Optimal Contracts of Public-Private Partnerships with Demand Risk

Daechang Kang *

The paper analyzes the service provision of infrastructure from the aspect of demand risk sharing. The society benefits more under the public-private partnership (PPP) than under government operation, because the government can transfer some risks to private firms through PPP. To reduce total cost, the government is more likely to apply PPP to projects with large risk factors. Using a two-period model, the paper examines the dynamic features of the optimal contract under the PPP. The optimal incentive scheme should be stronger during the second than the first period. As the performance target becomes lower, the incentive power increases in both periods with a higher increase in the first period. As the intertemporal externality becomes stronger, the incentive power increases in both periods with a higher increase in the second period. As the risk or risk aversion increases, the incentive power decreases in both periods, which resembles the static feature.

Keywords: Public-private partnerships, Incentive, Risk sharing, Intertemporal externality

JEL Classification: D8, H54, H57, L5

I. Introduction

Under public-private partnerships (PPPs), the central government or local authority arranges a long-term contract with a private firm for the service provision of building infrastructure. In this scheme, the firm takes responsibility for building infrastructure, financing investment, and operating the facilities for a fixed period. The government owns the facilities and the

* Research Fellow, Korea Institute for International Economic Policy, 246 Yangjaedaero, Seocho-gu, Seoul 137-747, Korea, (Tel) +82-2-3460-1166, (Fax) +82-2-3460-1044, (E-mail) dkang@kiep.go.kr. I am grateful to the anonymous referees for their helpful comments.

private firms gain profits by operating the facilities. With revenue generated from the operation, private firms can then recover the cost of investment and operation.[1] Under the traditional procurement of infrastructure service, private firms take responsibility only for building the facilities, whereas the government finances investment and operates the facilities. The main difference between the two schemes is the right of operation.

One of the main reasons for the existence of PPPs is the government budget pressure on infrastructure spending. PPP is generally applied to the building of infrastructure facilities such as roads, subways, seaports, airports, water system, and utilities. Currently, PPPs have also been applied to IT systems, leisure centers, hospitals, and school buildings, and have even extended to asylums, barracks, and prisons. Operating these facilities is regarded as the responsibility of the government, with the rationale of public good characteristics, strong externalities, or monopolistic inefficiency of the private provision of infrastructure. However, as the government budget for infrastructure investment becomes tighter, the governments have come up with the PPP system, through which private firms finance investment and the government delegates the right of operation to private firms for a fixed period, thereby allowing private firms to gain profits.

PPPs are being applied worldwide across Europe, Canada, the U.S., Latin America, and Asia, and their importance in the service provision of building infrastructure continues to increase (Malhotra 1997; Harris 2003; GAO 2008). Korea is one of countries that have witnessed a rapid expansion in PPPs. When PPPs first began in 1995, investment on infrastructure by PPP was merely 0.18% of that made through traditional government procurement; this ratio then increased to 17.4% in 2006. The portion is projected to increase further.[2]

Bundling the tasks of building, financing, and operating a project is a key feature of PPP. Task bundling allows private firms to form consortiums that include construction firms, facility management firms, and financial firms to run the project and through which the government enters into a contract for the project. One of the most important arguments for PPPs is that bundling the tasks enables private firms to provide better infrastructure service because they internalize the opportunity for efficiency. Thus,

[1] Ownership of properties belongs to the government, which makes PPP different from privatization. Shleifer (1998) discusses the ownership and operation between the government and the private sector from the contract perspective.

[2] Ministry of Planning and Budget (2007).

most studies focus on the bundling issue in a static model. Hart (2003) argues that PPP is better than the conventional unbundling provision if the quality of service is well specified. In a multi-tasking moral hazard model, task bundling is more efficient when there is positive externality of quality that improves the effort on cost-saving (Bennett and Iossa 2006; Iossa and Martimort 2008; Martimort and Pouyet 2008). However, Engel, Fischer, and Galetovic (2008) maintain that PPP does not ensure efficiency, because private finance does not save the distortionary taxation, and is equivalent to traditional procurement with regard to the burden of the public.

Another critical aspect of PPPs is the allocation of risks between the government and private firms. Private firms bear substantial risks because they take responsibility for the operation of the facilities for a very long period. The usual terms of the contract lasts over 20 years to even 50 years. In a PPP contract, the government specifies the service provision requirements as well as its basic standards, but it leaves it to the private firms to decide on how it must provide the service and meet the pre-specified standards. Thus, private firms have to take a risk while providing service under the PPP. The previous works do not address this aspect directly. Kang (2006) discusses the risk-sharing aspects and compares the various practices of PPPs in Korea.

The current paper examines the traditional government operation and PPP in terms of demand risk. We apply the moral hazard model to the PPP scheme.[3] We compare the operation level, expected total cost, and the expected social welfare under the two different operation schemes.

Recently in Korea, the first PPP for railroad service, the Incheon Airport Railroad, was transformed to traditional public operation due to the low demand. With an expected turnout of 200,000 passengers per day, the railroad between Incheon Airport and Seoul started operation in 2007; however, demand only realized about 7% of the projection and did not increase substantially. The Korean government had to compensate the private consortium for huge revenue losses with tax by the contract. The contract stipulates that if the realized demand is below 90% of the projected demand, the government pays a subsidy proportional to the difference of 90% of the projected demand and the realized demand. Thus, the Korean government ended up paying subsidy of about 100 billion Korean won to the private operator in 2008 and 160 billion Korean won in 2009.[4]

[3] The adverse selection problem is not that severe under PPPs because the government sets pre-specified qualifications for the contract.

