Testing the Capital Asset Pricing Model on Aggregate Data: with Special Reference to the United Kingdom*

Christopher Green
University of Wales, College of Cardiff

Seong L. Na
Hallym University

This paper utilizes the Sharpe-Lintner-Mossin-Black (SLMB) Capital Asset Pricing Model (CAPM) as a framework for studying the relationship between aggregate private sector portfolios and asset returns in the United Kingdom. Previous studies in the US have provided only partial support for the SLMB CAPM in this context; UK studies have generally rejected the model. Utilizing a comprehensive new monthly dataset covering 1972-85, we investigate possible reasons for the failure of the SLMB CAPM in the UK. We seek econometric evidence for possible violation of each of the three main assumptions of the model: one-period optimization; constant first and second moments of returns; and perfect markets. We concentrate particularly on the assumption of perfect markets as this has not previously been investigated in empirical work. We utilize a wide range of nested and non-nested tests for this purpose. We find that UK data are not consistent with a very simple model of certain market imperfections, but there is still ample evidence that market imperfections of several kinds have played an important role in determining UK asset returns. The assumptions of one-period optimization and constant first and second moments also appear unjustified. We indicate the direction that future research should take to improve our understanding of asset price formation in the UK.

I. Introduction

The Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lint-

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ner (1965), Mossin (1966), and Black (1972) (hereafter, SLMB) offers a simple and intuitively appealing theory of asset price formation. It can be summarized in three main assumptions (A1-A3) and three main predictions (P1-P3):

A1: Investors are one period mean-variance optimizers.
A2: They have identical views about the first and second moments of asset returns.
A3: Asset markets are perfect: assets are freely traded with no transaction costs.

P1: There is a linear relation between the expected return on an asset and its systematic risk (the "security market line").
P2: The slope of the security market line is equal to the excess return on the market portfolio of risky assets.
P3: The systematic risk (or beta coefficient) of an asset is measured solely by the ratio of its covariance with the return on the market portfolio to the variance of the return on the market portfolio.

Tests of the SLMB CAPM carried out by estimating the security market line have, mostly, rejected propositions P2 and P3. The security market line is typically flatter than predicted by theory, and with a higher intercept; and the expected return on a security can be explained by variables other than its beta. The methodology of such tests has been criticized by Roll (1977) who demonstrated that the properties of the security market line all follow directly from the mean-variance efficiency (MVE) of the market portfolio and are, therefore, not independently testable. Roll also argued that, in testing the CAPM, it was inappropriate to use proxies for the market portfolio as there is no necessary relation between the MVE of the true market portfolio and the MVE of any proxy. In response to these points, theories and empirical methods have been devised to relax the main assumptions of the SLMB CAPM. Efforts have also been directed at improving estimates of the market portfolio or at devising valid tests which do not explicitly utilize the market portfolio.

The relaxation of assumption A1 leads to the Inter-temporal Capital Asset Pricing Model (ICAPM). The Merton (1973)-Long (1974) version of this model has so far proven difficult to test, but Breeden's (1979) ICAPM has been extensively tested, beginning with Hansen and Singleton (1983). Assumption A2 has been not so much relaxed as clarified. Early tests of the SLMB CAPM assumed that the first and second moments of returns were not only known but also constant over time, an additional restriction which appears both unnecessary and implausible. Frankel and Dickens (1983) and Gibbons and Ferson (1985) devised testing procedures which allowed agents' expectations of returns to be conditional on available information and hence to vary over time. These were extended by Bollerslev, Engle and Wooldridge (1988) to allow also for conditional and hence time-varying variances. The new empirical methodologies have been marked by a shift of focus away from individual stock prices, studied in traditional tests, and towards the kind of more broadly-based but aggregative data which are familiar in macroeconomics. The "market portfolio" is, after all, a macroeconomic concept. Likewise Breeden's ICAPM involves the reformulation of an asset's beta in terms of the covariance of its return with aggregate consumption and therefore also admits of testing procedures utilizing aggregate time series data. Thus, the aim of many recent studies has been to explain the evolution of stock price indices (rather than individual stock prices) and their relationship to other asset price indices (such as bonds), and to economy-wide aggregate portfolios. As yet, however, it cannot be said that any of these different versions of the CAPM provides a fully satisfactory explanation of asset prices. Indeed, many would argue that other theories are better able to perform this task (for example, Ross 1978).

While the use of aggregate data in testing the CAPM solves certain problems, it also raises some new ones which have not as yet been considered. Many of these problems relate to assumption A3 and to the existence of imperfections in asset markets. The market portfolio includes, *inter alia*, stocks and bonds, a high proportion of which are held directly by life assurance and pension funds whose counterpart obligations consist of personal life insurance and pension policies. It is not clear that life and pension fund portfolios replicate the desired portfolios of policy holders or can be made to do so in the fashion of Modigliani and Miller. Moreover, the premature surrender of a life or pension policy as a portfolio adjustment is typically a prohibitively costly transaction. The market portfolio also includes foreign currency assets. However, in many countries, exchange controls have constrained foreign currency transactions by domestic residents. Levy (1978) has shown that in the presence of such constraints, an asset's systematic risk is measured by its own variance and not by its covariance with the market. Taxation of security returns creates imperfections. See Auerbach and King (1983) for an analysis. Even at a basic level, taxation clearly affects both the expected return on an asset and its variance, yet virtually all studies of the CAPM have utilized pre-tax rather than post-tax returns. A plausible estimate of the true market portfolio in an economy clearly calls for data with comprehensive coverage of the asset markets in that economy. However, *prima facie,* the more comprehensive this coverage, the more likely it is that the estimated market portfolio will include assets not traded in perfect markets. This in turn implies a violation of assumption A3. Unlike assumptions A1 and A2, this possibility has not been scrutinized closely in recent empirical work.

Virtually all recent tests of the CAPM have been carried out using US data. An exception is Green (1987, 1988a, 1989) who studied United Kingdom data using a methodology similar to Frankel and Dickens. This method involves the estimation of APEX (asset price expectation formation) equations, and it utilizes data on aggregate portfolios and security price indices. It delivers a test of the MVE of the market portfolio, and therefore of the SLMB CAPM; and an estimate of the market average coefficient of (relative) risk aversion. Over the period 1972-7, Green found the coefficient of risk aversion to be significantly negative. This is inconsistent with simultaneous portfolio diversification and MVE and is, therefore, suggestive of market inefficiencies and rejection of the SLMB CAPM.

In this paper, we examine a selection of issues involving the CAPM which have, so far, been comparatively neglected. Our overall objective is to clarify the reasons for the apparent rejection of the SLMB CAPM by UK data. For this purpose, we utilize a new and comprehensive dataset consisting of monthly observations on aggregate security returns and portfolios in the UK covering the period 1972-85. These data are a major extension and improvement of those used by Green in his earlier studies. On their own, these data would justify a further look at the CAPM in the UK. However, we do not simply seek to replicate an earlier study on new data. We conjecture that a principal reason for the rejection of the SLMB CAPM in the UK has to do with the existence of market imperfections and the corresponding failure of assumption A3. Our empirical tests therefore focus in particular on the role of transaction costs, restrictions on portfolio choices and associated regime changes (such as exchange control abolition), and the influence of taxation. However, we do not neglect assumptions A1 and A2 entirely. Thus, we examine the influence of consumption decisions on asset prices, but we do this within a transaction costs framework rather than a full intertemporal framework. We also allow expected returns to vary over time conditional on available information; and we test for a wide range of possible models of time-varying second moments.

Although the use of aggregate data in this work resolves, to some extent, the problem of calculating the market portfolio, it does raise a new difficulty not encountered in studies of individual stocks, and this concerns the measurement of asset returns. Whereas the return on the stock of one company is a largely unambiguous concept, the return on "equities" or "bonds" can be measured by a variety of indices and it is not obvious that one index is superior to another. Previous studies using aggregate data have resolved this problem by the *a priori* selection of one index corresponding to each asset. In this paper we begin by selecting our return indices *a priori*. However, we then employ a novel version of the P test (Davidson and MacKinnon, 1981) to test our choice of indices against a variety of plausible alternatives.

Our strategy in this paper is to set up a general and relatively eclectic empirical model of asset pricing which incorporates transaction costs and a variety of market imperfections. The model is *ad hoc* because there exists no overall theory of asset pricing in the presence of market imperfections. Nevertheless the model does encompass the SLMB CAPM as a special case. We estimate the general model over both the full 1972–85 period and a variety of sub-periods. We then test the restrictions implied by the SLMB CAPM. On the basis of Green's results for 1972–7, we expect the data to reject these restrictions and this turns out to be the case. We therefore carry out a wide range of specification tests on the model which are aimed at providing an account of the possible reasons for the rejection of the SLMB CAPM, concentrating in particular on the role of market imperfections. Our general model is sufficiently broad so that it encompasses both the SLMB CAPM and a range of market imperfections. We are therefore able to carry out some specification tests using a nested approach. The remaining tests are carried out using Lagrange Multiplier (LM) principles to test in the direction of interest. The results of these help define a research agenda for the further investigation of the determinants of UK asset returns.

