sampling frequency much higher than the bandwidth of the overlapped crosstalk signal, requiring large computational complexity.

In this paper, we propose a new scheme for cancellation of partially overlapped crosstalk signal with reduced implementation complexity. When the bandwidth of the overlapped crosstalk is small compared to that of the Tx signal, we can cancel the overlapped crosstalk signal in the baseband. With the use of appropriate interpolation and decimation filters, the proposed scheme can process the operation at a lower rate, significantly reducing the computational complexity.

Assume that the spectra of the Tx and Rx signal are partially overlapped for full-duplex operation as illustrated in Figure 1, where f_{C_L} and f_{C_H} denote the carrier frequency of the low-band and high-band signal in the DSL, respectively, and $f_e = f_{C_L} + \frac{f_{b_L}}{f_{b_L} + f_{b_H}}\left(f_{C_H} - f_{C_L}\right)$. Here f_{b_L} and f_{b_H} are the symbol rate of the low-band and high-band signal, respectively. The bandwidth overlapping with the use of the imperfect wire shielding results in the crosstalk signal in the received signal. The shadowed region in Figure 1 manifests the spectrum of the crosstalk signal partially overlapped with the received signal in the frequency domain.

We assume that the Tx signal is pulse-shaped using a square-root raised cosine filter with roll-off factor α. Then, the spectrum of the crosstalk signal is located in the frequency

$$f_{C_H} - \frac{f_{b_H}}{2}(1+\alpha) \le f \le f_{C_L} + \frac{f_{b_L}}{2}(1+\alpha). \quad (1)$$

As a measure of the amount of the spectral overlapping, we define by the carrier spacing ratio

$$\zeta \equiv \frac{2\left(f_{C_H} - f_{C_L}\right)}{\left(f_{b_L} + f_{b_H}\right)(1+\alpha)}. \quad (2)$$

This ratio ζ represents the frequency spacing between the center frequencies of the Tx and Rx signal normalized by the bandwidth of the Tx signal. If $\zeta < 1.0$, it implies that the two signals are overlapped in the frequency domain.

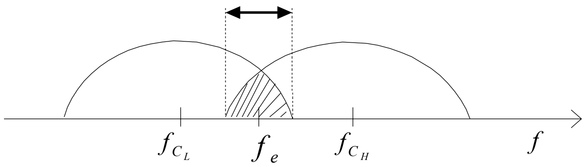


Figure 1. Spectral allocation when crosstalk is partially-overlapped.

The Nyquist rate IDXC has widely been used for crosstalk cancellation in wireline communications, where the interpolation ratio L of a polyphase structure should be determined such that [5]

$$L \geq \begin{cases} \left\lceil \dfrac{2f_{C_H} + f_{b_H}(1+\alpha)}{f_{b_L}} \right\rceil & ; \text{ in the premises} \\ \left\lceil \dfrac{2f_{C_H} + f_{b_H}(1+\alpha)}{f_{b_H}} \right\rceil & ; \text{ in the central office.} \end{cases} \quad (3)$$

Here $\lceil x \rceil$ denotes the largest integer less than or equal to x. When the crosstalk path has a duration of N symbols and a conventional least mean square (LMS) adaptation algorithm is employed in the premises [2], each L-polyphase sub-canceller requires $4N$ multiplications and accumulations (MACs) for filtering and adaptation of N complex-valued coefficients for each symbol duration. Thus, it requires a computational complexity of $8LNf_{b_L}$ MACs per second. As an example, consider the implementation of a symmetric DSL (SDSL) with a symbol rate of 256 Kbauds and ζ =0.83. For an crosstalk signal with a span of 80 microseconds, a conventional Nyquist rate IDXC requires a computational complexity of more than 640×10^6 MACs per second, which may not be practical for implementation with the use of conventional digital signal processors [5].

II. Computationally Efficient Crosstalk Canceller

When the Tx and Rx signals are partially overlapped in the frequency domain, the crosstalk signal is a passband signal whose bandwidth is relatively small compared to that of the Rx signal. Thus, the crosstalk signal can be processed at a lower rate by properly frequency down-shifting to the baseband. Figure 2 depicts the structure of the proposed crosstalk canceller (XC) comprising four blocks; the pre-processor, adaptive crosstalk canceller, post-processor and error feedback processor.

