# Non-overlapped interleaving for space-frequency trellis coded OFDM systems in frequency selective fading channel

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## Abstract

The performance of space-frequency trellis coded (SFTC) OFDM system is analyzed in frequency selective fading channel. Using a pairwise error probability upper bound expression, we design the code and interleaver that can maximize the diversity and coding gain. To alleviate the performance degradation due to the inter-antenna product terms, we propose a new SFTC scheme that employs non-overlapped interleaving for each antenna. A new code is optimized for the non-overlapped interleaving and a practical sub-optimal decoding algorithm is proposed. It is shown that the proposed scheme can substantially improve the performance compared to the conventional scheme that employs identical interleaving for all antennas.

## I. Introduction

The signal transmitted over a wireless channel suffers from a large fading due to constructive and destructive addition of signals propagated through different paths. The use of diversity has been considered as the most effective technique to overcome the detrimental effect of fading [1]. Space diversity is an effective scheme that can provide diversity gain irrespective of the signal condition [2]. Since the use of multiple antennas is practical only at the base station, transmit diversity is widely applied to the downlink.

The space-time code (STC) is one of the most promising techniques that can exploit the spatial diversity created by multiple transmit antennas [3][4]. It was shown that the space-time trellis code (STTC) and space-time block code (STBC) can provide significant performance improvement in flat fading channel. The application of STC to frequency selective fading channel can be achieved by combining the STC with OFDM because the OFDM signaling transforms a frequency selective fading channel into multiple flat fading channels. We consider the use of space-frequency trellis coded (SFTC) OFDM systems that employs the space-time trellis code (STTC) [3].

The performance of the SFTC-OFDM was analyzed in terms of the pairwise error probability (PEP) between the distinct codeword pair [5][6]. Although a code design criterion for maximum diversity and coding advantage was suggested [5], it was not clarified how both frequency and space diversity can be achieved in an optimal way because the analysis did not take into account the role of the interleaver.

In this paper, we analyze the performance of the SFTC-OFDM systems in frequency selective fading channel. Using a PEP upper bound expression, we consider the design of code and interleaver that maximizes the diversity and coding gain. To alleviate the performance degradation due to the inter-antenna product terms, we propose a new SFTC that employs non-overlapped interleaving for each antenna.

Following Introduction, the system model is described in Section II. The performance of the SFTC-OFDM is analyzed in Section III. Section IV describes the use of non-overlapped interleaving for each antenna. The performance of the proposed scheme is verified by computer simulation in Section V. Finally, conclusions are summarized in Section VI.

## II. System model

We consider an SFTC-OFDM communication system that employs $n_T$ transmit and $n_R$ receive antennas as depicted in Fig. 1. SFTC encoder transforms the user data into a codeword defined simultaneously in the space and frequency domain

[5][6]. The coded symbols are OFDM modulated and then transmitted through multiple antennas simultaneously. Note that frequency domain interleaving is incorporated to efficiently achieve the frequency diversity provided by the channel.

We assume that the channel between the *j*-th transmit and *i*-th receive antenna is frequency selective. The corresponding channel can be described by a tapped-delay line model with $L$ taps

$$h_n^{ij}(\tau) = \sum_{l=0}^{L-1} h_l^{ij} \delta(\tau - lT_s) \cdot \tag{1}$$

The gain of each path can be modeled as a zero mean complex Gauaussian random variable with variance $A_l$. It is further assumed that the channel gains between the different antennas or delays are uncorrelated.

At the receiver, the signal of each receive antenna is a superposition of $n_T$ transmitted signals propagated through independent frequency selective fading channels. Assuming perfect timing and frequency synchronization, the *k*-th sub-carrier signal through the *i*-th receive antenna can be represented after the fast Fourier transform (FFT) as

$$R_k^i = \sum_{j=1}^{n_T} H_k^{ij} C_{D^j(k)}^j + Z_k^i \tag{2}$$

where

$$H_k^{ij} = \sum_{l=0}^{L-1} h_l^{ij} e^{-j2\pi kl/N} \tag{3}$$

is the channel gain between the *j*-th transmit and the *i*-th receive antenna at the *k*-th sub-carrier and $C_{D^j(k)}^j$ is the coded symbol transmitted over the *k*-th sub-carrier of the *j*-th transmit antenna with interleaving $D^{j^{-1}}(k)$. The additive noise component $Z_k^i$ is a circularly complex Gaussian random variable with two-sided spectral density $N_o/2$. It is assumed that the noise components at different sub-carriers or receive antennas are mutually independent.