Our eclectic approach stands in contrast to virtually all previous work in this area which has emphasized the testing of very tightly specified models. However, a high proportion of this work has resulted in either outright rejection of the model (e.g. Frankel and Dickens) or acceptance of the model restrictions but rejection of LM-type specification tests, particularly for omitted variables (e.g. Bollerslev, Engle, and Wooldridge). Given the present uncertain status of the CAPM (see Ross) and the almost complete absence of UK research on the model, our view is that our eclectic approach is more useful than a further test of a very specific version of the CAPM which could well be rejected for any one of a large number of possible reasons.

The rest of the paper is organized as follows. In Section II we set out the theoretical model and consider in some detail the representation of market imperfections. In Section III we derive the formula for expected returns which follows from this representation, and show how it can be interpreted as a generalization of the SLMB CAPM. In Section IV we proceed from theory to specification of the regression model and consider the conditions needed to estimate the asset return equations. Estimation and testing of the model are carried out in two stages. First, we estimate a general version of the model and test those hypotheses which are nested within this general model. In Section V we outline the methodology involved in this process; the results are contained in section VI. The second step is to carry out a range of specification tests on the model to determine areas of weakness and thus to outline future research priorities. These tests and their results are described in Section VII. A final Section VIII contains some concluding remarks. Two appendices contain, respectively, some additional detail on the data used in this study and some results supplementary to those in the text.

II. The Theoretical Model

The notation is as follows:

A_t = Total transaction costs
C_t = Consumption
H_t = Labor income
T_t = Tax payments on labor and capital income
W_t = Wealth

p_t = Consumer price index

There are n assets indexed by $i = 1, \cdots, n$ with

X_{it} = Asset holdings at market value
q_{it} = Asset prices
r_{it} = Dividend price-ratios or running yields
$y_{it} = (p_t / p_{t+1})\ (r_{it} + q_{it+1} / q_{it})$ = Real asset returns
$y^e_{it} = (p_t / p^e_{t+1})\ (r_{it} + q^e_{it+1} / q_{it})$ = Expected returns
$X_t = (X_1, \cdots, X_n)'$
Q_t is a diagonal *matrix* of (q_{it} / q_{it-1})
R_t is a diagonal *matrix* of r_{it}
$y_t = (y_1, \cdots, y_n)'$

Σ is an $n \times n$ covariance matrix of real asset returns
I is the identity matrix
i is the sum (column) vector

Hence,

$D_t = i'R_{t-1}\ X_{t-1}$ = Income from capital
$E_t = y^e{}_t\ X'_t$ = Expected real wealth
$S_t = i'(X_t - Q_t\ X_{t-1})$ = Net acquisitions of assets (savings)
$G_t = i'(Q - I)X_{t-1}$ = Total capital gains
$V_t = X'_t \Sigma X_t$ = Variance of real wealth

As there is only one effective time period in the model, all portfolio and expenditure data (including those dated at $t-1$) are expressed in constant period t consumer prices. Where there is no ambiguity, the t subscripts will be dropped to economize on notation.

We derive formulae for asset returns using the standard model of a single "representative" consumer-investor[1] who chooses a vector of n assets (X) and consumption of a single homogeneous product (C) so as to maximize

$$U = U(C, E, V, A) \tag{1}$$

$$\text{subject to} \qquad i'X + C = W_{t-1} + G + H + D - T. \tag{2}$$

This set-up is analagous to that of Long's ICAPM. However, our specification of the utility function omits the covariances between

[1]The model can easily be extended to allow for agents who are heterogeneous in their attitudes towards risk. The only change would be that aggregate risk aversion would no longer be a constant but an average: equal to the harmonic mean of individual agents' risk aversion weighted by their shares in total wealth.

asset returns and the change in consumption goods prices. As we plan to assume that such covariances are constant over time, their inclusion would not materially affect the model. Instead, we include in the utility function transaction costs, the specification of which is considered below.[2] Long has given conditions under which inter-temporal optimization of lifetime utility can be decomposed into a sequence of independent one-period decisions such as that specified in (1) and (2) above. These conditions are not satisfied here because of the presence of transaction costs. We, nevertheless, do treat (1) and (2) as a one-period optimization problem. This amounts to an omission of future transactions from the regression model. However, we do not regard this omission as serious for the purposes of the present exercise.

The need for aggregative studies to focus more carefully on market imperfections is made starkly clear by the form of the standard inter-temporal budget constraint given in (2). Without further restrictions, this equation implies that all current income *and* wealth (on the right-hand side of (2)) are continuously and costlessly available for current consumption and portfolio adjustments. Given the important role of institutional savings schemes with severe withdrawal penalties, *prima facie*, this is an absurd representation. One standard method of modelling such restrictions is to introduce a set of inequality constraints on particular transactions. A second standard method is to follow Baumol-Tobin and assume that each transaction incurs an element of fixed costs. These procedures generate models which are typically characterized by neutral zones within which prices or other variables adjust without transactions taking place, as the return on a transaction is not sufficient to pay its (fixed) cost. When this cost threshold is breached, transactions involving a discrete jump in asset holdings generally occur. However, these features characterize the behaviour of individual agents. Different agents face different thresholds at which transactions

[2] The appearance of transaction costs as an argument in the utility function rather than as a separate "cost element" may appear unorthodox. It can be justified by first considering the following, presumably unexceptionable specification:

$$U = U(C - A^c,\ E - A^e,\ V) ; A = A^c + A^e \tag{1a}$$

Here, A^c are transaction costs included in measured consumption expenditure and $C - A^c$ is true consumption. Likewise, A^e are transaction costs paid "at source" to a broker, with $E - A^e$ being true expected net wealth. If formulation (1a) is acceptable, the inclusion of transaction costs as a separate argument, as in (1), can be regarded as an alternative, mathematically more convenient form of (1a).

attain a positive expected return. Thus, the larger is a given exogenous shock, the more agents will cross their respective thresholds and make transactions. This suggests that aggregate transactions will not replicate the discrete jumps to be expected in individual transactions, but will respond progressively and smoothly to changes in exogenous variables. This, in turn, provides a powerful argument for modelling transaction costs with aggregate data using techniques that generate smooth adjustment paths. The most convenient such technique is the quadratic costs of change model popularized by Sargent (1987). Quadratic costs are restrictive but they fit well with the SLMB CAPM as it, too, has a quadratic objective function. Green, in his earlier papers, used a quadratic costs of change model and found no evidence for transaction costs in this form. However, Green's model was concerned only with portfolio selection for given wealth and did not include consumption decisions. The inclusion of consumption in the utility function and corresponding modification of the budget constraint offers an opportunity to consider a far wider range of costs of change.

We consider four sources of costs of change defined by:

$$A = A_1 + A_2 + A_3 + A_4 \tag{3}$$

$$A_1 = (X - QX_{t-1})'\tilde{\Psi}(X - QX_{t-1});\ \tilde{\Psi} > 0;\ \tilde{\Psi} = \tilde{\Psi}' \tag{4}$$

A_1: These reflect the standard brokerage and information costs due to portfolio transactions at given asset prices.

$$A_2 = (X - X_{t-1})'\ \Phi(X - X_{t-1});\quad \Phi > 0;\ \Phi = \Phi' \tag{5}$$

A_2: These costs were introduced by Green (1988a). They are associated with changes in asset holdings, including those resulting from revaluations for which no transactions took place. They can be rationalized as being due to portfolio balancing behavior in which agents attach importance to a "balanced portfolio" *per se,* and therefore incur costs when asset holdings change. This seems to reflect most accurately certain aspects of the investment policies of life and pension funds.

$$A_3 = 2(X - QX_{t-1})'\,\theta\, C^*; \theta > 0; \quad ; C^* = (C,\text{-}H,T,\text{-}D)' \tag{6}$$

A_3: These costs are associated with the transfer of funds between the income and captial accounts. Friedman (1977) has emphasized that there may be differential costs as between allocating inflows of funds from the income account to asset purchases, and selling existing asset holdings for the same purpose. This hypothesis is reflected in the fact that the row sums of the cost matrix θ are not constrained to be equal to the row sums of the matrix of portfolio transaction costs ($\tilde{\Psi}$). More generally, θ is positive to reflect the hypothesis that it is relatively costly to liquidate assets to make good a shortfall in income but relatively inexpensive to convert additional income into assets.

$$A_4 = \Pi_1(\Delta C)^2 + 2\Delta C(-\Pi_2 \ \Delta H + \Pi_3 \ \Delta T - \Pi_4 \ \Delta D - \Sigma_i \ \Pi_{i+4} \ D_{it+1}); \tag{7}$$

$$\Pi_i > 0; \ \Delta C = C_t - C_{t-1} \ \text{(etc)}$$

A_4: The first four terms here are the standard quadratic costs of change applied to the income account. The remaining terms in (ΔC) (ΔD_{it+1}) are novel. D_{it+1} $(i = 1, \cdots, n)$ represents income from capital which will be paid next period. As there is typically a significant lag between the announcement and payment of dividends, D_{it+1} is known with certainty at time t. If agents know that income from capital will increase *next* period they may be more willing to increase *current* consumption for they will incur lower costs of change next period (through θ and the Π_i) in the event of unfavorable outcomes for labor income, taxation and asset prices. Conversely, a *future* fall in capital income may warrant a cut in *current* consumption. This represents the "bird-in-the-hand" hypothesis that agents may prefer dividends to capital gains on assets as the former can be transferred to the income stream at a lower transaction cost.