It was projected that the amount would increase over time and would reach 5 trillion Korean won by 2040. The public criticized the construction of the Incheon Airport Railroad *via* PPP as a waste of tax money; consequently, the Korean government took over the right of operation from the private consortium in 2009.

This incident raises many issues with regard to the demand risk of PPPs from the dynamic perspective. Did the demand turn out to be extremely lower than the projection because the private operator did not exert effort to provide better service? Was it due to the inadequately generous minimum revenue guarantee to the private operator? Was it because the private operator faced a too low incentive in the earlier stage? Was the contract dynamically optimal in terms of risk sharing between the government and the operator?

In order to examine the dynamic optimal contract of the PPP, we set up a two-period model and study how incentive power changes over time. The government tries to make the continuance of the private operation in the later period contingent on the performance in the earlier period as can be seen in the case of Incheon Airport Railroad. To capture this feature, we incorporate the performance evaluation into the model as an institutional factor. Considering the bundling efficiency of PPPs in a dynamic setting, the model takes into account intertemporal externality. The operation effort of the operator in the earlier period can increase the demand in the later and earlier periods. For example, the well-known quality of service in a leisure center in the earlier period is likely to boost the demand in the later period due to reputation or lock-in effect, although the operator does not induce the operation effort in the later period.

This paper is organized as follows. In Section 2, we set up a simple static model for the service provision of infrastructure with demand risk. We include government operation and PPP, and compare the performances of the two schemes in terms of expected social welfare and expected total cost. In Section 3, we analyze PPP in a dynamic environment by incorporating intertemporal externality and performance evaluation. We identify the incentive scheme over time, and examine the changes in the incentive scheme corresponding to changes in the environment. Finally, in Section 4, we conclude and summarize the findings of the paper.

[4] Ministry of Land, Transport, and Maritime Affairs (2009).

II. Simple Static Model

We suppose that the cost of building infrastructure facility is given fixed I. To focus on the risk sharing between the government and the operator, we assume that consumers have an inelastic demand for the service as follows:[5]

$$D(p)=\begin{cases} d+e+\varepsilon & \text{if } p\le c \\ 0 & \text{if } p>c \end{cases}, \tag{1}$$

where p denotes the price of service and c the reservation price. Consumers buy the service if the price is lower than or equal to the reservation price, otherwise they do not buy at all. It is assumed that random variable ε is normally distributed with mean 0 and variance σ^2. The variance captures the demand risk. e is the operation effort, and d is the base level of demand without effort. Here, it is assumed that operation effort e and shock ε are independent.[6]

For the account rule, we assume that the government collects revenue from consumers, uses a portion of it to cover the cost for management of the project, and spends the remaining revenue for consumers. Thus, the revenue net of the cost is social welfare. The revenue is approximately $R \approx c(d+e+\varepsilon)$, which is the value evaluated at the reservation price c for a sufficiently large base level of demand compared to variance, σ^2, and the expected revenue to $E(R)\approx c(d+e)$.

Inducing an operation effort costs the operator, we denote $m(e^2/2)$ to be the cost of the operation effort. The coefficient m accounts for efficiency by bundling effect. If a private firm operates the facilities, such firm could operate at a lower cost than the government with the same operation effort level due to the bundling opportunity. As the operation becomes less efficient, m becomes higher. Thus, m is lower for the PPP than for the government operation. We set m as 1 for the PPP, which is

[5] Thus, we do not analyze how the service price is determined.

[6] The effort of the private operator might affect the shock, but it would be very difficult for the private operator to affect the risk itself because the fluctuation arises from the behavior of the demanders. In the context of accounting reporting system, Son-Ku Kim (2005) analyzes a multi-period principal-agent model in which an agent can influence the variance as well as the mean of a firm's income stream. He shows that the contractual efficiency increases as the reporting frequency increases. See Kim (2005).

greater than or equal to 1 for the government operation.

The citizens of a country have to bear the demand risk. For simplicity, we use the mean-variance utility function for the expected utility. We assume the risk attitude of agents in the society to be as follows:

***Assumption* 1.** The private sector is risk-averse and the government is risk-neutral.

We denote r to be the common degree of risk-aversion of consumers and the private operator, and $\mathrm{Var}(x)$ variance of the variable x. We represent the risk cost as $r\{\mathrm{Var}(x)/2\}$. Then, we denote TC as the total cost to manage the project or the operation cost and risk cost, which is assumed to be financed by taxes. Social welfare represented as $W=c(d+e+\varepsilon)-TC$.

A. Government Operation

The government exerts operation effort to increase revenues and covers the operation cost and risk cost from the revenue. As the revenue decreases, consumers have less benefit from the government. Given that the risk to society is from the fluctuation of revenue and the government does not share the risk with the private firms under the government operation, the risk cost is represented as $r\{\mathrm{Var}(R)/2\}=r\{(\sigma^2c^2)/2\}$. The government itself does not cover the cost generated from a shock with insurance, but it makes people take all the cost from the shock by reducing the benefit. This means that risk cost forms part of the costs that society has to take. Thus, the total cost is $TC=m(e^2/2)+r\{(\sigma^2c^2)/2\}$.

The expected welfare is given by:

$$E(W)=c(d+e)-m\frac{e^2}{2}-r\frac{\sigma^2c^2}{2}. \tag{2}$$

The government chooses the operation effort to maximize the expected social welfare. By solving the first order condition, we have:

$$e=\frac{c}{m}. \tag{3}$$

The risk-neutral government does not consider the social demand risk for choosing the operation level, because it does not have the incentive to share the risk. The government induces less effort as the government operation becomes less efficient.

