Prediction of The Ultimate Strength of Composite Laminates Under In-Plane Loading Using A Probabilistic Approach

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SUMMARY

A numerical approach to predicting the ultimate strength of composite laminates is presented. In order to take into account the wide scatter in ply strengths, the basic strength distribution functions for the longitudinal, transverse and in-plane shear strengths were obtained using a two-parameter Weibull distribution. These functions, in conjunction with the Tsai-Hill failure criterion, were then used to predict the ultimate strength of the composites. Reasonable agreement was obtained between the predicted and experimental results, not only in respect of the ultimate strength, but also in relation to the stress-strain curves for the various composites.

INTRODUCTION

Fibre-reinforced composite materials have distinct advantages over traditional metallic materials in possessing low densities, high strength-to-weight ratios, and high modulus-to-weight ratios. For these reasons, they have emerged as a major class of structural material and are either used or being considered as substitutions for metals in many weight-critical components in aerospace, automotive, and other industries[1]. The difficulties in predicting the ultimate strength of composites comes from the fact that a wide scatter in ply strengths may enable composites to sustain the additional load even after a first ply failure. These factors have to be coped with in the design of laminar composite structures[2].

The scatter in measured ply strength values is greater than in stiffness. This is partly due to the failure behaviour of fibrous composites, whereby many interactions between failed fibres and the cracked matrix take place on a microscale.

There are three possible in-plane failure mechanisms on the macroscopic scale, associated with a ply composite. They are fibre, matrix, and shear failures. After the first ply failure in a laminate, in which failure is caused by any of three failure modes, the applied load is redistributed between the remaining intact plies. There is a redistribution of the loads as a result of the overall laminate stiffness reduction caused by the first ply failure. As the laminate stiffness is reduced, the load gets redistributed according to the relative stiffness of the intact plies. As the load increases, a second ply failure will occur, associated with the failure mode, so the laminate stiffness is further reduced, and the load is again redistributed according to the relative stiffness of the remaining intact plies. The load is further increased until a third ply failure occurs, and so on until the last ply failure occurs.

One method of simulating a failed ply in a laminate is to have the failed ply remaining in the same position in the laminate configuration, but to reduce the values of some of its elastic properties. This will reflect the practical case in which the failed ply is physically still there in the laminate configuration, but unable to carry any more load. The problem that arises is: by how much should these elastic values be reduced? Thus the predicted laminate strength depends on the reduction in the values. It is almost impossible to quantify this reduction for a general case of laminate configuration, in that the reduction will depend on the types of loading and the materials used[3,4,5,6]etc.

In this paper, the ultimate laminate strength was predicted using both a Weibull distribution (in order to account for the statistical behaviour of the ply strengths) and a simple ply-discount method, accounting for post-failure behaviour. Experimental and predicted results are compared and discussed.

NUMERICAL ANALYSIS AND RESULTS

Procedure for the determination of ultimate strength of laminate

The determination of the ultimate strength of a laminate requires an iterative procedure, taking into account the progressive failure of the plies in the laminate. The procedure used in this study for determining the successive loads between first ply failure and the ultimate strength of the laminate is as follows:

1. Give the mechanical loads or displacements.
2. Use classical lamination theory to find the midplane strains and curvatures[7].
3. Find the local stresses and strains in each ply under the assumed loads or displacements.
4. Compare the state of ply-by-ply stresses with the failure criterion.
5. When failure is predicted, degrade the stiffness of the failed ply or plies. This load is called the first ply failure load.
6. Apply the incremental loads or displacement, and go to step 2.
7. Repeat the above steps until all the plies in the laminate have failed. The load at which all the plies in the laminate have failed is called the ultimate strength of the laminate.

In this study, the total displacement, rather than the load, was applied in 40 equal increments. Displacement control has the merit that as the ultimate failure is reached, the load starts to decrease with increasing displacement. Therefore, the ultimate failure load is easily identified.

