Temporal Situations and a Representation of Concurrent Events

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The representation of time with the physical time parameter 't', often overspecifies the semantics of natural languages, because we cannot always map their temporal property on the abstract time axis.

In order to realize a more flexible time description, we extend temporal locations in situation theory to temporal situations which are defined as open sets in a topological space. With use of this concept, the relative distance of events and the relation between events are lucidly stated.

As a main field of application of temporal situations, we choose the problem of concurrent courses of events that is difficult to represent in terms of conventional concept of time.

In the first section we will discuss the problem of representation of time and situation theory, and in the following section we will formally define temporal situations. Thereafter we will apply this notion to the problem of course of events and concurrency, and we will consider to intertwine sequences on different time axes.

1. Introduction

The problem of semantics of time The semantic representation of time in natural languages has been dealt with, associated with the physical time parameter 't'. Namely, we have considered that time is one dimensional line which extends both to eternal past and to eternal future, and a point called 'now' moves along the line at a fixed speed. The interpretation of Allen (1984), that is one of typical time representation of natural languages, based on this view.

However, the introduction of parameter 't' seems too strong for natural languages, as is different from the case of physical equations; actually we cannot always map the temporal property of verbs or temporal anaphora on the time axis correctly.

Kamp (1979, 1986) proposed event calculus where he claimed that 'an instant' was relatively defined by all the known events. Though his DRT is advantageous as for the temporal relativity, it seems less powerful in dealing with properties of verbs which we will mainly pay attention to in this paper.

Temporal locations in situation theory We formalize temporal situations by situation theory, where we will realize another formalism of relative time.

The notion of a spatio-temporal location, or simply a location, in situation theory physically means the four-dimensional space of time and place. In the beginning stage of situation theory, situations and spatio-temporal locations were distinguished (Barwise & Perry (1983)), and the latter had no other meaning than physical time and locations though the former had been given a significant theoretical framework. Since then, the discussion with regard to the relation between situations and spatio-temporal locations is rarely mentioned and rather neglected, although Cooper (1985, 1986) had shown the situated representation of verb meanings based on Vendler classification (Dowty (1979)) with the notion of spatio-temporal location.

We have worked for the translation of verb aspects (Tojo (1990)), and have been in need to equip a more flexible representation of relative time. In this paper we propose to introduce temporal situations that can have a relative distance in time.

2. The Definition of Temporal Situation

In this section we will define temporal situations. Prior to that, we will settle several notions related to situation theory. An *infon* is a formal description of sentential information, which consists of a *relation* and a set of objects bound by that relation, often with *polarity* of 1/0: $\ll rel, \dot{x}>, pol>$. When *pol* is missing, we regard it 1. An event is a temporally anchored infon: $\ll l_t, <rel, \dot{x}>, pol>$.

2.1. Temporal Locations

Definition 1 (temporal location) *A temporal location l_t is a subset of the time space, and in case:*

$$<l_t, <rel, x>, 1>$$

then, for any time point $t \in l_t$, $\ll rel, \dot{x}>, 1>$ holds.

A temporal location is a part of the physical time space T. The first thought we may take is to regard temporal locations as temporal situations directly, however this idea fails by the following two reasons:

1. First, there is no consideration upon natural language sentences accompanying tense and aspect.
2. And also, the temporal location may not a continuous, uniform span of time.

For an example of the first reason, which time span supports or satisfies simple sentences such as:

John ran.
John is running.

How should we define the time when *John* actually *ran* or *is running*? The second reason comes from following sentences:

John eats a lunch box every day.
John used to drink too much in his youth.

Both of these two sentences mentions a kind of habituality. What time span should we regard supports the expression like '*used to*'? Namely we cannot always fix the shape of temporal locations in T.

2.2. Viewpoints and Perspectives

Reichenbach (1947) distinguishes following three kinds of temporal information:

- time of action
- time of reference
- time of speech

We will rephrase these notions in terms of our formalism though we are required to make slight changes. We can naturally replace the time of action for temporal locations, however the time of speech and that of reference are not easy. We regard that the time of speech is the epistemological 'now' which decides *tense* for every sentence. In this meaning, *utterance situation* should be the temporal point of view. The viewer pays attention to other parts of time which may be either a point or a span with some length. So what we deal with are:

- temporal location
- point of view
- scope or field of view

The last item of focusing scope should be used for the analysis of verb aspects though we will not detail the matter in this paper.