We obtain the expected total cost[7] and the expected social welfare from the government operation as follows:

$$E(TC) = \frac{1}{2}(\frac{1}{m} + r\sigma^2)c^2 \text{ and } E(W) = cd + \frac{1}{2}(\frac{1}{m} - r\sigma^2)c^2. \tag{4}$$

As the operation is less efficient, both the expected total cost and the expected social welfare are lower. As the risk factor increases, the expected total cost increases whereas the expected social welfare decreases.

B. PPP

We interpret the PPP contract in a principal-agent framework; therefore, we treat the government as a principal and the private operator as an agent of service provision. The operation effort is unverifiable and cannot be contracted upon because of the demand risk. When the demand is low, the private operator can assert that the market condition is not good, although it exerted sufficient operation effort. When demand is high, without the operation effort, the private operator can maintain that it is a result of the operation effort it has exerted.

The government and the private operator share risks by dividing the revenue between two parties. The risk-neutral government offers transfer, t, to the private operator to induce operation effort. With the transfer, the private operator covers the operation cost and risk cost for the service provision. Thus, the transfer in PPP scheme is comparable to the total cost in the government operation scheme. As the private operator is risk-averse, it pays the risk cost because of demand risk. The risk cost to the private operator is represented by $r\{\mathrm{Var}(t)/2\}$, where $\mathrm{Var}(t)$ denotes variance of the transfer. The utility of the operator is the transfer net of the disutility and the risk cost, $U = t - (e^2/2) - r\{\mathrm{Var}(t)/2\}$. For simplicity, we use the mean-variance utility function for the expected utility of the private operator given by $E(U) = E(t) - (e^2/2) - r\{\mathrm{Var}(t)/2\}$.

We consider a linear transfer scheme, $t = \alpha + \beta R$, which is known to be robust to the specifications (Holmstrom and Milgrom 1987). α is the fixed subsidy to the operator, and is paid regardless of the revenue level.

[7] The total cost is independent of the error term, because the government does not consider the risk when it chooses the operation level. Thus, the expected total cost is merely the total cost itself.

Meanwhile, β is the share of the revenue belonging to the private operator, where share $1-\beta$ goes to the government and share β captures the intensity of the incentive scheme. There is a trade-off between the incentive and risk sharing. Under a payment system of the user fee only characterized as $\alpha=0$ and $\beta=1$, the private operator has the strongest incentive to raise revenues, but bears all the demand risk. Under a payment scheme of availability only represented as $\alpha>0$ and $\beta=0$, the private operator receives a fixed fee from the government and the government takes all the risk; however, the private operator has no incentive to raise revenues.

The optimal linear contract maximizes the expected social welfare, making the private operator sign the contract and inducing the operation effort level that is best for the private operator. The government should provide the private operator with the expected utility greater than or equal to a certain level[8] to induce the private operator to take the project. The private operator does the best to protect its interest, taking into account the cost related to the operation effort and the risk as well as the share of the revenue. Thus, the contract can align the private operator's effort both to social welfare and its own interest.

The problem is that the government chooses α and β to maximize the expected social welfare as follows:

$$E(W) = -\alpha + (1-\beta)c(d+e). \tag{5}$$

st.

(i) The participation constraint is given by:

$$E(U) = \alpha + \beta c(d+e) - \frac{e^2}{2} - r\frac{\beta^2 c^2 \sigma^2}{2} \geq 0. \tag{6}$$

(ii) The incentive compatibility constraint is given by:

$$\max_{e'} E(U) = \alpha + \beta c(d+e) - \frac{e^2}{2} - r\frac{\beta^2 c^2 \sigma^2}{2}. \tag{7}$$

We set the reservation utility of the private operator as zero for simplicity. From the incentive compatibility constraint (7), the best operation

[8] For simplicity, the reservation level is set to 0 in the paper.

level is given as:

$$e = \beta c. \tag{8}$$

The operation effort is proportional to the incentive, because the private operator shares in the revenue of the PPP.

The optimal linear payment scheme pays only the reservation utility, because the participation constraint should be binding. Thus, α is determined from Equation 6 with the equality. By substituting α into Equation 5, we transform the expected social welfare as follows:

$$E(W) = c(d + e) - \frac{e^2}{2} - r\,\frac{\beta^2 c^2 \sigma^2}{2}. \tag{9}$$

By substituting the incentive compatibility constraint $e=\beta c$ (Equation 8) into Equation 9, we have the following expected social welfare:

$$\max_{\beta}\ c(d + \beta c) - \frac{(1 + r\sigma^2)}{2}\,\beta^2 c^2. \tag{10}$$

The government chooses β to maximize the expected social welfare, and α is determined by the participation constraint with β. The government then designs and offers a contract to implement the second-best level of effort given by:

$$\beta = \frac{1}{1 + r\sigma^2} < 1. \tag{11}$$

The optimal second-best effort level is given by:

$$e = \frac{c}{1 + r\sigma^2}. \tag{12}$$

At the margin, as one unit of operation effort increases, the expected social welfare increases by the fixed amount, c, regardless of the level of operation effort. On the other hand, as one unit of operation effort increases, the expected social welfare decreases in proportion to the level of operation effort exerted at the rate of effort-inducing cost, 1, and the rate of risk cost, $r\sigma^2$. This is because the operation effort costs disutility to the firm, and the government has to compensate it for risk sharing in order to induce the firm to exert effort. Thus, the term, $1+r\sigma^2$, represents the rate of the cost of the second-best contract. Share β then represents the ratio of welfare gain to the rate of welfare cost of the second best contract at the unit price of the service provision.