Assuming that the receiver has perfect knowledge of the channel gain, the maximum likelihood (ML) decoder selects the codeword that is closest to the received signal, i.e.,

$$\hat{\mathbf{C}}_{ML} = \arg\min_{\mathbf{C}} \sum_{i=1}^{n_R} \sum_{k=0}^{N-1} | R_k^i - \sum_{j=1}^{n_T} H_k^{ij} C_{D^j(k)}^j |^2 \cdot \tag{4}$$

where $N$ is the number of sub-carriers.

## III. Performance analysis

Let $P_e(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H})$ be the conditional PEP that the ML decoder selects the codeword $\hat{\mathbf{C}}$ when $\mathbf{C}$ is transmitted over the channel with gain $\mathbf{H}$. Then, it can be shown that

$$P_e(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H}) = Q\left(\sqrt{\frac{d^2(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H})}{2N_o}}\right). \tag{5}$$

where $d^2(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H})$ is the weighted squared Euclidean distance (WSED) given by

$$d^2(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H}) = \sum_{i=1}^{n_R} \sum_{k=0}^{N-1} \left| \sum_{j=1}^{n_T} H_k^{ij} E_{D^j(k)}^j \right|^2 \cdot \tag{6}$$

Here, $E_{D^j(k)}^j = C_{D^j(k)}^j - \hat{C}_{D^j(k)}^j$.

It can be shown using the Chernoff inequality [1] that the conditional PEP is bounded by

$$P_e(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H}) \le \exp\left\{-\frac{d^2(\mathbf{C}, \hat{\mathbf{C}} | \mathbf{H})}{4N_o}\right\}. \tag{7}$$

Averaging this upper bound over the probability density function (pdf) of the channel gain, we have

$$P_e(\mathbf{C}, \hat{\mathbf{C}}) \le \left(\prod_{l=0}^{n_T L-1} \frac{1}{1+\lambda_l/4N_o}\right)^{n_R} < \left(\frac{1}{4N_o}\right)^{-n_R r} \left(\prod_{l=0}^{r-1} \lambda_l\right)^{-n_R} \tag{8}$$

where $r$ and $\lambda_l$ are the rank and eigenvalue of $\mathbf{\Phi} = \mathbf{B}\mathbf{B}^H$. Here, $\mathbf{B}$ denotes the composite codeword difference matrix given by $\mathbf{B} = [\mathbf{B}^{1^T} \cdots \mathbf{B}^{n_T{}^T}]^T$, where $\mathbf{B}^j$ is the codeword difference matrix corresponding to the *j*-th transmit antenna

$$\mathbf{B}^j = \begin{bmatrix} \sqrt{A_0} E_{D^j(0)}^j & \cdots & \sqrt{A_0} E_{D^j(N-1)}^j \\ \vdots & \ddots & \vdots \\ \sqrt{A_{L-1}} E_{D^j(0)}^j & \cdots & \sqrt{A_{L-1}} E_{D^j(N-1)}^j e^{-j2\pi(N-1)(L-1)/N} \end{bmatrix} \tag{9}$$

Note that $\mathbf{B}$ is a function of the delay profile of the channel, code and interleaver.

Using the averaged upper bound of the PEP given by (8), the code and interleaver can be designed to provide the maximum diversity and coding gain: To achieve the maximum diversity, the matrix $\mathbf{B}$ should have a full rank, equal to $n_T L$, for any distinct codeword pair. To maximize the coding gain, the minimum of the product of non-zero eigenvalues taken over all distinct codeword pair should be maximized, which is equivalent to maximization of the minimum of $\det(\mathbf{B}\mathbf{B}^H)$ when the maximum diversity gain is achieved.