The input signal is first frequency down-shifted by an amount of f_o using a pre-processor, where

$$f_o = \begin{cases} f_{C_L} - f_e; & \text{in the premises} \\ f_{C_H} - f_e; & \text{in the central office.} \end{cases} \quad (4)$$

The frequency-shifted baseband-like crosstalk signal is input to the crosstalk canceller. Thus, the crosstalk canceller can process the input signal at a rate much lower than the conventional passband crosstalk canceller [7]. The crosstalk canceller can be implemented in the form of a polyphase filter scheme. The modified interpolation ratio K in the proposed scheme can be determined by

$$K = \lceil (1-\zeta)L \rceil \quad (5)$$

Note that the computational speed can be reduced by a factor of $\dfrac{L}{K}(\equiv R)$.

The post-processor converts the crosstalk canceller output into a passband signal using interpolation by a factor of R. The residual crosstalk signal is feedback to the crosstalk canceller as the error signal. Since the error signal is a passband signal, it needs to be frequency-shifted by an amount of $(f_o - f_{C_L})$ before being input to the crosstalk canceller. Note that the error signal needs to be decimated by a factor of R.

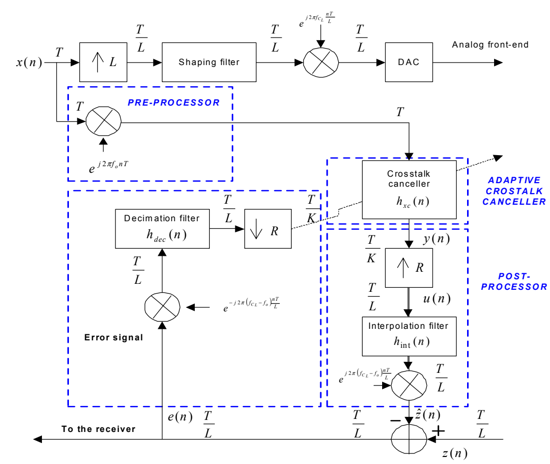


Figure 2. Block diagram of the proposed scheme.

III. Performance Analysis

Let $z(n)$ and $\hat{z}(n)$ be the crosstalk and the output of the crosstalk canceller, respectively, and $h_{xp}(n)$ and $h_{xc}(n)$ be the baseband equivalent impulse response of the crosstalk path and the crosstalk canceller, respectively. Assuming that the interpolation filter $h_{\text{int}}(n)$ is properly designed, $z(n)$ and $\hat{z}(n)$ in the premises modem can be represented as

$$\begin{aligned} z(n) &= \left[x(n) * h_{xp}(n)\right] e^{\frac{j2\pi f_{C_L} nT}{L}} + \nu(n) \\ \hat{z}(n) &= \left[u(n) * h_{\text{int}}(n)\right] e^{\frac{j2\pi (f_{C_L} - f_o) nT}{L}} \end{aligned} \quad (6)$$

where * denotes the convolution process, $\nu(n)$ is the additive noise term and $u(n)$ is an intermediate signal after the interpolation process,

$$u(n)=\begin{cases} y\left(\frac{n}{R}\right) & ;\ \text{when } n \text{ is an integer muptiple of } R \\ 0 & ;\ \text{otherwise} \end{cases} \quad (7)$$

where $y(n)$ is represented as

$$y(lK+m)=h_{xc}^{(m)}(l)*\left[x(l)e^{\frac{j2\pi f_o lT}{R}}\right]. \quad (8)$$

Here, $h_{xc}^{(m)}(l)$ denotes the impulse response of the m-th polyphase part of $h_{xc}(n)$, $n=lK+m$, $0\le m\le K\text{-}1$.

Assuming the use of equi-ripple interpolation and decimation filters, it can be shown that the power of the residual crosstalk is approximated as

$$\begin{aligned} E\left\{\,|e(n)|^2\right\} &= E\left\{\,|z(n)-\hat{z}(n)|^2\right\} \\ &\approx \sigma_v^2+\sigma_x^2\left[\delta_p^2+(R-1)\delta_s^2\right] \end{aligned} \quad (9)$$

where σ_x^2 and σ_v^2 are the power of the Tx signal $x(n)$ and background noise $v(n)$, respectively, and δ_p and δ_s are the maximum ripple magnitude of an equi-ripple filter in the passband and stopband, respectively.