The Tsai-Hill criterion was used as the failure criterion. According to the Tsai-Hill criterion, failure occurs if

$$\left(\frac{\sigma_1}{X}\right)^2 - \frac{\sigma_1\sigma_2}{X^2} + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2 \geq 1 \qquad (1)$$

where σ_1, σ_2 and τ_{12} are the ply longitudinal stress, ply transverse stress and in-plane shear stress, respectively and X, Y and S are the ply longitudinal strength, ply transverse strength and ply shear strength, respectively.

Once failure is predicted, the elastic properties of the ply are reduced, based on the ply-discount method as follows.

$$[E_1, E_2, G, \nu_{12}] \rightarrow [0.25E_1, 0.25E_2, 0.25G, 0.25\nu_{12}] \qquad (2)$$

where E_1, E_2, G, ν_{12} are the longitudinal modulus, transverse modulus, shear modulus and Poisson's ratio, respectively.

A parametric study was carried out to examine various stiffness reduction factors and the above values were chosen to achieve a realistic simulation of experimental results.

Procedure for determination of statistical strength distributions

Realistically, the quantities, X, Y, S in Equation (1) should be regard as random variables. Instead of a normal distribution, a more realistic representation of the strength variation of a composite is a two-parameter Weibull distribution and its cumulative density function is as follows[8].

$$F(x_i) = 1 - e^{-\alpha \cdot x_i^{\beta}} \qquad (3)$$

where x_i is the strength of the i-th specimen, α is the location parameter and β is the shape parameter, respectively.

The procedure for the determination of these two parameters is as follows.

Starting with the smallest number, arrange the observed strength values in ascending order, and assign the following probability of failure value for each strength.

$$F_i = \frac{i}{n+1} \tag{4}$$

where n is the total number of specimens tested, and i varies from 1 to n.

Thus, the plot of $\ln x_i$ versus $\ln[-\ln(1-F_i)]$ has been made using a linear least-square method to a fit a straight line to the data. The slope of this line is equal to β and its intersection with the $\ln x_i$ axis is equal to $\ln \alpha$.

All test specimens were prepared using UPN 116B carbon/epoxy prepreg from SKI Co. and the measurements of fundamental elastic properties were made at a crosshead speed of 1.3mm/min with a 3 tonne loadcell in the MTS machine. These properties are shown in Table 1.

Table 1 Fundamental elastic properties of unidirectional CFRP

($V_f = 0.6$)	
Longitudinal modulus, E_1	168.8 GPa
Transverse modulus, E_2	7.44 GPa
Shear modulus, G	6.89 GPa
Poisson's ratio, ν_{12}	0.25

The scales and shape parameters in the two-parameter Weibull distribution of the three strengths were obtained by using the linear least-square method presented in Table 2 and shown in Figures 1, 2 and 3, respectively. 22 specimens were tested to evaluate the parameters α and β for each X, Y, S. In these figures, R^2 represents the coefficient of determination.

Table 2 Weibull parameters for strength X, Y, S

	α	β
X	6.46E-20	5.92
Y	2.58E-12	6.78
S	1.06E-50	22.38

Once the values α and β for the longitudinal, transverse and in-plane shear strengths, respectively, were obtained, random numbers, F_i, were generated in the interval of (0, 1) and substituted into the following Equation (5) and a number of sets of (X, Y, S) random values, x_i, were formed.

$$x_i = \left[\frac{1}{\alpha}\ln[\frac{1}{1-F_i}]\right]^{\frac{1}{\beta}} \tag{5}$$

Figure 1 Determination of Weibull parameters for the longitudinal strength by the linear least-square method