We employ these differently specified costs of change as a convenient *ad hoc* representation of various, commonly hypothesized, forms of market imperfections. As will be seen, this specification enables us to test the SLMB CAPM within our more general model. However, we certainly would not claim to have included all possible forms of market imperfection in the model. Some of the imperfections to which we referred eariler are more easily tested for using LM-Type specification tests. Taxation and other policy imposed constraints fall into this category. Consideration of these and other imperfections is deferred to Section VII.

To generate easily interpreted closed-form solutions to the model, we make the standard assumption of constant relative risk aversion. In the context of (1), this implies:

$$-2U_VW / U_E = \gamma > 0; \ -2U_VC / U_C = \beta > 0; \quad -2U_AW / U_E = \delta > 0 \tag{8}$$

Here, $U_E = \partial U / \partial E$ (etc) and γ, β, δ, are the constant coefficients of relative risk aversion with respect to expected wealth, consumption and transaction costs respectively.

III. Derivation of Asset Return Equations

Choosing C and X to maximize (1) subject to (2), noting the definitions for E, V, and A yields as the $n+2$ first order conditions:

$$0 = U_E y^e + 2U_V \Sigma X + 2U_A(\Psi(X - QX_{t-1}) + \Phi(Q - I)X_{t-1} + \theta C^* - R\tau(\Delta C)) + \lambda i \tag{9}$$

$$0 = U_C + 2U_A(\theta'_{.1}(X - QX_{t-1}) + \Pi'_{.1}(\Delta C^{**})) + \lambda \tag{10}$$

$$0 = i'X + C - i'QX_{t-1} - H - D + T \tag{11}$$

where, $\Psi = \tilde{\Psi} + \Phi$

$\tau' = (\Pi_5, \cdots, \Pi_{n+4})$

$\theta'_{.1} = (\theta_{11}, \theta_{21}, \cdots, \theta_{n1})$

$\Pi'_{.1} = (\Pi_{11}, \Pi_{21}, \Pi_{31}, \Pi_{41}, \Pi_{51}, \cdots, \Pi_{n+4\,1})$

$C^{**} = (C, -H, T, -D, -D_{t\,t+1}, \cdots, -D_{n\,t+1})'$

λ is the Lagrange multiplier.

The main property of the n equations (9), common to all versions of the CAPM, is their linearity in expected returns (y^e). These equations can therefore be solved separately for y^e. First, multiply by W / U_E and rearrange, using the definitions (8):

$$Wy^e = \gamma \Sigma X + \delta \Psi(X - QX_{t-1}) + \delta \Phi(Q - I)X_{t-1} + \delta \theta C^* - \delta R \tau(\Delta C) - \lambda iW. \tag{12}$$

Deducting the nth equation from first $n-1$ gives:

$$\begin{aligned} W(y^e - iy^e) = {} & \gamma(\Sigma_{11} - i\Sigma_{n1})X_1 + \gamma(\Sigma_{1n} - i\sigma_{nn})X_n \\ & + \delta(\Psi_{11} - i\Psi_{n1})(X_1 - Q_1X_{it-1}) \\ & + \delta(\Psi_{1n} - i\psi_{nn})(X_n - Q_nX_{nt-1}) \\ & + \delta(\Phi_{11} - i\Phi_{n1})(Q_1 - I)X_{1t-1} \\ & + \delta(\Phi_{1n} - i\phi_{nn})(Q_n - I)X_{nt-1} \end{aligned} \tag{13}$$

$$+ \delta(\theta_1 - i\theta_n)C^* - \delta(R_1\tau_1 - R_n\tau_n i)(\Delta C).$$

With the vector matrix partitioning distinguishing the first $n-1$ rows and columns and the nth:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{1n} \\ \Sigma_{n1} & \sigma_{nn} \end{bmatrix}; \ \Psi = \begin{bmatrix} \Psi_{11} & \Psi_{1n} \\ \Psi_{n1} & \psi_{nn} \end{bmatrix}; \ \Phi = \begin{bmatrix} \Phi_{11} & \Phi_{1n} \\ \Phi_{n1} & \phi_{nn} \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_n \end{bmatrix} = \begin{bmatrix} \theta_{.1} & \theta_{.2} & \theta_{.3} & \theta_{.4} \\ \theta_{n1} & \theta_{n2} & \theta_{n3} & \theta_{n4} \end{bmatrix};$$

$$(R_1\tau_1 - R_n\tau_n i) = \begin{bmatrix} \tau_1 r_1 - \tau_n r_n \\ \vdots \\ \tau_{n-1} r_{n-1} - \tau_n r_n \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_n \end{bmatrix}; \quad X = \begin{bmatrix} X_1 \\ X_n \end{bmatrix}; \quad Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_n \end{bmatrix}; \quad R = \begin{bmatrix} R_1 & 0 \\ 0 & R_n \end{bmatrix}$$

We also define:

$$\begin{aligned} \hat{\Sigma} &= \Sigma_{11} - i\Sigma_{n1} - \Sigma_{1n} i' + ii'\sigma_{nn} \\ \hat{\Psi} &= \Psi_{11} - i\Psi_{n1} - \Psi_{1n} i' + ii'\psi_{nn} \\ \hat{\Phi} &= \Phi_{11} - i\Phi_{n1}\Phi_{1n} i' + ii'\phi_{nn} \end{aligned} \tag{14}$$

Utilizing the budget constraint (11) and the definitions (14) in (13) gives after some re-arrangement (including division by W)[3]:

$$\begin{aligned} y_1^e - iy_n^e = \ & \gamma(\Sigma_{1n} - i\sigma_{nn}) + \delta((\Psi_{1n} - i\psi_{nn}) \\ & - (\theta_{.1} - i\theta_{n1}))(F/W) + \gamma\hat{\Sigma}(X_1/W) \\ & + \delta\hat{\Psi}(X_1 - Q_1 W_{1t-1})/W + \delta\hat{\Phi}(Q_1 - I)(X_{1t-1}/W) \\ & + \delta(\theta_{.1} - i\theta_{n1} - (\theta_{.2} - i\theta_{n2}))(H + D)/W \\ & - \delta(\theta_{.1} - i\theta_{n1} - (\theta_{.3} - i\theta_{n3}))(T + \delta(\theta_{.2} \\ & - i\theta_{n2} - (\theta_{.4} - i_{n4}))(D/W) \\ & - \delta(R_1\tau_1 - R_n\tau_n i)(\Delta C)/W. \end{aligned} \tag{15}$$

[3]In utilizing the budget constraint (11) note that, in our data, there are no nominal gains or losses on the numeraire asset. Thus:

$$G = i'(Q_1 - I)X_{it-1} \text{ and } q_{nt}/q_{nt-1} = 1.$$

This does not however mean that numeraire is risk-free, because its real value can change stochastically through changes in consumer prices.

Equation (15) is the fundamental theoretical result of the analysis. It states that expected asset returns relative to the return on the numeraire (nth) asset can be expressed as linear combinations of: portfolio shares, net transactions and capital gains in the portfolio divided by wealth; labor and dividend incomes, and the product of the dividend price ratios and the change in consumption all divided by wealth. The share of the numeraire asset in wealth and the consumption-wealth ratio do not appear on the right-hand side as they have been eliminated using the budget constraint.[4]

The SLMB CAPM is represented by the terms involving the covariance matrix Σ. The ith row of $(\Sigma_{in} - i\sigma_{nn} + \hat{\Sigma} X_1 / W)$ gives the covariance of the excess return on the ith asset with the weighted average return on the market, and is therefore the numerator of the asset's beta coefficient. The denominator of beta is found from the coefficient of relative risk aversion (γ) which, at the optimum, is equal to the market price of risk, or the ratio of the excess return on the market to its variance. The remaining variables in the model reflect the impact of the different costs of change on asset returns. If these variables are all insignificant, the model reverts to the SLMB. An important feature of (15) is that variables from both the capital account *and* the income account help predict asset returns because of the hypothesis that there are costs involved in transfers between the income and capital accounts. This is to be contrasted both with the SLMB CAPM and with Breeden's ICAPM. In the latter, the change in aggregate consumption is a sufficient statistic for both income and capital account variables, because perfect captial markets allow continuous and costless transfers between the capital and income accounts.

IV. Specification of the Regression Model

Equations (15) are exact linear relations. They can be converted into a regression model by making two additional assumptions:

A4: Expectations are unbiased.

