Design of Pilot Pattern for Channel Estimation in OFDM Systems

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***Abstract* - Channel estimation is necessary for coherent detection in orthogonal frequency division multiplexing (OFDM) receivers. For accurate estimation, known pilot symbols are sent with data symbols at the transmitter. In general, these pilot symbols are placed with a regular pattern in two-dimensional (2-D) time-frequency grid. In a static additive Gaussian noisy channel, the channel estimation performance is known to depend not on the pilot pattern, but only on the pilot density. In time-variant frequency-selective fading channel, however, both the pilot pattern and the pilot density significantly affect the channel estimation performance. In this paper, we analytically design the pilot pattern so as to maximize the channel estimation performance when a piecewise linear interpolation filter is used for channel estimation in data-inserted region. Finally, the analytic design is verified by computer simulations.**

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) technique has been spotlighted for the last several years as a solution for the next generation wireless digital communication systems due to its high data rate transmission capability with high bandwidth efficiency and robustness to frequency selective fading [1]. Since the bandwidth of subcarrier in OFDM systems is determined such that each subcarrier experiences flat-like fading channel, the channel distortion can be compensated by a single-tap frequency domain equalizer. For equalization and coherent detection, the channel information should be estimated in the receiver.

The channel information can be estimated by using known pilot signal [2-4], which is inserted with a regular pattern in two-dimensional (2-D) time-frequency grid. Such a 2-D pilot pattern can be inserted in various ways with a specific shape such as block-, comb-, rectangular- and hexagonal-type [5]. If the channel is time-invariant additive white Gaussian noisy (AWGN) one with single path, it is reported that the channel estimation performance is only affected by the pilot density regardless of the pilot pattern [6]. In time-variant frequency-selective fading channels, however, the channel estimation performance is significantly affected by both the pilot pattern and pilot density [5, 7]. For example, when the channel varies fast with relatively small multipath delay spread, it would be advantageous to insert more pilot symbols in time direction than in frequency direction, and vice versa. Most of researches on channel estimation in OFDM systems have considered the use of some specific pilot patterns such as the block-type or comb-type with little analytic verification [2-4]. The channel estimation performance has been compared by simulation for various pilot patterns in some specific time-variant frequency-selective fading channels [5, 7]. The resulting pilot pattern may not provide the best channel estimation performance in other channel environments.

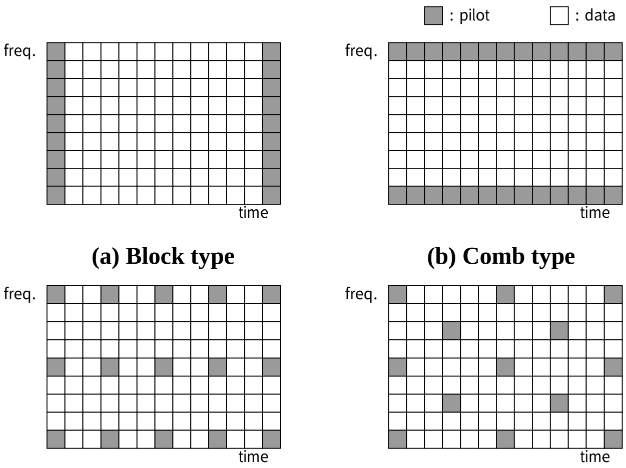
In this paper, we investigate the effect of the pilot pattern on channel estimation performance. We analytically design the optimum pilot pattern by use of the channel statistics. The optimum pilot spacing has been analytically derived for rectangular and hexagonal pilot patterns in terms of channel parameters such that the mean-squared-error (MSE) of the channel estimate is minimized. The optimum pilot pattern can be obtained by comparing the MSE of the channel estimate. For ease of analysis, the channel estimate for the data-inserted region has been obtained by using a piecewise linear interpolation technique [3].

Following Introduction, the problem formulation and vector notation of pilot pattern is described in Section Ⅱ. The 2-D correlation function of the channel is derived in Section Ⅲ. Using the channel correlation function and channel estimation performance metric, the pilot pattern is analyzed in Section Ⅳ. The analytic results are verified by computer simulation in Section V and the concluding remarks are summarized in Section Ⅵ.

