Monetary Allocation Mechanism under Asymmetric Information and Limited Communication

Seh-Jin Chang
Inha University

We consider the constitutional problem of choosing a social allocation mechanism of an endowment economy where individual endowments and preferences are subject to idiosyncratic shocks. Under idealistic environments, where individuals are honest or information and communication are free, the economy can implement the idealistic allocation mechanism which guarantees a Pareto optimal allocation or the first-best solution. Under asymmetric information (but with free communication), however, the economy should compromise with an suboptimal allocation mechanism, the second-best solution, due to the moral hazard and the adverse selection. This familiar conclusion of the incentive theory reverts when communication is not free. Then the economy should be satisfied with the third-best solution, where the pricing functions are linear again but money, serving as medium of communication, becomes essential.

I. Introduction

This paper is addressed to the fundamental question of monetary theory: "Why do we use money?" That money becomes redundant in the General Equilibrium theory invoked much efforts to integrate monetary and value theories.[1] A common direction is to introduce various frictions to nonmonetary exchanges (or conveniences of monetary exchanges) as in the Neo-Walrasian approach [Walras (1900, 1956), Patinkin (1965)], in the transactions cost approach [Jevons (1910), Tobin (1956), Niehans (1978)], in the cash-in-advance approach [Clower (1967), Lucas (1980)], or even in the

[1]The integration of monetary and value theories has been a proud claim by many eminent economists: see, for example, Walras (1900, pp. 9-10), Keynes (1936, pp. 7, 292), Patinkin (1965, p. 24), Clower (1967, p. 211).

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game-theoretic approach [Ostroy and Starr (1974), Radner (1980), Gale (1986)]. They all presuppose a market structure which would work perfectly without the imposed frictions. This paper departs from the tradition in that the allocation mechanism is to be chosen simultaneously, rather than after the presupposed market structure, so that money becomes fully integrated part of the market allocation mechanism. This approach sheds new lights on the nature of frictions which call for money as well as on markets themselves.

To be specific, we will consider a constitutional problem in which the society chooses, from scratch, a set of rules how to allocate endowed resources among individuals. The set of rules is to be chosen unanimously and unequivocally before any heterogenetic shocks are realized to individuals. Obviously, the choice depends on what can be called the allocative environment. Suppose that individuals behave noncooperatively [cf. Radner (1968), Feldman (1973), Madden (1975)] and that there exists asymmetric information about individual need (preference) and ability (endowment). The society then cannot achieve the first-best solution due to adverse selection and moral hazards [Mas-Colell (1980), Holmstrom (1982), Laffont and Maskin (1982)]. Under the second-best solution, however the pricing functions are, in general, nonlinear, precluding the market mechanism.

Rather neglected in the incentive theory is the fact that the chosen set of rules need be enforced by social agents, say the social planner himself (the unanimous public) who cannot survive beyond the constitutional period in our case. If the social planner delegates his authority to a group of social agents, there emerges a new problem of coordination among those agents. This additional problem may prevent the society from getting even the second-best solution. Under limited communication among social agents, however, pricing functions become linear again while money serves as medium of one-dimensional communication. The third-best solution is period-wise Pareto optimal but not in the Arrow-Debreu sense.

The remainder of the paper is organized as follows. The next section specifies the economy. Section III formulates the social planner's problem. Section IV solves the planner's problem. Section V considers the efficiency of the solution. Finally Section VI concludes the paper.

II. The Economy

A. The Physical Structure of the Economy

Throughout this paper, we shall consider the allocation problem of an endowment economy. There are a finite number l of (private) goods. The economy is composed of a continuum A of *ex ante* identical individual consumers indexed by $\alpha \in A$. Formally, we consider A as a measure space with the σ-algebra $\mathbf{A}$ generated by Borel sets of A and a measure ν on $\mathbf{A}$ such that $\nu(A) = 1$.

