Micro Adjustment Behavior and Macro Stability*

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In this paper we construct a framework for the analysis of the stability of capitalist economies. To this end, the behavior of economic agents (allocation of capital, loans, investment, production, customer credit, prices) is described in terms of adjustment. Agents make decision within disequilibrium and react to the observation of disequilibrium (what we call disequilibrium microeconomics). The conception of equilibrium is that of a long-term equilibrium with prices of production (normal equilibrium). We distinguish the stability of the system with respect to the relative values of the variables among industries (proportions) and the stability of the general level of activity (dimension). Capitalism appears very stable with respect to proportions and unstable with respect to dimension. The demonstration of a necessary and sufficient condition for stability is realized in a short-term approximation of the model. The introduction of non-linearities in the models of behavior allows for the existence and stability of two other equilibria (in addition to normal equilibrium): overheating and stagnation. An interpretation of the business cycle is given on this basis.

I. Introduction

The latter part of the term "neoclassical" refers to a *classical* school of thought at a preliminary stage of development of economic theory. "Classical" has been given so many different meanings, that much confusion still attaches to the word. In this paper, we will use the term "classics" in Karl Marx's original sense, to designate Adam Smith and David Ricardo, i.e., a school of thought which culminated in England with Ricardo's *Principles* (Ricardo 1817), about half a century before

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relative stickiness of reactions concerning the long run, we establish a necessary and sufficient condition for stability. Non-linearities in the behavior of economic agents are introduced in Part V. We show that a framework for the analysis of the business cycle is embodied in the model and can be made explicit. The fact that the modeling of individual behavior in terms of adjustment introduces a theory of the business cycle fulfills the project of providing macroeconomics with microfoundations.

II. The Model

Section *A* presents the general framework of analysis with a special emphasis on monetary mechanisms. Section *B* is devoted to the modeling of behavior in terms of adjustment. Section *C* introduces the relation of recursion which is obtained.

A. The General Framework of Analysis

The general guidelines governing the definition of the framework of analysis, are introduced in Subsection *A)*. In Subsection *B)*, we introduce the agents. The treatment of monetary and credit phenomena is made explicit in Subsection *C)*. Subsection *D)* discusses the impact of monetary constraints on behavior. Last, Subsection *E)* introduces a number of simplifying assumptions made to render the model manageable.

A) General Guidelines

In this model we first try to reproduce the framework in which the classical analysis of competition was originally developed. Capitalists and enterprises are the key agents in this paradigm. Capitalists allocate capital among enterprises. Profit rates are the crucial variables in this allocation.

The classics described capital mobility in terms of reaction to profitability differentials. We generalize this idea of reaction to disequilibrium, i.e., adjustment, to other decisions (production, prices, loans, etc.). We call this general application of the adjustment principle "disequilibrium microeconomics".

As a result of this extension of the classical adjustment procedure, we supplement the notion of long-term equilibrium with prices of production, with an original conception of short-term equilibrium in which the clearing of markets is achieved by quantities, instead of prices as is the case in a Walrasian model. It is the utilization rate of fixed capital which is adjusted in the short run.

In this analysis monetary phenomena are crucial, for two reasons:

1. The notion of the migration of capital refers to the movement of a purchasing power from one industry to another. The availability of purchasing power allows firms to invest these funds in fixed capital goods (and circulating inputs) to different extents, but fixed capital itself is never moved. Thus, the mobility of capital is a story concerning money flows. In particular, *firms are not limited in their expansion by diminishing returns* (they can open new units of production without restriction or exhaustion of natural resources), *but by the availability of funds* (what we call *the capital constraint*). This view of the development of production naturally leads to the adoption of constant return to scale in the modeling of technology.

2. A second theme, on which the classics are less explicit, is the effect of the issuance of money (monetary and credit mechanisms) on the overall level of activity. The existence and stability of the equilibrium are subject to specific conditions concerning *different types* of monetary phenomena. It is for this reason that these mechanisms will be modeled more explicitly than is traditional in this field.

B) The Agents

Five groups of agents interact in the model: Wage earners, Capitalists, Enterprises, Banks, and the State.

Wage earners sell their labor power to enterprises and produce. In exchange, they receive a wage in cash. On the basis of this purchasing power, they consume. As consumers, they can buy on credit.

Capitalists control the movements of capital (the cash flow of enterprises, i.e., depreciation allowances and profits). They subtract from these sums a given fraction for their personal consumption. The remainder is re-invested. They allocate capital among different enterprises. With respect to consumption, their behavior is not different from that of wage earners. They can also buy on credit.

Enterprises organize production and make decisions concerning prices and outputs. They pay wages and transfer the cash flow to capitalists. They receive funds from the capitalists and make investment decisions. They can borrow from the banks for all of these operations: 1) Production and transfer of funds to capitalists, in the short run, and 2) Investment, with respect to the long run. They sell on credit to final consumers.

In the model, *banks* only deal with enterprises which deposit, withdraw funds, and borrow. No distinction is made between commercial banks and a central bank. The behavior of the banking system in the model is supposed to account globally for the various components of the system.

The *state* plays a double role in the model through monetary and demand policies. This last agent is not considered before Section III. *D*.

C) Money and Credit

For clarity we will distinguish two forms of money, cash and deposits, and restrict the use of each component to one group of agents: consumers hold cash and enterprises hold bank deposits. The exact form of the several types of transactions have no consequence in the model. This is the case, for example, for financial transfers between capitalists and enterprises.

Three types of credits are considered: customer credit, loans for investments, and loans for the other transaction of enterprises.

1. Enterprises can sell on credit to final consumers (wage earners and capitalists). This credit is granted for commercial reasons, to stimulate sales (to cut inventories and increase the capacity utilization rate). We assume that these offers are always accepted by customers.

2. Banks lend to enterprises which are willing to invest beyond the possibilities available from the allocation of capital by capitalists. The quantity of loans is the result of a negotiation between enterprises and banks. The desire by firms to borrow is measured in the model by their capacity utilization rate.

3. Enterprises can always withdraw funds from the banks (short-term borrowings) to make payments to other enterprises and to capitalists, or to pay wages. They are never restricted in this respect. They deposit all cash that they receive from the sale of their output in the bank. As a result of this treatment of short-term borrowing, reciprocal interenterprise credit is abstracted from.

D) Liquidity and Capital Constraints

It is implicit in the above presentation of monetary mechanisms that two different channels of funds are considered in the model. The first channel corresponds to investment, and the second to short-term transactions.

Enterprises can only invest on the basis of funds allocated by capitalists or loans from the banks specifically ear-marked for this purpose. Thus, in the enterprise's balance sheet, the sum *equity + loans for investment* is always equal to the value of fixed capital. Enterprises are always constrained by the availability of funds (under normal circumstances, profitable investment opportunities always exist). They experience what we call the "*capital constraint*". The existence of this capital constraint is a prominent feature of the classical analysis. The concept of "capital mobility" has no meaning if investment decisions are made in the absence of such a constraint.

The second channel corresponds to short-term transactions. Enterprises could be subject to the existence of a *liquidity constraint*, in the sense that they might be obiliged, for example, to scale down their activity because of a lack of liquidity. We abstract from this constraint. Thus, all decisions concerning short-term management (prices, outputs, customer credit) are made only on the basis of physical variables: capacity utilization rates and ratio of inventories, in the model.

E) Technical Assumptions

We make of number of technical assumptions in order to make the model accessible:

1. Only two industries exist. A fixed capital good is produced in industry one, and a consumption good is produced in the second industry. There are no physical circulating inputs. Production is obtained from labor and fixed capital. For the actual use of one unit of fixed capital, production can be symbolically represented as follows:

$$1 \text{ (\textit{unit of fixed capital})} + l^i \text{ (\textit{units of labor})}$$
$$\rightarrow b^i \text{ (\textit{units of good } } i\text{)} + (1 - \delta^i)\text{(\textit{unit of fixed capital}).}$$

Parameter δ^i measures the loss of productive power of fixed capital during one period. If this unit of fixed capital is used only at a rate $0 \leq u^i \leq 1$ (capacity utilization rate), production is modeled as follows:

$$1 \ (\textit{unit of fixed capital}) + l^i u^i \ (\textit{units of labor})$$
$$\rightarrow b^i u^i \ (\textit{units of good } i) + (1 - \delta^i)(\textit{unit of fixed capital}).$$

The capacity utilization rate is an important variable in the description of disequilibrium from the point of view of production (cf. Figure 1 below). This variable is absent from traditional microeconomics, but is often considered in macroeconomic models. If the stock of fixed capital in enterprise i is denoted K_t^i, production is equal to: $Y_t^i = K_t^i b^i u_t^i$, and the quantity of labor employed is $K_t^i l^i u_t^i$.

