Bankruptcy and Unemployment in Optimal Wage Employment Contracts*

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Optimal wage employment contracts under asymmetric information have shown that, if the firm is risk neutral and has the informational advantage over the risk averse workers, the second best contract prescribes underemployment if and only if the workers view leisure as an inferior good, and overemployment if and only if leisure is a normal good. In this paper it is shown that if leisure is a normal good and that if the firm wants to avoid bankruptcy, the optimal layoff contract, with optimal severance payment, may prescribe unemployment. A stronger unemployment result is obtained in the absence of severance pay. Finally, it is shown that, if the contracts specify both the working hours and the number of empolyed workers, the optimal contract may exhibit unemployment with respect to both the number of workers and the working hours, provided few technological preconditions are satisfied.

I. Introduction

The recent literature on optimal wage employment contracts under asymmetric information could not provide an unambiguous explanation of (non-Walrasian) involuntary unemployment. In the generally examined framework of multiplicative uncertainty in production with productivity information private to the firm, it has been shown that the second best contracts predict under or overemployment depending primarily on how workers view leisure. In particular, if the firm is risk neutral, optimal wage employment contracts under asymmetric information prescribe overemployment if and only if leisure is a normal good and underemployment if and only

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if leisure is an inferior good. This was first shown independently by Chari (1983) and Cooper (1983) in the context of the pure work-sharing contracts and by Brown-Wolfstetter (1985) in the context of the pure layoff contracts.

Given that it is natural to assume leisure to be a normal good, it is important to search for more robust microeconomic explanation of (non-Walrasian) involuntary unemployment. There are several directions of research which promise important results. Brown and Wolfstetter (1984) provide an interesting example. Combining the possibilities of worksharing and layoff, they concur with the unemployment result. Using a worksharing framework Ghosh (1984) demonstrated that if the compensation to the workers is constrained by an exogenous lower limit, the second best contracts may prescribe underemployment, even when leisure is a normal good.

Limitations on the firm's liability provide another important exception to the general results of the wage-employment contracts literature. If the firm is required to avoid bankruptcy, then even if leisure is a normal good the firm may not be able to "overemploy" the workers, because the wage offers may turn out to be infeasible in view of the bankruptcy constraint faced by the firm.

The purpose of this paper is to establish, *inter alia,* that if the bankruptcy constraint is effective, the second best contracts may prescribe underemployment even if leisure is a normal good. Kahn and Scheinkman (1985) have derived a similar result, however, ignoring the possibility of layoffs and limiting their analysis only to the worksharing contracts. The analysis of this essay is more general, since it considers the standard layoff contracts and also combines the possibilities of layoff and worksharing.

The plan of the paper is as follows. In Section II, we analyze the pure layoff contracts with optimal severance pay. It is shown that in the presence of active bankruptcy constraint the second best contract may exhibit underemployment even if leisure is a normal good. Section III considers the pure layoff contracts without the provision of severance payments. It is then established that, in the absence of severance pay, if the bankruptcy constraint is effective, the second best contract always exhibits underemployment. In Section IV, we develop the generalized contracts that combine the possibilities of worksharing and layoffs. It is shown that if some technological requirements are met, the second best contract may exhibit underemployment with respect to both the number of workers and the working hours, even if leisure is a normal good. Concluding remarks are made in Section V.

II. Pure Layoff Model with Severance Pay

A. Assumptions and Specifications of the Model

Consider a risk neutral, price taking firm entering into wage-employment contracts $C = \{n(\theta), w(\theta), b(\theta), t(\theta)\}$ with N homogeneous workers. The stipulated wage $w(\theta)$ is paid to each employed worker. The workday is institutionally fixed and chosen as the time unit. $n(\theta)$ is the number of employed workers. The workers are chosen randomly, through a lottery, for layoff. Hence $t(\theta) \equiv n(\theta)/N$ is the probability that a particular worker will be retained. The firm pays, each of the $(N - n)$ laid off workers the severance pay $b(\theta)$ (this assumption will be relaxed in Section III). Besides the severance pay, each laid off worker collects exogenous unemployment compensation e, provided by the government. The revenue of the firm is subject to a productivity (or demand) shock θ, so that the ex post profit of the firm is given by:

$$\pi(\theta) = \theta f(n(\theta)) - w(\theta)n(\theta) - b(\theta)(N - n(\theta))$$

where $f(\cdot)$ is a smooth, concave production function. The random shock θ is assumed to be a continuous random variable, defined over $[\underline{\theta}, \bar{\theta}]$, with the positive density function $g(\theta)$. The corresponding cumulative distribution function is denoted by $G(\theta)$. At the time the contracts are made, θ is unknown. All that is known to the firm and the workers is $g(\theta)$. However, when the final production decisions are made, θ is known with certainty (perhaps only to the firm). It is at this point that the contract is applied.