Let Ξ be the set of individual characteristics. The set Ξ consists of the set of individual endowments, X, and the set of preference parameters, Θ, so that $\Xi = X \times \Theta$. We assume that both X and Θ are nonempty and compact in R^l_+.

Let $Z = R^l_+$ be the set of (technically feasible) individual consumptions. The utility function $u : Z \times \Theta \to R$ satisfies "standard" properties in that:

(U1) u satisfies the EU axioms;
(U2) u is continuous on $Z \times \Theta$; and
(U3) For each $\theta \in \Theta$, $u(\cdot, \theta)$ is strictly increasing, strictly concave and continuously differentiable with respect to z, and $\partial u(\cdot, \theta) / \partial z_j$ is strictly increasing in θ_j for $j = 1, \cdots, l$.

Given $\theta \in \Theta$, individual α's preference $\gtrsim$ on consumption is given by $u(\cdot, \theta)$. By (U3), $\gtrsim$ invariably satisfies the nonsatiation and a strict risk aversion.

Let $(\Omega, \mathbf{F}, \mu)$ be the fundamental probability space of states. Individual characteristics are assigned by a stochastic rule

$$\xi : A \times \Omega \to \Xi, \tag{1}$$

where ξ is $\mathbf{A} \times \mathbf{F}$-measurable. We assume

(S1) $\{\xi(\alpha, \cdot)\}$ is a continuum of *iid* random variables;

(S2) $\xi(\alpha, \cdot)$ and $\xi(\cdot, \omega)$ have the same distribution almost everywhere (a.e.) with respect to the product measure $\nu \times \mu$;[2]

[2] The Condition (S2) is nothing but the law of large numbers, which would be implied by (S1), if A is countably infinite. Judd (1985) showed that the law of large numbers fails, in general, for a continuum of iid random variables with atomless (i.e. absolutely continuous) measures, but that Conditions (S1) and (S2) can be assumed without logical inconsistency. Note, however, that we did not restrict the measure ν to be atomless, hence there can be

(S3) $x(\alpha,\cdot)$ and $\theta(\alpha,\cdot)$ are mutually independent.

Let $F: X \rightarrow [0, 1]$ and $G: \Theta \rightarrow [0, 1]$ denote (identical) distributions for random variables $x(\alpha,\cdot)$ and $\theta(\alpha,\cdot)$ respectively.

Since individuals are identical except their characteristics or realized shocks, we can treat as if they are indexed by shocks only, suppressing their unique indices, $\alpha \in A$. Hence, the physical structure of economy is completely characterized by $(Z, X, \Theta; u, F, G)$, or (u, F, G) for short.

B. The Allocation Problem in the "Honest" Economy

By assumption, our economy (u, F, G) invariably "suffers" from the scarcity of commodities so that it should solve the allocation problem somehow. Treating individuals with the same characteristics identically, we define an *allocation* by an integrable function $z: X \times \Theta \rightarrow Z$, which assigns a consumption $z(x, \theta)$ to each individual with realized shocks (x, θ).

We suppose that all individuals gather in a large agora to choose an allocation unanimously, *before any heterogenetic shocks are realized.* To economize our wordings, we shall call the homogeneous public at agora, distinguished from individuals after shocks are realized, the *social planner.*[3]

The social planner tries to maximize the expected utility of *ex ante* identical individuals, or, equivalently, the *mean* utility of *ex post* heterogeneous individuals:

$$W(z) = \iint u[z(x, \theta), \theta]\, dF(x)\, dG(\theta) \tag{2}$$

An allocation z is said to be *attainable* if it satisfies

$$\iint z(x, \theta)\, dF(x)\, dG(\theta) = \bar{x}, \tag{3}$$

where $\bar{x} = \int x dF(x) < \infty$.[4] Obviously the attainability is the minim-

a finite number of individual "types" [cf. Hammond (1979), Gale (1980)]. If A is finite, then (S2) implies a joint probabilistic restriction.