2. The two industries are identical with respect to technology (l^i, b^i, and δ^i are independent from i) and their behavior.
3. No nonreproducible resources are considered, with the exception of labor which is assumed available.
4. The real wage $\bar{w}$ is given (a number of units of the consumption good). We define $w = \bar{w}l$, the real wage for one unit of fixed capital used at full capacity.
5. All profits are spent by capitalist for consumption. Only depreciation allowances are re-allocated. Thus, when equilibrium prevails, the system is in simple reproduction (only replacement investment exists). In a disequilibrium, a net positive or negative accumulation can be observed because of loans for investment which respond to disequilibrium on capacity utilization rates.
6. All capitalists are identical (or only one capitalist exists).
7. All final consumers have the same propensity to consume, α, with respect to the amount of cash that they hold. Thus, they can be aggregated.
8. Production and markets follow one another ($\cdots \rightarrow$ market $(t-1)$ $\rightarrow$ production $t \rightarrow$ market $t \rightarrow \cdots$).
9. All interest rates are equal to zero.

B. The Modeling of Behavior

The general principle in the modeling of adjustment behavior can be represented as follows:

$\cdots \rightarrow$ *Evidence of disequilibrium* $\rightarrow$ *Modification of behavior* $\rightarrow \cdots$

In this section we first recapitulate the various disequilibria to which agents are sensitive and, then, present the behaviors of the agents, in terms of adjustment, for the following decisions: allocation of capital, loans for investment, investment, production, customer credit, final consumption, and prices.

A) Evidences of Disequilibrium

Three marks of disequilibrium play a role in the model:

1. *Disequilibrium on profit rates.* The capitalists are sensitive to profitability differentials. In this model in which two industries are considered this disequilibrium is measured by $(r_t^1 - r_t^2)$, in which r_t^i is the rate of profit of enterprise i.

2. *Disequilibrium between supply and demand.* For accounting reasons the difference between supply, i.e., output plus inventories transmitted $Y_t^i + S_t^i$, and demand D_t^i is equal to the new stock of inventories:

$$S_{t+1}^i = S_t^i + Y_t^i - D_t^i . \tag{1}$$

   We define the following ratio of inventories:

$$s_t^i = S_t^i / K_t^i b. \tag{2}$$

   Because of the constant fluctuations of demand, enterprises do not seek to reduce their inventories to zero, but to maintain a certain ratio, $\bar{s}$, of inventories to sales. The variable which measures the disequilibrium between supply and demand is $s_t^i - \bar{s}$.

3. *Disequilibrium in the utilization of productive capacities.* Also as a result of the fluctuations of demand, enterprises do not attempt to use their respective capacities at one hundred percent. They tend to maintain a certain "normal" vaule, $\bar{u}$, of this ratio. The disequilibrium on the capacity utilization rate is measured by $u_t^i - \bar{u}$.

We call $\bar{u}$ and $\bar{s}$ the "target" or "normal" values of u and s.

B) The Allocation of Capital

Profits of enterprise i are equal to *Sales — Wages — Depreciation allowances*: $\pi_t^i = Y_t^i p t^i - W_t^i - A_t^i$. With $W_t^i = K_t^i w u_t^i p_t^2$ and $A_t^i = K_t^i \delta p_t^1$, one obtains:

TABLE 1
NOTATION

Indexes:	
i	Index of a firm(superscript): $i = 1$, fixed production good, and $i = 2$, consumption good
t	Index of the period (subscript)
Variables:	
A	Depreciation allowances
ΔC	Net customer credit
K	Stock of fixed capital
κ	Gross capital accumulated during a period (a purchasing power)
$\Delta\kappa$	Net loans for investment
M^f	Stock of money held by final consumers before consumption
M	Stock of money held by final consumers after consumption
p	Price
π	Profits
r	Rate of profit
$\rho(X)$	Rate of growth of variable X
$s,\bar{s}$	Ratio of inventories, Target ratio of inventories
$u,\bar{u}$	Capacity utilization rate, Target capacity utilization rate
v	Speed of circulation of money
W	Total real wages
x	Relative prices, $x = p^2/p^1$
Y	Output, $Y = Kbu$
y	Relative stocks of fixed capital, $y = K^2/K^1$
z	Normalized stock of money, $z_{t+1} = M_{t+1}/K^2_{t+1}p^2_t$
Parameters:	
α	Propensity to consume
b	Output obtained with one unit of fixed capital for $u = 1$
δ	Proportion of the productive capacity of fixed capital lost in one period
l	Quantity of labor combined with one unit of fixed capital for $u = 1$
w	Real wage
$\bar{w}$	Real wage per unit of fixed capital for $u = 1 (\bar{w}=lw)$
Reaction Coefficients:	
β	Price decision (sensitivity to s)
ε	Decision to produce (sensitivity to s)
γ	Allocation of capital (sensitivity to profit rate differentials)
ω	Borrowings for investment (sensitivity to u)
φ	Lending to consumers (sensitivity to s)
σ	Decision to produce (stickiness of u)
θ	Defined in equation (9) (condition for stability in dimension)

$$\pi_t^i = K_t^i(bu_t^i p_t^i - wu_t^i p_t^2 - \delta p_t^1).$$

The profit rate, r_t^i, of enterprise i which is used by capitalists in the allocation of capital is defined as the ratio of profits over the stock of fixed capital. Recall that this is the only fraction of capital financed by capitalists. One has:

$$r_t^i = \frac{\pi_t^i}{K_t^i p_t^1} = u_t^i \frac{bp_t^i - wp_t^2}{p_t^1} - \delta.$$

As a result of the assumption of simple reproduction, all profits are devoted to consumption and only depreciation allowances are reallocated. The value of the amount of capital which is reallocated to all enterprises in period $t + 1$ is:

$$\kappa_{t+1} = A_t^1 + A_t^2 = (K_t^1 + K_t^2)\delta p_t^1 = K_t \delta p_t^1,$$

where K_t denotes total fixed capital in t. Notice that, in this equation, κ denotes a purchasing power and K a quantity of goods.

The amounts of capital κ_{t+1}^1 and κ_{t+1}^2 allocated to enterprises 1 and 2 are determined as follows:

$$\kappa_{t+1}^1 = (\delta K_t^1 + \gamma K_t(r_t^1 - r_t^2))p_t^1,$$
$$\kappa_{t+1}^2 = (\delta K_t^2 + \gamma K_t(r_t^2 - r_t^1))p_t^1.$$

The new capital κ_{t+1}^i allocated to enterprise i is added to the previous amount of equities existing in this enterprise. If the rates of profit r_t^1 and r_t^2 are equal, depreciation allowances, $\delta K_t^i p_t^1$, return to the enterprises (no mobility of capital). In the general case, this amount is modified as a result of the existence of a profitability differential. γ is a reaction coefficient which accounts for the intensity of the reaction to disequilibrium. This coefficient, as all other reaction coefficients in this study, is normalized and without dimension. (The model is homogenous with respect to prices and quantities.) One can verify that $\kappa_{t+1} = \kappa_{t+1}^1 + \kappa_{t+1}^2$.

C) Loans for Investment

The value of net loans to enterprise i made available for market t for the purchase of capital goods to be used for production $t + 1$ is denoted $\Delta\kappa_{t+1}^i$. The disequilibrium on the capacity utilization rate is the variable which is considered in this decision:

$$\Delta\kappa_{t+1}^i = \omega K_t^i p_t^1 (u_t^i - \bar{u}). \tag{3}$$

The quantity of loans is an increasing function of the capacity utilization rate. In this equation ω is a reaction coefficient which models the sensitivity of the bank to, $(u_t^i - \bar{u})$, the disequilibrium in the capacity utilization rate. This coefficient is without dimension. The total $K_t^i p_t^1$ represents the value of the stock of fixed capital already hold by the enterprise.