We then have that:

[4]In (15) income from labor (H) and capital (D) have been arranged so that the right-hand side variable are total income ($H + D$) and capital income (D). Empirically, it is more convenient to work with these data than with labour and capital income separately. Clearly this will not affect the empirical results.

$$y_1 - iy_n = y_1^e - iy_n^e + \varepsilon_1 - i\varepsilon_n . \tag{16}$$

$\varepsilon' = (\varepsilon_1' \varepsilon_n)$ is a vector of white noise errors with covariance matrix $\Lambda = \mathrm{E}(\varepsilon\varepsilon')$

Replacing the expectations $y_1^e - iy_n^e$ in (16) with the expressions given by (15) yields a set of stochastic equations:

$$\begin{aligned} y_1 - iy_n = {} & \gamma(\Sigma_{1n} - i\sigma_{nn}) + \delta\{\Psi_{1n} - i\psi_{nn} - (\theta_{.1} - i\theta_{n1})\} \\ & (F/W) + \gamma\hat{\Sigma}(X_1/W) + \delta\hat{\Psi}(X_1 - Q_1X_{1t-1})/W \\ & + \delta\hat{\Phi}(Q_1 - I)(X_{1t-1}/W) \\ & + \delta\{\theta_{.1} - i\theta_{n1} - (\theta_{.2} - i\theta_{n2})\}(H+D)/W \\ & - \delta\{\theta_{.1} - i\theta_{n1} - (\theta_{.3} - i\theta_{n3})\}\ (T/W) \\ & + \delta\{\theta_{.2} - i\theta_{n2} - (\theta_{.4} - i\theta_{n4})\}\ (D/W) \\ & - \delta(R_1\tau_1 - R_n\tau_n i)\ (\Delta C)/W + \varepsilon_1 - i\varepsilon_n . \end{aligned} \tag{17}$$

These are not, however, regression equations unless we also assume:

A5: Expected asset returns and their errors are orthogonal.

Under these assumptions, equations (17) do constitute regression equations which can be estimated by ordinary least squares (OLS).

The left-hand side variables in the regression model are actual asset returns between time t and $t+1$ and the right-hand side variables are all dated at time t or earlier. The matrices $\hat{\Sigma}$, $\hat{\Psi}$, $\hat{\Phi}$, and θ and the vector τ are the parameters to be estimated. Given the optimality properties of least squares, this linear combination of variables constitutes a rational forecast of asset returns between t and $t+1$, conditional on information at time t and, of course, conditional on the validity of the theoretical model of asset returns. It is for this reason that the present set-up can be interpreted as a model of conditional expectations and that the regression equations are dubbed APEX (asset-price expectation formation) equations.

Assumption (A4) is relatively inocuous. Provided the explanatory variables in (17) do form a sub-set of those which help predict asset returns, the regression model will deliver unbiased forecasts. See Gibbons and Ferson(1985). If, however, there are omitted variables, the forecast will be inefficient and, as explained in Section V, this could affect one of the tests on the model. Assumption (A5) is stronger. Practically, it implies that the only source of random variation in the model is that arising from errors in forming price expectations. Suppose, as is common, the mean-variance model were used as a basis for estimating asset demands. These could be written:

$$X_1/W = (y^e - iy^e)B_1 + ZB_2 + u \tag{18}$$

and, where Z is a matrix of all other variables than X and y^e, B_1 and B_2 are matrices of co-efficients, and u is a vector of white noise errors.

If asset markets clear, we have:

$$y_1^e - iy_n^e = (X_1/W)B_1^{-1} - ZB_2B_1^{-1} - uB_1^{-1}. \tag{19}$$

Now utilizing (19) in (16) with assumption (A4) gives

$$y_1^e - iy_n = (X_1/W)B_1^{-1} - ZB_2B_1^{-1} + (\varepsilon_1 - i\,\varepsilon_n - uB^{-1}). \tag{20}$$

This is clearly not a regression model, because of the correlation between X_1 and the residuals. One possibility is to check for the exogeneity of X_1 using LM tests. See Engle (1982). The difficulty with this approach is that it would require estimation of the model for X_1 (18) which is simply the inverse of (20) and therefore cannot be used to construct a valid exogeneity test in this context. The alternative we have adopted is to estimate (17) under assumptions A4 and A5, but to carry out a range of specification tests on the estimated model. In particular, we test for the existence of omitted variables which may also be valid instruments for X_1, especially lagged values of y_1-iy_n. If X_1 or any of the regressors are not exogenous in (17), this will appear as mispecification. Tests for the omission of lagged values of y_1-iy_n are obviously equivalent to the standard LM tests for autocorrelation (Godfrey, 1978).

The data used in estimating (17) are described in detail by Green and Na (1988a). A summary is given in the appendix to this paper. In brief, the data are monthly and run from 1972:7 to 1985:6. The model consists of four assets: bonds, equities, foreign currency denominated assets and "liquid assets", the latter being a residual category consisting mainly of short-term capital-certain assets and acting as the numeraire. The portfolio data consist of the asset holdings of the consolidated UK private sector: including financial institutions, but excluding the monetary authorities, central government and overseas. There are, therefore, three regression equations corresponding to equation (17). They explain three rates of return relative to the numeraire: on bonds, equities and foreign currency assets. The three equations share a mostly common set of explanatory variables. However, the dividend yield variables (R_t) are unique to each equation, and the equity equation has an additional share dilution term (see Appendix A for details).

V. Estimation and Testing Procedures

The general model consists of the three equations corresponding to (17) without parameter restrictions imposed, together with 11 seasonal dummies which, initially, are appended to each equation. The seasonal dummies were included because it was thought likely that either the portfolio or income-consumption variables would exhibit seasonality. If so, seasonal dummies are needed to get unbiased estimates of the parameters of interest, even if asset returns do not themselves contain a seasonal element. Unrestricted estimation of the general model was carried out using maximum likelihood on all three equations simultaneously. As these equations do not contain identical explanatory variables, a systems estimation procedure is more efficient than single equation estimation.

Tests of the SLMB CAPM and of the role of costs of change are carried out by testing the restrictions implied by the parameter matrices in (17) and by testing the significance of individual variables. Many of the parameter restrictions are cross-equation restrictions which require a systems method of estimation under the null hypothesis. We therefore used maximum likelihood to estimate all the different restricted versions of the model on the complete dataset. This enabled us to follow a uniform testing procedure. Single linear restrictions were tested using the usual t test. Multiple restrictions were tested using the Edgeworth-adjusted likelihood ratio statistic. This is given by the product of minus twice the log of the likelihood ratio and $(T - K + R/2 - 1) / T$

where T = Number of observations
K = Number of exogenous variables in the model
R = Number of restrictions.

The unadjusted likelihood ratio statistic is asymptotically distributed as $\chi^2(R)$ but is known to be too large in small samples. The Edgeworth adjustment produces a statistic which is approximately the correct size. The critical value of the χ^2 statistic is unaffected by the adjustment (see Evans and Savin, 1982). In general, the critical value of test statistics was set at the 95% level but we report exact significance levels to permit the reader to form her own judgement about our tests.

The testable restrictions on the model parameters are those associated with quadratic costs of change and with the SLMB CAPM. Costs of change imply a relatively weak set of restrictions. The matrices $\hat{\Psi}$ and $\hat{\Phi}$ should be positive, semi-definite, and every non-zeno element in θ and τ should be strictly positive. In addition, $\hat{\Psi}$ and $\hat{\Phi}$ should be symmetric as they are each associated with a quadratic form. δ is not identified, but this does not affect the tests of these restrictions. The budget constraint means that individual elements of θ are also not separately identified. However, the rank ordering of any row of $\theta : \theta_{i1}, \cdots, \theta_{i4}$ can, in principle, be established. All the cost matrices may include any patterns may be more plausible than others. Given that income tax is mostly deducted at source in the UK, it also makes sense to test for the equality: $\theta_{.2} - i\theta_{n2} = \theta_{.3} - i\theta_{n3}$. This states that pre-tax income $(H+D)$ and tax payments (T) do not have a differential effect in predicting asset returns. If this restriction is accepted, only after-tax income appears in the model. Clearly, if $\hat{\Psi}$, $\hat{\Phi}$, θ, and τ are all zero, then transaction costs, as measured, are negligible.

The SLMB CAPM implies a somewhat stronger set of restrictions. The covariance matrix $\hat{\Sigma}$ must be symmetric and positive semi-definite. In addition, if expectations are rational, $\hat{\Sigma}$ must obey further restrictions. $\hat{\Sigma}$ represents agents' subjective second moments. Under rational expectations, the subjective second moments coincide with the true second moments. The latter can be deduced from (16) and are given by $\hat{\Lambda} = \Lambda_{11} - i\Lambda_{n1} - \Lambda_{1n}i' + ii'\lambda_{nn}$. (Here, Λ is partitioned conformably to Σ). Moreover, Λ is just the residual covariance matrix of the regression model (17). Thus, under rational expectations, $\hat{\Sigma} = \hat{\Lambda}$. This equality is satisfied if the matrix of co-efficients $(\gamma\hat{\Sigma})$ is proportional to the covariance matrix of regression residuals $\hat{\Lambda}$, with the factor of proportionality (γ) equal to the co-efficient of relative risk aversion. Indeed, when this restriction is not imposed, γ is unidentified as it appears indistinguishably in the product $\gamma\hat{\Sigma}$. In this framework, the conditional second moments are assumed constant over time following Frankel and Dickens. Moreover, as the estimate of γ depends on the estimate of $\hat{\Lambda}$, it makes a difference if the model does not represent the exact process generating asset price expectations. If there are omitted variables, the predictions will be inefficient, $\hat{\Lambda}$ will be too large and the estimate of γ will be both inefficient *and* biased (downwards). In the case of this one parameter, therefore, unbiased estimation does require specification of the true model rather than just a sub-set of the variables which help predict asset returns.

