Pilot Symbol Initiated Iterative Channel Estimation and Decoding for QAM Modulated OFDM Signals

Jong-Ho Lee, Yong-Hwa Kim and Seong-Cheol Kim
Institute of New Media and Communications
School of Electrical Engineering and Computer Science
Seoul National University, Seoul, Korea
Email: {dowar,yhkim,sckim}@maxwell.snu.ac.kr

Jae Choong Han
Dept. of Infromation and Communications
Myong Ji University, Yongin, Kyunggido, Korea
Email: jhan@mju.ac.kr

Abstract—**In this paper, we consider the iterative channel estimation and decoding algorithms for quadrature amplitude modulation (QAM) modulated orthogonal frequency division multiplexing (OFDM) signals. We first describe the optimal sequence estimation based on the expectation-maximization (EM) algorithm. Using some approximations, we propose a sub-optimal iterative channel estimation and decoding algorithm, which is shown to have reduced computational complexity. The bit error rate (BER) performances of the proposed algorithm are evaluated using computer simulations. The results show that the proposed algorithm performs nearly as well as the optimal EM algorithm, and outperforms the conventional minimum mean square error (MMSE) estimator.**

I. Introduction

Othogonal frequency division multiplexing (OFDM) is well known to be a useful technique for high rate data transmission over a frequency selective fading channel. However, for coherent detection of OFDM signals, fading compensation techniques are required to mitigate amplitude and phase distortions due to the multipath channel fading. The fading compensation technique becomes even more crucial when quadrature amplitude modulation (QAM) is used for OFDM systems.

A channel estimation technique using periodically inserted pilot symbols in the data stream is well known to provide a reliable way to mitigate the distortions. Many channel estimation schemes for the detection of OFDM signals are reported in the technical literatures [1]-[6]. In [6], two different estimators using pilot symbols are investigated. One is the deterministic maximum likelihood (ML) estimator assuming the channel impulse response (CIR) is unknown but deterministic. The other is the minimum mean square error (MMSE) estimator assuming random CIR.

The application of the expectation-maximization (EM) algorithm is known to provide the ML estimate over random channel under some conditions [7], [8]. In this paper, in order to obtain ML estimate assuming random CIR, we first describe the application of the EM algorithm for QAM modulated OFDM signals in a frequency selective fading channel. Then, we propose a sub-optimal iterative channel estimation and decoding algorithm for QAM modulated OFDM signals with some approximations. However, the initial estimate for the iteration of the proposed algorithm is obtained using pilot symbols and deterministic ML estimator studied in [6]. The BER performances of the proposed algorithm are evaluated using computer simulations and compared with those of other algorithms.

II. System Model

Let N be the number of subcarriers and $K = 2N_\alpha + 1$ be the number of parallel data symbols to be transmitted. The data symbol sequence vector can be expressed as

$$\mathbf{s} = [s(-N_\alpha), \cdots, s(0), \cdots, s(N_\alpha)]^T. \quad (1)$$

Note that $N - K$ subcarriers at the edges of the spectrum are not used. One OFDM symbol is formed using the inverse discrete Fourier transform (IDFT) of the data symbol vector $\mathbf{s}$, inserting cyclic prefix to avoid the intersymbol interferences [9]. After removing cyclic prefix and sampling at the data symbol rate in the OFDM receiver, the received signals are given as

$$\mathbf{y} = \mathbf{SFh} + \mathbf{n}. \quad (2)$$

where $\mathbf{h} = [h(1), h(2), \cdots, h(L)]^T$ denotes the channel impulse response and $\mathbf{S} = diag[s(-N_\alpha), \cdots, s(0), \cdots, s(N_\alpha)]$ is a diagonal symbol matrix. Moreover, the channel response in the frequency domain $\mathbf{H} = [H(-N_\alpha), \cdots, H(0), \cdots, H(N_\alpha)]^T$ can be given as

$$\mathbf{H} = \mathbf{Fh}. \quad (3)$$

The additive white Gaussian noise vector $\mathbf{n}$ has zero mean and the covariance matrix of $\sigma_n^2\mathbf{I}$. The entries of discrete Fourier transform (DFT) matrix $\mathbf{F}$ in (2) and (3) are given by

$$[\mathbf{F}]_{k,l} = e^{-j\frac{2\pi k(l-1)}{N}} \quad (4)$$

where $|k| \le N_\alpha$ and $1 \le l \le L$.