D) Investment

Gross investment, I_{t+1}^i, of enterprise i, purchased on market t, to be used for production in period t, is equal to what is allowed by the allocation of capital and the loans from the bank:

$$I_{t+1}^i = \frac{\kappa_{t+1}^i + \Delta\kappa_{t+1}^i}{p_t^1}.$$

Total demand, D_t^1, for the fixed capital good is the sum of the investments of the two industries:

$$D_t^1 = I_{t+1}^1 + I_{t+1}^2 = \delta K_t + \omega(K_t^1 \ (u_t^1 - \bar{u}) + K_t^2 \ (u_t^2 - \bar{u})). \tag{4}$$

The first term δK_t corresponds to replacement. The second term expresses the impact of disequilibrium on the capacity utilization rates. One has:

$$K_{t+1}^i = (1 - \delta) K_t^i + I_{t+1}^i.$$

If the two capacity utilization rates are equal to their target value ($u_t^1 = u_t^2 = \bar{u}$), then $K_{t+1} = K_t$, since $I_{t+1}^1 + I_{t+1}^2 = \delta K_t$.

E) Production

The decision to produce is equivalent to the decision of the new capacity utilization rate, since the stock of capital is given and $Y_t^i = K_t^i b u_t^i$. This decision is made on the basis of the previous value of u and the disequilibrium on the level of inventories:

$$u_{t+1}^i - \bar{u} = \sigma(u_t^i - \bar{u}) - \varepsilon(s_t^i - \bar{s}), \tag{5}$$

where the reaction coefficients σ and ε have no dimension, since u and s themselves have no dimension. This equation shows, for example, that enterprises will react to inventories larger than normal by diminishing their capacity utilization rate. The degree of this sensitivity to stockpiling is measured by ε.

Reaction coefficient σ models the stickiness of the capacity utilization rate, with $0 \leq \sigma \leq 1$. The first term, $\sigma(u_t^i - \bar{u}^i)$, expresses two different types of phenomena: 1) We make an implicit assumption on the demand function which the enterprise confronts: it is subject to random shock and only returns progressively to normal levels (autoregressive shocks), 2) In addition to traditional production costs, the enterprise incurs disequilibrium costs, such as cost of stockpiling or the cost of changing production. The existence of this latter cost induces a degree of stickiness in the decision to produce. A more detailed analysis of these mechanisms can be found in our other contribution to this conference, entitled "The Rationality of Adjustment Behavior in a Model of Monopolistic Competition" (Duménil and Lévy 1989b), which is devoted to the derivation of equation (5) (and equation (8) below).

The decision to produce determines the amount of wages (paid before production occurs):

$$W_{t+1}^i = K_{t+1}^i \, w u_{t+1}^i \, p_t^2 .$$

F) Customer Credit

The credit to consumer is only granted by industry 2. Its variation, ΔC_t, is modeled as:

$$\Delta C_t = \varphi K_t^2 p_t^2 (s_t^2 - \bar{s}). \tag{6}$$

If inventories are large, then ΔC_t is also large. In this equation φ is a reaction coefficient without dimension. The dimension of ΔC_t is given by $K_t^2 p_t^2$ (since this purchasing power has been created in order to reabsorb, $bK_t^2 \rho_t^2 (s_t^2 - \bar{s})$, the unvoluntary inventories of industry 2).

The variation of the general level of prices could be added as an argument in this equation as well as in equation (3) above, to model the reaction of monetary authorities to inflation or deflation. For example, inflation would result in a tightening of credit conditions.

G) Final Consumption

At the end of the preivous period, final consumers, capitalists and workers, hold a given stock of money M_t. Their purchasing power, M_t, is increased by the new wages and profits distributed, plus consumer credit:

$$M_t^f = M_t + \pi_t^1 + \pi_t^2 + W_{t+1}^1 + W_{t+1}^2 + \varphi K_t^2 p_t^2 (s_t^2 - \bar{s}).$$

Their propensity to consume is defined as a given fraction α of this purchasing power. They spend αM_t^f and hoard $(1 - \alpha) M_t^f =$

M_{t+1} until the next market. Thus, D_t^2, the demand for consumption goods on market t is:

$$D_t^2 = \frac{\alpha M_t^f}{p_t^2} = \frac{\alpha}{1-\alpha} \frac{M_{t+1}}{p_t^2} . \tag{7}$$

For simplicity, we abstract from the fact that the decisions of final consumers is influenced by their backlog of debts to enterprises.

H) Prices

The new ratios of inventories s_{t+1}^i are determined by equations (1) and (2), and the demand equations (4) and (7). Prices are corrected according to the disequilibrium between supply and demand:

$$p_{t+1}^i = p_t^i (1 - \beta (s_{t+1}^i - \bar{s})), \tag{8}$$

in which β is a reaction coeffcient without dimension. The disequilibrium in capacity utilization rates or the cost of inputs could also be included as arguments in this function.

C. The Relation of Recursion

On the basis of the above equations, it is possible to derive a relation of recursion which defines the value of the variables in period $t+1$ as a function of this value in period t.

Nine variables are involved in this recursion: u^i, s^i, p^i, and K^i for $i = 1,2$, plus M. *A priori* a double continuum of equilibria exists as a result of the indeterminacy of the general levels of both prices and quantities. It is, therefore, possible to reduce the number of variables to 7:

1. The two capactiy utilization rates u^i and two ratios of inventories s^i, which already have no dimension.
2. Relative prices: $x = p^1/p^2$.
3. Relative stocks of fixed capital: $y = K^1/K^2$.
4. The normalized stock of money: $z_{t+1} = M_{t+1}/K_{t+1}^2 p_t^2$.

One can add the two auxiliary variables, $\rho(p)$, the rate of change of average prices, and $\rho(K)$, the rate of variation of the total stock of fixed capital, to these 7 variables (u^1, u^2, s^1, s^2, x, y, z).

Using this notation the recursion can be written as shown in the following table.

TABLE 2
THE RELATION OF RECURSION

$$u_{t+1}^1 = \bar{u} + \sigma(u_t^1 - \bar{u}) - \varepsilon(s_t^1 - \bar{s})$$

$$u_{t+1}^2 = \bar{u} + \sigma(u_t^2 - \bar{u}) - \varepsilon(s_t^2 - \bar{s})$$

$$s_{t+1}^1 = (s_t^1 + u_t^1 + \frac{1-\delta}{b}\frac{1+y_t}{y_t})/(1 + \gamma\frac{1+y_t}{y_t}(r_t^1 - r_t^2) + \omega(u_t^1 - \bar{u})) - \frac{1+y_t}{by_t}$$

$$s_{t+1}^2 = (s_t^2 + u_t^2)/(1 - \gamma\frac{1+y_t}{y_t}(r_t^1 - r_t^2) + \omega(u_t^2 - \bar{u})) - \frac{\alpha}{1-\alpha}\frac{1}{b}z_{t+1}$$

$$x_{t+1} = x_t(1 - \beta(s_{t+1}^1 - \bar{s}))/(1 - \beta(s_{t+1}^2 - \bar{s}))$$

$$y_{t+1} = (y_t + \gamma(1 + y_t)(r_t^1 - r_t^2) + \omega y_t(u_t^1 - \bar{u}))/(1 - \gamma(1 + y_t)(r_t^1 - r_t^2) + \omega(u_t^2 - \bar{u}))$$

$$z_{t+1} = (1-\alpha)(\frac{z_t}{1-\beta(s_t^2-\bar{s})} + y_t x_t r_t^1 + x_t r_t^2 + \varphi(s_t^2-\bar{s}))/(1-\gamma(1 + y_t)(r_t^1 - r_t^2) + \omega(u_t^2-\bar{u})) + (1-\alpha)w(y_{t+1}u_{t+1}^1 + u_{t+1}^2)$$

with $r_t^1 = u_t^1(b - \frac{w}{x_t}) - \delta$ and $r_t^2 = u_t^2(b - w)\frac{1}{x_t} - \delta$

III. Dimension and Proportions

In the study of a dynamic model such as that presented in Part I, one must distinguish between the issues of the existence of a equilibrium and its stability. Section *A* is devoted to equilbrium and introduces the study of stability. In the stability problem, we distinguish two aspects: proportions and dimension. Section *B* focuses on stability in dimension, i.e., on the ability of the system to achieve a normal utilization of resources (in the model, only the normal use of fixed capital: $u = \bar{u}$, since we abstract from the labor market). Section *C* is devoted to stability in proportions, i.e., to the capability of the system to maintain appropriate relative values of the variables (relative prices, relative proportions of capitals and outputs among industries). In Section D, we introduce the notion of a "shift" of normal equilibrium, and discusss in this context the effects of demand and monetary policies. Section E concludes this second part by general considerations concerning the stability of capitalist economies.