Clearly, there are a large number of restrictions to test and there is no unique order in which the restrictions can be nested within one another. However, all the restrictions are nested within the general model. In testing the validity of these restrictions, we explored a variety of test routes but in no case did the test route affect the accept-reject decision at the 95% level. Therefore, in our results Section VI we report only a single set of test statistics. These follow the route which appears to be the most logical.

VI. Results of Hypothesis Tests and Parameter Estimation

The seasonal dummies, which are essentially nuisance variables, proved to be significant only in the equity equation. Their significance here may be attributable to the use of Tobin's Q as the price of capital rather than a stock market index. Other preliminary hypotheses which proved easily acceptable were as follows:

H1: The share dilution term in the equity equation was insignificant (1 restriction).

H2: $\tau_n = 0$ in all equations (3 restrictions). These are the coefficients associated with $R_n i(\Delta C)/W$.

H3: $\theta_{.2} - i\theta_{n2} = \theta_{.3} - i\theta_{n3}$ (3 restrictions). This implies that only after-tax income helps predict asset returns.

H4: $\Psi_{1n} - i\psi_{nn} - (\theta_{.1} - i\theta_{n1}) = 0$ (3 restrictions). These are the coefficients associated with (S/W).

H5: $(\theta_{31} - \theta_{n1}) - (\theta_{32} - \theta_{n2}) = 0$; $(\theta_{32} - \theta_{n2}) - (\theta_{34} - \theta_{n4}) = 0$ (2 restrictions).

These are the coefficients associated with $(H + D - T)/W$ and with (D/W), respectively, in the foreign currency equation only.

These 12 restrictions were accepted individually and in combination and appear innocuous, although a rationale for H4 in terms of the parameters is not immediately obvious. Estimation under these restrictions and excluding seasonals from the bond and foreign currency equations yields a "simplified model".

In Figure 1, we give the results[5] of the major hypothesis tests on

[5]In the presentation of results, the equations are assumed to be ordered and the parameters correspondingly numbered as: i) bonds ii) equity and iii) foreign currency denominated.

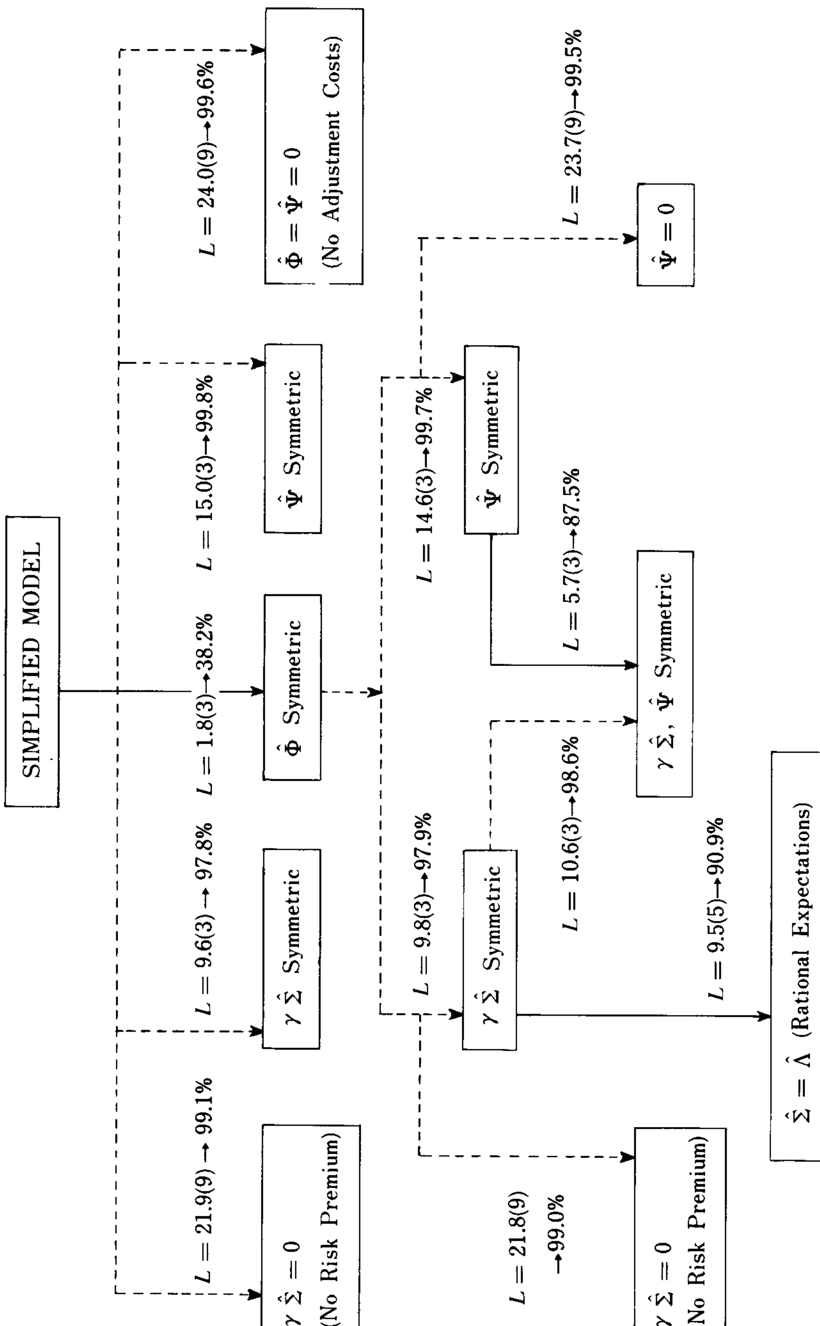

FIGURE 1 RESULTS OF TESTS OF CAPM

the cost matrices and of the SLMB CAPM using as null the simplified model. Here, each arrow corresponds to a test for which we give successively the value of the Edgeworth-adjusted likelihood ratio (χ^2) statistic; the degrees of freedom of the test and the significance level of the statistic. Continuous lines linking hypotheses show restrictions which are accepted by the data; broken lines give restrictions which are rejected. We accept the symmetry of the cost matrix $\hat{\Phi}$, although this is mainly because most of its elements are individually insignificant. However, the symmetry of $\hat{\Psi}$ and of $\hat{\Sigma}$ are both rejected. If the symmetry of $\hat{\Sigma}$ is imposed, the rational expectations restrictions ($\hat{\Sigma} = \hat{\Lambda}$) are accepted.[6] Clearly, though, this acceptance is invalidly conditioned on the symmetry of $\hat{\Sigma}$. Overall, these tests indicate rejection of the SLMB CAPM and the exact form of costs of change investigated in this model. Because this rejection is so clear-cut, we have not explicitly estimated the risk-aversion coefficient (γ). Rejection of the model is to be expected in view of Green's earlier results. However, rejection does not mean that the explanatory variables are insignificant. On the contrary, the hypotheses that there is no portfolio-related risk premium ($\gamma\hat{\Sigma} = 0$) and that transactions variables are insignificant are all decisively rejected.

Table 1 gives the parameter estimates for the most parsimonious "theory-based" model (equivalent to the "simplified model" with symmetry imposed on $\hat{\Phi}$). This is the most parsimonious model accepted by the data following the test routes in Figure 1. Although some of the parameters in this model are individually insignificant, it is clear that portfolio, transactions and income-consumption variables each contribute significantly in some measure to explaining asset returns. This is consistent with Bollerslev, Engle and Wooldridge who reported that both portfolio and consumption variables helped explain US asset returns. There are some sign anomalies among the co-efficients in the portfolio and transactions variables. Four of the nine on-diagonal coefficients are incorrectly signed negative. However, only one of these is significant: that on the share of bonds in the bond yield equation. The rank ordering of the elements of θ is sensible. The costs of converting assets into consumption are largest followed by the costs of converting labor income into assets. The costs of converting capital income into assets are the lowest of the three. Finally, the elements of τ_1 are all

[6]These restrictions were tested with a Wald test.