[3]Since the notion "social planner" is also used in quite different contexts, we shall clarify differences to prevent possible confusions. First, his preference is naturally derived rather than exogenously given (usually additively separably with some weight function). Second, he cannot break or threat to break the society's budget constraints (even if it is socially desirable, say to make individuals honest), since his "move" should be subgame-perfect [Holmstrom (1982, Section 2)]. Third, he literally vanishes after making the "plan," rather than remaining to execute his plan physically. In fact, the contracting public itself may be a conceptualization of some evolutional force of institutions.

[4]We shall use the convention of element-wise inequalities for vectors, e.g. $x < y$ if and

al restriction to be imposed on the social planner.

Since the social planner *literally* vanishes after drafting a plan, there should exist "social agents," to whom the authority to execute the plan can be delegated. Such delegation generally involves an incentive problem of its own. We shall *neglect* this incentive problem simply by assuming that the social agents are unconditionally loyal to the social planner, i.e. to the society, so that we can concentrate on the incentive problem of individual consumers. For simplicity, we also assume that the social agents incur no (social) costs.[5]

Given the availability and unconditional loyalty of the free social agents, whether an attainable allocation can be implemented depends basically on the social agents' "ability" to process information and make designated physical transfers, hence the informational structure of the society and the individuals' ability to "cheat" against social agents. The restriction on "available" allocations from these considerations, in addition to the resource or attainability constraints, can be called the *implementability constraints*.

The implementability constraints may well be nonbinding, if the realization of individual shocks is public, if (free) monitoring or auditing is available to the social agents, or if individuals are "honest." We shall call such a case an *allocatively idealistic* economy, or simply an *honest* economy.

The social planner of an honest economy (u, F, G) would maximize (2) subject to (3). The unique solution z^I is given by (3) and

$$\frac{\partial u(\cdot, \theta)}{\partial z_j} = \lambda_j \quad j = 1, \cdots, l, \tag{4}$$

where λ's are the Lagrangean multipliers for the resource constraints. We shall say that the solution z^I is (*absolutely*) *idealistically optimal, absolutely optimal,* or simply *idealistic*. Note that λ's are constants with idiosyncratic shocks so that z^I depends only on θ. This rationalizes the idealism, "from each according to his ability; to each according to his need." What is questionable in this idealism is *not* its desirability *but* its practicality, i.e. its total negligence of the implementability constraints.[6]

only if $x_j < y_j$ $\forall j$. Note that we have used the *equality* rather than the weak inequality to conform with that the social planner cannot use group punishment [see footnote 3]. When confusion may arise, we shall refer to (3) as the *strict* attainability condition.

[5]These assumptions are conventional in virtually all incentive literature, though such assumptions are usually implicit [Laffont and Maskin (1982, footnote 4)].

[6]Though it is well-recognized that the socialistic idealism involves serious incentive

C. The Allocative Environment of the Economy

In general, however, the "ability" of available social agents is less than perfect so that the idealistic solution is not implementable. The following specification of the allocative environment restricts the ability of the social agents, or what can be enforced by them.

(E1) *Private Shocks* : Individual shocks, both x and θ, are private, and no monitoring of auditing of them is available at any cost.

This asymmetry of information is standard and provides the fundamental restriction for the social agents' information. The following two assumptions provide much stricter restriction on the social agents' ability to process information.

(E2) *Specialized, Localized, Redundant, Complete Social Agents* : Following Alchian (1977), we restrict the perception skills of social agents, assuming that each social agent can correctly recognize the quality (and quantity) of *one and only one* commodity, *according to* his specialization. Moreover, following Lucas (1980) and Townsend (1986), we assume that they cannot communicate directly with each other. We also assume that there exist infinitely many specialized social agents for each commodity $j = 1,\cdots,l$.

To visualize the situation, we presume that each social agent is "dispatched" to a fixed place, called "store," after receiving detailed instructions from the social planner. Then, we may say that each "market" consists of infinitely many stores, though *market j* is nothing but the collection of fixed places where a group of social agents specializing in commodity j execute their part of instructions loyalfully.