From the discussion on the feature of the optimal contract, it is obvious that, in order to balance the trade-off between incentive and risk sharing, the incentive power of the optimal contract should be reduced corresponding to the higher demand risk and risk aversion of the private operator. We can interpret this feature in a dynamic setting where demand risk is becoming smaller because of learning by doing. As the private operator runs the project, both the private operator and the government know the demand situation better. The private operator should bear the demand risk more and the incentive increases as time goes on.

We obtain the expected transfer and the expected social welfare from the PPP as follows:

$$E(t)=\frac{1}{2}\frac{1}{(1+r\sigma^2)}c^2 \text{ and } E(W)=cd+\frac{1}{2}\frac{1}{(1+r\sigma^2)}c^2. \tag{13}$$

As the risk factor increases, both the expected transfer and expected social welfare decrease.

C. Government Operation vs. PPP

As mentioned earlier, we aim to evaluate government operation and PPP in terms of the expected social welfare and expected total cost. The expected social welfare is the primary criterion for comparing the two schemes. However, considering that the PPP has been conceived to avoid budget pressure on infrastructure spending, the size of the expected total cost of each scheme is also critical in choosing a scheme for a specific project.

The society benefits more from PPP than from government operation. The expected social welfare is higher under PPP than under government operation, regardless of the operation level. By considering $m\geq 1$, this point can be seen easily when the expected social welfare in Equations 4 and 13 are compared. This is due to the fact that the government can reduce the risk cost by transferring some of the demand risk to private firms under the PPP, which it cannot do under traditional government operation. Using Equations 3 and 8, we can identify the incentive difference of the two schemes in terms of risk sharing. Under PPP, private firms take into account the demand risk when they choose their operation level.

***Proposition* 1.** The society benefits more from PPP than from traditional government operation.

From the proposition, we can see that PPP is preferable to traditional government operation because of the advantages of risk sharing and the bundling effect. Both this paper and earlier works show the same result of the welfare comparison, although this paper evaluates two schemes in the aspect of risk sharing, whereas earlier research have done so in terms of bundling effect. It should be noted that the welfare comparison in this work is not dependent on the bundling effect. Thus, even when there is no bundling efficiency, which is represented as $m=1$, the expected social welfare is higher under PPP than under government operation. The welfare gain from the PPP stems from risk sharing and not from the bundling effect.

Another important issue related to the operating scheme is identifying under which scheme the expected total cost is lower compared with others. In practice, when the government chooses an operating scheme for infrastructure service provision, it compares two schemes in terms of the expected total costs rather than the expected social welfare. The relative size of the expected total cost is dependent on operation efficiency and risk factor, as we can see from the total costs shown in Equations 4 and 13. If $m \leq 1+r\sigma^2$, the expected total cost is always lower under PPP than under government operation. We can interpret $m \leq 1+r\sigma^2$ to mean that the risk factor is more significant than operation efficiency. This means that when the risk factor is considerably large, the government can substantially reduce the expected total cost by transferring the demand risk to private firms. If $m > 1+r\sigma^2$, the relative size of the expected total costs is indefinite. For a fairly small risk factor, the total cost could be higher under PPP than under government operation. This is because the government pays the private firm a considerably large risk cost in order to induce a high operation effort from the private firm. In this case, the government could decrease expected total cost by operating the facilities itself. This implies that the government is more likely to apply PPP to projects with high risk factor under the budget pressure.

III. Dynamic Optimal Contract under PPP

PPP has a long-term contract feature and requires operation effort from the private operator to enhance demand in the earlier period and ensure a long-lasting effect on the demand in the later period. The effort of the private operator in the earlier period can have a positive effect on demand in the later period. The government would then want to have the project to be continued in the later period for as long as possible. Knowing that the project is likely to be long term, the private operator may not exert sufficient operation effort during the earlier period. Responding to that possibility, the government tries to induce effort in the earlier period by adopting performance evaluation. Thus, the contract can take into account the intertemporal externality of the operation effort and the performance evaluation.

We can imagine two probable positions for the incentive scheme over time. On one hand, the private operator may argue that the project can end earlier than the desired time point if the environment is insecure in the early stage, because they cannot meet the costs with unexpected low demand. Thus, they can demand more security in the early period to continue the operation. On the other hand, the government may maintain that if the private operator does not exert sufficient effort in the earlier period, the project can terminate earlier than the preferred time point due to low demand. Therefore, the government may induce the private operator to face the stronger incentive to keep on the project. Both of the positions are reasonable. The paper analyzes which position is supported by the dynamic optimal contract.

A. Model

Assume that there are two periods of the contract. The demand for the service provision in the first and second periods is as follows:

$$D_1(p) = \begin{cases} d + e_1 + \varepsilon & \text{if } p \le c \\ 0 & \text{if } p > c \end{cases},$$
$$D_2(p) = \begin{cases} d + \lambda e_1 + e_2 + \varepsilon & \text{if } p \le c \\ 0 & \text{if } p > c \end{cases}, \tag{14}$$

where $D_1(p)$ and $D_2(p)$ denote the first and second period demands, respectively, and e_1 and e_2 denote the first and second period operation efforts of the private operator, respectively.

The operation effort in the first period affects the second period demand as well as the first period demand. Here λ captures the degree of the intertemporal externality of the first period operation effort. We assume that for the second period demand, the first period operation effort is effective, but it is no more effective than the second period operation effort. Thus, we set $0<\lambda<1$.[9] The intertemporal externality is the dynamic feature of the positive externality of the bundling in the static model.

The government offers the linear transfer scheme of two periods, $(\alpha_1+\beta_1 R_1,\ \alpha_2+\beta_2 R_2)$. Here, the transfer to the private operator in the second period is contingent on the performance evaluation. If the private operator passes the performance evaluation in the first period, the private operator continues with the project and obtains the transfer in the second period. If the private operator fails, the government takes over the right of operation and the private operator gets nothing in the second period. We assume that the performance target of the period one is set and the probability to pass the performance review is given δ, with $0<\delta<1$. We capture the intensity of the performance target with the probability δ. The higher the performance target, the lower the probability of the continuance.