II. PROBLEM FORMULATION

Fig. 1 depicts various pilot patterns on a 2-D time-frequency grid. The first two patterns are continuous ones and the last two patterns are scattered ones. Since the scattered patterns can include the continuous patterns by adjusting the spacing between pilot symbols according to the channel characteristics, we will consider the last two patterns.

The regularly arranged pilot symbols can be represented by two basis vectors, as shown in Fig. 2 which will be represented by the Cartesian coordinates: the time axis by abscissa and frequency axis by ordinate. Assume that the pilot symbols are inserted in a regular format at position $(0,0)$, (x_1, y_1), (x_2, y_2) and (x_1+x_2, y_1+y_2)



(a) Block type **(b) Comb type**

(c) Rectangular **(d) Hexagonal**

Fig. 1. Pilot arrangement on a 2-D grid.

as in Fig. 2. This pilot arrangement can be characterized by two basis vectors, $\mathbf{v}_1 = [x_1 \quad y_1]^T$ and $\mathbf{v}_2 = [x_2 \quad y_2]^T$.

The pilot density can be represented in terms of the number of pilot symbols in some fixed area of 2-D time-frequency grid. Since the pilot density D is inversely proportional to the pilot spacing, it can be defined as the inverse of the area of parallelogram formed by $\mathbf{v}_1$ and $\mathbf{v}_2$, i.e.,

$$D = \frac{1}{|\det(\mathbf{V})|} = \frac{1}{|x_1 y_2 - x_2 y_1|}. \tag{1}$$

where $\mathbf{V} = [\mathbf{v}_1 \quad \mathbf{v}_2]$ is a 2×2 matrix representing the pilot pattern.

The goodness of the pilot pattern can be evaluated by its channel estimation performance with the constraint of a fixed pilot density. Hence, we define a metric J from the mean-squared-error (MSE) of the channel estimate by

$$J = E\left\{ \left| H[n,k] - \hat{H}[n,k] \right|^2 \right\} \tag{2}$$

where $H[n,k]$ is the frequency response of the n-th OFDM block and k-th subcarrier, and $\hat{H}[n,k]$ is its estimate. We will find an optimum pilot pattern that can result in a lowest metric J for a fixed pilot density D.

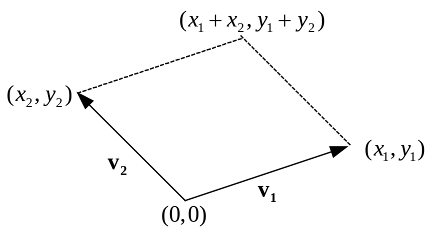


Fig. 2. Pilot pattern in a vector form.

III. CHANNEL CORRELATION FUNCTION

The channel impulse response (CIR) of a time-variant multipath channel can be written as

$$h(t,\tau) = \sum_{i=0}^{L-1} \alpha_i(t)\delta(\tau - \tau_i) \tag{3}$$

where τ_i and $\alpha_i(t)$ are the delay spread and complex channel gain of the *i*-th path at time *t*, respectively, $\delta(\tau)$ is a Dirac delta function and L is the number of multipaths. The channel gain $\alpha_i(t)$ can be modeled as a complex Gaussian random process which is independent for other propagation paths. The frequency response $H(t, f)$ of the channel can be obtained by taking the Fourier transform of (3) and the 2-D channel correlation function $r_H(dt, df)$ for time offset dt and frequency offset df can be represented by multiplication of the time- and frequency-autocorrelation function [4]

$$\begin{aligned} r_H(dt, df) &= E\{H(t+dt, f+df)H^*(t, f)\} \\ &= r_t(dt) r_f(df) \end{aligned} \tag{4}$$

where $r_t(dt)$ is the normalized time-autocorrelation function of $\alpha_i(t)$ and $r_f(df) = \sum \sigma_k^2 e^{-j2\pi df \tau_k}$ is the frequency autocorrelation function which is determined by the multipath channel delay profile. Here, σ_i^2 denotes the normalized average power of the *i*-th path signal, satisfying $\sum \sigma_i^2 = 1$.