TABLE 1
ESTIMATES OF THE RESTRICTED THEORY-BASED MODEL

	YG Coefficient (t-value)	*YE* Coefficient (t-value)	*YF* Coefficient (t-value)
Constant	0.136	−0.752	−0.204
	(0.59)	(−1.65)	(−1.50)
G_t / W_t	−0.791	0.603	0.390
	(−2.81)	(1.19)	(2.07)
E_t / W_t	−0.043	0.282	0.070
	(−0.19)	(0.68)	(0.52)
F_t / W_t	−0.924	0.293	0.395
	(−2.51)	(0.46)	(1.58)
$(G_t - g_t G_{t-1}) / W_t$	3.151	0.606	−1.531
	(3.28)	(0.43)	(−2.32)
$(E_{t-1} - e_t E_{t-1}) / W_t$	−0.714	0.039	0.784
	(−1.52)	(0.06)	(2.36)
$(F_t - f_t F_{t-1}) / W_t$	0.423	−1.954	−0.11
	(0.65)	(−2.11)	(−0.24)
$(g_t - 1)G_{t-1} / W_t$	−0.129	0.046	−0.113
	(−0.46)		
$(e_t - 1)E_{t-1} / W_t$	0.046	0.575	0.05
	(0.40)	(3.44)	
$(f_t - 1)F_{t-1} / W_t$	−0.113	0.05	−0.150
	(−0.51)	(0.58)	(−0.18)
$(H_t + D_t - T_t)W_t$	0.687	5.247	
	(1.76)	(4.05)	
D_t / W_t	49.371	26.19	
	(4.77)	(1.56)	
$(\Delta C_t)R_{gt} / W_t$	−2.762		
	(−1.37)		
$(\Delta C_t)R_{et} / W_t$		−23.776	
		(−2.69)	
$(\Delta C_t)R_{ft} / W_t$			−2.489
			(−1.58)
R^2	0.27	0.36	0.11

Note : Log-Likelihood = 859.6.
For explanation of notation see Appendix B.

correctly-signed and generally significant, providing evidence for the "bird-in-the-hand" hypothesis.

Although a wide range of theoretical restrictions are rejected by the data. it is clear from Table 1 that zero restrictions on certain coefficients will be accepted. In all, a further 12 zero restrictions can be validly imposed on the restricted theory-based model. The model with these additional restrictions we call our "data-based" model and its parameter estimates are given in Appendix B. However, they are not substantively different from the parameters of the theory-based model.

VII. Specification Tests

To explore further the possible reasons for the rejection of the SLMB CAPM, and the quadratic costs of change model we carried out three sets of specification tests. These consisted of:

1) Analysis of residuals using LM tests
2) Chow tests for constancy of the parameter estimates
3) Analysis of different indices of asset returns using modified *P* tests.

In conducting these tests the appropriate null hypothesis is not always apparent. For example, the most general model is a natural null hypothesis against which to perform LM tests for autocorrelation. However, a general model is necessarily over-parameterized and, if this over-paramaterization involves mis-specification, autocorrelation can be absent from the general model only to reappear as the model is simplified. Testing for autocorrelation using as null only one version of the model is, therefore, virtually meaningless. We have carried out our tests using a wide range of null hypotheses. To save space, we report only one version of each test, but we comment briefly if changing the null hypothesis or other parameters of the test had a significant effect on its outcome.

The general form of the LM test for *residual autocorrelation* is given by Godfrey. We tested each equation for simple third-order, simple twelfth-order and vector first-order autocorrelation. Let the general autocorrelation structure of the model be:

$$u_t = \sum_{i=1}^{n} S^i u_{t-i} + v_t. \tag{21}$$

with u_t being a 3×1 vector of errors
v_t being a 3×1 vector of white noise

each $S^i = [s^i_{jk}]$ is a 3×3 parameter matrix $j = 1,\cdots,3$; $k = 1, \cdots, 3$

The null hypothesis is $S^i = 0 \;\forall i$

TABLE 2
RESULTS OF LAGRANGE MULTIPLIER TESTS ON GENERAL MODEL

TEST: χ^2 (degree of freedom)	*YG* χ^2 value (significance %)	*YE* χ^2 value (significance %)	*YF* χ^2 value (significance %)
LM3: $\chi^2(3)$	7.67	3.37	2.72
	(94.78)	(66.0)	(51.7)
LM1: $\chi^2(3)$	4.28	3.55	2.33
	(77.00)	(68.60)	(49.0)
LM12: $\chi^2(12)$	20.47	24.45	21.72
	(94.1)	(98.2)	(95.9)
ARCH3: $\chi^2(3)$	3.81	8.56	1.74
	(71.8)	(96.45)	(36.8)
ARCH1: $\chi^2(3)$	4.11	2.08	2.31
	(75.1)	(44.0)	(48.8)
HETERO 1: χ^2	28.59	72.56	47.74
(32, 33, 32)	(36.0)	(99.9)	(96.33)
HETERO 2: χ^2 (152)	154.93	155.18	151.10
	(58.0)	(58.7)	(49.6)

Note: LM3 = Simple 3rd-order autocorrelation.
LM1 = Vector 1st-order autocorrelation
LM12 = Simple 12th-order autocorrelation.
ARCH3 = Simple 3rd-order autoregressive conditional heteroscedasticity
ARCH1 = Vector 1st-order autoregressive conditional heteroscedasticity.
HETERO1 = Breusch and Pagan test for heteroscedasticity.
HETERO2 = White test for heteroscedasticity.

Simple third-order autocorrelation: $S^i_{jj} = 0,\ i = 1,\cdots, 3;\ \forall j$
Simple twelfth-order autocorrelation: $S^i_{jj} = 0,\ i = 1,\cdots, 12;\ \forall j$
Vector first-order autocorrelation: $S^i_{jk} = 0,\ i = 1;\ \forall jk$

Tests for *Autoregressive Conditional Heteroscedasticity* (ARCH) are given by Engle (1982b) and we tested each equation for simple third-order and vector first-order ARCH processes. These are defined analogously to the autocorrelations given above. Tests for general forms of *heteroscedasticity* are given by Breusch and Pagan (1979) and by White (1980). To implement the former, the variables in the regressor set were used to characterize the form of heteroscedasticity. To implement White's test we deleted the cross-products of the dummy variables with the variables of interest to make it possible to run the auxiliary regression and the degrees of freedom were adjusted accordingly.

The results of these LM tests on the residuals of the general model (Table 2) show no evidence of autocorrelation apart from twelfth-order in the equity equation which could be attributable to a seasonal factor such as the "January effect". There is a little more evidence of different forms of heteroscedasticity, but this too is primarily in the equity equation. Repeating these tests on more restricted versions of the model did not significantly alter their outcomes, apart from a tendency to find rather more autocorrelation of different forms in the equity equation. Re-estimation of all versions of the model using White's "heteroscedasticity-consistent" method to calculate the covariance matrices did not alter the outcomes of any of the t tests. On balance, it seems clear that non-constant second moments deserve further investigation, especially in the equity equation. Overall, though, these tests are hardly sufficient to claim that there is major mis-specification in the general model.

Chow (1960) tests for within-sample parameter constancy were carried out on all three equations simultaneously using the Edgeworth-adjusted likelihood ratio statistic and taking as null hypothesis the restricted "theory-based" model. We examined stability across three separate dates on which observable regime changes took place:

1) Oct.-Nov. 1979: Abolition of Exchange Controls.
2) Jun.-Jul. 1980: Abolition of the Supplementary Special Deposit Scheme.
3) Oct.-Sep. 1981: Introduction of revised monetary controls on banks.

These dates are clustered together and follow closely the 1979 change of government which, in itself, represented a substantial regime change. Nevertheless, the results (Table 3) are quite clear-cut: they reject constancy across exchange control abolition but accept it across the other two dates. Furthermore, it transpired that the main differences in the parameter estimates between the first and second sub-periods were in the foreign currency equations for the exchange control test. Post-abolition there was a marked increase in the individual significance of the portfolio and transactions variables in this equation. Thus, despite the confluence of several regime changes, it appears reasonable to assert that exchange control abolition was the

TABLE 3
RESULTS OF CHOW TESTS FOR PARAMETER CONSTANCY

	Restricted Theory-Based Model χ^2 (31) Value (significance %)	Restricted Data-Based Model χ^2 (18) Value (significance %)
(A) Exchange control abolition	46.43 (96.25)	19.50 (63.7)
(B) Abolition of supplementary special deposits	47.17 (89.5)	21.68 (75.2)
(C) Revised monetary controls	39.79 (86.6)	23.45 (82.65)

most important.[7]

P tests provide a framework for comparing two non-nested models using either one as null. See Davidson and MacKinnon (1981). The standard *P* test involves a comparison between two different models of a common endogenous variable(s). Here, we have a given model of the endogenous variable—the rate of return: but we have several different variables which, in principle, provide an imperfect measure of the "true" rate of return. In short, we encounter a particular kind of measurement error problem. Let the true (single equation) model be:

$$y = X\beta + e. \tag{22}$$

The true values of y are unknown but two alternative measures of y exist:

$$z_0 = y + V_0, \tag{23}$$
$$z_1 = y + V_1. \tag{24}$$

With V_0 and V_1, being white noise errors with the usual properties, including *Plim* $T^{-1}y'V_0 =$ *Plim* $T^{-1}y'V_1 = 0$. Combining (22), (23), (24) gives two regression hypotheses:

$$\mathrm{H}_0\text{: } z_0 = X\beta_0 + e_0;\ e_0 = e + V_0, \tag{25}$$
$$\mathrm{H}_1\text{: } z_1 = X\beta_1 + e_1;\ e_1 = e + V_1. \tag{26}$$

[7]In addition we re-estimated Green's earlier model over his 1972-7 data period using our new data. Although there were change in the estimated values of parameters as compared with the earlier estimates, the results of all the hypothesis tests were the same as in his study. This also suggests that structural changes may be partly responsible for rejection of the SLMB CAPM.