The optimal linear contract maximizes the expected social welfare over two periods, making the private operator sign the contract and inducing the best operation effort level for each period for the private operator. The government should provide the private operator with the expected utility that is greater than or equal to 0 in each period in order to induce the private operator to take the project. If the expected utility of the first period is less than 0, the private operator may not want to be involved with the project. If the expected utility of the second period is less than 0, the private operator may try to stop operating the facilities even though it passes the performance evaluation after the first period. The private operator performs what is best for its interest over two periods, taking into account the intertemporal externality of the first

[9] It is possible that the first period effort is more effective than the second period effort for the second period demand. In this case, the first period effort is dominant, and the importance of the second period effort becomes negligible. Thus, the contract is very likely to be renewed towards the second period as a reward for the high demand in the second period, even though the performance in the first period is poor. Thus, the dynamic contract of the two periods is not that critical for inducing operation effort.

period operation effort.

The problem occurs when the government chooses α_1, β_1, α_2, and β_2 to maximize the expected social welfare as follows:

$$\max_{\alpha,\beta_1,\alpha_2,\beta_2} -\alpha_1 + (1-\beta_1)c(d+e_1) + \delta\left[-\alpha_2 + (1-\beta_2)c(d+\lambda e_1 + e_2)\right]. \tag{15}$$

st.

(i) The participation constraints are given by:

$$\begin{aligned} & E_1(U) = \alpha_1 + \beta_1 c(d+e_1) - \frac{e_1^2}{2} - r\frac{\beta_1^2 c^2 \sigma^2}{2} \geq 0, \\ \text{and } & E_2(U) = \alpha_2 + \beta_2 c(d+\lambda e_1 + e_2) - \frac{e_2^2}{2} - r\frac{\beta_2^2 c^2 \sigma^2}{2} \geq 0. \end{aligned} \tag{16}$$

(ii) The incentive compatibility constraint is given by:

$$\begin{aligned} & \max_{e_1,e_2} \alpha_1 + \beta_1 c(d+e_1) - \frac{e_1^2}{2} - r\frac{\beta_1^2 c^2 \sigma^2}{2} + \\ & \delta\left[\alpha_2 + \beta_2 c(d+\lambda e_1 + e_2) - \frac{e_2^2}{2} - r\frac{\beta_2^2 c^2 \sigma^2}{2}\right]. \end{aligned} \tag{17}$$

From the incentive compatibility constraints, we can obtain the first order conditions as follows:

$$\begin{aligned} & e_1 = \beta_1 c + \delta\lambda\beta_2 c \quad \text{of} \quad e_1, \\ \text{and} \quad & e_2 = \beta_2 c \quad \text{of} \quad e_2. \end{aligned} \tag{18}$$

The optimal linear payment scheme pays just the reservation utility for each period, because the participation constraint should be binding. Thus, α_1 and α_2 are determined from Equation 16 with the equality. By substituting α_1 and α_2 into Equation 15, we transform the expected social welfare as follows:

$$\max_{\beta_1,\beta_2} c(d+e_1) - \frac{e_1^2}{2} - r\frac{\beta_1^2 c^2 \sigma^2}{2} + \delta\left[c(d+\lambda e_1 + e_2) - \frac{e_2}{2} - r\frac{\beta_2^2 c^2 \sigma^2}{2}\right]. \tag{19}$$

By substituting the incentive compatibility conditions (18) into the welfare function (19), we have the expected dynamic social welfare function given by:

$$\max_{\beta_1,\beta_2} c(d + c(\beta_1 + \delta\lambda\beta_2)) - \frac{c^2(\beta_1 + \delta\lambda\beta_2)^2}{2} - r\,\frac{\beta_1^2 c^2\sigma^2}{2}$$
$$+\delta\left[c\left\{d + \lambda c(\beta_1 + \delta\lambda\beta_2) + \beta_2 c)\right\} - r\,\frac{\beta_2^2 c^2\sigma^2}{2}\right] \quad (20)$$

The government chooses β_1 and β_2 to maximize the expected social welfare, and α_1 and α_2 is determined by the participation constraint β_1 and β_2.

Next, we obtain the first order conditions of β_1 and β_2 for the maximization as follows:

$$1 - (\beta_1 + \delta\lambda\beta_2)e_1 - r\beta_1\sigma^2 + \delta\lambda = 0 \;\text{ of } \beta_1, \quad (21)$$

and

$$\lambda - \lambda(\beta_1 + \delta\lambda\beta_2) + \delta\lambda^2 + 1 - \beta_2 - r\sigma^2\beta_2 = 0 \;\text{ of } \beta_2. \quad (22)$$

Solving the two first order conditions of β_1 and β_2, we have β_1 and β_2 as follows:

$$\beta_1 = \frac{1 + r\sigma^2 + \delta\lambda r\sigma^2}{(1 + r\sigma^2)^2 + \delta r\sigma^2\lambda^2}, \quad (23)$$

and

$$\beta_2 = \frac{1 + r\sigma^2 + \lambda r\sigma^2 + \delta\lambda^2 r\sigma^2}{(1 + r\sigma^2)^2 + \delta r\sigma^2\lambda^2}. \quad (24)$$