If we denote T_s by the OFDM symbol duration and Δf by the subcarrier spacing, the 2-D channel correlation function $r_H[n,k]$ in a discrete form for the *n*-th symbol and *k*-th subcarrier spacing can be written as

$$r_H[n,k] = r_t(nT_s) r_f(k\Delta f) \equiv r_t[n] r_f[k]. \tag{5}$$

Note that $r_H[0,0] = 1$. From the Jakes' channel model [8], the discrete-time autocorrelation function $r_t[n]$ for the *n*-th symbol spacing can be written as

$$r_t[n] = J_0(2\pi f_d n T_s) \tag{6}$$

where f_d is the maximum Doppler frequency and $J_0(x)$ is the zeroth order Bessel function which can be represented in the polynomial form as [8]

$$J_0(x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i}}{2^{2i}(i!)^2}. \tag{7}$$

Since the cosine and sine function can be expanded in a polynomial form by the Taylor's series as

$$\cos(x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i}}{(2i)!}, \quad \sin(x) = \sum_{i=0}^{\infty} \frac{(-1)^i x^{(2i+1)}}{(2i+1)!} \tag{8}$$

$r_H[n,k]$ can be approximated by a second-order polynomial for sufficiently small *n* and *k*.

IV. DESIGN OF PILOT PATTERN

The MSE of the channel estimate depends on the interpolation scheme for channel estimation of the data-inserted region. The optimum interpolation can be performed by 2-D minimum mean squared error (MMSE) filter [4], but it is not realizable. We consider a piecewise linear interpolation filter as a practical solution since it is widely used [3] for its simplicity and useful for analytic derivation.

We can consider the use of the average MSE or the maximum MSE as the channel estimation performance metric. When the pilot density is low and the area to be considered becomes large, the former may not be a good choice due to increased computational complexity. In practice, when the channel falls in a deep null, the latter becomes a dominant factor affecting the receiver bit error rate (BER) performance. Moreover, since the linear interpolation scheme considered in this paper does not produce an abrupt change in the adjacent channel estimate, the channel estimation performance is evaluated in terms of the maximum value of the MSE.

The rectangular pilot pattern corresponds to the case when $x_2 = y_1 = 0$, $x_1 = x_r$ and $y_2 = y_r$ in Fig. 2. The channel estimate at an arbitrary position (a, b), where $0 < a < x_r$ and $0 < b < y_r$, can be obtained by linear interpolation as

$$\begin{aligned} \hat{H}[0,b] &= \frac{y_r - b}{y_r}\tilde{H}[0,0] + \frac{b}{y_r}\tilde{H}[0, y_r], \\ \hat{H}[x_r, b] &= \frac{y_r - b}{y_r}\tilde{H}[x_r, 0] + \frac{b}{y_r}\tilde{H}[x_r, y_r], \\ \hat{H}[a,b] &= \frac{x_r - a}{x_r}\hat{H}[0,b] + \frac{a}{x_r}\hat{H}[x_r, b]\ . \end{aligned} \tag{9}$$

where $\tilde{H}[x, y]$ denotes the temporary least-square (LS) channel estimate [9] at pilot position (x, y). Denoting $J(0, b)$ by the MSE of channel estimate $\hat{H}[0,b]$, we can find that the value b maximizing $J(0, b)$ is $y_r / 2$ by $\partial J(0,b)/\partial b = 0$. Similarly, it can be shown that the value a maximizing the MSE in the third step of interpolation is $x_r / 2$. Therefore, the maximum MSE occurs at $(x_r / 2,\ y_r / 2)$, i.e., the center of the square region. The corresponding maximum MSE is

$$\begin{aligned} J_{r,\max} &= E\left\{\left| H\left[\frac{x_r}{2},\frac{y_r}{2}\right] - \frac{1}{4}\left(\hat{H}[0,0] + \hat{H}[x_r,0] + \hat{H}[0,y_r] + \hat{H}[x_r,y_r]\right)\right|^2\right\} \\ &\approx r_H[0,0] - \mathrm{Re}\left\{ r_H\left[\frac{x_r}{2},\frac{y_r}{2}\right] + r_H\left[\frac{x_r}{2},-\frac{y_r}{2}\right]\right\} \\ &\approx 1 - 2\left(1 - \frac{1}{4}(f_d \pi T_s)^2 x_r^2\right)\sum_{i=0}^{L-1}\sigma_i^2\left(1 - \frac{(\pi\Delta f \tau_i)^2}{2} y_r^2\right). \end{aligned} \tag{10}$$

Since the linear interpolation scheme uses only the adjacent symbols whose distance is sufficiently small, the second-order polynomial approximation in the last line of (10) is valid.