Clearly OLS on (25) and (26) will deliver unbiased estimates of the true parameters β, but these estimates will differ from one another. The question is whether non-nested tests can be used to compare these estimates more systematically; in effect, to compare the value of the information about y contained in z_0, z_1? Green (1988b) showed that a modified version of the P test could indeed be used for this purpose. We summarize his results briefly.

Consider the artificial model

$$(1-\alpha)z_0 + \alpha z_1 = X\beta + u. \tag{27}$$

$\alpha/(1-\alpha)$ can be regarded as the relative weight to be put on z_1/z_0 in estimating the true of value of y, which is, in turn, determined by the X variables. A test for the truth of H_0 could, in principle, be carried out if the parameter α could be identified, estimated, and a t test performed on this estimate. A consistent estimate of α can be obtained as follows. First estimate (25) and (26) by OLS and retrieve the residuals: $\hat{e}_0$, $\hat{e}_1$, respectively. Then run the auxiliary regression:

$$\hat{e}_0 = Xb_0 + (\hat{e}_0 - \hat{e}_1) + u_0. \tag{28}$$

It can be shown that the estimated α from this regression is a consistent estimate of α in (27). The asymptotic variance of α is however "too large" in a certain sense and the t test on α will therefore tend to accept H_0 too frequently. Clearly though the test can be used to reject H_0. See Green for further details. In practice, we utilized this test in a system of equations, and we therefore used the Edgeworth-adjusted likelihood ratio statistic to test the significance of a vector of parameters analagous to α.

As with any non-nested test this modified P test can be performed either with H_0 as null using $\hat{e}_0$ as regressand or with H_1 as null using $\hat{e}_1$ as regressand. The test only gives an unambiguous result if either H_0 is accepted against H_1 *and* H_1 is rejected against H_0 or if H_0 is rejected against H_1 *and* H_1 is accepted against H_0. This is inherent in any non-nested testing procedure. If one model is superior to the other we expect an unambiguous test outcome. However, it is possible that both models are false or that both models are partly true; in either of these circumstances, a non-nested test should have an ambiguous outcome.

Table 4 reports three sets of modified P tests using as a basis the restricted theory-based model. The first uses the return on consols in place of a calculated average return on government bonds. This is

TABLE 4
RESULTS OF MODIFIED P TESTS ON DEFINITIONS OF ASSET RETURNS USING RESTRICTED THEORY-BASED MODEL

Null Hypothesis	$YG0$ vs $YG1$ $\chi^2(3)$ value (significance %)	$YF0$ vs $YF1$ $\chi^2(3)$ value (significance %)	$Y0$ vs $Y1$ $\chi^2(9)$ value (significance %)
$YG0$, $YF0$, $Y0$	0.76	0.48	306.4
	(61.3)	(50.0)	(100.0)
$YG1$, $YF1$, $Y1$	24.77	60.31	124.1
	(99.9)	(99.9)	(100.0)

Note: $YG0$ = Bond return measured as a weighted average.
$YG1$ = Bond return measured as yield on $3\frac{1}{2}$% War Loan.
$YF0$ = Exchange rate measured as official dollar-sterling rate.
$YF1$ = Exchange rate measured as investment dollar rate through Oct. 1979.
$Y0$ = All returns measured pre-tax.
$Y1$ = All returns measured post-tax.

a partial check on the wrong sign we found in the bond equation. The second uses an alternative definition of the exchange rate in place of the official sterling-dollar rate. In view of the Chow test results, we constructed a series which was equal to the investment dollar rate through Oct. 1979 and the (unique) sterling-dollar rate thereafter. In both cases, the original definition rejects the new definition but is not itself rejected by the new definition. Therefore, subject to the power considerations of the test, our preferred definition is unambiguously superior. The exchange rate result is a little surprising, but could be due, in part, to the fact that our foreign currency portfolio is a net portfolio and does not discriminate between assets according to whether they were subject to the investment dollar premium. Further tests on the definition of the equity and foreign currency returns produced the same unambiguous acceptance of our original definitions and are not reported here. The third test reported in Table 4 compares the use of pre-tax returns in all equations simultaneously with the use of post-tax returns. Our calculations of post-tax returns are detailed in a companion paper but, in brief, they utilize estimated median tax rates and allow for indexation relief of capital gains. This test indicates overwhelming rejection in each direction. It suggests to us that pre-tax and post-tax returns are measuring different phenomena, and it underscores the need to take into account taxation in modelling asset returns. We have not done so in this paper because the use of after-tax returns raises a variety of new problems. These are discussed in Green and Na (1988b).

VIII. Conclusions

Our study confirms that a relatively simple version of the SLMB CAPM is not capable of explaining aggregate UK monthly asset returns over the period since 1972. The main theoretical restrictions associated with the SLMB CAPM are rejected by the data; in addition variable other than the market portfolio clearly help explain asset returns.

Referring back to assumptions A1-A3 of the SLMB CAPM, we see first that, although we did not adopt an inter-temporal framework, we did find both consumption and income variables to be significant determinants of asset returns, thus suggesting that assumption A1 does need to be relaxed. Our diagnostic tests suggested that it may be worthwhile to follow Bollerslev, Engle and Wooldridge and further relax assumption A2 to allow for time-varying covariances of asset returns as well as time-varying means. We are also concerned at our inability to eliminate entirely the residual autocorrelation from the model, although, as noted above, 12th-order autocorrelation could be another manifestation of the well-documented "January effect".

A major part of our findings concerns assumption A3 and the role of market imperfections. On the one hand, several of the restrictions implied by the quadratic costs of change representation of market imperfections were rejected by the data. We are, however, in good company in finding that models of asset returns do reject theoretically plausible restrictions. On the other hand, the quadratic costs of change model did prove effective in selecting variables that help explain asset returns. Portfolio transactions, income, consumption, and dividend yields all contributed significantly to explaining asset returns, even if the parameters associated with these variables did not always coincide with theoretical expectations. Moreover, the rank ordering of the elements of the martrix (θ) of costs of conversions between income and capital accounts did conform with intuition and the evidence in favour of the "bird-in-hand" hypothesis was notable. The specification tests showed clearly the importance of taxation as a factor influencing asset returns and, albeit with more equivocation, that exchange controls were probably a relevant distorting factor.

Our results underline the need to relax assumption A3 as well as A1 and A2 in order to arrive at a fully satisfactory account of UK asset returns. The basic SLMB CAPM is clearly not adequate for this purpose. It is partly a matter of semantics as to whether we describe a model which relaxes assumptions A1–A3 as "The Capital Asset Pricing Model". Clearly it is not the SLMB version, although we would be inclined to the view that any model of asset returns based on mean-variance optimization or its inter-temporal equivalent will turn out to be a member of the same family.

However, our results also suggest that modelling market imperfections is not likely to prove easy. The quadratic costs of change model clearly has its limitations although, because of its convenience, we would not wish to abandon it entirely. A main characteristic of market imperfections is that they often impose constraints which change the structure of expected returns and of variances and covariances which investors face. We conjecture therefore that there is likely to be an interaction between relaxing assumptions A2 and A3. If investor constraints and their impact on the first and second moments of returns are fully spelled out in a model, it is possible that a relatively simple underlying model may prove consistent with the data and capable of explaining a signicantly higher proportion of the variation in asset returns than was the case in the present study. We plan on pursuing this argument in future research.

Appendix A

Notes on the Data

These notes are brief; the reader is referred to Green and Na (1988a) for full details.

1. *Rates of Return*

As far as possible the rates of return (y_t) are those recorded between the last working day of successive months. They are defined in the text as:

$$y_{it} = (p_t / p_{t+1})\ (r_{it} + q_{it+1} / q_{it}). \tag{A1}$$

p_t is the UK retail price index. The asset prices (q_t) and dividend yields (r_t) are calculated separately for each security. For *liquid* *assets* $q_t =$ 1 and r_t is the Local Authority 3 month rate adjusted to a one-month basis. For *bonds*, q_t is the ratio of the market value of the outstanding stock of bonds to its face value and r_t is the average coupon. See Green (1988c) for details. For *foreign currency assets*, q_i is the sterling-dollar spot exchange rate and r_t the 3 month Euro-dollar rate adjusted to a one-month basis.

The calculation for *equities* is more complex as it must allow for share dilution when new equity is sold in existing firms, and for share enhancement when net new capital is constructed out of retained earnings. The nominal rate of return to existing equity holders (μ) is given by:

$$\mu = (M_{t+1} - M_t - m_t + D_t) / M_t \qquad \text{(A2)}$$

where $M_t =$ Market value of Equity
$m_t =$ Value of net new shares (dilution)
$D_t =$ Dividend payments.

The market value of Equity can, in turn, be written:

$$M_t = q_{et} \ \rho_t \ K_{t-1}(1 - n) \qquad \text{(A3)}$$

where $q_{et} =$ Tobin's q(the price of Equity)
$\rho_t =$ The price of capital goods
$K_t =$ The stock of physical capital with
$n =$ The depreciation rate.