## IV. Non-overlapped interleaving

The WSED is a distance metric that determines the error performance of the ML detector. It can be expanded by

$$d^2(\mathbf{C},\hat{\mathbf{C}}\,|\,\mathbf{H}) = \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{k=0}^{N-1}\left|H_k^{ij}E_{D^j(k)}^{j}\right|^2 + \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{j'\neq j}\sum_{k=0}^{N-1}H_k^{ij}E_{D^j(k)}^{j}(H_k^{ij'}E_{D^{j'}(k)}^{j'})^* \quad (12)$$

where the first term is the distance metric when the signals from different transmit antennas are separable at the receiver and the second term is the cross product of the signals from different antennas. It can be shown that

$$\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{k=0}^{N-1}\left|H_k^{ij}E_{D^j(k)}^{j}\right|^2 = \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{k=0}^{N-1}|E_{D^j(k)}^{j}|^2\sum_{l=0}^{L-1}|h_l^{ij}|^2 + \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{l}\sum_{m\neq l}h_l^{ij}h_m^{ij^*}\sum_{k=0}^{N-1}|E_{D^j(k)}^{j}|^2 e^{-j2\pi k(\tau_l-\tau_m)/N} \quad (13)$$

where the first term is called diversity extractor because it aggregates the energy of multi-path components and the second term is called diversity disturber since it corresponds to the sum of random gain fluctuations. The magnitude of diversity disturber can be made small by elaborately designing the code and interleaver [7].

Similarly, it can be shown that

$$\sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{j'\neq j}\sum_{k=0}^{N-1}H_k^{ij}E_{D^j(k)}^{j}(H_k^{ij'}E_{D^{j'}(k)}^{j'})^* = \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\sum_{j'\neq j}\sum_{l=0}^{L-1}\sum_{m=0}^{L-1}h_l^{ij}h_m^{ij'^*}\sum_{k=0}^{N-1}E_{D^j(k)}^{j}E_{D^{j'}(k)}^{j'^*}e^{-j2\pi(l-m)k/N} \quad (14)$$

Since the inter-antenna cross product terms only contribute the diversity disturber, they should be minimized to improve the diversity performance of the SFTC-OFDM system.

To eliminate the inter-antenna cross product terms, we consider the use of non-overlapped interleaving instead of identical interleaving for all the transmit antennas. With non-overlapped interleaving, $n_T$ symbols generated by the trellis encoder are mapped onto different sub-carriers so that the symbols pertaining to short error events are spectrally separated. This ensures that $E_{D^j(k)}^{j}E_{D^{j'}(k)}^{j'^*}=0$, completely eliminating the inter-antenna product terms. The interleavers can be made non-overlapped by introducing an offset to the interleaver input index of each antennas and passing them to the same interleaver $I(k)$, i.e., $I^j(k)=I(k+(j-1)S)$. Here, $I^j(k)$ is the interleaver for the *j*-th antenna and $S$ is the minimum index offset between the antennas. By introducing an offset of $S$, it can be ensured that the inter-antenna product terms are completely eliminated for error events shorter than $S$.

Fig. 2 depicts how the interleaving scheme affects diversity gain of a two-antenna SFTC-OFDM system employing a code with the shortest error event of length $d_{e,\min}$. Since the diversity gain of the SFTC-OFDM system is determined by the number of sub-carriers with distinct code symbols, the diversity gain can be at most $d_{e,\min}$ when identical interleaving is employed. However, with non-overlapped interleaving, there can be as many as $2d_{e,\min}$ different sub-carriers, yielding substantial increase of potential diversity gain.

Note that conventional Viterbi algorithm cannot be used for the ML decoding of SFTC-OFDM with non-overlapped interleaving. This is mainly due to the fact that each transmit antenna carries branch symbols pertaining to a state transition using different sub-carriers of multiple transmit antennas. As an alternative, we construct a sub-optimal decoding algorithm by modifying the branch metric calculation for conventional Viterbi decoder. The modified branch metric is given by

$$BM(b) = \sum_{i=1}^{n_R}\sum_{j=1}^{n_T}\min_{\mathbf{C}\in\Omega_{C^r(b)}}|R_{k+(r-1)S}^{i} - \sum_{j=1}^{n_T}H_{k+(r-1)S}^{ij}C^j|^2 \quad (15)$$

where $\mathbf{C}=[C^1C^2\cdots C^{n_T}]^T$ and $\Omega_{C^r(b)}$ denotes a set of all $n_T$-tuples with the *r*-th element equal to $C^r(b)$.