III. Iterative Channel Estimation and Decoding

In this section, we first describe the application of the EM algorithm for iterative channel estimation and decoding for QAM modulated OFDM signals. Then, the optimal EM algorithm is modified to the sub-optimal iterative algorithm using some approximations.

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A. Iterative Algorithm based on EM algorithm

The EM algorithm is an iterative two-step algorithm that includes the expectation step and maximization step. The EM algorithm iterates until the estimate converges. Each step of the EM algorithm for OFDM is summarized below.

The expectation step requires the evaluation of the likelihood function given by

$$Q(\mathbf{s} \mid \mathbf{s}^i) = \sum_{k=-N_\alpha}^{N_\alpha} Re\left[y^*(k) s(k) \sum_{l=l}^{L} [\mathbf{F}]_{k,l} m_1^i(l) \right] - \frac{1}{2}|s_k|^2 \sum_{l=1}^{L}\sum_{m=1}^{L} [\mathbf{F}]_{k,l}[\mathbf{F}]^*_{k,m} m_2^i(l,m). \quad (5)$$

where

$$\begin{aligned} \mathbf{m}_1^i &= [m_1^i(1), m_1^i(2), \cdots, m_1^i(L)]^T \\ &= E[\mathbf{h} \mid \mathbf{y}, \mathbf{s}^i] \end{aligned} \quad (6)$$

and

$$\begin{aligned} \mathbf{m}_2^i &= \begin{bmatrix} m_2^i(1,1) & m_2^i(1,2) & \cdots & m_2^i(1,L) \\ m_2^i(2,1) & m_2^i(2,2) & \cdots & m_2^i(2,L) \\ \vdots & & \ddots & \vdots \\ m_2^i(L,1) & m_2^i(L,2) & \cdots & m_2^i(L,L) \end{bmatrix} \\ &= E[\mathbf{h}\mathbf{h}^\dagger \mid \mathbf{y}, \mathbf{s}^i]. \end{aligned} \quad (7)$$

The vector $\mathbf{s}^i$ is the data symbol sequence estimate at the i-th iteration and $\dagger$ denotes conjugated transpose. The conditional first moment $\mathbf{m}_1^i$ can be expressed as

$$\mathbf{m}_1^i = \mathbf{R}^i \mathbf{F}^\dagger \left(\mathbf{S}^i\right)^* \mathbf{y} \quad (8)$$

where

$$\mathbf{R}^i = \left[\sigma_n^2 \mathbf{R}_h^{-1} + \mathbf{F}^\dagger \left(\mathbf{S}^i\right)^* \left(\mathbf{S}^i\right) \mathbf{F}\right]^{-1} \quad (9)$$

and $\mathbf{R}_h = E[\mathbf{h}\mathbf{h}^\dagger]$. The conditional second moment $\mathbf{m}_2^i$ can be expressed as

$$\mathbf{m}_2^i = \sigma_n^2 \mathbf{R}^i + \mathbf{m}_1^i \left(\mathbf{m}_1^i\right)^\dagger \quad (10)$$

The maximization step, which is used to generate the i+1-th data symbol sequence estimate, can be represented by

$$\mathbf{s}^{i+1} = \arg\max_s Q(\mathbf{s} \mid \mathbf{s}^i). \quad (11)$$

The iteration continues until the data symbol sequence estimate converges.