A. Equilibrium and Stability

In this model, there exists an equilibrium which we call *normal equilibrium.* It corresponds to the classical notion of a long-term equilibrium with price of production:

1. The utilization of capacity rates and the ratios of inventories to sales reach their target value: $u^i = \bar{u}$ and $s^i = \bar{s}$.
2. The rates of profits are equal, $r^i = \bar{r} = \bar{u}(b-w) - \delta$ and, prices are equal to production prices. As a result of the assumption of an identical technology within the two industries, prices are equal: $x = 1$ or $p^2 = p^1$.
3. The proportions of capital between the two industries are given by: $y = \bar{y} = \delta/(b\bar{u}-\delta)$.
4. The amount of money held by final consumers is given: $z = \bar{z} = b\bar{u}(1-\alpha)/\alpha$.
5. There is no growth ($\rho(K) = 0$) and no variation of the general level of prices ($\rho(p) = 0$).

It is possible to check the empirical relevance of this classical conception of long-term equilibrium: rates of profit tend to gravitate around a uniform rate (cf, for example, Ehrbar and Glick 1988a, 1988b, for the U.S. economy).

In what follows, the analysis will focus on the stability of this equilibrium (cf. III. *B. A)* and III. *C*) and dynamics in a vicinity of equilibrium (cf. III. *B. B)* and III. *B. C*)).

Since the model is non-linear, the uniqueness of the equilibrium is not guaranteed, and other equilibria can also exist. The consideration of such equilibria is interesting, but it would be inconsistent to engage in such an analysis while preserving the linear forms of the behavioral equations (non-linearities are introduced in V. *A*).

In order to study the local stability of an equilibrium such as that introduced above, it is necessary to determine the Jacobian matrix which expresses the linear approximation of the recursion in the vicinity of equilibrium. Stability is insured if the modulus of all the eigenvalues are smaller than 1. The polynomial characteristic of the Jacobian is denoted $P(\lambda)$:

$$P(\lambda)=\begin{vmatrix} \lambda-\sigma & 0 & \varepsilon & 0 & 0 & 0 & 0 \\ 0 & \lambda-\sigma & 0 & \varepsilon & 0 & 0 & 0 \\ \begin{matrix}-1+A(b-w)\\ +(\bar{s}+\frac{\bar{u}}{\delta})\omega\end{matrix} & -A(b-w) & \lambda-1 & 0 & Ab\bar{u} & \frac{1-\delta-\lambda}{b\bar{y}^2} & 0 \\ -B(b-w) & \begin{matrix}-1+B(b-w)\\ +(\bar{s}+\bar{u})\omega\end{matrix} & 0 & \lambda-1 & -Bb\bar{u} & 0 & \frac{\alpha}{1-\alpha}\frac{\lambda}{b} \\ 0 & 0 & \beta\lambda & -\beta\lambda & \lambda-1 & 0 & 0 \\ -C(b-w)-\omega\bar{y} & C(b-w)+\omega\bar{y} & 0 & 0 & -Cb\bar{u} & \lambda-1 & 0 \\ -D(b-w)+E\bar{y} & \begin{matrix}D(b-w)+E+\\ (\frac{b}{\alpha}-w(1+\bar{y}))\omega\bar{u}\end{matrix} & 0 & -\bar{z}\beta-\varphi & -Db\bar{u} & E\bar{u}+\delta & \lambda-1+\alpha \end{vmatrix}$$

$$\begin{aligned} \text{With } A &= (\bar{s}+\frac{\bar{u}}{\delta})\frac{b\bar{u}}{\delta}\gamma \\ B &= (\bar{u}+\bar{s})(1+\bar{y})\gamma \\ C &= (1+\bar{y})^2\gamma \\ D &= \bar{u}(\frac{b}{\alpha}-w(1+\bar{y}))(1+\bar{y})\gamma \\ E &= w(1-\lambda)-b\,. \end{aligned}$$

$P(\lambda)$ is a polynomial of the seventh degree, whose analysis is difficult. In what follows we will demonstrate a number of properties of this general formalism. In Part IV a full treatment of stability will be developed for the short-term approximation of the model.

B. Stability in Dimension

In this section, the analysis focuses on a necessary condition for stability: $P(1)>0$. The limit for stability corresponds to $P(1)=0$, i.e., one eigenvalue is equal to 1. We study the dynamics of the economy when the dominant eigenvalue is close to 1. In Subsection *A)*, we express the stability condition and determine the eigenvector associated with the eigenvalue equal to 1. It is shown that the condition $P(1)>0$ refers to what has been called above, the stability in dimension of the economy. Subsection *B)* develops the interpretation of these results by demonstrating the existence of a number of relationships among the variables when the dominant eigenvalue is in the vicinity of 1. In Subsection *C)* we contend that this vicinity is a basic feature of capitalism, which is consequently very unstable with respect to dimension, and provide some elements of empirical verification of this view.

A) The Condition for Stability in Dimension

With F denoting a constant independent from reaction coefficients:

$$F = \bar{y}(\bar{s}+\frac{\bar{u}}{\delta}) + (1-\alpha)(\bar{s}+\bar{u}) + \frac{w}{b}(1+\bar{y}) - \frac{\bar{u}}{\alpha},$$

condition $P(1)>0$ can be written:

$$\theta<1 \text{ with } \theta = \sigma+\varepsilon b\frac{\omega F+\alpha}{\varphi+\beta\bar{z}}. \tag{9}$$

We call the "stability frontier" the situation in which θ is equal to 1. In simpler models, condition (9) is also a sufficient condition to insure that no real eigenvalue larger than 1 can exist (as is the case in Part IV). However, we are not yet capable of demonstrating this result in the general form of the model used here.

We now make the very important assumption, which will be discussed below in III. *B. C)*, that the dominant eigenvalue is close to 1, and consider the associated eigenvector V. This vector accounts for the primary characteristics of the dynamics in a vicinity of the equilbrium ($u^1 = u^2 = \bar{u}$, $s^1 = s^2 = \bar{s}$, $x = 1$, $y = \bar{y}$, and $z = \bar{z}$), when the economy is close to the stability frontier ($\theta = 1$). One obtains:

$$V = \begin{bmatrix} 1 \\ 1 \\ -\frac{1-\sigma}{\varepsilon} \\ -\frac{1-\sigma}{\varepsilon} \\ 0 \\ \frac{b\bar{y}^2}{\delta}((\bar{s}+\frac{\bar{u}}{\delta})\omega - 1) \\ \frac{b(1-\alpha)}{\alpha}(1-\omega(\bar{s}+\bar{u})) \end{bmatrix} \tag{10}$$

This vector shows that, in the vicinity of the stability frontier, and in a vicinity of equilibrium, the following properties hold:

$$u^1 - \bar{u} \simeq u^2 - \bar{u} \quad \text{or} \quad u^1 \simeq u^2$$

$$s^1 - \bar{s} \simeq s^2 - \bar{s} \quad \text{or} \quad s^1 \simeq s^2$$

$$x - 1 \simeq 0 \quad \text{or} \quad p^1 \simeq p^2 .$$

From this follows that the proportions between the two industries are not disrupted when the limits for stability are overstepped (i.e., when θ becomes larger than 1). This corresponds to the fact that the stability in proportions of the economy is not dependent on condition (9). What this condition expresses could be studied in a model with only one commodity. *It is for this reason that we relate condition* (9) *to the stability in dimension of the economy.*

B) Three Basic Relations

From the value of the above eigenvector (cf. equation (10)), it is possible to derive other relationships among the variables in a vicinity of equilibrium and for θ close to 1. These relationships account for the features of the economy when it is drawn out of equilibrium. Such departures from equilibrium can correspond to: 1) The usual gravitation process, 2) The Business cycle, and 3) The effect of policies attempting, for example, to achieve a level of capacity utilization different from normal (for example, to fight unemployment). We denote u the common value of u^1 and u^2, and s the common value of s^1 and s^2:

1. The following relationship holds:

$$(1-\sigma)(u-\bar{u}) + \varepsilon(s-\bar{s}) = 0 . \tag{11}$$

   In the plane (u, s), this defines a trade-off which goes through $(\bar{u}, \bar{s})$.

2. There is a positive or negative relationship between the capacity utilization rate and the ratio of the stocks of capital of the two industries. It can be represented in the plane (y, u) by a positive relation or a trade-off. The sign of the relationship depends on the value of ω. A likely expectation would be the observation of a positive relation (a larger u associated with a larger investment in the industry producing investment goods). This situation corresponds to: $\omega > 1/(\bar{s} + \bar{u}/\delta)$.