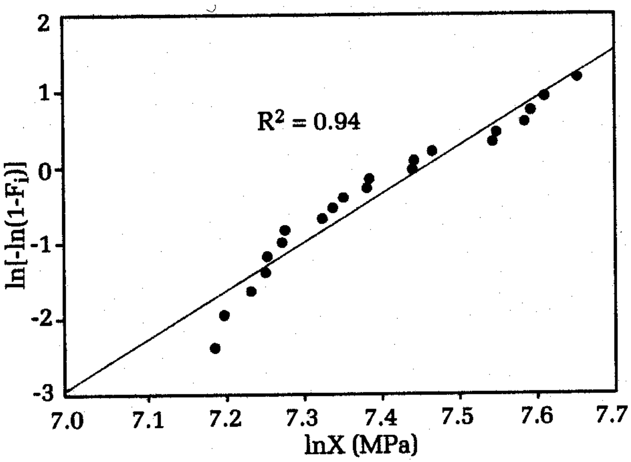


Figure 2 Determination of Weibull parameters for the transverse strength by the linear least-square method

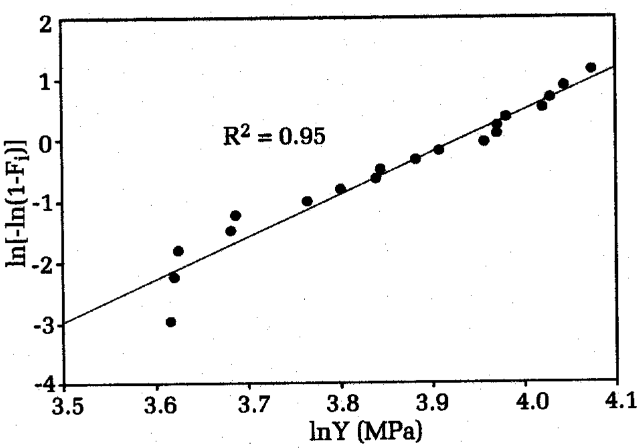
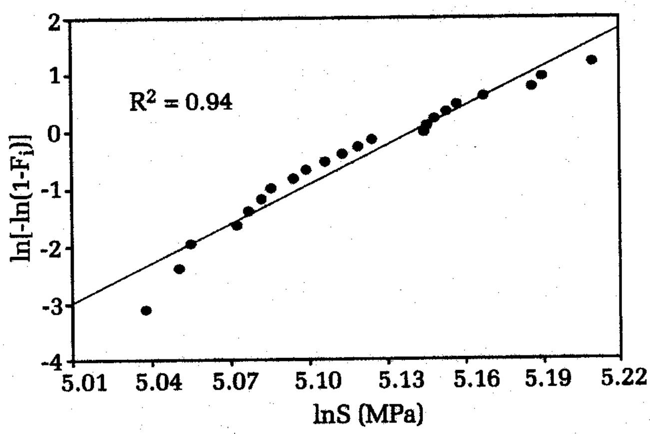


Figure 3 Determination of Weibull parameters for the shear strength by the linear least-square method



At each displacement increment for each ply, 300 sets of (X, Y, S) random values were generated. Therefore, even plies with the same ply angle in any laminate may have different sets of (X, Y, S) random values and different failure loads. The ply failure is assumed to occur when the probability of failure reaches 50%, according to the Tsai-Hill failure criterion, with an increase in displacement. That is, a ply is considered to have failed if Equation (1) is satisfied in more than 150 out of 300 trials.

The strength prediction procedure described above can be illustrated in a program flow chart as shown in Figure 4.

To verify the program described above for predicting laminate strength under in-plane loading, the ultimate strength of two stacking sequences of laminates, $[0_2/\pm45]s$, $[0_2/\pm60]s$ were examined.

The numerical predictions of the ultimate strength of $[0_2/\pm45]s$ and $[0_2/\pm60]s$ laminates, together with the stress-strain curves were compared with the experimental results for two laminates, as shown in Figures 5 and 6. It is clear that the numerical prediction fits reasonably with the experimental results, and that the shape of the stress-strain curve of the composites fits the experiments well.