(E3) *Free Shopping* : Individual consumers have no other way to alter their possessions than through the agents, though they can conceal or dispose them freely. However, following Lucas (1980), we assume that they can visit freely any set of stores in any order (including "repeated" visits or no visit to stores in the same mar-

problems, at least since the socialist controversy, it does not necessarily mean that our economists discarded the idealistic solution or the honest economy entirely. To the contrary, it is rather surprising that many standard economic tools, not mentioning the pure planning problem, are based on the honest economy, implying the idealistically optimal allocation, though it is frequently described merely as Pareto optimal (or "efficient"). See Supplementary Notes B and G in the full version which is available on request.

ket), about which the social agents have no control or information.

Thus far, our assumptions — private shocks, specialized perception skill, no communication, free (private) shopping by individuals — invariably restricted the informational capability of the social agents. The following provides an exit from the otherwise inevitable result of autarky:

(E4) *The Existence of Money* : There exists a currency which can be produced costlessly for the society. The currency is individually uncounterfeitable, universally recognizable (regardless of specialization), divisible, durable, transportable and concealable. Initial currency holdings of individuals are identically zero.

Being divisible, the currency can represent any nonnegative real number. The universal recognizability, together with transportability and durability, makes the currency a promising candidate for the medium of communication across space and time. The concealability means that the message can be scrambled by the messengers (consumers) — but only unidirectionally due to the uncounterfeitability.

This completes the specification of the allocative environment. Unlike the case for physical structure, a compact mathematical representation seems unavailable, though a mathematical formalization is not impossible. To economize our wordings, we shall call such an economy an (*informationally*) *decentralized* economy. In contrast, the economy in the usual mechanism theory, where the information is grantedly centralized (no localization) and individual visits are controlled (no free shopping) will be called the (*informationally*) *centralized* economy.[7]

III. The Social Planner's Problem

A. Allocation Mechanisms and the Clower Arrangements

We confine our attention on the following type of arrangements of local submechanisms, which is to be specified by $\{S^j, \delta_j, \gamma_j\}_{j=1}^{l}$, where $\delta_j : S^j \rightarrow R$ and $\gamma_j : S^j \rightarrow R$ denote net distributions of com-

[7]Note that the decentralization or centralization referred here is informational, rather than "institutional," i.e. they refer whether the information should be processed at local levels or can be centralized. This usage might be confusing with the usual convention, referring whether the decision-making units are multiple or not [e.g. Radner (1982)]. In the latter sense, our economy is *always* decentralized, as much as there are more than one individuals.

modity j and the currency respectively.

(a) Perceive individual's signal s^j;
(b) Request to submit max$[0, -\gamma_j(s^j)]$ units of currency and max$[0, -\delta_j(s^j)]$ units of commodity j;
(c) Observe submittances;
(d) If the visitor submits not less than requested (in both), then distribute max$[0, \gamma_j(s^j)]$ units of currency and max$[0, \delta_j(s^j)]$ units of commodity j.
(e) Otherwise, do nothing.

The central features of the above arrangement is that each individual is qualified to receive something [(d)], only if he is *able* as well as *willing* to submit as requested. Otherwise, he is *penalized* [(e)], in the sense that he would be worse off — regardless of his private shocks, hence in a way that the planner *knows* he would. As a result, individual possessions shall be kept *nonnegative* throughout the shopping period, under any specification of $\{S^j, \delta_j, \gamma_j\}$. We shall call the above type of arrangements the *Clower class arrangements*. Correspondingly, we shall call the arrangement with the instructions (a) through (d) specified by $\{S^j, \delta_j, \gamma_j\}$ the *Clower arrangement with* $\{S^j, \delta_j, \gamma_j\}$, or simply "the" arrangement $\{S^j, \delta_j, \gamma_j\}$.