For constant envelope modulation scheme such as PSK, the second moment of the channel impulse response $\mathbf{m}_2^i$ is not necessary [8]. Moreover, $\mathbf{R}^i$ in (9) can be pre-computed and used to obtain the first moment $\mathbf{m}_1^i$. For M-ary QAM signaling, however, $\mathbf{R}^i$ in (9) should be calculated with matrix inversion at each iteration. The required inversion makes the implementation of the optimal EM algorithm impractical. In the next subsection, we propose a sub-optimal algorithm to reduce the computational complexity.

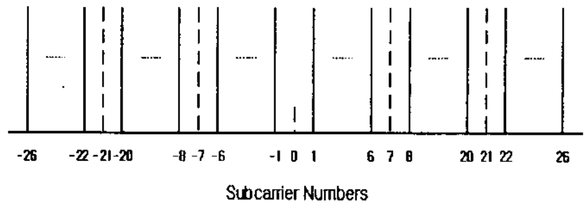

Fig. 1. Subcarrier frequency allocation

B. Sub-optimal Iterative Algorithm

Given i-th sequence estimate $\mathbf{s}^i$, which is assumed to be the same as the transmitted data symbols, we define the normalized received signal vector $\mathbf{y}'$ which is given by

$$\mathbf{y}' = \left(\mathbf{S}^i\right)^{-1} \mathbf{y} = \mathbf{F}\mathbf{h} + \mathbf{n}'. \quad (12)$$

where $\mathbf{n}' = \left(\mathbf{S}^i\right)^{-1} \mathbf{n}$. Here, we assume that for M-ary QAM scheme, $\mathbf{n}'$ is an additive white Gaussian noise vector which has the variance of

$$\sigma_{n'}^2 = \frac{1}{M} \sum_{m=1}^{M} \frac{\sigma_n^2}{|s_m|^2} = \beta \sigma_n^2 \quad (13)$$

where s_m is the m-th possible symbol. Variance scaling factor can be shown to have values of $\beta = 1$ for PSK signaling, $\beta = 1.8889$ for 16-QAM signaling, and $\beta = 2.6854$ for 64-QAM signaling when the average symbol energy is normalized to unity [10]. Using this approximation, $\mathbf{R}^i$ in (9) can be substituted by $\left(\mathbf{R}'\right)^i$ as given in the following equation

$$\left(\mathbf{R}'\right)^i = \left[\sigma_{n'}^2 \mathbf{R}_h^{-1} + \mathbf{F}^\dagger \mathbf{F}\right]^{-1}. \quad (14)$$

Note that the inverse operation in (14) can be computed in advance because it does not depend on the i-th data symbol sequence estimate $\mathbf{s}^i$. Therefore, the complexity of the receiver structure is reduced so much that its implementation is practical.

The conditional moment $\mathbf{m}_1^i$ and $\mathbf{m}_2^i$ at the i-th iteration can be easily evaluated using

$$\mathbf{m}_1^i = \left(\mathbf{R}'\right)^i \mathbf{F}^\dagger \mathbf{y}' \quad (15)$$

$$\mathbf{m}_2^i = \sigma_{n'}^2 \left(\mathbf{R}'\right)^i + \mathbf{m}_1^i \left(\mathbf{m}_1^i\right)^\dagger. \quad (16)$$

C. Initialization

The initial estimate for our iterative algorithm is obtained using pilot symbols located at pilot subcarriers and the deterministic ML estimator. In each OFDM symbol, a total of J subcarriers are dedicated to the pilot symbols. The received signals for pilot symbols at known locations, $\{p_1, p_2, \cdots, p_J\}$, can be expressed as

$$\mathbf{y}_p = \mathbf{S}_p \mathbf{F}_p \mathbf{h} + \mathbf{n}_p \quad (17)$$

where $\mathbf{S}_p = diag[s(p_1), s(p_2), \cdots, s(p_J)]$ is a known diagonal pilot symbol matrix and $\mathbf{F}_p$ is a matrix with entries of

$$[\mathbf{F}_p]_{p_j,l} = e^{-j\frac{2\pi p_j(l-1)}{N}} \qquad 1 \le j \le J,\ 1 \le l \le L. \quad (18)$$