Figure 4 Flow chart for predicting ultimate strength

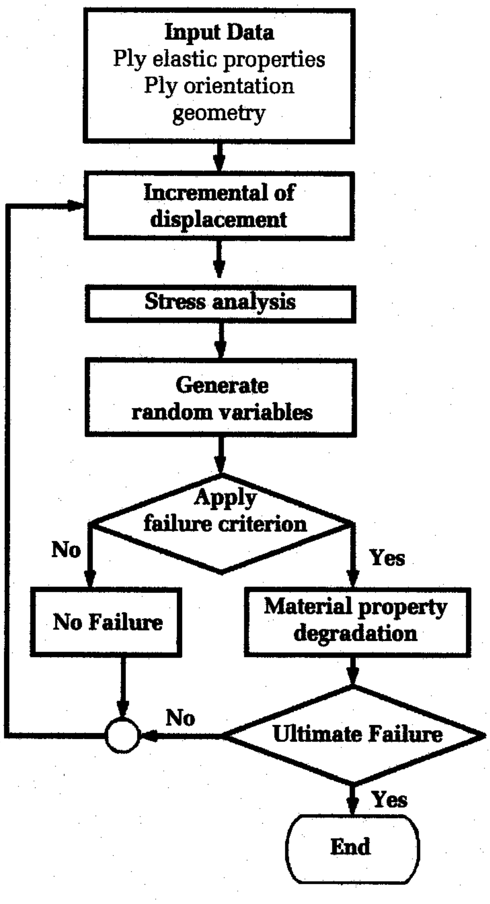


Figure 5 Comparison of numerical prediction of stress-strain curves up to ultimate strength with experimental results for a $[0_2/45/\text{-}45]_s$ laminate

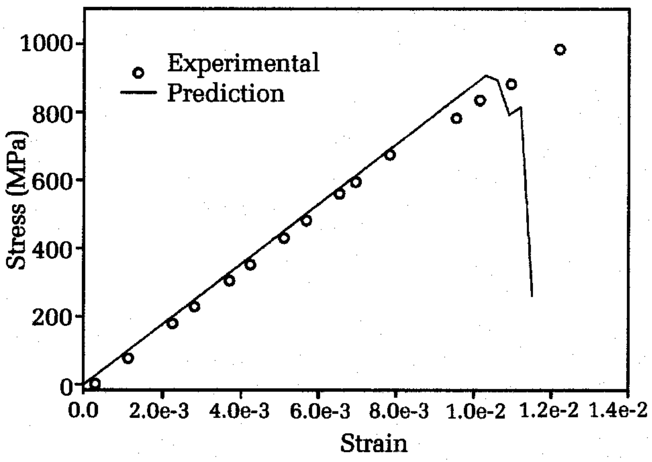
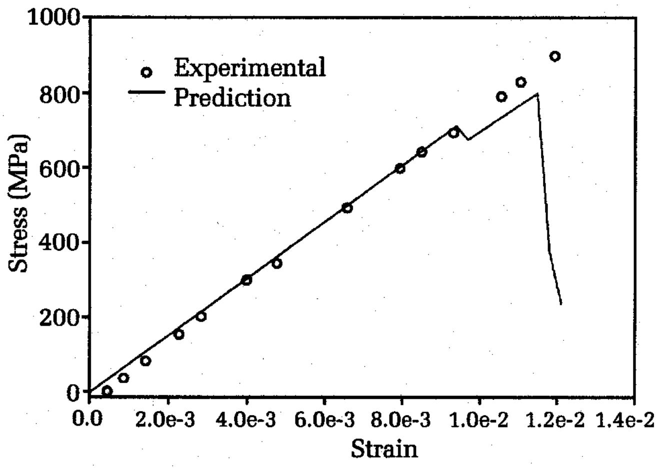


Figure 6 Comparison of numerical prediction of stress-strain curves up to ultimate strength with experimental results for a $[0_2/60/\text{-}60]_s$ laminate



CONCLUSIONS

An approach to predicting the ultimate strength of laminar composites having a wide scatter in ply strength was presented. Distribution functions for the longitudinal, transverse and in-plane shear strengths were obtained, and used to predict the ultimate strength of the various stacking sequences of the composites.

A comparison between the numerical and experimental results shows reasonable agreement in the prediction of ultimate strength and stress-strain curves of laminate composites.

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