3. There is also a relationship between u and z, the normalized stock of money. However, rather than z, it is more intuitive to consider the speed of circulation of money v:

$$v_t = \frac{Y_t^1 p_t^1 + Y_t^2 p_t^2}{M_t}.$$

The relationship between v and u is:

$$v - \bar{v} = (u - \bar{u}) \frac{\alpha}{1-\alpha} \frac{b\delta}{(b\bar{u} - \delta)^2} (\omega (\bar{s} + \frac{\bar{u}}{\delta} + \frac{\bar{s} + \bar{u}}{\bar{y}}) - 1).$$

In the plane (v, u), it corresponds either to a positive relation or a trade-off. If ω is sufficiently large that a positive relation exists between y and u, then the relationship between v and u is also positive (a larger u is associated with a larger speed of circulation of money).

C) The Stability Frontier

It is an obvious characteristic of capitalism that it is very unstable in dimension. This is reflected in the constant oscillation of the capacity utilization rate (as shown in Figure 1 for manufacturing industries in the U.S. after World War II). The observation that capitalism is very unstable in dimension is equivalent to the state-

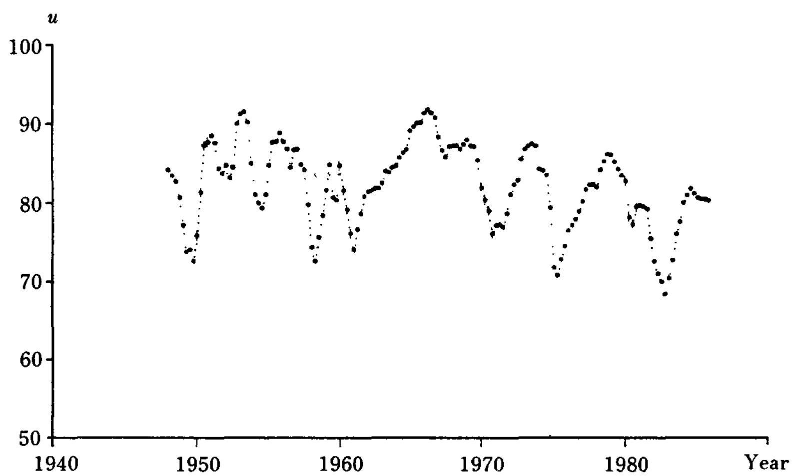

FIGURE 1
CAPACITY UTILIZATION RATE
MANUFACTURING INDUSTRIES, QUARTERLY DATA (1948–1985)

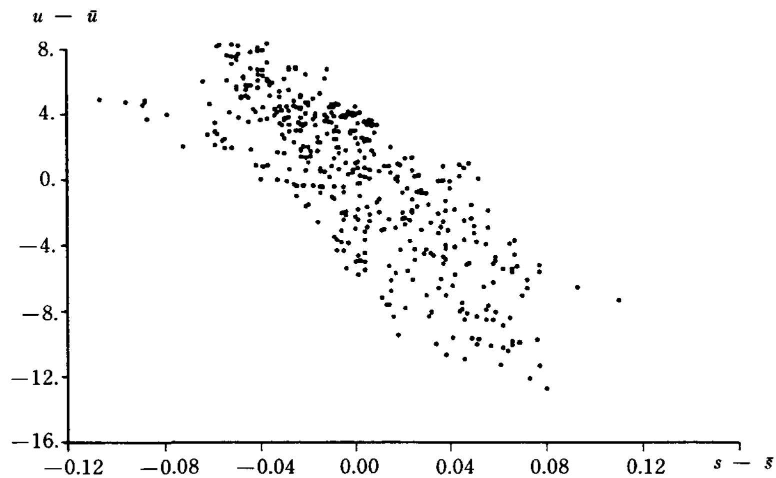

FIGURE 2
PLOT OF u AGAINST s (RESIDUAL AROUND THE TRENDS)
MANUFACTURING INDUSTRIES, JAN. 1950 - DEC. 1985

ment that θ is always in a vicinity of 1, sometimes below and sometimes above this limit.

It is possile to verify empirically in a straightforward manner that the dominant eigenvalue is close to 1, by investigating the relationship between the variables when the economy moves out of equilibrium. This is done in Figure 2, which represents an empirical verification of equation (11). This figure displays the relationship between u and s for the period 1950–1985, using monthly data. (The trends of the variables have been subtracted). It is evident from this figure that a trade-off exists. This observation proves that: 1) The economy is constantly maintained in a vicinity of the stability frontier, 2) Equation (5) is a good model of the decision to produce (equation (11) is a straightforward consequence of equation (5)).

C. Stability in Proportions

In Subsection *A)* we introduce the condition for stability in proportions. Then, in Subsections *B)* and *C)* we show that stability in proportions cannot be obtained if reaction coefficients ε or ω are equal to zero. This is equivalent to stating that a minimum value of these two coefficients is required for stability to be insured.

A) The Condition for Stability in Proportions

The system is unstable with respect to proportions if there exist two complex conjugate eigenvalues of modulus, υ, larger than 1. It is easy to verify that, under such conditions, the dominant component of each variable, X_t, revolves around equilibrium:

$$X_t \simeq \upsilon^t X \cos(\frac{t}{T} - a),$$

in which T, a, and X are three constants.

In a simple model such as described by Duménil and Lévy (1987d) or in the short-term approximation of the model in Part IV, it is possible to express analytically the condition that no complex conjugate eigenvalues of modulus larger than 1 exist. In other models (Duménil and Lévy 1987c, 1989a), one can only show that a non-empty set of values of the parameters exists for which stability is insured. Below we will only demonstrate two properties which we consider important in the analysis of the stability of capitalism: *if* $\varepsilon = 0$ *or* $\omega = 0$, *stability in proportions cannot be obtained.*

B) Instability in Proportions for $\varepsilon = 0$

If $\varepsilon = 0$, $P(\lambda)$ can be factored: $P(\lambda) = (\lambda - \sigma)^2 p(\lambda)$. Then σ appears as an eigenvalue twice. The 5 other eigenvalues are the zeros of polynomial $p(\lambda)$, and it is not possible, in the general case, to determine these zeros explicitly. In order to study these eigenvalues, we resort to the *perturbation* calculation (cf. Wilkinson 1965, Ch. 2).

The eigenvalues only depend on the product $\psi = \beta\gamma$, and not on the value of β and γ, considered separately. For $\psi = 0$, $\lambda = 1$ is an eigenvalue with a multiplicity of 3, and the two remaining eigenvalues satisfy $(\lambda - 1)(\lambda - 1 + \alpha) + \frac{\alpha}{1-\alpha}\frac{\varphi}{b}\lambda = 0$. The moduli of these two eigenvalues are always smaller than 1.

We then develop the equation $p(\lambda) = 0$ in the vicinity of $\psi = 0$. The moduli of the two remaining eigenvalues remain strictly smaller than 1 and, consequently, do not pose any problem concerning stability. The three other eigenvalues, which are equal to 1 if $\psi = 0$, remain in the vicinity of 1, and must be studied. These eigenvalues are functions of the parameter $\psi: \lambda = \lambda(\psi)$. We assume that we can develop $\lambda(\psi)$ in the vicinity of $\psi = 0$, as a series in $\psi^{1/3}$:

$$\lambda(\psi) = 1 + \mu\psi^{1/3} + \mu_2(\psi^{1/3})^2 + \mu_3(\psi^{1/3})^3 + \cdots.$$

It is possible to calculate μ, μ_2, μ_3,$\cdots$ by, first, substituting the expressions of λ, as a function of ψ, for its value in $p(\lambda)$ and, second, nullifying all the coefficients in the series in $\psi^{1/3}$ thus obtained. A set of equations is then determined which allows the calculation of μ, μ_2, μ_3, $\cdots$. Variable μ is a zero of a polynomial of degree 3, i.e., the multiplicity of the eigenvalue $\lambda = 1$ if $\psi = 0$. This equality justifies the assumption made in the equation above concerning the development of $\lambda(\psi)$ for $\psi = 0$ (cf. Wilkinson 1965, Th.2, Ch.2, p. 65).