2. 3. Points and Sets in the Space

One of our objectives is to regard temporal situations as an open set in a topological space and to discuss the relations between them such as inclusion in terms of topology. We will assume all the temporal points of view as the background of the temporal space, namely the space itself becomes the same one as the physical time space T. However we assume no metric topology a priori in it. This means that at the initial stage there is no chronological order nor the distance between two points. It is a set of events that decides temporal relations in this space. In order to distinguish T from the non-metric space, we denote the latter T^*. When a set of open sets of temporal situations O:

$$O=\{o_1, o_2, o_3, \cdots\}$$

was given, we say we introduced topology with regard to time into T^*. If two points exist in a certain o_i but there is not any o_j which includes only one of them, we can regard these two points are not distinguishable temporally, according to the *axiom of separation*[1].

[1] Axiom of Separation: (T_0) for two distinct points x and y, there is a neighborhood of one point, for example x, that does not contain the other point y (Kolmogorov). (T_2) for two distinct points x and y, there are a neighborhood of x U and that of y V that satisfies $U \cap V=\phi$ (Hausdorff).

2. 4. Temporal Situations and the Support Relation

Definition 2 (temporal situation) *A temporal situation is a set of viewpoints, and is a set in T^*. For any σ of present tense:*

$$S_t \models \sigma$$

iff σ is satisfiable from every point of view in S_t.

In trivial cases temporal situations exactly correspond to their temporal locations. However, our definition is a slight extension of the concept of temporal location in the following meaning. Firstly we can give a solution to the temporal representation of habituality. We can regard *habitual* as a bead thread of that activity as in fig. 1. However, considering the perspective which satisfies that habituality, we can take the closure of all the beads in fig. 1 for S_t:

$$S_t = Cl(\bigcup l_t)$$

Secondly we can define a temporal situation of some infon with a relation in past tense, as a temporal area posterior to the event; from a point of view of that area the infon can be mentioned in past tense. We will return the representation of past after we introduced *chronological order* in situations.

Next, we will define the partial relation between temporal situations.

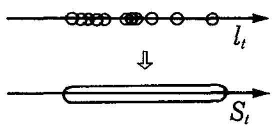

Figure 1: Habitual.

2. 5. Relations in Situations

Definition 3 (partiality relation) *We adopt the normal subset relation $\sqsubseteq$ between temporal situations $S_t \in T$ to represent the partiality relation $\sqsubseteq$.*

$$St_1 \subseteq St_2 \Leftrightarrow_{def} St_1 \sqsubseteq St_2$$

We will use $\sqsubseteq$, observing the convention of situation theory, to avoid further notational confusion. Note that:

$$St_1 \sqsubseteq St_2,\ St_2 \models \sigma \not\Rightarrow St_1 \models \sigma$$

This means that this partiality relation violates the persistency. For example, a set of verbs of *accomplishment* of Vendler classification (Dowty (1979)) does not satisfy this relation.

What we need to do next, is to fix a unique temporal situation for an event.

Definition 4 (induced temporal situation)
The maximal temporal situation which supports σ with regard to the relation $\sqsubseteq$:

$$\cup[S_t \mid S_t \models \sigma]$$

is called the temporal situation induced by the infon σ, and we denote it S_t^σ.
We will equip ourselves with another order between temporal situations.

Definition 5 (chronological order) *If all the points in St_1 precedes those in St_2 temporally, we write it:*

$$St_1 \preceq St_2$$

Example: (tense) If a temporal situation S_t satisfies:

$$S_t^\sigma \preceq S_t$$

We can view σ in past tense from S_t .(fig. 2) The maximal temporal situation which gives past tense to σ becomes the span from the ending point of σ to the eternal future.

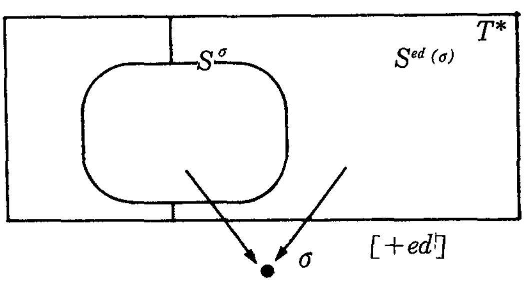

Figure 2: Viewpoints.

Example: (aspect) We will consider verb aspects of progressive and perfect, as an example of representation by temporal situation. Fig. 3 explains this matter: assume that the temporal location l_t for σ is some duration on the time axis, then *progressive* can be explained that a viewer paid attention to the inside of the duration, and *perfect* to the completing point of that event.