It is assumed that the firm wants to avoid possible bankruptcy. Hence the firm is faced with a bankruptcy constraint which stipulates that at any state θ the total payment made by the firm to the workers cannot exceed the revenue of the firm plus the scrap value of the firm's capital stock (which is assumed to be fixed all through) and the financial wealth of the owners of the firm. These are combined together and represented by $\bar{\pi}$. Thus the bankruptcy constraint can be formulated as:

$$\theta f(n(\theta)) - w(\theta)\, n(\theta) - b(\theta)\, (N - n(\theta)) + \bar{\pi} \geq 0 \quad (1)$$

(for a similar treatment of bankruptcy constraint, see Kahn and Scheinkman 1985).

All workers have identical preferences characterized by a strictly quasi-concave, state independent utility function, U, defined over income and work. We denote the employed worker's ex post utility by:

$$u\ (w) \equiv U\ (w,\ 1),\ with\ u' > 0,\ u'' < 0 \tag{2}$$

and the laid off worker's ex post utility by:

$$\hat{u}\ (b) \equiv U\ (b + e,\ 0),\ with\ \hat{u}' > 0,\ \hat{u}'' < 0. \tag{3}$$

The worker's reservation wage r is defined as the income supplement which exactly compensates workers for their opportunity cost of employment i.e., r is defined such that

$$U\ (r + b,\ 1) = U\ (b + e,\ 0) \equiv u\ (r + b) \tag{4}$$

(for a detailed discussion of a similar treatment of the reservation wage, see Brown and Wolfstetter 1985).

Since the workers are expected utility maximizers they do not enter into a contract with a particular firm, unless they are guaranteed to receive the level of (expected) utility $\bar{U}$ which they may receive if they decide to leave this firm and enter into a contract with some other firm. Hence the worker's ex ante (expected) utility constraint under the contract is given by:

$$\mathrm{E}_{\theta}\ [(n(\theta)/N)\ u(w(\theta)) + (N - n(\theta)/N)\ \hat{u}(b\ (\theta))] \geq \bar{U}. \tag{5}$$

Since the labor market is competitive (5) holds with strict equality.

It is assumed that, at the time of production, only the firm can observe the realization of θ. The worker cannot observe θ. He (she) has to accept the firm's word for its realized values. Thus the value $\tilde{\theta}$ reported by the firm may not coincide with true θ. Hence if a contract gives the firm an incentive to "cheat" (i.e., misreport θ), the firm will do so without risking detection. Consequently, without any loss of generality, we may assume, following Harris and Townsend (1982), that the worker will require the contract to be "truth telling" (incentive compatible) i.e., the worker accepts only the contract which makes telling the truth the firm's own best response. The "truth telling" (or incentive compatibility) constraint is given by:

$$\theta = \underset{\theta}{\operatorname{argmax}}\ [\theta f(n(\tilde{\theta})) - w(\tilde{\theta})\ n(\tilde{\theta}) - b(\tilde{\theta})\ (N - n(\tilde{\theta}))],\ \forall\ \theta. \tag{6}$$

As is customary (6) is replaced by the following first order condition:

$$[\theta f'(n(\theta)) - w(\theta) + b(\theta)]\, n'(\theta) - n(\theta)\, w'(\theta) - (N - n(\theta))\, b'(\theta) = 0. \tag{7}$$

Assuming that the corresponding second order condition is satisfied, it is clear that the incentive compatibility further requires

$$n'(\theta) > 0, \ \forall\ \theta \text{ for which } N > n(\theta) \tag{8}$$

and

$$n'(\theta) = 0, \ \forall\ \theta \text{ for which } N = n(\theta). \tag{9}$$

Since only the incentive compatible contracts are considered, the bankruptcy constraint, which must hold for all θ, takes a very simple form. Note that if (7) holds, profit must be increasing in θ.

$$\begin{aligned}\frac{d\pi}{d\theta} &= f(n(\theta)) + [\theta f'(n(\theta)) - w(\theta) + b(\theta)]\, n'(\theta) - \\ &\quad n(\theta)\, w'(\theta) - (N - n(\theta))\, b'(\theta) \\ &= f(n(\theta)) > 0.\end{aligned}$$

Therefore, if the bankruptcy constraint is not binding at $\hat{\theta}$ it is not binding for any $\theta > \hat{\theta}$. Hence, without loss of generality, (1) can be replaced by

$$\underline{\theta} f(\underline{n}) - \underline{w}\underline{n} - \underline{b}(N - \underline{n}) + \bar{\pi} \geq 0 \tag{10}$$

where, $\underline{n} \equiv n(\underline{\theta})$, $\underline{w} \equiv (w(\underline{\theta}))$, $\underline{b} \equiv (b(\underline{\theta}))$.