The term of lower degree in $\psi^{1/3}$ in the development of $p(\lambda(\psi))$ is:

$$(\psi^{1/3})^3 \frac{\alpha}{1-\alpha} \frac{\varphi}{b} (\mu^3 + b\bar{u}^2).$$

In this expression $(\psi^{1/3})^3$ is factored out and corresponds to the three eigenvalues $\lambda = 1$ if $\psi = 0$. This expression is nullified if μ is one of the three cubic roots of $-b\bar{u}^2$. Among these three roots, 2 are complex conjugate and have a positive real part. Thus, 2 among the 3 eigenvalues, $\lambda = 1 + \mu\psi^{1/3} + \cdots$, are complex conjugate and have a modulus larger than 1 for small values of ψ. This is equivalent to stating that the system is unstable in proportions.

C) Instability in Proportions for $\omega = 0$

A similar demonstration can be made for $\omega = 0$, using the perturbation calculation of eigenvalues. We develop around $\psi = \beta\gamma = 0$. If $\psi = 0$, two eigenvalues are equal to 1. If ψ is small, these two eigenvalues can be expressed as $\lambda = 1 \pm iG\psi^{1/2}$, in which G is a positive and real constant. Their moduli are larger than 1, and the system is unstable in proportions.

D. The Shift of Normal Equilibrium

In Section *A* above, we refered to the notion of a "normal equilibrium," for which both the capacity utilization rate and the ratio of inventories are equal to their target values. This property of equilibrim ($u^i = \bar{u}$ and $s^i = \bar{s}$) only holds if the credit mechanisms and the formation of consumption demand are not biased. Such biases correspond in the model to the existence of a constant in equations (3), (6), or (7), representing the facts that $\Delta\kappa$ and ΔC are different from zero whereas capacity utilization rates or ratios of inventories are normal, and that demand has an exogenous component. This absence of bias is closely related to monetary and demand policies:

1. A first function of monetary policy, in the broad sense of the term, is to avoid the cumulative indebtness of agents (enterprises, consumers), i.e., in the model, the absence of constant term in equations (3) and (6), which guarantees that no such development can occur provided that the economy gravitates around normal equilibrium.

2. A second function of monetary policy is to insure the stability of the general level of prices. The existence of constant terms in equations (3) and (6) would result in the shift of normal equilibrium with a rate of variation of the general level of prices different from zero.

3. In equation (7), demand for consumption is determined on the basis of the amount of money held by final consumers. Their stock of purchasing power grows as a result of the inflow of income and customer credit, and diminished with the expenses. Demand policy can be interpreted on this basis as an exception to this rule of financing of expenses out of income. This is equivalent to adding a constant term in equation (7).

The existence of such biases has two different consequences:

1. Normal equilibrium still exists, but is *shifted* upward or downward. This is an important property of the model. The existence of equilibrium is not subject to the absence of bias in credit and demand mechanisms.

2. These disequilibria bear on the wealth of economic agents. For brevity, we assume the existence of a public deficit. In this case, it is the state which is going into debt. This growing indebtness is compensated for by two mechanisms: 1) The increase of the general price level reduces the real value of the stock of money held by final consumers (cf. equation (8)), 2) The high level of demand (i.e., low inventories) diminishes the amount of customer credit (cf. equation (6)).

The notion of the shift of normal equilibrium is related to the analysis above of the trade-off between u and s(cf. III. *B. B)*): *the shift occurs along the trade-off*. Assume that an extra amount of consumption from the state, G_t, is financed by a deficit. We assume that G_t can be expressed as a given fraction g without dimension of total consumption when normal equilibrium prevails: $G_t = bK_{t+1}^2 g$. The introduction of this exogenous demand only modifies the equation for s_{t+1}^2 in the recursion(cf. Table 2). Equilibrium is shifted. We denote $(u^{1*}, u^{2*}, s^{1*}, \cdots)$ the new equilibrium values of the variables and X, the vector $(u^{1*} - \bar{u}, u^{2*} - \bar{u}, s^{1*} - \bar{s}, \cdots)$. If θ is close to 1, then the dominant term of X is:

$$X = \frac{H}{1-\theta} gV, \tag{12}$$

where V is the eigenvector associated with $\lambda = 1$ (cf. equation (10)), and H, a positive constant. This equation shows that the shift occurs along the dominant eigenvector.

Equation (12) can be interpreted as a demand multiplier. As a result of the approximation of the model, it tends to infinity when $\theta \to 1$. In any case, without the approximation, this multiplier would be large in a vicinity of $\theta = 1$. This property reflects the fact that the least shock on aggregate demand would result in considerable variations of the general level of activity. Such circumstances render demand policy very difficult to manage.

E. *Stability and Instability in Capitalism*

In this section we briefly discuss the results obtained above and in other earlier models (cf. in particular, Duménil and Lévy 1987d, 1987c, 1989a).

It is not possible to provide an overall judgement concerning the stability of capitalist economies, without distinguishing between what we have called dimension (the general level of activity) and proportions (the relative values of variables among industries). Capitalism is very stable only in this second respect. It is a system in which capitals are allocated properly, rationing is scarce, and excessive inventories rare. If such disequilibria exist, the sanction comes very quickly from the market, and the enterprise or the capitalists are eliminated promptly. Conversely, capitalism is very unstable with respect to dimension. Paraphrasing president Hoover, one can contend that "recession is always around the corner".

The first aspect, the stability of proportions is the strong point of capitalist economies. It is used as basis of propagenda: private enterprise and free markets are efficient. It is not possible to ignore the second aspect, however, the fragility of dimension. This evident weakness of the system was often attributed, following the same interpretation, to the imperfection of real capitalism, but never to its nature. It is contended that capitalism does not actually conform to its concept, and therefore something like a business cycle can exist: markets are imperfect, information is asymetrical, prices are sticky, workers are unionized, etc.

In this apology for capitalism the two aspects of the stability problem are juxtaposed, but not articulated. *It is our contention that the same mechanisms which account for the stability of proportions also account for the fragility of dimension.* This is evident from the examination of the stability conditions:

1. *Condition* (9) *shows that the values of* ε *or* ω *cannot be too large, without pushing the economy beyond the limits of its stability in dimension.* Recall that ε measures the sensitivity of enterprises to excessive or deficient stockpiling, and ω the degree to which borrowing for investment is dependent on the deviation of the capacity utilization rate from its target value. If they are too intense, these two first mechanisms jeopardize the stability of the system. In the denominator, $\beta\bar{z}$ models the stabilizing properties of the real balance effect. However, for reasons which go beyond the limits of this study, we do not believe that this mechanism is important, in the actual stabilization of the economy. On the contrary the direct action of enterprises on the availability of purchasing power, which is represented by φ, corresponds in our opinion to a simple form of the prominent stabilizing mechanisms in capitalism. It is easy to contrast the actions of ε and φ with respect to stability in dimension. Consider, for example, a situation in which enterprises are stockpiling. A large ε means that this bloating of inventories provokes a further reduction of demand since firms are scaling down their activity. Through ε deficient demand can become cumulative. Conversely, in the case of φ, enterprises, confronted with the same deficient demand strive to diminish their inventories by creating a purchasing power — a correcting mechanism. The first mechanism (ε) is procyclical, whereas the second one (φ) is countercyclical.

2. Although the conditions for stability in proportions have not been made fully explicit, it has been shown in III. *C. B)* and III. *C. C)* that no stability in proportions can be obtained with ε or ω equal to zero. This is an illustration of a more general property: *substantial values of* ε *and* ω *are favorable to the stability in proportions of the system.* A large value of ε guarantees in the

short run that production will not be maintained if outlets are not sufficient or, conversely, that diminishing inventories provoke a strong acceleration of production. In a similar manner, a large ω allows for a fast correction of the inappropriate allocation of fixed capital. The speed of the mobility of capital is restricted by the availability of capital set-free, since capital engaged in fixed capital or inventories cannot be displaced. Loans for investment are not subject to this restriction and, within disequilibrium, the mass of capital (a purchasing power) can be extended, or restricted, and responds vigorously to local requirements.

The contradiction between the two aspects of the stability problem can be formulated as follows: *large ε and ω are favorable to stability in proportions and, simultaneously, jeopardize stability in dimension.* Moreover, it is possible to show that the optimal individual values of the two reaction coefficients do not coincide with the optimal compromise concerning the overall stability of the system.