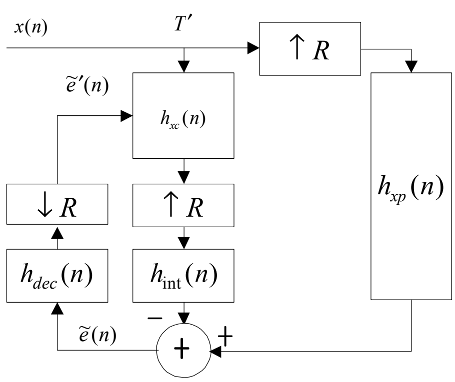


Figure 3. Baseband-equivalent block diagram of the proposed scheme.

The proposed crosstalk canceller can be described by a baseband equivalent one as depicted in Figure 3, where $T'=\frac{T}{K}$. In Figure 3, the baseband equivalent residual crosstalk is obtained as

$$e(n)\ =e(n)e^{j\frac{2\pi\left(f_o-f_{C_L}\right)nT}{L}}. \quad (10)$$

It can be represented in the frequency domain as

$$\begin{aligned} E(j\omega) &= \Im\{e(n)\} \\ &= X(j\omega R)\left[H_{xp}(j\omega)-H_{xc}(j\omega R)H_{\text{int}}(j\omega)\right] \end{aligned} \quad (11)$$

where $\Im\{x\}$ denotes the Fourier transform of x and $X(j\omega)$, $H_{xp}(j\omega)$, $H_{xc}(j\omega)$ and $H_{\text{int}}(j\omega)$ denote the Fourier transform of the input signal $x(n)$, baseband equivalent crosstalk path $h_{xp}(n)$, crosstalk canceller $h_{xc}(n)$ and interpolation filter $h_{\text{int}}(n)$, respectively. The error signal $e'(n)$ for updating the crosstalk canceller can be obtained by decimating $e(n)$ by a factor of R. The error signal can be represented in the frequency domain,

$$\begin{aligned} E'(j\omega) &= \frac{1}{R}\sum_{k=0}^{R-1}E\left(j\frac{\omega-2\pi k}{R}\right)H_{dec}\left(j\frac{\omega-2\pi k}{R}\right) \\ &= \frac{1}{R}\sum_{k=0}^{R-1}X\left(j\frac{\omega-2\pi k}{R}\cdot R\right)H_{dec}\left(j\frac{\omega-2\pi k}{R}\right)\left[H_{xp}\left(j\frac{\omega-2\pi k}{R}\right)-H_{xc}\left(j\frac{\omega-2\pi k}{R}\cdot R\right)H_{\text{int}}\left(j\frac{\omega-2\pi k}{R}\right)\right] \\ &= X(j\omega)\left[\frac{1}{R}\sum_{k=0}^{R-1}H_{dec}\left(j\frac{\omega-2\pi k}{R}\right)H_{xp}\left(j\frac{\omega-2\pi k}{R}\right)-\frac{H_{xc}(j\omega)}{R}\sum_{k=0}^{R-1}H_{\text{int}}\left(j\frac{\omega-2\pi k}{R}\right)H_{dec}\left(j\frac{\omega-2\pi k}{R}\right)\right] \end{aligned} \quad (12)$$

where $H_{dec}(j\omega)$ is the Fourier transform of the decimation filter $h_{dec}(n)$.

Because the error signal $e'(n)$ can only be extracted from $e(n)$ using the decimation filter $h_{dec}(n)$, the use of a conventional LMS algorithm may not provide reliable convergence performance. This problem can be alleviated using a so-called filtered-X algorithm [6-7], which has widely been applied to artificial cardiac operations as auxiliary pre- and post-processing filters. Let us define by an auxiliary filter $h_{aux}(n)$ and modified crosstalk path $h'_{xp}(n)$ in the frequency domain

$$\begin{aligned} H_{aux}(j\omega) &\equiv \frac{1}{R}\sum_{k=0}^{R-1}H_{\text{int}}\left(j\frac{\omega-2\pi k}{R}\right)H_{dec}\left(j\frac{\omega-2\pi k}{R}\right) \\ H'_{xp}(j\omega) &\equiv \frac{1}{R}\sum_{k=0}^{R-1}H_{dec}\left(j\frac{\omega-2\pi k}{R}\right)H_{xp}\left(j\frac{\omega-2\pi k}{R}\right). \end{aligned} \quad (13)$$

The modified input signal $x'(n)$ for adaptation of the filtered-X LMS algorithm can be obtained by

$$x'(n)=\sum_{i=0}^{N_{aux}-1}h_{aux}(i)x(n-i) \quad (14)$$

where N_{aux} denotes the tap size of $h_{aux}(n)$. Assuming that the inverse of $h_{aux}(n)$, $h_{aux}^{-1}(n)$, exists, a block diagram of the proposed scheme can be depicted as in Figure 4, where $h''_{xp}(n)=h'_{xp}(n)*h_{aux}^{-1}(n)$.