B. Characterization of the Optimal Contract

The optimal second best contract C^{**} can now be viewed as the solution to the following maximization program:

$$\max_{n(\theta),w(\theta),b(\theta)} \mathrm{E}_\theta\, [\theta f(n(\theta)) - w(\theta)\, n(\theta) - b(\theta)\, (N - n(\theta))] \tag{11}$$

subject to

$$\mathrm{E}_\theta\, [(n(\theta)/N)\, ((u\, (w(\theta)) - \hat{u}\, (b\, (\theta))) + \hat{u}(\theta))] = \bar{U} \tag{12}$$

$$n(\theta) \leq N \tag{13}$$

$$[\theta f'(n(\theta)) - w(\theta) + b(\theta)]\, n'(\theta) - n(\theta)\, w'(\theta) - (N - n(\theta))\, b'(\theta) = 0 \tag{7}$$

$$\underline{\theta} f\, (\underline{n}) - \underline{w}\underline{n} - \underline{b}\, (N - \underline{n}) + \bar{\pi} \geq 0 \tag{10}$$

where (13) is the labor force (or layoff) constraint.

Define the augmented objective functional as:

$$L \equiv \{[\theta f(n) - wn - b(N-n)]\, g + \delta\,[(n/N)(u-\hat{u}) + \hat{u}]\, g + y(\theta)[n(N-n) - x^2(\theta)] + p(\theta)[(\theta f' - w + b)n' - nw' - (N-n)b']\}. \tag{14}$$

Following Valentine's formulation (see Pierre 1969), layoff constraint (13) has been transformed into an equality constraint $n(\theta)[N - n(\theta)] - x^2(\theta) = 0$, $\forall\, \theta$. $x(\theta)$ is the auxiliary slack variable, $y(\theta)$ the associated multiplier. δ and $p(\theta)$ are the multipliers associated with the expected utility constraint (12) and the incentive compatibility constraint (7) respectively.

The Euler-Lagrange equations, from the standard optimal control technique, are given by:

$$(\theta f' - w + b)\, g + \bar{\delta}\,(u - \hat{u})\, g + y\,(N - 2n) = p'\,(\theta f' - w + b) + pf' \tag{15}$$

$$(1 - \bar{\delta}\,\hat{u}')\, g = p' \tag{16}$$

$$(1 - \bar{\delta}\, u')\, g = p' \tag{17}$$

$$xy = 0 \tag{18}$$

where $\bar{\delta} \equiv \delta / N$.

(18) corresponds to the auxiliary variable $x(\theta)$. Note that since the non-zero domains of $x(\theta)$ and $y(\theta)$ are mutually exclusive, the layoff range $K = \{\theta \mid n(\theta) < N\}$ is characterized by $y(\theta) = 0$. The transversality conditions are given by (see Kamien and Schwartz 1981):

$$p(\underline{\theta}) > 0 \tag{19}$$

if the bankruptcy constraint is active at $\underline{\theta}$;

$$p(\underline{\theta}) = 0 \tag{20}$$

if the bankruptcy constraint is inactive at $\underline{\theta}$;

$$\text{and} \quad p(\bar{\theta}) = 0. \tag{21}$$

Brown and Wolfstetter (1985) have emphasized the role of severance payments in the wage employment contracts. Since severance payments increase wealth in the unemployed state, it changes the willingness of the worker to pay for leisure (unless leisure is neutral) and consequently the reservation wage r changes. The following theorem (originally proved by Brown and Wolfstetter 1985, here included for the sake of completeness) brings out these interrelations.

Theorem 1

For optimal layoff contracts with optimal severance pay

$$\hat{u}\,(b) \equiv U\,(b+e,\,0) \gtreqless U\,(w,\,1) \equiv u\,(w) \tag{22}$$

if and only if leisure is normal, neutral or inferior.

Proof: Utilizing (4), it follows from (16) and (17)

$$u'\,(w) = \hat{u}'\,(b) \equiv u'\,(b + r\,(b))\,[1 + r_b]$$

(where r_b is the partial derivative of r with respect to b). Hence (22) is true if and only if $r_b \gtreqless 0$. The assertion follows immediately by noting that $r_b \gtreqless 0$ according as leisure is normal, neutral or inferior.

Before analyzing the employment policy under the second best contract, it is helpful to consider the employment policy in the layoff range under the first best contract C^* (when both the firm and the workers can freely observe realization of θ), in order to characterize the efficient or inefficient employment policy. Note that C^* can be obtained by omitting the incentive compatibility constraint (7) from the above maximization program. Consider the layoff range K. Substituting $y(\theta) = p(\theta) = 0$ it follows from the above Euler Langrange equations that under C^* (in the layoff range) the employment variation satisfies the following efficiency condition:

$$\theta f'\,(n) - \hat{w} = 0 \tag{23}$$

where $\hat{w} \equiv w(\theta) - b(\theta) - (u - \hat{u}/u')$.
In view of the above efficiency condition it is now straighforward to characterize the inefficient employment policy. In the layoff range overemployment (underemployment) is characterized by $\theta f'(n) - \hat{w} <(>)\ 0$.