By substituting (23) and (24) into (18), we have e_1 and e_2 as follows:

$$e_1 = \frac{1 + r\sigma^2 + \delta\lambda r\sigma^2 + \delta\lambda\left[1 + r\sigma^2 + \lambda r\sigma^2 + \delta\lambda^2 r\sigma^2\right]}{(1 + r\sigma^2)^2 + \delta r\sigma^2\lambda^2}\,c, \quad (25)$$

$$e_2 = \frac{1 + r\sigma^2 + \lambda r\sigma^2 + \delta\lambda^2 r\sigma^2}{(1 + r\sigma^2)^2 + \delta r\sigma^2\lambda^2} c. \tag{26}$$

As the intertemporal externality becomes negligible, the share in the two periods becomes similar and approaches the static share. This means that $\lim_{\lambda \to 0} \beta_1 = \lim_{\lambda \to 0} \beta_2 = \frac{1}{1 + r\sigma^2}$. In case there is no correlation over time, the contract can be implemented as the static case. This refers to the so-called false dynamics.[10]

The denominator of β_1 and β_2, which is the rate of the welfare cost of the second-best contract, is the sum of the compound of the stationary rate of the cost of the second-best contract in two periods, $(1 + r\sigma^2)^2$, and the dynamic welfare cost. The discounted rate of risk cost in period one affects the second period transfer, $\delta r\sigma^2\lambda^2$. The numerator of β_1, which is the welfare gain in the first period evaluated at the welfare cost, is the sum of the stationary rate of the cost of the second-best contract in the first period, $1 + r\sigma^2$, and the dynamic welfare gain or the discounted risk cost of the first period affecting the second period transfer, $\delta\lambda r\sigma^2$. The numerator of β_2, which is the welfare gain in the first period evaluated at the welfare cost, is the sum of the stationary rate of the cost of the second-best contract in the second period, $1 + r\sigma^2$, and the dynamic welfare gain or the sum of the risk cost in period one affecting the first period transfer evaluated at the second period transfer, $\lambda r\sigma^2$, and the discounted risk cost in period one affecting the second period transfer, $\delta\lambda^2 r\sigma^2$.

As we compare the incentive power of the first and second periods of the dynamic optimal contract, $\beta_1 < \beta_2$ as

$$\beta_1 - \beta_2 = \frac{r\sigma^2\lambda\left[\delta(1 - \lambda) - 1\right]}{(1 + r\sigma^2)^2 + \delta r\sigma^2\lambda^2} < 0.$$

***Proposition* 2.** The optimal incentive is more high-powered in the second period than in the first period.

One of the main goals of the dynamic contract is to induce the first period operation level to be sufficiently beneficial from the dynamic externality. By setting the incentive in the second period to be higher than

[10] Laffont and Tirole (1993) and Laffont and Martimort (2002) provide detailed descriptions of the effects of different correlations over time on the contract.

in the first period, the contract can provide gain from the dynamic externality with lower risk cost. If the incentive in the first period is higher than or equal to that in the second period, the operation effort level is higher in the first period than in the second period based on Equation 18. It means that in the first period, the incentive should be high, but the risk cost should also be high. However, if the government sets the incentive in the first period to be lower than that in the second period, it can ensure sufficient operation effort level with much lower risk cost in the first period. In this way, the government could balance the gain of the dynamic externality and the risk cost dynamically.

The dynamic incentive scheme is the same as in Rogerson's (1985). The underlying factor is the dynamic externality in this paper and the difference of the risk-averseness for the principal and the agent in Rogerson's paper.

B. Comparative Static for the Dynamic Optimal Contract

We examine how the incentive power of the contract responds to the changes of performance target, intertemporal externality, and risk factor. The sign and relative size of the changes of incentive are critically dependent on how the intertemporal externality affects the operation effort and the revenue in two periods.

The incentive in both periods should increase as the probability of passing the performance evaluation increases as follows:

$$\frac{\partial \beta_1}{\partial \delta} = \frac{r\sigma^2\lambda(1-\lambda+r\sigma^2)(1+r\sigma^2)}{\left[(1+r\sigma^2)^2+\delta r\sigma^2\lambda^2\right]^2} > 0, \tag{27}$$

and

$$\frac{\partial \beta_2}{\partial \delta} = \frac{r\sigma^2\lambda^2(1-\lambda+r\sigma^2)r\sigma^2}{\left[(1+r\sigma^2)^2+\delta r\sigma^2\lambda^2\right]^2} > 0. \tag{28}$$

The lower performance target could increase revenue much more than the total cost in both periods. As the probability of passing the performance evaluation becomes higher, the private operator increases the operation effort at the same risk cost in the first period. The revenue increases as the operation effort level increases in the first period. Thus, there is a room to increase the risk cost in the first period, although the operation cost increases. As the operation effort in the first period increases, the social welfare increases in the second period at the same risk cost because of intertemporal externality. Thus, there is also a room to increase the risk cost in the second period despite the increase in operation cost. Therefore, the incentive should be higher both in the first and second periods to raise social welfare, corresponding to the lower performance target.

We compare the relative changes in the incentives in two periods to the changes in the probability of passing the performance evaluation. There should be a higher increase in incentive in the first period than in the second period. This is expressed as:

$$\frac{\partial \beta_1}{\partial \delta} - \frac{\partial \beta_2}{\partial \delta} = \frac{r\sigma^2 \lambda (1-\lambda + r\sigma^2)(1 + (1-\lambda) r\sigma^2)}{\left[(1 + r\sigma^2)^2 + \delta r \sigma^2 \lambda^2\right]^2} > 0. \tag{29}$$

***Proposition* 3.** As the performance target becomes lower, the optimal incentive power increases in both periods, resulting in a higher increase in the incentive in the first period.