The coefficient of the proposed crosstalk canceller can be updated using the filtered-X LMS method as [7]

$$\mathbf{h}_{xc}(n+1)=\mathbf{h}_{xc}(n)+\mu\mathbf{x}'(n)e'(n) \quad (15)$$

where

$$\mathbf{x}'(n)=\left[x'(n)\ \ x'(n-1)\ \ x'(n-2)\ \ \quad x'(n-N+1)\right]^T.$$

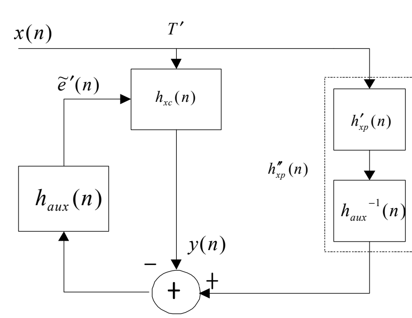


Figure 4. A modified block diagram of the proposed scheme.

Defining the tap error vector at time n by

$$\boldsymbol{\varepsilon}(n) = \mathbf{h}_{xc}(n) - \mathbf{h}''_{xp}(n), \tag{16}$$

it can be shown that

$$\begin{aligned}\boldsymbol{\varepsilon}(n+1) = \boldsymbol{\varepsilon}(n) - \mu \sum_{i=0}^{N_{aux}-1} \sum_{j=0}^{N_{aux}-1} h_{aux}(i) h_{aux}(j) \mathbf{x}(n-j) \mathbf{x}^T(n-i) \boldsymbol{\varepsilon}(n-i) \\ + \mu \sum_{i=0}^{N_{aux}-1} \sum_{j=0}^{N_{aux}-1} h_{aux}(i) h_{aux}(j) \mathbf{x}(n-j) v(n-i)\end{aligned} \tag{17}$$

Assuming that the input signal $x(n)$ is an uncorrelated random process, it can be shown that

$$E\{\boldsymbol{\varepsilon}(n+1)\} = E\{\boldsymbol{\varepsilon}(n)\} - \mu\sigma_x^2 \mathbf{I}_N \sum_{i=0}^{N_{aux}-1} h_{aux}^2(i) E\{\boldsymbol{\varepsilon}(n-i)\} \tag{18}$$

where $\mathbf{I}_N$ denotes an $(N \times N)$ -dimensional identity matrix. For each element of $E\{\boldsymbol{\varepsilon}(n)\}$, the characteristic equation is given by

$$z - 1 + \mu\sigma_x^2 \sum_{i=0}^{N_{aux}-1} h_{aux}^2(i) z^{-i} = 0 \tag{19}$$

Thus, the adaptation parameter μ should be determined such that the roots of the characteristic equation of (19) are inside the unit circle to guarantee the stability [8-9].

Defining the covariance matrix by

$$\mathbf{R}_{\boldsymbol{\varepsilon}}^{(i,j)}(n) = E\left\{\boldsymbol{\varepsilon}(n-i)\boldsymbol{\varepsilon}^T(n-j)\right\}, \tag{20}$$

it can be shown that

$$\begin{aligned}\mathbf{R}_{\boldsymbol{\varepsilon}}^{(0,0)}(n+1) = \mathbf{R}_{\boldsymbol{\varepsilon}}^{(0,0)}(n) - \mu\sigma_x^2 \sum_{i=0}^{N_{aux}-1} h_{aux}^2(i) \mathbf{R}_{\boldsymbol{\varepsilon}}^{(i,0)}(n) - \mu\sigma_x^2 \sum_{k=0}^{N_{aux}-1} h_{aux}^2(k) \mathbf{R}_{\boldsymbol{\varepsilon}}^{(0,k)}(n) \\ - \mu^2 \sum_{i=0}^{N_{aux}-1} \sum_{j=0}^{N_{aux}-1} \sum_{k=0}^{N_{aux}-1} \sum_{l=0}^{N_{aux}-1} h_{aux}(i) h_{aux}(j) h_{aux}(k) h_{aux}(l) \\ \cdot E\left\{\mathbf{x}(n-j)\mathbf{x}^T(n-i)\mathbf{R}_{\boldsymbol{\varepsilon}}^{(i,l)}(n)\mathbf{x}(n-l)\mathbf{x}^T(n-k)\right\} \\ + \mu^2 \sigma_x^2 \sigma_v^2 \mathbf{I}_N \sum_{i=0}^{N_{aux}-1} \sum_{j=0}^{N_{aux}-1} h_{aux}^2(i) h_{aux}^2(j).\end{aligned} \tag{21}$$