C. *Non-Walrasian Employment Distortion under the Second Best Layoff Contracts with Optimal Severance Payment.*

To derive the inefficiency of the employment policy under the second best contract the following lemmas are needed.

Lemma 1

Consider the layoff range K. Assume leisure to be a normal good and that the bankruptcy constraint is active at $\underline{\theta}$. Then there does not exist any $\hat{\theta} \in K$ such that $p(\hat{\theta}) > 0$ with $p'(\hat{\theta}) = 0$ and $p''(\hat{\theta}) < 0$.

Proof: Suppose that there exists some $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $p(\hat{\theta}) > 0$, $p'(\hat{\theta}) = 0$ and $p''(\hat{\theta}) < 0$. Now differentiate (16) totally with respect to θ and evaluate the resulting expression at $\hat{\theta}$ to obtain

$$\frac{d}{d\theta}[u'(w(\hat{\theta}))] > 0. \tag{24}$$

(24) implies the $w'(\hat{\theta}) < 0$. Hence $\hat{\theta}$ must belong to K because if not (i.e., if $\hat{\theta}$ belongs to the full employment range $F \equiv \{\theta \mid N = n(\theta)\}$) by (9) and (7), $w'(\hat{\theta}) = 0$.

Now since $w'(\hat{\theta}) < 0$, it follows from (7) that

$$[\hat{\theta}f'(n(\hat{\theta})) - w(\hat{\theta}) + b(\hat{\theta})]\, n'(\hat{\theta}) - (N - n(\hat{\theta}))\, b'(\hat{\theta}) < 0. \tag{25}$$

But from (17) one may obtain that $b'(\hat{\theta}) < 0$. Hence (25) combined with (8) leads to

$$\hat{\theta}f'(n(\hat{\theta})) - w(\hat{\theta}) + b(\hat{\theta}) < 0. \tag{26}$$

However, by assumption $p(\hat{\theta}) > 0$ and $p'(\hat{\theta}) = 0$ and that leisure is a normal good. Thus utilizing *Theorem* 1 one can easily obtain from (15) that:

$$\hat{\theta}f'(n(\hat{\theta})) - w(\hat{\theta}) + b(\hat{\theta}) > 0. \tag{27}$$

But (27) contradicts (26). Hence the proof is complete.

Lemma 2

Consider the layoff range K. Assume leisure to be a normal good and that the bankruptcy constraint is active at $\underline{\theta}$. Then there does not exist any $\hat{\theta} \in K$ such that $p(\hat{\theta}) = 0$ with $p'(\hat{\theta}) \neq 0$ and $p(\theta) = 0$ for all $\theta > \hat{\theta}$, $\theta \in K$.

Proof: Assume on the contrary that there exists some $\hat{\theta} \in$ K such that $p(\hat{\theta}) = 0$ with $p'(\hat{\theta}) \neq 0$ and $p(\theta) = 0$ for all $\theta > \hat{\theta}$. Choose θ^* ($> \hat{\theta}$) arbitrarily close to $\hat{\theta}$ so that θ^* also belongs to K. Thus at θ^*, $p(\theta^*) = p'(\theta^*) = p''(\theta^*) = 0$. By the same line of reasoning as outlined in *Lemma* 1, it therefore follows that $w'(\theta^*) = b'(\theta^*) = 0$. And hence, since $\theta^* \in K$, $n'(\theta^*) > 0$,

$$\theta^*f'(n(\theta^*)) - w(\theta^*) + b(\theta^*) = 0 \tag{28}$$

But since leisure is a normal good, (27) evaluated at θ^*, still holds and this contradicts (28).

The result on employment distortion is now summarized in the following theorem.

Theorem 2

Consider the layoff range K. Assume leisure to be a normal good and that the bankruptcy constraint is active at $\underline{\theta}$. Then either (1) there exists a $\theta^* \in K$ such that the second best contract exhibits unemployment $\forall\, \theta > \theta^*$ (in the layoff range K) and overemployment $\forall\, \theta > \theta^*$ (with the possible exception at the upper end point), or (2) the second best contract exhibits unemployment $\forall\, \theta \in K$ (with the possible exception at the upper end point).

Proof: Since the layoff range is characterized by $y(\theta) = 0$, substitute $y(\theta) = 0$ in (15) and combine it with (16) to yield

$$\theta f' - \hat{w} = pf'/\tilde{\delta} g u' \tag{29}$$

Hence unemployment (overemployment) is characterized by $p > (<)$ 0. The rest of the proof follows directly from *Lemma* 1 and 2.

Thus *Theorem* 2 establishes the existence of non-Walrasian unemployment (in the sense of excessive layoffs) even when leisure is a normal good. It shows that the firm may not be able to "overemploy" the workers since some wage offers may turn out to be infeasible in view of the bankruptcy constraint. Consequently, it is possible for unemployment to occur for "lower" states of nature or even for all the states of nature (with the exception of the highest state of the nature).