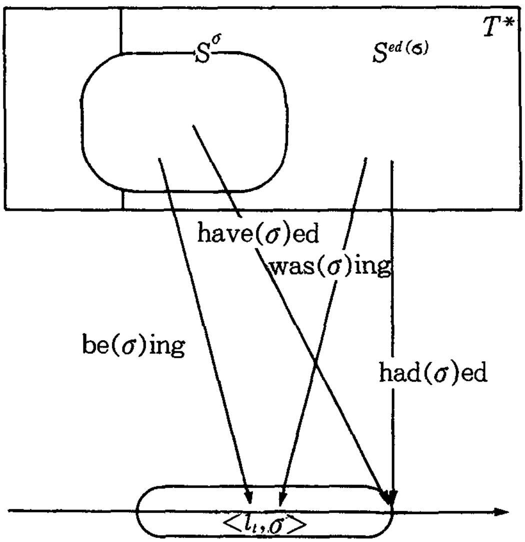

Figure 3: Aspects

2.6. Temporal Situations Mapped to Sets of Events

As is often the case in situation theory, we often need to interpret situations as sets of infons. We use a functor $\triangle$(Fernando (1990)) to denote a mapping from a situation to a set of infons:

$\triangle$: situation $\rightarrow$ set of infons

Definition 6 (infon set) $\triangle S_t$ *means a set of events that supported by* S_t.
For example, for some σ_1 and σ_2, if:

$S_t^{\sigma_1} \sqsubseteq S_t^{\sigma_2}$

then:

$\sigma_1 \in \triangle S_t^{\sigma_2}$

and:

$$\triangle S_l^{\sigma 1} \subseteq \triangle S_l^{\sigma 2}$$

Or, in more plain words:

$$\triangle S_l^{\sigma 2} = \{\sigma_1, \sigma_2, \cdots\}$$

(Hereafter we abbreviate $S_l^{\sigma i}$ as S^i, and a temporal situation S_l with the suffix i as S_i, as far as no confusion.)

3. Concurrency and Topology

3. 1. Course of Events

A course of events (we abbreviate *coe* hereafter) is an ordered set of infons that are temporally anchored. For example, for a sequence of temporal locations:

$\{l_1, l_2, l_3, l_4 \cdots\}$

if we can assume a chronological order $\preceq$ in them like:

$$l_1 \preceq l_2 \preceq l_3 \preceq l_4 \cdots$$

then:

$$< l_1 < rel_1, \dot{x} >, 1 >$$
$$< l_2 < rel_2, \dot{x} >, 1 >$$
$$< l_3 < rel_3, \dot{x} >, 1 >$$
$$< l_4 < rel_4, \dot{x} >, 1 >$$
$$\vdots$$

can be regarded as a coe. Our first problem is to introduce this chronological order properly in a set of events. However as we discussed in the previous section, temporal locations l_i is not adequate for the objects to be ordered. (Recall that l_i might not be a continuous span with rigid boundary.) Instead in this section, we difine a coe in terms of temporal situations.

Definition 7 (temporally relevant) *If two temporal situations S_i and S_j satisfy:*

$$S_i \preceq S_j \text{ or } S_i \cap S_j \neq \phi$$

those S_i and S_j are called temporally relevant. Otherwise they are temporally irrelevant.

Definition 8 (course of events) *For a set of events:*

$$\{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \cdots\}$$

if any two of their induced temporal situations S^i and S^j are temporally relevant, they are called a course of events.

If $\{\sigma_1, \sigma_2, \cdots\}$ is a coe, events of σ_i's must be ordered on a time axis according to the row of i though there may be some overlapping. This formalization is easy to extend to multi-axes time.

3. 2. Multi-axes Time and Concurrency

We can interpret the time of multi-axes for the human cognition feature where some events are ordered properly though the sequence cannot always be knitted with another sequence into one string, viz. we can only ordered things locally and partially. At the same time we can interpret this multi-axes as concurrency. Hereafter we will discuss this partial ordering and the mergence of multiple sequences of events.

The knowledge representation for concurrent events becomes harder because we need to consider the length of time span of one event as well as the order between them. For example, if one event is represented as a dot on a line the concurrency is easy to represent, however if we regard their time span the simultaneity expands in many ways. This distinction of variety in overlapping is actually redundant from our point of view: we do not always need to specify the real shape of temporal location. What we need to do is to establish a formalization which satisfies minimal constraints that are mentioned in natural language sentences. The constraint means such a temporal requirement as "a certain time point t_i must precede another time point t_j". We will discuss this constraint and how to intertwine multiple sequences of events into one in the following subsection.