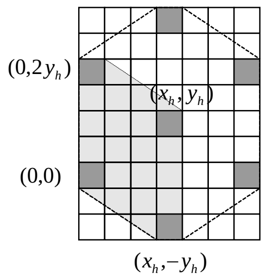


Fig. 3. Hexagonal pilot pattern.

From the definition of the pilot density D in (1), it can be seen that

$$y_r = \frac{1}{Dx_r}\ . \tag{11}$$

Thus, the optimum value of x_r and y_r minimizing $J_{r,\max}$ are given by

$$x_{r,opt} = \sqrt{\frac{\sqrt{2}\Delta f \tau_{rms}}{f_d T_s D}}, \qquad y_{r,opt} = \sqrt{\frac{f_d T_s}{\sqrt{2}\Delta f \tau_{rms} D}} \tag{12}$$

where τ_{rms} is the root-mean-squared (RMS) delay spread defined as $\sqrt{\sum \sigma_i^2 \tau_i^2}$.

The hexagonal pilot pattern can be formed as in Fig. 3, by making $x_1 = -x_2 = x_h$ and $y_1 = y_2 = y_h$. It can be shown that the maximum MSE occurs at point $(x_h / 2, y_h / 2)$, which is the center of the shaded region in Fig. 3. As in the rectangular pilot pattern, it can be shown that

$$y_h = \frac{1}{2Dx_h} \tag{13}$$

for a given pilot density D. Thus, the optimum pilot spacing minimizing the maximum MSE $J_{h,\max}$ is given by

$$x_{h,opt} = \sqrt{\frac{\sqrt{3}\Delta f \tau_{rms}}{\sqrt{2} f_d T_s D}}, \qquad y_{h,opt} = \sqrt{\frac{f_d T_s}{\sqrt{24}\Delta f \tau_{rms} D}}\ . \tag{14}$$

Let $\min\{J_{r,\max}\}$ and $\min\{J_{h,\max}\}$ be the minimum values of the maximum MSE. Then, since $\sum \sigma_i^2 = 1$ and $\Delta f = 1/T_s$, it can be shown that

$$\begin{aligned} &\min\{J_{r,\max}\} - \min\{J_{h,\max}\} \\ &\quad = \frac{\pi^2 f_d}{32D}\sqrt{\sum_{i=0}^{L-1}\sigma_i^2\tau_i^2}\left(8\sqrt{2}\left(2-\sqrt{3}\right) - \frac{\pi^2 f_d}{D}\sqrt{\sum_{i=0}^{L-1}\sigma_i^2\tau_i^2}\right). \end{aligned} \tag{15}$$

To be free from aliasing, it can be seen from the Nyquist sampling theorem that the pilot spacing should satisfy [2]

$$x_r \le \frac{1}{2 f_d T_s}\ , \quad y_r \le \frac{1}{2\tau_{\max}\Delta f} \tag{16}$$

where $\tau_{\max}$ is the maximum delay spread of the propagation channel. Since the pilot density D should be bounded by

$$D = \frac{1}{x_r y_r} \ge 4 f_d \tau_{\max}, \tag{17}$$

it can be shown that

$$\frac{\pi^2 f_d}{D}\sqrt{\sum_{i=0}^{L-1}\sigma_i^2\tau_i^2} \le \frac{\pi^2}{4\tau_{\max}}\sqrt{\sum_{i=0}^{L-1}\sigma_i^2\tau_i^2} \le \frac{\pi^2}{4\tau_{\max}}\sqrt{\sum_{i=0}^{L-1}\sigma_i^2\tau_{\max}^2} = \frac{\pi^2}{4} < 8\sqrt{2}\left(2-\sqrt{3}\right). \tag{18}$$

This implies that the use of a hexagonal pattern can provide better channel estimate that the use of a rectangular pattern.