Then, the residual crosstalk power can be represented as

$$E\left\{ |e(n)|^2 \right\} = \sigma_v^2 + trace\left\{\mathbf{R}_{\boldsymbol{\varepsilon}}^{(0,0)}(n)\right\}\sigma_x^2. \tag{22}$$

IV. SIMULATION RESULTS

The performance of the proposed scheme is evaluated in an SDSL environment, where the symbol rate is 128 Kbauds and the data rate is 768 Kbps. Other simulation parameters and requirements for the scheme summarized in Table 1.

Figure 5 depicts the steady-state performance of the proposed XC and conventional IDXC with the analytic result. It can be seen that the crosstalk suppression of the proposed XC is limited to approximately 50 dB mainly due to the stopband ripple of the interpolation and decimation filters. If these filters are more strictly designed, better crosstalk suppression can be achieved.

Figure 6 depicts the learning curve of the proposed crosstalk canceller with the use of linear-phase interpolation filters with different group delay τ_{int}. The learning curve of the conventional IDXC is also depicted for comparison. It can be seen that linear-phase interpolation filters can increase the out-of-band suppression by increasing the tap size (i.e., group delay), resulting in the increase of the convergence time.

Figure 7 depicts the optimum K in the proposed crosstalk canceller according to the bandwidth expansion. It can be seen that the proposed crosstalk canceller can process at a lower rate if the expansion of the spectrum is small, significantly reducing the computational load. The proposed scheme in the premises requires a computational complexity of $8KNf_{b_L}$ MACs per second for adaptive crosstalk cancellation and $2KN_{aux}f_{b_L}$ MACs per second for interpolation. Thus, the total computational complexity κ_{tot} can empirically be represented in terms of the tap size of an equi-ripple filter [5]

$$\begin{aligned}\kappa_{tot} &= \left[8KN + 13.7KR\log_{10}(R\eta)\right] f_{b_L} \\ &= \left[8N\left\lceil(1-\zeta)L\right\rceil + 13.7L\log_{10}\left(\frac{\eta L}{\left\lceil(1-\zeta)L\right\rceil}\right)\right] f_{b_L} \quad \text{(MAC)}\end{aligned} \tag{23}$$

where η denotes the required amount of crosstalk suppression.

Figure 8 depicts the computational complexity of the proposed XC. It can be seen that the proposed scheme needs approximately half the computational load of the conventional scheme. The smaller the bandwidth of the partially overlapped crosstalk signal, the less the computational load is required.

Table 1. Simulation condition

Condition	Details
Target service	768 Kbps at 10^{-7} BER
Loop type	CSA-1 loop
f_{b_L} f_{b_H}	128 Kbaud 128 Kbaud
Training sequence	4-QAM
f_{C_L} f_{C_H}	128 KHz 256 KHz
α	20 %
ζ	0.83
K	1
L	8
δ_s	55 dB
Signal constellation (Data mode)	64-QAM
Required crosstalk suppression	41.7 dB
N	30

V. Conclusions

In this paper, we have also proposed a new structure for cancellation of partially overlapped single-carrier crosstalk signal. The proposed scheme employs interpolation and decimation filters to process the crosstalk cancellation in the baseband, significantly reducing the computational burden. The performance of the proposed crosstalk canceller is analytically evaluated and verified by computer simulation. The proposed crosstalk canceller can be applied to bandwidth expansion in high speed data transceivers such as the SDSL systems and the single-carrier ADSL systems.

References

[1] W. Y. Chen, *DSL simulation techniques and standards development for digital subscriber line systems*, MTP, 1998.

[2] D. Messerschmitt, "Crosstalk cancellation in speech and data transmission," *IEEE J. on Selected Areas in Commun.*, Mar. 1984, pp. 283-297.

[3] S. B. Weinstein, "A passband data-driven crosstalk canceller for full-duplex transmission on two-wire circuits," *IEEE Trans. Commun.*, July 1977, pp. 654-666.