As the incentive increases in both periods, the operation effort level increases in both periods. Suppose that the incentive increases at the same rate in two periods. There is a higher increase in the operation level in the first period than in the second period because of intertemporal externality. Thus, there is a higher increase in revenue in the first period than in the second period even with the same increase in the risk cost of the two periods. Furthermore, there is a higher increase in social welfare because of the increase in operation effort level of the first period more than of the second period. Thus, there is more room to increase the risk cost in the first period than in the second period. Therefore, the government should raise the incentive higher in the first period than in the second period.

We also check out how the incentive power of the contract changes as the degree of intertemporal externality varies. The incentive should increase in the second period as follows:

$$\frac{\partial \beta_2}{\partial \lambda} = \frac{r\sigma^2 \left[(1 + 2\delta\lambda)(1 + r\sigma^2)^2 - 2\delta\lambda(1 + r\sigma^2) - \delta\lambda^2 r\sigma^2\right]}{\left[(1 + r\sigma^2)^2 + r\sigma^2 \delta \lambda^2\right]^2} > 0. \tag{30}$$

The value in the parenthesis of the numerator of $\partial\beta_2/\partial\lambda$ is a quadratic in λ and has its maximum at the value larger than 1. The value in the parenthesis of the numerator of $\partial\beta_2/\partial\lambda$ at $\lambda=0$ is $(1+r\sigma^2)^2>0$. The value in the parenthesis of the numerator of $\partial\beta_2/\partial\lambda$ at $\lambda=1$ is $(1+2\delta)(r\sigma^2)^2+(2+\delta)r\sigma^2+1$ and positive. Thus, $\partial\beta_2/\partial\lambda$ is positive.

The larger intertemporal externality causes the social welfare in the second period to be higher directly without any change in the operation effort level. Thus, the incentive in the second period should increase because the social welfare gain outweighs the increase in the total cost.

However, the change in incentive power in the first period is indefinite for the change in intertemporal externality. We can show this using the following equation:

$$\frac{\partial\beta_1}{\partial\lambda}=\frac{\delta r\sigma^2\left[(1+r\sigma^2)^2-2\lambda(1+r\sigma^2)-\delta\lambda^2 r\sigma^2\right]}{\left[(1+r\sigma^2)^2+r\sigma^2\delta\lambda^2\right]^2}. \tag{31}$$

The value in the parenthesis of the numerator of $\partial\beta_1/\partial\lambda$ is a quadratic in λ and has its maximum at the value larger than 1. The value in the parenthesis of the numerator of $\partial\beta_1/\partial\lambda$ at $\lambda=0$ is $(1+r\sigma^2)^2>0$. The value in the parenthesis of the numerator of $\partial\beta_1/\partial\lambda$ at $\lambda=1$ is $(r\sigma^2)^2-\delta r\sigma^2-1$, which is indefinite.

As the intertemporal externality increases, the operation effort level increases in the first period without any change in the risk cost. However, the operation effort level increases in line with the increase in the incentive in the second period. This could lead to an excessive increase in the operation cost in the first period. In this case, the incentive in the first period should be lower to offset the increase in the operation cost in the same period. Moreover, if the increase in revenue outweighs the increase in the operation cost, then the incentive in the first period should be higher in order to increase the social welfare.

We compare the relative changes in incentives in the two periods to the changes in intertemporal externality. There should be a higher increase in incentive in the second period than in the first period, as the intertemporal externality increases as follows:

$$\frac{\partial\beta_1}{\partial\lambda}-\frac{\partial\beta_2}{\partial\lambda}=\frac{r\sigma^2\left[-\delta(\delta-1)r\sigma^2\lambda^2-2\delta(1+r\sigma^2)^2\lambda+(\delta-1)(1+r\sigma^2)^2\right]}{\left[(1+r\sigma^2)^2+r\sigma^2\delta\lambda^2\right]^2}<0. \tag{32}$$

***Proposition* 4.** As the degree of the intertemporal externality increases, the optimal incentive power increases in the second period. The incentive in the first period increase less than that in the second period.

In the case of a decrease in incentive in the first period, an obviously higher relative increase of the incentive in the second period can be observed. Consider the case of the increase in the incentive in the first period. The increase in intertemporal externality increases the revenue of the second period directly and indirectly through the increase in operation effort in the first period. However, it increases revenue in the first period only indirectly through the increase in operation effort in the same period. Thus, in this case, in order to increase social welfare, there is more room to increase the risk cost in the second period than in the first period. The increase of the incentive should, therefore, be higher in the second period than in the first period.

Now, we investigate how the incentive power of the optimal contract responds to the changes in demand risk and risk attitude of the private operator. As expected, the incentive power decreases as the risk or risk aversion increases in both periods. This is because the essence of the contract is to balance the incentive to induce operation effort and risk sharing. Thus, incentive power should be adjusted inversely to the risk or risk aversion. This result has the same feature as that of static optimal contract given by:

$$\frac{\partial \beta_1}{\partial (r\sigma^2)} = \frac{\left[(\delta\lambda - 1)(1 + r\sigma^2)^2 - 2\delta\lambda r\sigma^2(1 + r\sigma^2) - \delta\lambda^2\right]}{\left[(1 + r\sigma^2)^2 + r\sigma^2\delta\lambda^2\right]^2} < 0, \tag{33}$$

$$\frac{\partial \beta_2}{\partial (r\sigma^2)} = \frac{\left[(\lambda - 1) - 2r\sigma^2 - (1 + \lambda + \delta\lambda)(r\sigma^2)^2\right]}{\left[(1 + r\sigma^2)^2 + r\sigma^2\delta\lambda^2\right]^2} < 0. \tag{34}$$

***Proposition* 5.** As the risk or risk aversion increases, the optimal incentive power decreases in both periods.

However, the relative change in incentive power is indefinite for the change in the risk or risk aversion expressed as:

$$\frac{\partial\beta_1}{\partial(r\sigma^2)}-\frac{\partial\beta_2}{\partial(r\sigma^2)}=\frac{\left[-\lambda(1+\delta(\delta-1)+\lambda(r\sigma^2)^2\right]}{\left[(1+r\sigma^2)^2+r\sigma^2\delta\lambda^2\right]^2}. \tag{35}$$

If the risk or risk aversion is relatively large ($r\sigma^2 > 1$), the incentive should decrease less in the first period. When the risk or risk aversion is large, the incentive is small in the second period. However, the incentive in the first period is very small because the incentive should be smaller in the first period than in the second period. Thus, there is not much room to reduce the incentive in the first period.