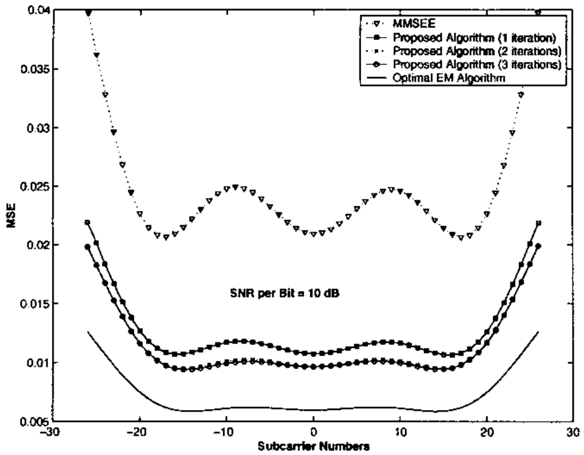

Fig. 2. MSE vs subcarrier numbers, 16-QAM

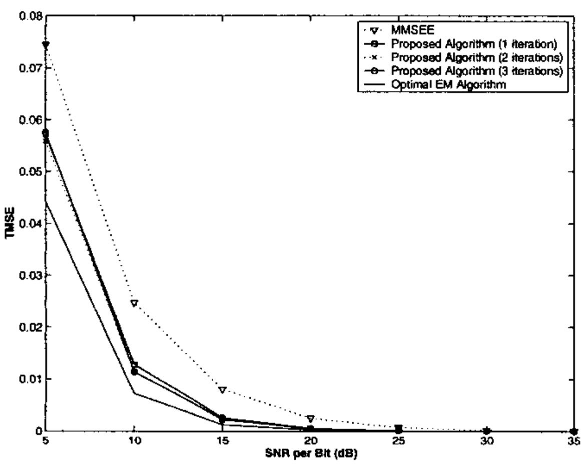

Fig. 3. TMSE vs SNR per bit, 16-QAM

We define $\mathbf{y}'_p$ similarly in (12), which is given as

$$\mathbf{y}'_p = (\mathbf{S}_p)^{-1}\,\mathbf{y}_p = \mathbf{F}_p\mathbf{h} + \mathbf{n}'_p \tag{19}$$

where $\mathbf{n}'_p = (\mathbf{S}_p)^{-1}\,\mathbf{n}$. To obtain the initial estimate by deterministic ML estimator, it is assumed that $\mathbf{h}$ is a deterministic but unknown [6]. Then, the deterministic ML estimate of $\mathbf{h}$ is given as

$$\hat{\mathbf{h}} = \left(\mathbf{F}_p^{\dagger}\mathbf{F}_p\right)^{-1}\mathbf{F}_p^{\dagger}\mathbf{y}'_p. \tag{20}$$

IV. Performance Analysis

The performance of the channel estimation methods can be expressed by the mean square error (MSE) at the k-th subcarrier

$$MSE(k) = E\left[|\hat{H}(k) - H(k)|^2\right] \tag{21}$$

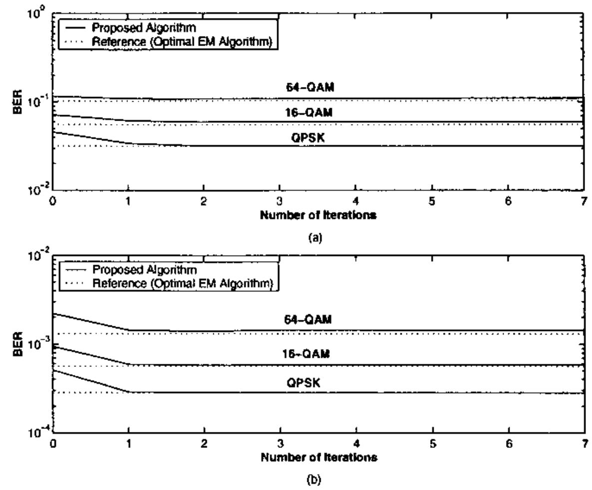

Fig. 4. BER vs the number of iterations. (a) SNR per bit = 10dB, (b) SNR per bit = 30dB