If the bankruptcy constraint is never active, then the present analysis reduces to the standard characterization of the optimal layoff contracts (see for example, Brown and Wolfstetter 1985). In this case the relevant transversality conditions are (20) and (21). If leisure is normal, following the same reasoning as in *Lemma* 1 and 2, it is easy to see that in the layoff range $p(\theta) < 0$. Hence in brief:

Corollary 1

Consider the layoff range K. Assume leisure to be a normal good and that the bankruptcy constraint is never active. Then the second best contract exhibits overemployment $\forall\, \theta \in K$ (with the possible exceptions at the end points).

III. Pure Layoff Contracts without Severance Payments

It turns out that severance payments play a very important role in the non-Walrasian employment distortion. However, in view of the fact that the practice of providing severance pay is not at all widespread in the non-union sector, it is important to analyze the effect of bankruptcy constraint in the absence of severance payments. In the absence of severance payments, the firm's profit is given by:

$$\pi = \theta f(n(\theta)) - w(\theta)\, n(\theta).$$

The laid off worker's ex post utility is given by:

$$\hat{u} \equiv U(e, 0). \tag{30}$$

The worker's reservation wage $\hat{r}$ is defined as the wage level such that

$$\hat{r} = u^{-1}(\hat{u}). \tag{31}$$

$(w - \hat{r})$ is the "capital gain" earned by the worker who is not selected for layoff. The incentive compatibility constraint can be written as the following first order condition:

$$[\theta f'(n(\theta)) - w(\theta)]\, n'(\theta) - n(\theta)\, w'(\theta) = 0. \tag{32}$$

Note that because of the corresponding second order condition (8) and (9) still hold. In view of the incentive compatibility condition (32) the bankruptcy constraint can be reformulated as:

$$\underline{\theta} f(\underline{n}) - \underline{w}\underline{n} + \bar{\pi} \geq 0. \tag{33}$$

The optimal second best contract can now be viewed as the solution to the following maximization program:

$$\max_{n(\theta), w(\theta)} E_{\theta}[\theta f(n(\theta)) - w(\theta)\, n(\theta)] \tag{34}$$

subject to

$$E_{\theta}[(n(\theta)/N)(u - \hat{u}) + \hat{u}] = \bar{U} \tag{35}$$

$$n(\theta) \leq N \tag{13}$$

$$[\theta f'(n(\theta)) - w(\theta)]\, n'(\theta) - n(\theta)\, w'(\theta) = 0 \tag{32}$$

$$\theta f(\underline{n}) - \underline{w}\underline{n} + \bar{\pi} \geq 0. \tag{33}$$

The objective functional is given by:

$$L \equiv [\theta f(n) - wn]\, g + \delta\ [(n/N)(u - \hat{u}) + \hat{u}]g + y(\theta)[n(N - n) - x^2(\theta)] + p(\theta)[(\theta f' - w)\, n' - w'n]. \tag{36}$$

The Euler–Lagrange equations are given by:

$$(\theta f' - w)g + \bar{\delta}(u - \hat{u})g + y(N - 2n) = \dot{p}(\theta f' - w) + pf' \tag{37}$$

$$(1 - \bar{\delta}\, u')\, g = p' \tag{38}$$

$$xy = 0.$$

As before, the layoff range $K = \{\theta \mid n(\theta) < N\}$ is characterized by $y(\theta) = 0$ and the transversality conditions are the same as (19) ~ (21).

From a first-order Taylor's series expansion of $u(\hat{r})$ around the optimal w, it is easy to see that in the layoff range efficient employment is characterized by $\theta f' - \hat{r} = 0$. Unemployment (overemployment) is said to occur when $(\theta f' - \hat{r} >(<) 0$. The following theorem summarizes the main result regarding the non-Walrasian employment distortion.

Theorem 3

Consider the layoff range K. In the absence of severance payments, if the bankruptcy constraint is active at $\underline{\theta}$, the second best contract prescribes unemployment for all θ (except $\bar{\theta}$).

Proof: (37) and (38) along with $y = 0$ implies:

$$\theta f' - \hat{r} = pf' / \bar{\delta} u' g. \tag{40}$$

Hence it is required to prove that $p(\theta) > 0$, $\forall\, \theta \in K$ (except $\bar{\theta}$).

First, we establish that $p(\theta)$ can never be negative in the layoff range. To see this, assume on the contrary that there exists some θ for which $p(\theta) < 0$. Thus (40) implies that

$$\theta f' - \hat{r} < 0$$

and consequently,

$$\theta f' - w < 0. \tag{41}$$

But (19) and (21) imply that there exists some $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ at which $p(\theta)$ reaches a minimum, i.e., at $\hat{\theta}$, $p'(\hat{\theta}) = 0$, $p''(\hat{\theta}) > 0$. Now differentiating (38) with respect to θ one may easily obtain that:

$$\left.\frac{du'}{d\theta}\right|_{\hat{\theta}} < 0 \tag{42}$$

and hence, $w'(\hat{\theta}) > 0$. This establishes that $\hat{\theta} \in K$, because if $\hat{\theta}$ would belong to the full employment range, F, $n'(\theta) = 0$ and consequently, in view of the incentive compatability constraint (32), $w'(\hat{\theta}) = 0$. Hence combining (32) and (42), we get,

$$\hat{\theta} f' - w > 0. \tag{43}$$

But (43) contradicts (41) (evaluated at $\hat{\theta}$). Thus in the layoff range $p(\theta) \geq 0$.