IV. Conclusion

Demand risk is an important feature of the service provision of building infrastructure regardless of the operation scheme. In particular, PPP is profoundly associated with demand risk because of the long-term contract. Thus, a critical aspect of any PPP contract is the allocation of demand risk between the government and the private operator. The earlier studies focused on the bundling effect and did not deal with risk sharing characteristics. Thus, this paper analyzes the service provision of infrastructure in terms of demand risk.

From the risk-sharing perspective, we characterize the traditional government operation and PPP with a simple static model. Under government operation, the government considers operation efficiency and not demand risk when it chooses the operation level. The government induces less operation effort because the government operation is less efficient. On the other hand, under PPP, the government transfers some of the demand risks to the private operator, and the incentive for the private operator is determined inversely by the risk factor.

We compare government operation and PPP in terms of expected social welfare and expected total cost. The society benefits more from PPP than from government operation, because there is opportunity of risk sharing between the government and the private operator through PPP. The result is the same as that obtained in previous works focusing on the bundling effect of PPP. This paper finds another supporting argument for PPP that differs from those obtained by earlier research. Considering the expected total cost structure, the government is likely to apply PPP rather than government operation to the projects with a high demand risk.

Taking into account the performance evaluation, we derive the dynamic optimal contract for the PPP in a two-period model with intertemporal externality. The optimal incentive is such that the private operator bears less risk in the earlier period and the incentive is high in the later period. This level of incentive can be attributed to dynamic externality, which works favorably on the first period effort. Thus, there is room to reduce the risk cost while inducing sufficient operation level by setting the incentive lower in the first period.

The paper analyzes the response of the incentive in each period to the change in the dynamic environment. As the performance target becomes lower, the incentive power increases in both periods. The incentive should increase more in the first period than in the second period, because the increase in the incentive of the first period can cause a higher raise in revenue more at the same risk cost. As the intertemporal externality becomes stronger, the incentive power increases in the second period. The incentive should increase more in the second than in the first period, because the increase in intertemporal externality can boost the revenue in the second period both directly and indirectly. As the risk factor increases, the incentive power decreases in both periods. This result resembles the static feature.

(Received 14 July 2010; Revised 24 May 2011; Accepted 16 June 2012)

References

Bennett, J., and Iossa, E. "Building and Managing Facilities for Public Services." *Journal of Public Economics* 90 (Nos. 10-11 2006): 2143-60.

Engel, E., Fischer, R., and Galetovic, A. The Basic Public Finance of Public-Private Partnerships. Cowles Foundation Discussion Paper No. 1618, 2008.

GAO. Highway Public-Private Partnerships. Testimony GAO-08-1052T, July 24, 2008.

Harris, C. Private Participation in Infrastructure in Developing Countries Trends, Impact, and Policy Lessons. World Bank Working Paper No. 5, The World Bank. 2003.

Hart, O. "Incomplete Contracts and Public Ownership: Remarks and an Application to Public-Private Partnerships." *Economic Journal* 113 (No. 486 2003): C69-76.

Holmstrom, B., and Milgrom, P. "Aggregation and Linearity in the Provision of Intertemporal Incentives." *Econometrica* 55 (No. 2 1987): 303-28.

Iossa, Elisabetta, and Martimort, D. The Simple Micro-Economics of Public-Private Partnerships. CEIS Working Paper No. 139, 2008.

Kang, D. An Analysis on Public-Private Partnerships in Korea and Discussion on Development of the System. Unpublished Paper, National Assembly Budget Office, 2006 (in Korean).

Kim, Son-Ku. "Optimal Reporting Frequency in Agencies." *Seoul Journal of Economics* 18 (No. 1 2005): 1-19.

Laffont, J. J., and Martimort, D. *The Theory of Incentives: The Principal-Agent Model.* Princeton and Oxford: Princeton University Press, 2002.

Laffont, J. J., and Tirole, J. *A. Theory of Incentives in Procurement and Regulation.* Cambridge, Mass: MIT Press, 1993.

Malhotra, Anil K. "Private Participation in Infrastructure: Lessons from Asia's Power Sector." *Finance&Development* 34 (No. 4 1997): 33-5.

Martimort, D., and Pouyet, J. "To Build or Not to Build: Normative and Positive Theories of Public-Private Partnerships." *International Journal of Industrial Organization* 26 (No. 2 2008): 393-411.

Ministry of Land, Transport, and Maritime Affairs. "Equity Transfer of Incheon Airport Railroad." Press Release, March 31, 2009 (in Korean).

Ministry of Planning and Budget. "Performance of PPPs for 10 Years in Korea." Press Release, August 8, 2007 (in Korean).

Rogerson, W. "Repeated Moral Hazard." *Econometrica* 53 (No. 1 1985): 69-76.

Shleifer, A. "State versus Private Ownership." *Journal of Economic Perspectives* 12 (No. 4 1998): 133-50.