3. 3. Intersectioning

Assume that:

$\{\sigma_1, \sigma_2, \sigma_3, \cdots\}$

is a coe, then we can get a set of open set:

$$s=\{S^1, S^2, S^3, S^4, \cdots\} \tag{1}$$

We will consider to introduce a topology with this set of temporal situations s; the topology $O_3(T^*)$ must satisfy the *axiom of topological space* so that it becomes a set of all the members of s together with all of their intersections and unions, with T^* and ϕ.

Definition 9 (introduced topology) *If s is, a set of temporal situations, then the superset of s O_3 that satisfies* :

$$\begin{aligned}
&\forall S^i, S^j \in s: \\
&S^i \in O_3(T^*), S^j \in O_3(T^*) \\
&S^i \cap S^j \in O_3(T^*) \\
&S^i \cup S^j \in O_3(T^*) \\
&T^* \in O_3(T^*), \phi \in O_3(T^*)
\end{aligned}$$

is called the introduced topology *by s*.

We can calculate a set of open sets, or temporal situations, that is finer-grained than the original s, making use of intersection mentioned in the above definition. Namely if two sets of S_1 and S_2 are overlapping, they are divided into three temporally consecutive sets, as is shown in fig. 4.

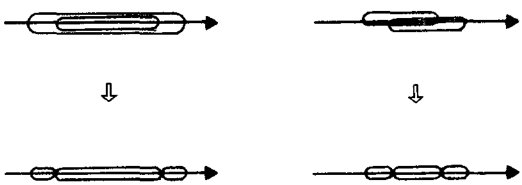

Figure 4: Intersectioning

Assume that a coe has temporal situations of (1). Then we can calculate a finer-grained sequence of sets:

$$s' = \{s_1, s_2, s_3, s_4, \cdots\} \tag{2}$$

where:

$$\forall s_i, s_j \in s' : s_i \preceq s_{i+1}, s_i \cap s_j = \phi$$

Namely s' becomes a sequence of disjoint sets though some of s_i's may be identical to S_i's in (1).

3. 4. Temporal Constraint

Assume that two coe's:

$$\{\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \cdots\}$$
$$\{\sigma_{21}, \sigma_{22}, \sigma_{23}, \sigma_{24}, \cdots\}$$

are happening concurrent, viz. on two independent time axes though there are several constraints on time between them such that:

- The beginning time of σ_{21} must precede the beginning time of σ_{13}.
- σ_{14} must be temporally included in σ_{23}.
- The finishing time of σ_{13} is earlier than the beginning time of σ_{24}.

⋮

Our goal is to intertwine these two strings so as to satisfy given constraints of the form:

$t_i \in coe_1 \preceq t_j \in coe_2$

Note that any t_i must interleave at joint points of the sequence (2).

3.5. Intertwining

We consider here to satisfy a constraint by induced topology. A constraint $t_1 \preceq t_2$ on different axes is diagrammed as in fig. 5.

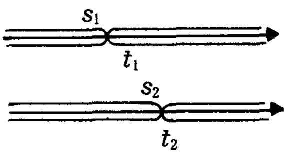

Figure 5: Contraint

We regard that to satisfy the constraint is to induce a topology of s_1 into s_2 in fig. 5, so as to realize a continuous mapping between two time axes.

We can consider that bringing a new topology namely a set of open sets O_2, into the original O_1 as a simple addition:

$O = O_1 \cup O_2$

however we may need to introduce topology locally in one open set or to replace one open set for other sets. In order to meet this requirement, we will formalize the application of the concept of *induced topology*[2].

Fig. 6 explains this matter: firstly the topology of O_1 is given, where an event is viewed as one point (or a set which has no internal set). However the induction of the topology of by f, the point is replaced for an open set $f^{-1}(O_1)$ in the figure, and we can view the inside of it as $f^{-1}(O_2)$, where O_2

[2] Given a space X, a topological space $(Y, O(Y))$, and a mapping $f: X \to Y$, we can define the *induced* topology $\{f^{-1}(O) \mid O \in O(Y)\}$ in X.