Note, however, that there exists *no* presumption in the Clower arrangement of this general form that money should be used in a nontrivial sense. γ_j may well be identically zero for all j, for instance. Note also that, even if money is used and even if there exist parametric relations between money and commodities, there exists no intrinsic restriction to these relations. In fact, it can be shown, if we can go a little deeper, that the Clower class arrangements are flexible enough to represent all "enforcible" arrangements, in a proper sense, under our assumptions.[8] Henceforth, we shall confine our attention on the Clower class arrangements.

B. Individual Optimization under a Clower Arrangement

A Clower arrangement $\{S^j, \delta_j, \gamma_j\}_{j=1}^{l}$ deterministically assigns a consumption $\tilde{z} \in Z$ for each individual $(x, \theta) \in X \times \Theta$ according to his chosen behavior. With the nonsatiation assumption ($u' > 0$), rational individuals would never violate provision (e) ["credible"

[8]See Proposition 4.2 and preceding discussion in the full version.

threats]. Fix a Clower arrangement $I = \{\delta_j, \gamma_j\}$. We shall say a consumption $\tilde{z} \in Z$ is *strictly feasible* for individual (x, θ), if there exist some behaviors through which $\tilde{z}$ can be obtained without violating (e). Let $F_I(x, \theta)$ represent the set of strictly feasible consumptions of individual (x, θ). We shall say that an allocation z is *incentive compatible* if z satisfies

$$z(x, \theta) \in \arg\max\ u(\tilde{z}, \theta) \quad \text{s.t. } \tilde{z} \in F_I(x, \theta) \tag{5}$$

for almost all (x, θ).

Under any Clower arrangement $\{S^j, \delta_j, \gamma_j\}_{j=1}^{l}$, the only relevant individual behaviors are: i) What stores are to be visited in what order and ii) What individual shocks are to be announced in each visit.[9] We shall call the first decision a *shopping schedule*, represented by an element σ of the set Σ of finite sequences in $J = \{1, \cdots, l\}$, so that σ_n denotes the market visited at the nth shopping. We shall denote by σ_j^{-1} the subset of shoppings (indexed by positive integers) made in market j, i.e. $\sigma_j^{-1} = \{n \in \mathbf{N} : \sigma_n = j\}$. We shall call the latter decision a *reporting plan*, represented by an element $\rho = \{s^n\}$ of the set $\mathbf{P}$ of finite sequences in $\tilde{S}$. A *shopping plan* is a pair (σ, ρ) of a shopping schedule and a reporting plan which are conforming, i.e. having the same "length." Hence, the set B of relevant individual behaviors are completely specified by $B = \Sigma \times \mathbf{P}$, where it is understood that the latter two should be conforming. A shopping plan is (*strictly*) *feasible for* an individual with (x, θ) *under* a Clower arrangement $\{S^j, \delta_j, \gamma_j\}$, if he can submit as much as requested, i.e. avoid the penalty provision (e), during the whole course of shopping. We may confine our attention on the (strictly) feasible shopping plans.

Given an arrangement $\{S^j, \delta_j, \gamma_j\}_{j=1}^{l}$, an individual with a realized shock (x, θ) tries to find a pair of a shopping schedule $\sigma \in \Sigma$ and a reporting plan $\rho \in \mathbf{P}$, thereby a consumption $\tilde{z} \in Z$, which solves

$$\max_{\tilde{z}, \sigma, \rho} u(\tilde{z}, \theta) \tag{P1}$$

$$\text{s.t. } \tilde{z}_j = x_j + \sum_{n \in \sigma_j^{-1}} \delta_j(s^n) \quad j = 1, \cdots, l \tag{6}$$

[9] The submittance decision about how to response to requests in each visit is immaterial, since rational individuals shall submit exactly as requested in the face of penalization and nonsatiation. By the same spirit, we ignore individual decisions about free disposal.