It has already been demonstrated that the modeling of economic phenomena in terms of adjustment can be applied to socialist economies in frameworks of analysis different from those appropriate to the study of capitalism (Kornai 1979, 1980; Kornai and Martos 1973, 1981; Simonovits 1986). An interesting issue in this respect is to discuss the relationship between the two aspects of the stability problem in these economies. The use of the concept of "shortage economy" in Kornai's work suggests that the hierarchy established with respect to capitalism, instability in dimension and stability in proportions, should be reversed in the case of socialist economies which could be characterized as stable in dimension and unstable in proportions.

IV. Short Term vs. Long Term

In the description of our general framework of analysis (cf. II. *A. A)*), the distinction between short- and long-term equilibria has been introduced. Long-term equilibrium corresponds to the classical conception of equilibrium with prices of production. Concerning the short run, the generalization of the classical treatment of competition leads to the definition of a short-term equilibrium in which the clearing of the markets is obtained by quantities (instead of prices as is traditional), i.e., the establishment of capacity utilization rates to levels corresponding to demand.

In Section A, the notion of a short-term approximation of the model is introduced. In this framework, and at the expense of two further technical assumptions, it is possible to show that condition (9) is a necessary and sufficient condition for stabiliy. This demonstration is made in two steps in Sections B and C.

A. The Short-term Approximation of the Model

This part is devoted to the analysis of the stability of the model under the assumption that the reaction coefficients involved in the modeling of long-term decisions are small in comparison to coefficients considered in short-term decisions:

1. In the first group (long-term), we include γ for the mobility of capital, and ω for loans for investment.
2. The second group (short-term) comprises ε and $1 - \sigma$ considered in the decision to produce, φ for customer credit, and α which models the propensity to spend of final consumers.
3. Some ambiguity exists concerning the price decision (coefficient β). In the demonstration below, this coefficient has been classified in the first group, since we believe that the equilibration of the commodity market in the short-terms is basically realized by adjustment of the capacity utilization rate. However, this assumption is not essential to the properties shown in the remainder of this part.

The difference between the two types of reactions is, in our opinion, a prominent characteristic of capitalist or socialist economies. It introudces, as will be shown below, the idea that the problem of stability in the *long run* is only that of proportions, whereas, in the *short run,* the two aspects, proportions and dimension, coexist. Therefore, the issue of *dimension* appears as specific of the short run.

The study of the stability of normal equilibrium, in which the polynomial $P(\lambda)$ of the seventh degree is implied in the general case, becomes simpler in the short-term approximation of the model. This investigation will be conducted in two stages:

1. We will first study the "short-term *limit* of the model" for $\gamma = \omega = \beta = 0$. In this case the Jacobian can be separated into three factors:

$$P(\lambda) = (1-\lambda)^2 p_2(\lambda) p_3(\lambda)$$

in which $p_2(\lambda)$ and $p_3(\lambda)$ are polynomials of the second and third degree respectively. The two first eigenvalues $\lambda = 1$ correspond to the long-term dynamics which has been neutralized: $x = p^1/p^2$ and $y = K^1/K^2$ are constant. We will show in Section B that the polynomial $p_2(\lambda)$ can be associated with short-term proportions and $p_3(\lambda)$ with short-term dimension. Under rather weak conditions, the modulus of the zeros of $p_2(\lambda)$ and $p_3(\lambda)$ are smaller than 1, if $\theta < 1$.

2. In the second stage, we will actually consider the "short-term *approximation* of the model", assuming that γ, ω, and β are small in comparison to other reaction coefficients, and resorting again to the method of perturbation (cf. Wilkinson 1965). The five eigenvalues which were within the unit circle remain in it. We will show in Section C, that the two eigenvalues which were equal to 1 in the short-term limit, move back within the circle. *This is equivalent to the demonstration of the stability of the whole process: short-term and long-term, for proportions and dimension.*

Globally, we will prove the following theorem:

Theorem

If the two following conditions are staisfied:

$$\frac{\varphi}{b} < \alpha(1-\alpha) \tag{13}$$

$$1-\sigma < \alpha, \tag{14}$$

if the three long-term reaction coefficients γ, ω, and β are positive but remain small enough, and if $\varepsilon > 0$, then $\theta < 1$ (cf. equation (9)) is a necessary and sufficient condition for stability.

B. The Short-term Limit

If $\omega = \gamma = \beta = 0$, the Jacobian can be written: $P(\lambda) = (1-\lambda)^2$ $p_2(\lambda)p_3(\lambda)$, with:

$$p_2(\lambda) = (\lambda-1)(\lambda-\sigma) + \varepsilon$$

$$p_3(\lambda) = \begin{vmatrix} \varepsilon + (\lambda-1)(\lambda-\sigma) & \dfrac{\alpha}{1-\alpha}\dfrac{\lambda}{b} \\ -(w(1-\lambda) - b)\varepsilon - \varphi(\lambda-\sigma) & \lambda - 1 + \alpha \end{vmatrix}.$$

The modulus of the two zeros of $p_2(\lambda)$ are smaller than 1 if $\varepsilon + \sigma < 1$.

Polynomial $p_2(\lambda)$ is, in fact, the Jacobian of the following model of partial equilibrium in which a single enterprise, i, is considered and demand, d^i, is exogenous:

$$u^i_{t+1} = \bar{u} + \sigma(u^i_t - \bar{u}) - \varepsilon(s^i_t - \bar{s})$$

$$s^i_{t+1} = s^i_t + u^i_t - d^i.$$

This model corresponds to a specific problem which refers to what could be called short-term "individual stability" or "stability in proportions".

The study of the three zeros of $p_3(\lambda)$ is more difficult. For simplicity we will assume that conditions (13) and (14) are satisfied and determine the intervals of ε in which the three eigenvalues have a modulus smaller than 1. The limits of these intervals necessarily correspond to one of the three following conditions: 1) One eigenvalue is equal to 1, 2) Two complex conjugate eigenvalues have a modulus equal to 1, or 3) One eigenvalue is equal to -1.

1. *One eigenvalue is equal to* 1

The values of ε for which one eigenvalue is equal to 1 satisfy $p_3(1) = 0$. One obtains:

$$\varepsilon = \frac{\varphi(1-\sigma)}{ab}$$

This value corresponds to condition (9): $\theta = 1$, which is reduced to the above equation for $\omega = \beta = 0$.

2. *Two complex conjugate eigenvalues have a modulus equal to* 1

Polynomial $p_3(\lambda)$ can be written: $p_3(\lambda) = \lambda^3 + A\lambda^2 + B\lambda + C$. The condition under consideration is equivalent to: $1 + C(A-C) - B = 0$. A long but simple calculation shows that $1 + C(A-C) - B$ is always larger than 0 if conditions (13) and (14) are satisfied, and if:

$$0 \leq \varepsilon \leq \frac{\varphi(1-\sigma)}{ab} \tag{15}$$

3. *One eigenvalue is equal to* -1

This limit is associated with $p_3(-1) = 0$, which is satisfied if:

$$\varepsilon = (1+\sigma)\frac{2(1-\alpha)(2-\alpha) - \alpha\frac{\varphi}{b}}{\alpha(1-2\frac{w}{b}) - (2-\alpha)(1-\alpha)}$$

Condition (13) is a sufficient condition insuring that this value of ε does not belong to the interval defined by (15).

It is easy to verify that, if $\varepsilon = 0$, then the three eigenvalues have a modulus smaller than 1. This remains true in the whole interval defined by (15), since we showed above that for all ε in this interval, it is impossible to have two complex conjugate eigenvalues with modulus equal to 1, or one eigenvalue equal to -1.

If $\frac{\varphi(1-\sigma)}{ab} < \varepsilon$, there is necessarily an odd number of real eigenvalues larger than 1, since an eigenvalue can only reach the real axis beyond 1 in two manners: 1) Crossing the limit $\lambda = 1$, but this occurs only for $\varepsilon = \frac{\varphi(1-\sigma)}{ab}$, or 2) When two complex conjugate eigenvalues become real.

Therefore, if conditions (13) and (14) are satisfied, (15) is a necessary and sufficient condition for the three eigenvalues to have a modulus smaller than 1.

C. *The Short-term Approximation*

We now assume that γ, ω, and β are infinitely small, and define β' and γ' by: $\beta = \beta'\omega$ and $\gamma = \gamma'\omega$. We consider the two eigenvalues which were equal to 1 when these coefficients were equal to zero in the above section, and use the method of perturbation with $\lambda = 1 + \mu\omega$. We substitute the values of γ, β, and λ, as functions of ω in the Jacobian $P(\lambda)$. The dominant term in ω in $P(\lambda)$ is:

$$L\omega^2\left(\varepsilon - \frac{(1-\sigma)\varphi}{ab}\right)\begin{vmatrix} 1-\sigma & \varepsilon & 0 \\ 0 & \beta' & \mu \\ \gamma'(b-w) + \dfrac{\delta/b+\mu}{\bar{u}(1+\bar{y})} & 0 & \gamma' b\bar{u} \end{vmatrix}$$

in which L is a positive constant. The factor ω^2 corresponds to the two eigenvalues which are equal to 1 if $\omega = 0$.