We now establish, following the same line of reasoning as in *Lemma* 2, that there does not exist any $\tilde{\theta} \in K$ such that $p(\tilde{\theta}) = 0$ and $p(\theta) = 0$, $\forall \theta > \tilde{\theta}$, $\theta \in K$.

Finally, we need to prove that except $\tilde{\theta}$ there cannot exist any θ in layoff range for which $p(\theta) = 0$. Assume on the contrary that such θ (say θ^*) exists. In view of the above arguments it is now clear that the p function reaches a minimum at θ^*. This can be ruled out by following the same reasoning as in the first part of the proof. Hence, the proof of the theorem is complete.

IV. A Generalized Model Allowing for Layoff and Worksharing

A. Assumptions and Specifications of the Model

We now consider generalized contracts which combine layoffs and worksharing. The risk neutral firm is assumed to enter into comprehensive wage-employment contracts $C = \{w(\theta), n(\theta), h(\theta), b(\theta), t(\theta)\}$ with N homogeneous workers. The stipulated wage $w(\theta)$ is paid to each employed worker who is required to work for $h(\theta)$ hours. In other words, $w(\theta)$ is the total wage income of each employed worker. As before it is assumed that at the time the contracts are made, θ is unknown. All that is known to the firm and the workers is the positive density $g(\theta)$. However, when the final production decisions are made the firm can observe the realization of θ. It is at this point that the contract is applied. Profit of the firm is given by:

$$\pi = \theta f(n(\theta), h(\theta)) - w(\theta)\, n(\theta) - b(\theta)\, (N - n(\theta))$$

where $f(n, h)$ is the smooth neoclassical production function with $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, $f_{22} < 0$, $f_{12} > 0$, $f_{11}f_{22} - (f_{12})^2$ 0, $f(0, h) =$

$f(n, 0) = 0$, where f_j denotes the partial derivative of $f(\cdot)$ with respect to the j-th variable. Further restrictions on the technology may be imposed as they become necessary.

As before, the firm is faced with the bankruptcy constraint:

$$\theta f(n(\theta), h(\theta)) - w(\theta)\, n(\theta) - b(\theta)\, (N - n(\theta)) + \bar{\pi} \geq 0. \quad (44)$$

All workers have identical preferences characterized by a strictly quasi-concave, state independent utility function U defined over income and work: U (income, work).

To simplify the calculation it is assumed that the utility function is additively separable, which, however, also guarantees that leisure is a normal good. U exhibits $U_1 > 0$, $U_2 < 0$, $U_{11} < 0$, $U_{22} < 0$, $U_{12} = 0$. For convenience denote the employed worker's utility by:

$$u \equiv U\,(w(\theta), h(\theta)) \quad (45)$$

and the unemployed worker's utility by:

$$\hat{u}\,(b) \equiv U\,(b(\theta) + e, 0)). \quad (46)$$

The reservation wage $\hat{\hat{r}}$ is defined such that:

$$U\,(\hat{\hat{r}} + b, n) = U\,(b + e, 0). \quad (47)$$

The incentive compatibility constraint is given by:

$$\theta = \underset{\tilde{\theta}}{\operatorname{argmax}}\ [\theta f\,(n(\tilde{\theta}), h(\tilde{\theta})) - w(\tilde{\theta})\, n(\tilde{\theta}) - b(\tilde{\theta})\, (N - n(\tilde{\theta}))], \ \forall \theta \quad (48)$$

(48) is replaced by the corresponding first order condition:

$$[\theta f_1(n(\theta), h(\theta)) - w(\theta) + b(\theta)]n'(\theta) + \theta f_2(n(\theta), h(\theta))\, h'(\theta) - n(\theta)\, w'(\theta) - [N - n(\theta)]\, b'(\theta) = 0. \quad (49)$$

Assuming that the corresponding second order condition is satisfied, it is easy to see that the incentive compatibility further requires

$$f_1 n'(\theta) + f_2 h'(\theta) > 0. \quad (50)$$

As before, due to the incentive compatibility condition, the bankruptcy constraint assumes the following form:

$$\underline{\theta} f\,(\underline{n}, \underline{h}) - \underline{w}\underline{n} - \underline{b}\,(N - \underline{n}) + \bar{\pi} \geq 0. \quad (51)$$