where $\hat{H}(k)$ denotes the frequency-domain channel estimate at the k-th subcarrier and $|k| \le N_\alpha$. The total mean square error (TMSE) can be also expressed by

$$TMSE = \frac{1}{2N_\alpha + 1}\sum_{k=-N_\alpha}^{N_\alpha} MSE(k). \tag{22}$$

In our iterative algorithms, the MSE at the i-th iteration can be evaluated by

$$MSE^i(k) = E\left[|\hat{H}^i(k) - H(k)|^2\right] \tag{23}$$

where

$$|\hat{H}^i(k) - H(k)|^2 = \sum_{l=1}^{L}\sum_{m=1}^{L}[\mathbf{F}]_{k,l}[\mathbf{F}]^*_{k,m}m_2^i(l,m) - 2Re\left[\sum_{l=1}^{L}[\mathbf{F}]_{k,l}m_1^i(l)H^*(k)\right] + |H(k)|^2. \tag{24}$$

In (24), $m_1^i(l)$ and $m_2^i(l,m)$ are given in (8) and (10) for the optimal EM algorithm. Moreover, the conditional moments for the proposed sub-optimal algorithm are shown in (15) and (16). The TMSE at the i-th iteration is given by

$$TMSE^i = \frac{1}{2N_\alpha + 1}\sum_{k=-N_\alpha}^{N_\alpha} MSE^i(k). \tag{25}$$

V. Simulation

The proposed algorithm for iterative channel estimation and decoding for OFDM signals is based on the EM algorithm. To obtain initial estimate, we use pilot symbols and MLE that is relatively simple [6].