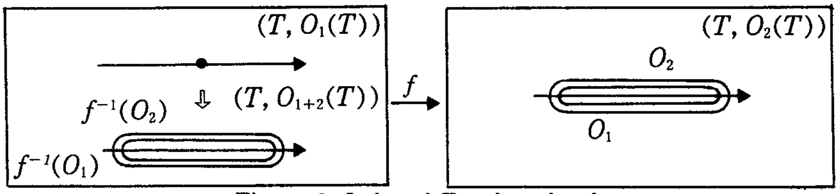

Figure 6: Induced Topology by f

is an internal set of O_1 in $(T, O_2(T))$.

As was discussed previously, we could acquire constraints of partially ordered timse:

$\{t_{i1} \preceq t_{j1},\ t_{i2} \preceq t_{j2},\ t_{i3} \preceq t_{j3},\ \cdots\}$

These relations can be represented as a diagram of multi-axes, as in fig. 7.

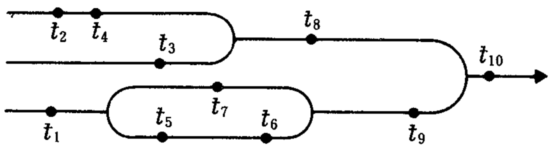

Figure 7: Branching Time

We denote Pt_i for all the past preceding the point t_i. We articulate the time space T^* by P_t. The calculus of "which set overlaps which set" is done by inducing a topology of one axis to another topology of another axis, as a calculation of addition and replacement of open sets, or temporal situations.

In the diagram in fig. 8 is the division by P_t, according to the time order-

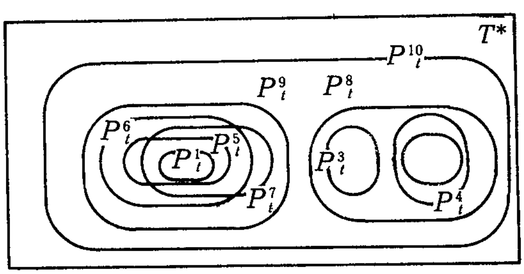

Figure 8: Division of T^*

ing in fig. 7. In that figure, a part of open sets that has open to the eternal past in that topology are shown, and every event can be articulated and be separated by the temporal order.

4. Discussion

We difined temporal situations and introduced topology in them, regarding temporal situations as open sets, so as to represent relative time. We can make use of several topological concepts such as axiom of topology, axiom of separation, and induced topology, to clarify the features of semantics of time in natural languages.

We have applied temporal situations to course of events and their concurrency. The problem of parallelism of activities or states are often hard to represent formally, because the information of overlapping of events cannot be decided rigidly from given natural languages. Our position is that the things which are expressed indefinitely in natural language must be formalized to the expression including indefiniteness, and from this postion we have tried to exclude the concept of metrics of physical parameter. Though we used the parameter named 't', they are not physical ones but relative ones and the distance in time is only decided relatively from the given set of events.

Although we can claim that this topological view of time is adequate for modeling the cognition of time, the calculation of intertwining multiple sequences should be remained as a virtual one. Actually it is much difficult to collect all right information of time precedence $t_1 \preceq t_2$ in T^* from a given paragraph, as a general problem of natural language processing.

In this paper we have mentioned the example of tense and aspect of verbs, where we introduced a point of view and focusing scope. At this stage we target further consideration on temporal features of verbs and aspects so as to extend our analysis on inter-event relations.

References

Allen, J. F. (1984) 'Towards a General Theory of Action and Time,' *Artificial Intelligence*.

Barwise, J. and J. Perry (1983) *Situations and Attitudes*, MIT Press.

Cooper, R. (1985) 'Aspectual Classes in Situation Semantics,' *Technical Report* CSLI-84-14C, Center for the Study of Language and Information.

Cooper, R. (1986) 'Tense and Discourse Location in Situation Semantics,' *Linguistics and Philosophy* 9(1): 17-36.

Dowty, D. (1979) *Word Meaning and Montague Grammar*, D. Reidel.

Fernando, T. (1990) 'Some Remarks on the Mathematics of Situation Theory,' *Technical Report* ICOT-IR, Institute for New Generation Computer Technology.

Hinrichs, E. (1986) 'Temporal Anaphora in Discourse,' *Linguistics and Philosophy* 9 (1): 17-36.

Kamp, H. (1979) 'Events, Instants, and Temporal References,' In *Semantics from Different Points of View*, Springer Verlag, pp. 376-417.

Reichenbach, H. (1947) *Elements of Symbolic Logic*, University of California Press, Berkeley.

Tojo, S. (1990) 'Type-theoretical Analysis of Complex Verb Generation,' In *Proc. of the 13th International Conference on Computational Linguistics*, Universitay of Helsinki.

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