The determinant is a polynomial of the second degree in μ:

$$\mu\left(\mu + \frac{\delta}{b} + \gamma'(b-w)(1+\bar{y})\bar{u}\right) + \beta'\gamma'\frac{1-\sigma}{\varepsilon}\, b\bar{u}^2(1+\bar{y})$$

If $\varepsilon > 0$ and if $\beta'\gamma' > 0$, then this polynomial has two zeros μ_1 and μ_2 which are either: 1) Real and negative, or 2) Complex conjugate with a negative real component. In both cases, the modulus of the two eigenvalues $\lambda_i = 1 + \mu_i\omega$ $(i = 1,2)$ are smaller than 1, if ω, and therefore also β and γ, remain sufficiently small. This calculation is valid only if ε is strictly positive.

Thus, if conditions (13) and (14) are satisfied, if ω, γ, and β remain sufficiently small, the system can only be destabilized if one eigenvalue becomes equal to 1 (instability in dimension): *Condition* (9) *is a necessary and sufficient condition for stability.*

V. Instability in Dimension

In this last part we will briefly describe what happens in the economy when the condition for the stability in dimension of the system is trespassed (condition (9): $\theta > 1$), using the results obtained in other studies (cf. Duménil and Lévy 1987a, 1987b). Since the point at issue only concerns dimension, it is a useful simplification to consider models with only a single commodity. The notion of non-linear behavior is introduced in Section *A*. Section *B* presents the two equilibria, different from normal equilibrium, which exist and are stable when $\theta > 1$, overheating and stagnation. Section *C* briefly suggests how the pattern of the three equilibria can be used to build a theory of business cycle.

A. Non-Linearities

Although the model presented in Part II is not linear, all behavioral equations (cf II. *B*) have been defined in a linear form. This modeling of behavior is appropriate in a vicinity of a stable normal equilibrium. If normal equilibrium is not stable, and the economy strays away from this situation, this simple representation of behavior cannot be maintained. An unstable linear model only "explodes" (such as $x_{t+1} = \lambda x_t$, for $\lambda > 1$). As was shown in Subsection III. *B. C)* and Figure 2, the economy is always in a vicinity of $\lambda = 1$, and it is therefore necessary to introduce non-linearities in the modeling of behavior.

At least three sources of non-linearities are already implied in the model:

1. In the modeling of the decision to produce (cf. II. *B. E)*), the capacity utilization rate u is confined between 0 and 1. As u moves closer to one, for example, the same reduction of the ratio of inventories s results in smaller increases of u. This behavior

would reflect the fact that costs increase more than proportionaly, when u is close to 1.

2. The issuance and destruction of money through credit mechanisms are not symmetrical. It is difficult and dangerous to destroy money by way of a tight monetary policy, since enterprises can seek new loans just in order to pay their matured debts. The denial of these demands can lead to a cumulative spiral downward.

3. It is well known that the propensity to consume becomes smaller as income is increased.

In the next section we will consider the effect of the modeling of such nonlinearities.

B. The Configuration of Equilibria: The Pitchfork

The number of equilibria depends on the value of θ. If $\theta < 1$, only one equilibrium exists: normal equilibrium. If $\theta > 1$, three equilibria exist: normal equilibrium and two other equilibria:

1. An equilibrium in which $u > \bar{u}$ and $s < \bar{s}$, which we call *overheating*.

2. An equilibrium in which $u < \bar{u}$ and $s > \bar{s}$, which we call *stagnation*.

The stability of these equilibria also depends on the value of θ. For $\theta < 1$, only normal equilibrium is stable. For $\theta > 1$, only overheating and stagnation are stable.

Concerning overheating and stagnation, not only their existence and stability are subject to the value of θ, but also the equilibrium values of the variables. As θ increases, the distance between normal equilibrium and the two other equilibria grows wider. If θ is slightly above 1, the capacity utilization rates and ratios of inventories which are attained in the two equilibria are close to normal. For lage values of θ, these same variables reach levels which can differ considerably from normal.

Consider, for example, u. It is possible to represent this configuration graphically, as shown in Figure 3. The image of the stable equilibria is that of a pitchfork with two prongs whose distance from normal equilibrium increases with θ.

An important property of this configuration is that it is maintained if a shift, as defined in Section III. *D* occurs. The whole pitchfork is shifted upward or downward, including overheating and

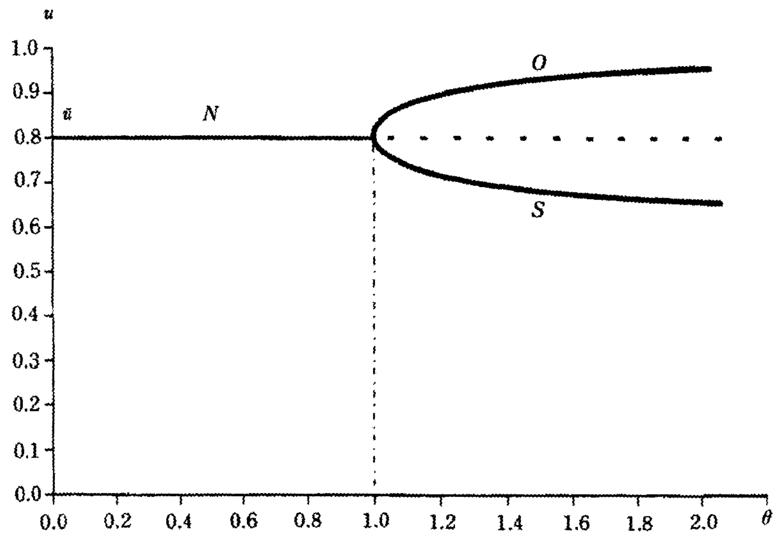

FIGURE 3
THE PITCHFORK
THE CONFIGURATION OF EQUILIBRIA

This figure represents the configuration of the equilibria with respect the value of the capacity utilization rates obtained, as functions of θ. Stable equilibria are represented by a dark line and the unstable normal equilibrium for $\theta > 1$, by a doted line. *N*, *O*, and *S* respectively denote "Normal equilibrium", "Overheating", and "Stagnation". The gap between overheating and stagnation increases with θ.

stagnation. For large values of θ the economy can remain in a stagnation, but a demand policy makes stagnation less stagnating. In a vicinity of $\theta = 1$, the exact form of the fork is altered, and the analysis of the precise consequences of a demand policy, beyond the simple notion of the lifting of the economy, becomes complex.

C. The Business Cycle

It is important to stress that this configuration of the three equilibria provides a basis for a theory of the business cycle. We interpret the business cycle as a succession of switches from one stable equilibrium to another. For example, the traditional decennial 19th century cycle implies a variation of θ and corresponds to a succession of stagnation, normal equilibrium, and overheating, and over again. The recession corresponds to the sudden destabilizing of overheating and the plunge into stagnation. However, other forms of

TABLE 3
LIST OF FIGURES AND SOURCES

1. Capacity Utilization Rate, Manufacturing Industries, Quarterly Data(1948–1985). IPXMCA.
2. Plot of u Against s (Residual Around the Trends), Manufacturing Industries, Jan. 1950–Dec. 1985 u = IPXMCA and s = IVM3/MFGSM. u and s are detrended using a polynomial of the second degree in time. The variables used are the residuals.
3. The Pitchfork, The Configuration of Equilibria.

The three series used in Figures 1 and 2 were obtained from the Citibase Tape (Citicorp 1986):

Capacity Utilization Rate (Manuf.)	1948–1985	IPXMCA
Invent. of Finished Goods (Manuf.)	1946–1985	IVM3
Shipments (Manuf).	1947–1985	MFGS

the business cycle, such as those which prevailed since World War II, can be analyzed as constant flips from overheating to stagnation (cf. Figure 1), for an approximately constant value of θ.

How this model can be used to build a theory of business cycle has been shown in various other studies with factual illustrations borrowed from the example of the U.S. economy since the civil war (cf. Duménil and Lévy 1988a, 1988b, 1988c, 1988d).

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