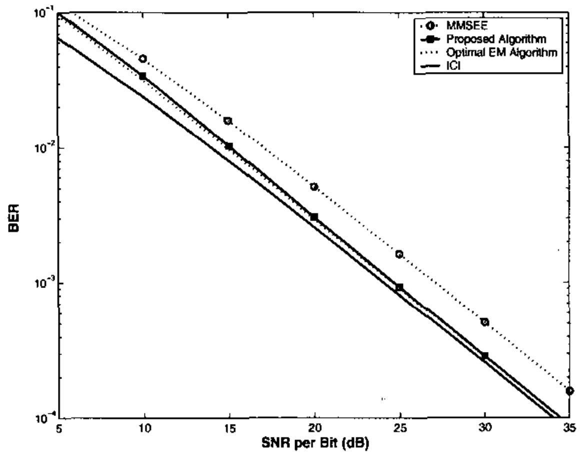

Fig. 5. Comparison of BER performances, QPSK

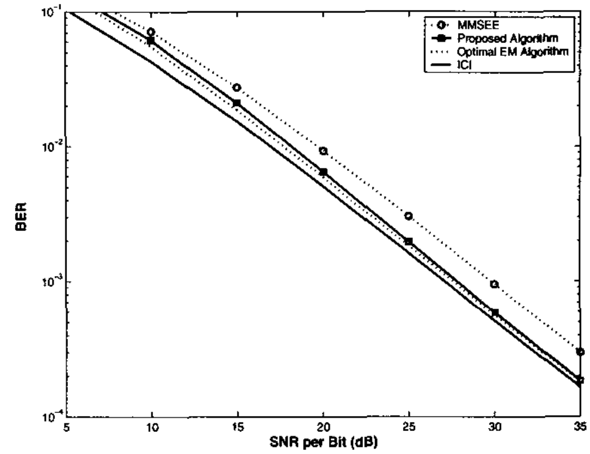

Fig. 6. Comparison of BER performances, 16-QAM

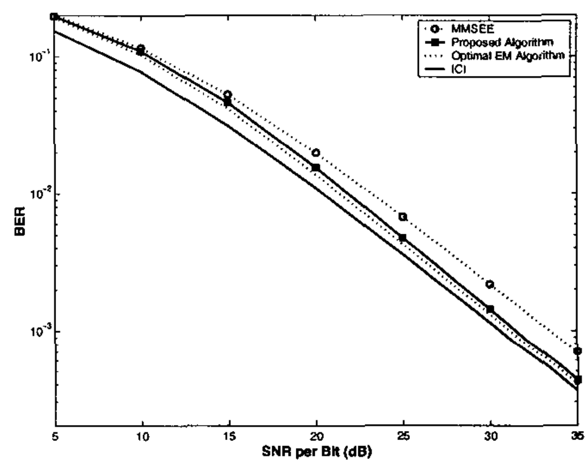

Fig. 7. Comparison of BER performances, 64-QAM

A. System Parameters

The system parameters of the simulation environments correspond to the IEEE 802.11a physical layer standard, which is summarized as follows [11].

- The DFT size N is 64.
- The number of modulated subcarriers equals to 52, that is, $N_\alpha = 26$. The subcarrier number is $0 < k \leq 26$ except the 0-th subcarrier.
- Four pilot subcarriers ($J = 4$) are located at $k = -21, -7, 7$ and 21 shown in Fig. 1 ($p_1 = -21, p_2 = -7$, $p_3 = 7$ and $p_4 = 21$).
- The OFDM subcarriers are modulated using QPSK, 16-QAM and 64-QAM.

It is assumed that the channel impulse response has L taps and the amplitude of each path varies independently according to Rayleigh distribution with exponentially decaying power delay profile, that is,

$$E[|h(l)|^2] = exp\left(-\frac{(l-1)}{10}\right), \qquad l = 1, 2, \cdots, L. \quad (26)$$

We consider the power delay profile of $L = 4$ that equals to the number of pilot subcarriers in our simulations. It is also assumed that the guard time is large enough to eliminate the intersymbol interferences.

B. Performance Evaluation

The MSE and TMSE of the proposed algorithm in (23) and (25) are evaluated for 16-QAM scheme in the simulation environment described in the above subsection and compared with those of MMSEE and the optimal EM algorithm shown in Fig. 2 and 3. In the optimal iterative algorithm, the number of iterations was not limited for the sequence estimate to converge. It is observed that MSE and TMSE of the proposed algorithm is improved at all subcarriers. Fig. 4 shows the variation of BER performances according to the number of iterations of the proposed algorithm. It is observed that one iteration after obtaining the initial estimate is enough to improve the BER performance of the proposed algorithm. Therefore, we limit the number of iteration to one.

The BER performances of the proposed algorithm for QPSK, 16-QAM and 64-QAM are compared with those of MMSE estimator, the optimal EM algorithm with unlimited iterations and the ideal channel information (ICI) case. The simulation results are shown in Fig. 5, 6 and 7. The results indicate that the proposed algorithm outperforms the conventional algorithm in all ranges of the SNR and performs nearly to the optimal EM algorithm.

VI. Conclusion

In this paper, we discuss the difficulties in the implementation of the optimal EM algorithm for QAM modulated OFDM signals. Using some approximations, we propose an EM based sub-optimal two-step iterative channel estimation and decoding algorithm with reduced computational complexity especially for QAM modulated OFDM signals.

The proposed algorithm starts iteration obtaining initial estimate using pilot symbols and deterministic ML estimator. The mean square error, the total mean square error and the BER performances are evaluated according to the number of iterations. It is seen that the number of iteration can be limited to one after obtaining the initial estimate that is enough to improve the BER and MSE performance of the proposed algorithm. The simulation results of BER performances indicate that the proposed algorithm outperforms the conventional algorithm in all ranges of SNR and performs near optimally.

ACKNOWLEDGMENT

This work was supported by ITRC and Brain Korea 21 Project in 2002.

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