V. NUMERICAL RESULTS

To verify the analytic results, the performance is evaluated in terms of the MSE of the channel estimate and the receiver bit error rate (BER) with the use of four pilot patterns: block-type, comb-type, rectangular and hexagonal. As a rule of thumb, the pilot density is set to a value of four times minimum required value in (17) [2]. We assume that multipath channel has an exponential decaying power profile [10] with equi-spaced multipath delay, each of which experiences independent Rayleigh fading. Table 1 summarizes the parameters of the OFDM system and propagation channel.

Fig. 4 depicts the MSE and BER performance, where the optimum pilot spacing is x_r = 12, y_r = 5 for rectangular pattern and x_h = 10, y_h = 3 for hexagonal pattern. The ideal channel estimator (i.e., using known CIR) is also considered for comparison. Keeping the pilot density constant, the block- and comb-type correspond to the case when x_r = 60, y_r =1 and x_r = 1, y_r = 60, respectively. It can be seen that the use of a hexagonal pattern provides the best channel estimation performance, while the use of block- and comb-type is worst.

VI. CONCLUSION

In this paper, we have examined the effect of the pilot signal pattern on channel estimation in OFDM systems. To see the dependence of channel estimation performance on the pilot pattern, we considered the use of a linear interpolation scheme for channel estimation in data-inserted region. For a given pilot density with rectangular and hexagonal patterns as a general one, the optimum pilot spacing are analytically derived in the MMSE sense as a function of the channel parameters. The results show that the hexagonal pilot pattern can provide better channel estimation performance than the rectangular one and it is also verified by computer simulation.

REFERENCES

[1] R. van Nee and R. Prasad, *OFDM wireless multimedia communications*, Artech House, 2000.

[2] P. Hoeher, S. Kaiser and P. Robertson, “Two-dimensional pilot-symbol-aided channel estimation by Wiener filtering,” *Proc. ICASSP’ 1997*, pp. 1845-1848, 1997.

[3] F. Said and H. Aghvami, “Linear two dimensional pilot assisted channel estimation for OFDM systems,” *IEE Conference on Telecommunications*, pp. 32-36, 1998.

[4] Y. Li, “Pilot-symbol-aided channel estimation for OFDM in wireless systems,” *IEEE Trans. Veh. Tech.*, vol. 49, pp. 1207-1215, July 2000.

[5] M. J. F. – G. Garcia, S. Zazo and J. M. Paez-Borrallo, “Pilot patterns for channel estimation in OFDM,” *Electronics Letters*, vol. 36, pp. 1049-1050, June 2000.

[6] R. Negi and J. Cioffi, “Pilot tone selection for channel estimation in a mobile OFDM system,” *IEEE Trans. Consum. Electron*, vol. 44, pp. 1122-1128, Aug. 1998.

[7] F. Tufvesson and T. Maseng, “Pilot assisted channel estimation for OFDM in mobile cellular systems,” *Proc. VTC’ 1997*, pp. 1639-1643, 1997.

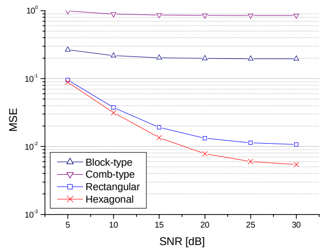
[8] W. C. Jakes, *Microwave mobile communications*, New York, Wiley, 1974.

[9] S. M. Kay, *Fundamentals of statistical signal processing: vol. 1 Estimation theory*, Prentice-Hall, 2002.

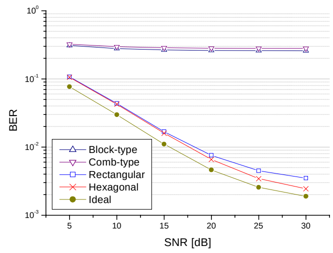
[10] G. Halls, “HIPERLAN radio channel models and simulation results,” RES10TTG 93/58.

Table 1. OFDM system parameters and channel model

FFT size	512
Subcarrier spacing (Δf)	125 kHz
Symbol duration (T_s)	8 μs
Subcarrier modulation	QPSK
Number of multipaths	12
Multipath delay spread	31.25 *ns* equi-spaced
Multipath power profile	Exponential decaying
Maximum Doppler frequency	1250 Hz ($f_d T_s$ =0.01)



(a) MSE



(b) BER

Fig. 4. MSE and BER performance.