Suppose also, that a fraction (λ_t) of new investment is financed by share dilution then:

$$m_t = \lambda_t \ q_{et} \ \rho_t(K_t - K_{t-1} \ (1 - n)). \qquad \text{(A4)}$$

Combining (A2), (A3), (A4) gives

$$1 + \mu = \frac{q_{et+1} \ \rho_{t+1} \ K_{t+1}}{q_{et} \ \rho_t \ K_t} - \frac{\lambda_t(K_t - K_{t-1} \ (1 - n))}{K_{t-1} \ (1-n)} + r_{et} \qquad \text{(A5)}$$

with $r_{et} =$ The dividend yield on equity (on a one-month basis).

By definition the real (gross) return on equity (y_{et}) is then:

$$y_{et} = (p_t / p_{t+1})(1 + \mu). \qquad \text{(A6)}$$

Substituting from (A5) into (A6) gives the appropriate expression for the equity return. More precisely, y_{et} is the return on captial available in the equity market. Since λ_t is unknown we actually used $((q_{et+1} \ \rho_{t+1} \ K_{t+1} / q_{et} \ \rho_t \ K_t) + r_{et})$ as the left-hand side variable and entered $(K_t - K_{t-1} \ (1 - n)) / (K_{t-1}(1 - n)) = Z_t$ as an additional explanatory variable in the equity equation. This is the share dilution term to which reference is made in the text. In principle, the coefficient on Z_t estimates the average proportion of share dilution over the period of the model. In fact, this coefficient, though correctly signed, was always insignificant. Therefore the estimated "restricted" models explain the zero share dilution ($\lambda = 0$) return on equity.

2. *Portfolio and Transactions Data*

Monthly data were constructed using as a basis Green's (1984) estimates and the Bank of England Bulletin Tape. The foundations of these estimates are the monthly balance sheets of banks, and money supply and counterparts data. Some data, particularly for the overseas sector, was interpolated from quarterly. As indicated in the text, the balance sheets are those of the consolidated private sector. Assets are valued using the same prices as those used in the rate of return calculations.

Foreign currency assets consist of all net holdings of assets denominated in currencies other than sterling. They include UK portfolio investment overseas and borrowing, but exclude direct investment which is assumed to be indistinguishably represented in the equity of UK compaines. *Equity* consists of the market value of the stock of all UK companies (including financial companies) calculated in the standard way by captializing the dividend stream. This is, however adjusted upwards on account of UK firms' overseas earnings from foreign direct investment, and downwards on account first of payments of profits overseas reflecting foreign direct investment in the UK, and second of payments of dividends overseas reflecting foreign portfolio investment in the UK. *Bonds* consist of the outstanding stock of UK government bonds in market hands including index-linked. Debentures are included with equity. *Liquid assets* are essentially the residual items. Considering that financial institutions are consolidated with the private sector, bank loans and deposits are mainly offsetting. Thus liquid assets consist chiefly of high-powered money, national savings, and bills: Treasury Bills; Tax certificates; and Local Authority Temporary Money.

Stocks and flows are related through the identity:

$$X_{it} - X_{it+1} = (X_{it} - X_{it-1}\ q_{it} / q_{it-1}) + (q_{it} / q_{it-1} - 1)X_{it-1} \quad \text{(A7)}$$

the first term on the right-hand side being net transactions and the second net captial gains.

3. *Income and Expenditure*

Capital income was calculated directly using the identity

$$D_{it} = r_{i\,t-1}\ X_{i\,t-1} \tag{A8}$$

for each asset. The remaining data is available quarterly from the national accounts. These were converted to monthly by interpolation using appropriate benchmarks. Simple interpolations produces monthly data which are equivalent to flows at a quarterly rate. This is undesirable as it introduces smoothing into the data. Accordingly, the interpolation was done to produce monthly flows at monthly rates. Thus, for example, estimated consumption in Juanuary, February, March each year was interpolated (using retail sales) in such a way that the sum of the January, February and March estimates were constrained to equal national accounts consumption in the first quarter. This procedure was used for all similar interpolations.

Appendix B

Supplementary Parameter Estimates

1. *Notation*

For purposes of presenting the parameter estimates we use the following notation:

YG, YE, YF = One month returns on bonds (G), equities (E), and foreign currency assets (F) less the one month return on liquid assets.

G_t, E_t, F_t = Market values of private sector holdings of bonds (G), equities (E), and foreign currency assets (F).

W_t = Private net worth ($= G_t + E_t + F_t +$ Liquid Assets).

g_t, e_t, f_t, = Gross captial gains on bonds, equities and foreign currency assets defined as the ratios $q_{it} / q_{i\,t-1}$, $i = g, e, f$.

Z_t = Net investment as a proportion of the net captial stock.

R_{gt}, R_{et}, R_{ft}, R_{lt} = Dividend price ratio of bonds, equities, foreign currency assets, and liquid assets.

TABLE B1
ESTIMATES OF THE GENERAL, UNRESTRICTED MODEL

	YG Coefficient (t-value)	*YE* Coefficient (t-value)	*YF* Coefficient (t-value)
Constant	0.118	−0.833	−0.287
	(0.30)	(−1.35)	(−1.02)
G_t / W_t	−0.726	0.778	0.454
	(−1.71)	(1.22)	(1.49)
E_t / W_t	−0.047	0.360	0.167
	(−0.13)	(0.66)	(0.66)
F_t / W_t	−0.863	(0.473)	0.463
	(−1.68)	(0.61)	(1.26)
$(G_t - g_t G_{t-1}) / W_t$	3.346	1.377	−1.444
	(3.40)	(0.93)	(−2.07)
$(E_t - e_t E_{t-1}) / W_t$	−1.211	0.446	0.460
	(−1.39)	(0.35)	(0.73)
$(F_t - f_t F_{t-1}) / W_t$	0.260	−1.971	−0.277
	(0.37)	(−1.93)	(−0.55)
$(g_t - 1)G_{t-1} / W_t$	−0.261	−0.530	−0.143
	(−0.76)	(−1.06)	(−0.58)
$(e_t - 1)E_{t-1} / W_t$	0.019	0.592	0.047
	(0.15)	(3.07)	(0.49)
$(f_t - 1)F_{t-1} / W_t$	−0.280	−1.707	−0.138
	(−0.23)	(−0.96)	(−0.16)
Z_t		−1.346	
		(−0.30)	
S_t / W_t	0.512	−0.185	0.241
	(0.70)	(−0.17)	(0.46)
$(H_t + D_t) / W_t$	0.803	5.419	0.309
	(0.74)	(3.38)	(0.39)
T_t / W_t	0.174	−4.239	−0.588
	(0.14)	(−2.37)	(−0.67)
D_t / W_t	42.269	14.822	0.847
	(3.07)	(0.71)	(0.08)
$(\Delta C_t)R_{gt} / W_t$	4.540		
	(0.49)		
$(\Delta C_t)R_{et} / W_t$		0.912	
		(0.05)	
$(\Delta C_t)R_{ft} / W_t$			−10.025
			(−2.03)
$(\Delta C_t)R_{lt} / W_t$	−5.220	−15.421	2.731
	(−0.69)	(−1.72)	(0.69)
R^2	0.34	0.39	0.18

Note: Log-Likelihood = 876.8.

TABLE B2
ESTIMATES OF THE RESTRICTED DATA-BASED MODEL

	YG Coefficient (*t*-value)	*YE* Coefficient (*t*-value)	*YF* Coefficient (*t*-value)
Constant	0.111	−0.430	−0.126
	(1.25)	(−4.55)	(−2.01)
G_t / W_t	−0.750	0.351	−0.295
	(−3.84)	(1.52)	(2.13)
F_t / W_t	−0.876		0.284
	(−3.33)		(1.43)
$(G_t - g_t G_{t-1}) / W_t$	2.641		−1.346
	(3.25)		(−2.18)
$(E_t - e_t E_{t-1}) / W_t$	−0.794		0.782
	(−1.96)		(2.44)
$(F_t - f_t F_{t-1}) / W_t$		−2.060	
		(−2.71)	
$(e_t - 1)E_{t-1} / W_t$	0.611	0.557	
	(1.71)	(3.96)	
$(H_t + D_t - T) / W_t$	0.750	3.987	
	(1.89)	(6.38)	
D_t / W_t	47.751	27.331	
	(4.85)	(1.82)	
$(\Delta C)R_{gt} / W_t$	−2.821		
	(−1.44)		
$(\Delta C)R_{et} / W_t$		−15.218	
		(−2.50)	
$(\Delta C)R_{ft} / W_t$			−2.463
			(−1.62)
R^2	0.260	0.355	0.101

Note: Log-Likelihood = 854.7.

We also use the following notation which is identical to that used in the theoretical model.

S_t = Real net acquisitions of assets (savings)
H_t = Real pre-tax labor income
D_t = Real pre-tax capital income
T_t = Income tax payments
C_t = Real consumption

The estimated parameters of the seasonal and data dummies which are included in the regression models are not shown in these tables. They can be obtained from the authors on request.

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