B. Characterization of the Optimal Contract

The optimal second best contract C^{**} can now be viewed as the solution to the following maximization program:

$$\max_{n(\theta), h(\theta), w(\theta), b(\theta)} \mathrm{E}_\theta [\theta f(n(\theta), h(\theta)) - w(\theta)\, n(\theta) - b(\theta) \\ (N - n(\theta))] \tag{52}$$

subject to

$$\mathrm{E}_\theta [(n(\theta)/N)(u - \hat{u}) + \hat{u}] = \bar{U} \tag{53}$$

$$n(\theta) \leq N \tag{12}$$

$$[\theta f_1 (n(\theta), h(\theta)) - w(\theta) + b(\theta)]\, n'(\theta) + \theta f_2 (n(\theta), \\ h(\theta))\, h'(\theta) - n(\theta)\, w'(\theta) - (N - n(\theta))\, b'(\theta) = 0 \tag{49}$$

$$\underline{\theta} f(\underline{n}, \underline{h}) - \underline{w}\underline{n} - \underline{b}(N - \underline{n}) + \bar{\pi} \geq 0 \tag{51}$$

Define the objective functional as:

$$L \equiv [\theta f(n(\theta), h(\theta)) - w(\theta)\, n(\theta) - b(\theta)(N - n(\theta)]\, g(\theta) + \\ \delta [(n(\theta)/N)(u - \hat{u}) + \hat{u}]\, g(\theta) + y(\theta)[n(\theta)(N - n(\theta)) - \\ x^2(\theta)] + p(\theta)[(\theta f_1(n(\theta), h(\theta)) - w(\theta) + b(\theta))n'(\theta) + \\ \theta f_2(n(\theta), h(\theta))\, h'(\theta) - n(\theta)w'(\theta) - (N - n(\theta))\, b'(\theta)]. \tag{52}$$

The Euler-Lagrange equations are given by:

$$(\theta f_1 - w + b)\, g + \bar{\delta}\, (u - \hat{u})\, g + y(N - 2n) \\ = p'(\theta f_1 - w + b) + p f_1 \tag{53}$$

$$\theta f_2 g + \bar{\delta}\, n\, u_2 g = p' \theta f_2 + p f_2 \tag{54}$$

$$(1 - \bar{\delta} u_1)\, g = p' \tag{55}$$

$$(1 - \bar{\delta} \hat{u}')\, g = p' \tag{56}$$

where $\bar{\delta} \equiv \delta/N$. The transversality conditions are the same as (19) ~ (21).

As in Section II, here also, severance pay plays an important role in the wage employment contracts. The following theorem emphasizes the interrelation between the severance pay, reservation wage and worker's attitude towards leisure (The proof is similar to *Theorem* 1 and hence omitted).

Theorem 4

For optimal wage employment contract characterized by (53) ~ (56) with optimal severance pay

$$U(b + e, 0) \equiv \hat{u} \gtreqless u \equiv U(w, h)$$

if and only if leisure is normal, neutral or inferior.

C. Optimal Contract and Employment Distortion

Consider the layoff range $K = \{\theta \mid n(\theta) \leq N\}$. Substituting $y(\theta) = p(\theta) = 0$, it follows from (53) ~ (56) that the first best contract C^* prescribes employment variations according to the following efficiency conditions:

$$\theta f_1 - \hat{\hat{w}} = 0, \ \forall\ \theta \in K \tag{57}$$

where $\hat{\hat{w}} = w - b - (u - \hat{u}/u_1)$ and,

$$(1/n)\ \theta f_2 + u_2/u_1 = 0, \forall\ \theta \in K \tag{58}$$

In view of these efficiency conditions in the layoff range, un(over)-employment with respect to n can be characterized by $\theta f_1 - \hat{\hat{w}} >$ $(<)$ 0, while under-(over-) employment with respect to h can be characterized by $(1/n)\ \theta f_2 + u_2/u_1 >(<) 0$.

We now turn to the analysis of employment implications of the second best contract. The following theorem summarizes a fundamental result.

Theorem 5

In the layoff range, the second best contract may prescribe under, efficient or over-employment with respect to both n and h.

Proof: Combine (53) and (55) and utilize $y = 0$ to obtain

$$\theta f_1 - \hat{\hat{w}} = (pf_1/\bar{\delta} gnu_1). \tag{59}$$

Similarly, combine (54) and (55) and utilize $y = 0$ to obtain

$$(1/n)\ \theta f_2 + u_2/u_1 = (pf_2/\bar{\delta} gnu_1). \tag{60}$$

Since $p \gtreqless 0$ the assertion follows immediately from (57) and (60).

It will be useful for further analysis to note the following definitions and characterization of the production technology.

Difine η as the elasticity of the marginal product of h with respect to n,

$$\eta = (f_{12}/f_2)\ n.$$

It is clear that a high value of η, such as, greater than 1 (note that since f_{12} has been assumed to be positive, $\eta > 0$), implies that the two factors n and h are "significantly cooperative."