[4] J. J. Werner, "Effects of channel impairments on the performance of an in-band data-driven crosstalk canceller," *AT&T Tech. Journal*, Jan. 1985, pp. 91-113.

[5] A. V. Oppenheim, *Digital signal processing,* Prentice Hall, 1980.

[6] E. Bjarnason, "Analysis of the filtered-X LMS algorithm," *IEEE Trans. Speech, Audio Process.*, Nov. 1995, pp. 504-514.

[7] M. Rupp, "Robust FxLMS algorithms with improved convergence performance," *IEEE Trans. Speech, Audio Process.*, Jan. 1998, pp. 78-85.

[8] G. Long and F. Ling, "The LMS algorithm with delayed coefficient adaptation," *IEEE Trans. Commun.,* Sept. 1989, pp. 1397-1405.

[9] G. Long and F. Ling, "Corrections to 'the LMS algorithm with delayed coefficient adaptation," *IEEE Trans. Commun.,* Jan. 1992, pp. 230-231.

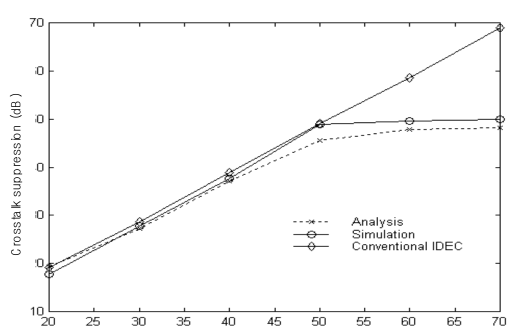
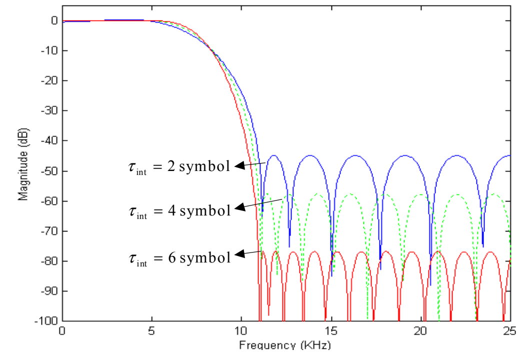
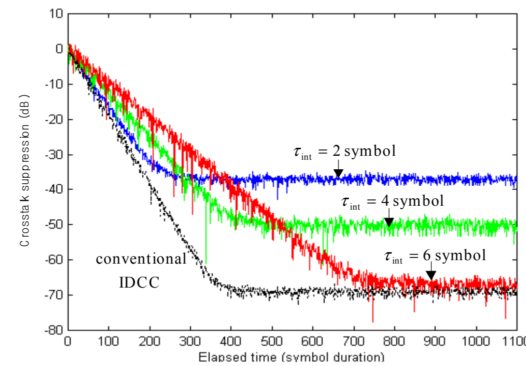


Figure 5. Steady-state performance of the proposed scheme.



(a) Design of error feedback filter (linear-phase)



(b) Convergent statistics

Figure 6. Convergence property of the proposed scheme.

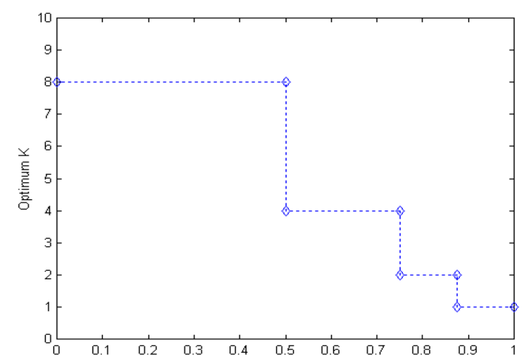


Figure 7. Optimum K according to the bandwidth.

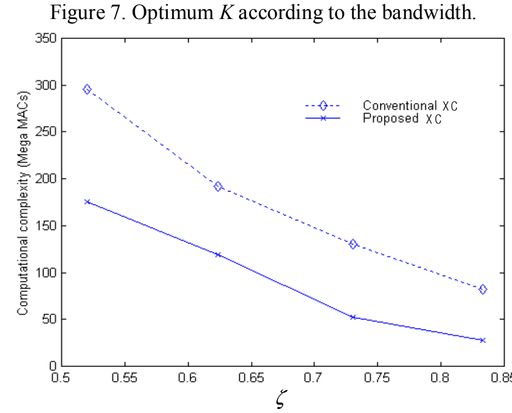


Figure 8. Comparison of the computational complexity.