## V. Performance evaluation

To evaluate the performance of the proposed scheme, we consider an SFTC-OFDM system with two transmit and one receive antennas in frequency selective fading channel. The OFDM signal has 64 sub-carriers and a CP of length 8. We assume that the channel gain is unchanged within a packet consisting of 16 OFDM symbols but it varies independently between the packets. The SFTC employs two 16-state QPSK codes whose generator matrix is given by

$$\mathbf{G}_{Tarokh} = \begin{bmatrix} 1 & 2 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 2 & 0 \end{bmatrix} \quad (16)$$

and

$$\mathbf{G}_{new} = \begin{bmatrix} 2 & 1 & 3 & 3 & 3 & 2 \\ 2 & 3 & 3 & 2 & 1 & 1 \end{bmatrix}. \quad (17)$$

where $\mathbf{G}_{Tarokh}$ is a code designed by Tarokh [3] and $\mathbf{G}_{new}$ is a code optimized for non-overlapped interleaving.

Fig. 3 depicts the packet error rate (PER) performance of the SFTC-OFDM with different interleaving schemes. The performance of three interleaving schemes (i.e., no interleaving, identical block interleaving and non-overlapping interleaving) is compared when they are combined with $\mathbf{G}_{Tarokh}$ and $\mathbf{G}_{new}$. Performance evaluation is carried out in channels with small selectivity ($\tau_{rms}=1$) and large selectivity ($\tau_{rms}=4$). It can be seen that the use of non-overlapped interleaving with $\mathbf{G}_{new}$ provides a substantial performance improvement over the use of the identical interleaving, yielding an SNR gain of 2.5~2.9dB at PER $10^{-2}$. It can also be seen that the SNR gain increases as the channel fading becomes more selective. Analytic results indicate that further performance gain can be obtained if the ML decoding algorithm is optimized for SFTC-OFDM with non-overlapped interleaving.

## VI. Conclusions

The performance of the SFTC-OFDM system has been analyzed by averaging the PEP upper bound over the fading channel. It has been shown that the code and interleaver should be co-designed so that the minimum of $rank(\mathbf{B})$ and $\det(\mathbf{BB}^H)$ is maximized. We have proposed a new non-overlapped interleaving scheme to alleviate the performance degradation due to the inter-antenna cross product terms. With the proposed non-overlapped interleaving, the coded symbols pertaining to the same error event are spectrally separated for each transmit antenna. Simulation results indicate that the use of non-overlapped interleaving significantly outperforms the use of conventional identical interleaving.

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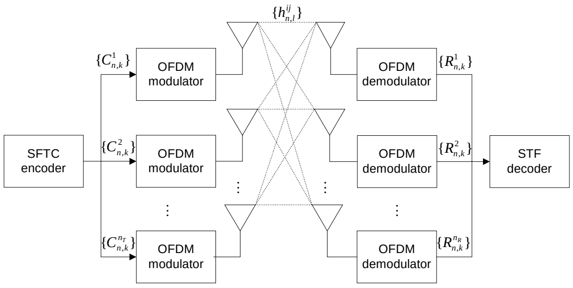



Fig. 1 Space-frequency trellis coded OFDM system

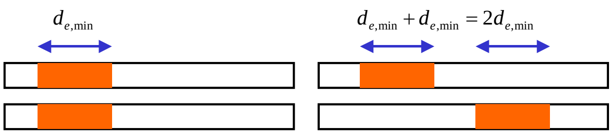
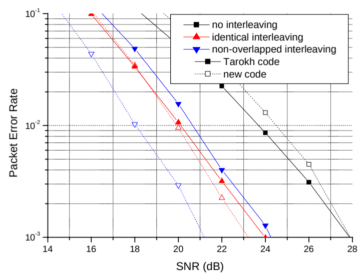
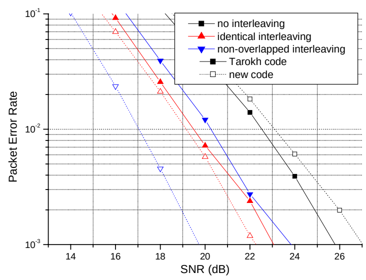



Fig. 2 Effect of interleaving schemes on the diversity gain




(a) $\tau_{rms}=1$




(b) $\tau_{rms}=4$

Fig. 3 PER performance with different interleaving schemes