Difine ε_n as the output elasticity with respect to n,

$$\varepsilon_n = (f_1/f)\ n.$$

Also note that if the production function f is linear homogeneous, it implies the following important relationship involving the elasticity of substitution (σ):

$$\sigma = f_1 f_2 / f_{12} f.$$

To determine the direction of the employment distortion under the second best contract the following lemmas are needed.

Lemma 3
Consider the layoff range K. If there exists $\hat{\theta} \in K$ such that $p'(\hat{\theta}) = 0$, the optimal contract prescribes, at $\hat{\theta}$, $n' > 0$, $h' > 0$, provided (1) $\eta > 1$, or alternatively (2) f is linear homogeneous with $\varepsilon_n > \sigma$.

Proof: Define
$$\begin{aligned} \alpha &= f_{11}\ (\hat{\theta} g - p) \\ \beta &= f_{12}\ (\hat{\theta} g - p) + \bar{\delta}\, u_2 g \\ \mu &= f_{22}\ (\hat{\theta} g - p) + \bar{\delta}\, h u_{22} g \end{aligned}$$
(all the expressions are evaluated at $\hat{\theta}$)

Through straightforward calculations it can be shown that $n'(\hat{\theta})$ and $h'(\hat{\theta})$ have the same sign if (a) α and β are of opposite signs and (b) β and μ are of opposite signs. But evaluating (54) at $\hat{\theta}$ it is readily seen that $(\hat{\theta} g - p) > 0$. Thus $\alpha < 0$ and $\mu < 0$. Again utilizing (54) β can be written as:

$$\beta = \bar{\delta}\, u_2 g\ (1 - (f_{12}/f_2)\ n) = \bar{\delta}\, u_2 g\ (1 - \eta)$$

Hence if conditions (1) or (2) is satisfied, $\beta > 0$. The assertion now follows immediately from (50).

Lemma 4
Consider the layoff range K and assume that the bankruptcy constraint is active $\underline{\theta}$. Then there does not exist any $\hat{\theta} \in K$ such that $p(\hat{\theta}) > 0$, $p'(\hat{\theta}) = 0$ and $p''(\hat{\theta}) < 0$, provided that the technological precondition (1) or (2) holds.

Proof: Suppose, on the contrary, that there exists some $\hat{\theta} \in K$ $(\underline{\theta}, \bar{\theta})$ such that $p(\hat{\theta}) > 0$, $p'(\hat{\theta}) = 0$ and $p''(\hat{\theta}) < 0$. Differentiating (55) with respect to θ and evaluating the resulting expression at $\hat{\theta}$ it is easy to obtain that $w'(\hat{\theta}) < 0$. It is also readily seen that $\hat{\theta} \in K$. Now from (55) and (56) it can be concluded that $b'(\hat{\theta}) < 0$. Utilizing *Lemma* 3, (49) then yields:

$$(\hat{\theta} f_1 - w + b) < 0. \tag{61}$$

Since $p(\hat{\theta}) > 0$, and leisure is a normal good, (59) in conjunction with *Theorem* 5 implies

$$(\hat{\theta} f_1 - w + b) > 0$$

But (62) contradicts (61) and hence the proof is complete.

Lemma 5

Consider the layoff range K and assume that the bankruptcy constraint is active at $\underline{\theta}$. Then there does not exist any $\hat{\theta} \in K$ such that $p(\hat{\theta}) = 0$ with $p'(\hat{\theta}) \neq 0$ and $p(\theta) = 0$ for all $\theta > \hat{\theta}$, $\theta \in K$, provided that the technological precondition (1) or (2) holds.

Proof: The proof is similar to *Lemma* 2 and hence omitted.

Theorem 6

Consider the layoff range K and assume that the bankruptcy constraint is active at $\underline{\theta}$. Suppose that the technology is such that either $\eta > 1$ or f is linear homogeneous with $\varepsilon_n > \sigma$. Then either (1) there exists a $\hat{\theta} \in K$ such that the second best contract exhibits underemployment with respect to both n and h, $\forall\, \theta < \hat{\theta}$, $\theta \in K$, and overemployment, $\forall\, \theta > \hat{\theta}$, $\theta \in K$ (with the possible exception at $\bar{\theta}$), or (2) the second best contract exhibits underemployment with respect to both n and h, $\forall\, \theta \in K$ (with the possible exception at $\bar{\theta}$).

Proof: Follows from *Theorem* 5 and *Lemma* 4 and 5.

V. Conclusion

Using an asymmetric information framework, this paper has attempted to provide an explanation of unemployment even when the workers view leisure as a normal good. Our results show that if the firm faces a bankruptcy constraint, the second best contract may predict unemployment in the sense of excessive layoffs. In the context of generalized contracts, allowing for both layoffs and work-sharing, it is also shown that there may be underemployment with respect to hours of work along with excessive layoffs. However, in this case, the empirical significance of the technological preconditions remain to be examined.

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