IMPLEMENTATION OF DISCONTINUOUS GALERKIN METHOD IN GAS DYNAMICS

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ABSTRACT

Discontinuous Galerkin (DG) method is tested and valuated in this paper in Euler equations of gas dynamics. For one- and two-dimensional problems, this method shows promising results. However, to acquire same quality of results, DG demands about 7 times more CPU time than MUSCL of the finite volume method. Moreover, removing oscillation is tough for this scheme, so we've tested several limiters. In this paper, detail procedure to achieve high-order of accuracy is analyzed followed by a comparison study with MUSCL in terms of accuracy and efficiency. Lastly, some current issues of DG are introduced.

INTRODUCTION

Discontinuous Galerkin method has raised great interest and has been developed during the past twenty years[1,2]. This method which can be regarded as a mixture of the finite volume, the finite element methods, and Riemann problems shows satisfactory results not only in hyperbolic problems include wave propagation but also in elliptic problems such as viscous problem in gas dynamics. DG method is largely based on the finite element method in that it actually adopts a number of characters of the finite element method to form a high-resolution numerical scheme. First, DG reconstructs high-order of accuracy only by information within the elements, without those of neighboring elements as the finite volume method does. Second, numerical flux of an edge is not a constant value but a changing one depends on inner distribution of properties. However, in that it permits to have discontinuous properties in the boundaries of elements and fluxes are computed by various flux schemes, DG has feature of the finite volume method, too. High order of accuracy is attainable in DG by means of high-order polynomial within elements. Owing to its ability to construct high order of accuracy without information of neighboring elements, this scheme is easy to handle unstructured meshes as well as it can keep its accuracy even near the boundary of computational domain.

However, in order to make DG an attractive candidate for high resolution, viscous, compressible flow solutions, this method should be improved further. DG requires more CPU cost and more memory than MUSCL[3], and optimizing oscillation eliminating algorithm is not available yet[4]. Furthermore, about its fast convergence problem, there is still much room for improvement.

GENERAL PROCEDURE

Unsteady, compressible Euler equation is first multiplied with test functions W, and flux integrals are parted through Green theorem.

$$\int_\Omega \frac{\partial U}{\partial t} W d\Omega - \int_\Omega \left(\vec{F}\cdot\vec{\nabla}\right) W d\Omega + \int W\vec{F}\cdot d\vec{S} = 0 \tag{1}$$

Approximated solution is represented in terms of the node solutions u_l and the shape functions φ_l at each node. By replacing test functions with interpolation functions at each node, we can get Galerkin method formulation. Furthermore, when discontinuities are permitted in the boundaries of elements, the flux of the last term of (1) represents numerical flux. Therefore, for one-dimensional problems,

$$U(x,t) = \sum_{l=0}^{k} u_l \varphi_l(x),\; W = \sum_{j=0}^{k} \varphi_j(x) \tag{2}$$

$$\sum_{l=0}^{k} \frac{du_l}{dt} \int_{\Omega_j} \varphi_l \varphi_j d\Omega - \int_{\Omega_j} \vec{F}(U)\left(\varphi_j\right)_x d\Omega + \hat{f}(U)\varphi_j \Big|_{x_{i-1/2}}^{x_{i+1/2}} = 0. \tag{3}$$

On unstructured mesh problems, interpolation functions are defined as the finite element method does – functions are based on each node. However, on structured mesh or one dimensional problems, owing to their straightforward directionality, Legendre polynomial is used. With Legendre's orthogonality, equation (3) can be deformed as follow.

$$\sum_{l=0}^{k} \left(\frac{du_l}{dt} + \frac{2l+1}{\Delta x}\left(- \int_{\Omega_j} \vec{F}(U)\left(\varphi_l\right)_x d\Omega + \hat{f}(U)\varphi_l \Big|_{x_{i-1/2}}^{x_{i+1/2}} \right)\right) = 0 \tag{4}$$

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