$$x_{\sigma_n}^{n-1} + \delta_{\sigma_n}(s^n) \geq 0 \quad n = 1,\cdots, |\sigma| \tag{7}$$

$$m^{n-1} + \gamma_{\sigma_n}(s^n) \geq 0 \quad n = 1,\cdots, |\sigma|, \tag{8}$$

where the sequence of possessions $\{x^n, m^n\}$ is given recursively by

$$x_j^0 = x_j,$$

$$x_j^n = \begin{cases} x_j^{n-1} + \delta_j(s^n) & \text{if } \sigma_n = j \\ x_j^{n-1} & \text{otherwise} \end{cases} \tag{9}$$

for $j = 1,\cdots,l$ and

$$m^0 = 0$$

$$m^n = m^{n-1} + \gamma_{\sigma_n}(s^n) \tag{10}$$

for $n = 1,\cdots, |\sigma|$.

Equations (6) through (10) define the set of strictly feasible consumptions, $F_l(x, \theta)$. Hence it follows: An allocation z is *incentive compatible under* a Clower arrangement $\{S^j, \delta_j, \gamma_j\}$, if and only if, given $\{S^j, \delta_j, \gamma_j\}$, $z(x, \theta)$ solves (P1) a.e.

Considering that money is intrinsically valueless for individuals, the nonnegativity condition in (8), taken together with (10), is of special importance. It requires that a certain set of signals should be "verified" by submitting money, hence by sufficient possession of *preacquired currency holdings*. We shall call the nonnegativity conditions in (8) the *Clower constraints.* Needless to say, it is what led us to refer the underlying arrangement as the Clower arrangement in the first place.

C. The Social Planner's Problem

Now we are ready to state the planner's problem with specialized, localized agents and free shopping. The planner tries to find an attainable, implementable allocation $z : X \times \Theta \rightarrow Z$ and a supporting (enforcible) Clower arrangement $\{S^j, \delta_j, \gamma_j\}$ such that the expected utility of *ex ante* homogeneous public at agora (hence, the mean utility of *ex post* individuals) is maximized. Formally, the planner solves

$$\max_{z,\delta,\gamma} W(z) = \iint u[z(x, \theta), \theta]\, dF(x)\, dG(\theta) \tag{P2}$$

$$\text{s.t.} \quad \iint z(x, \theta)\, dF(x)\, dG(\theta) = \bar{x} \tag{11}$$

$$z(x, \theta) \text{ solves (P1) under } \{\delta_j, \gamma_j\} \text{ a.e.} \tag{12}$$

Naturally, the condition (12) is called the *incentive compatibility constraints*. The following definitions are also natural:

Definition 1
We shall call the solution $\{S^{*j}, \delta_j^*, \gamma_j^*\}_{j=1}^{l}$ of (P2) the *optimal allocation arrangement* and z^* the (*implementably*) *optimal allocation* for the decentralized economy (u, F, G).

D. The Revelation Principle and the Decomposability of Incentive Compatibility Constraints

We notice that the tautological result of the revelation principle facilitates the analysis of conventional incentive mechanisms greatly, by reducing the incentive compatibility condition into the "truth-telling" condition, which is much easier to "visualize." We are seeking for a similar tautology, extended to the decentralized economy, to deal with the cumbersome incentive compatiblity condition (12).

Define the set $\hat{\Sigma}$ of *restricted schedules* by

$$\hat{\Sigma} = \{\sigma \in \Sigma : |\sigma_j^{-1}| = 1,\ j = 1,\cdots,l\}.$$

$\hat{\Sigma}$ is the set of schedules in which a consumer visits each market $j = 1,\cdots,l$ once and only once. For each element $\sigma \in \hat{\Sigma}$, there corresponds a permutation π on J such that π_j is the single element of σ_j^{-1}. Hence π_j denotes the order in which market j is visited.

Given an arrangement, each individual with (x, θ) solves for his optimal strategy (σ, ρ). Applying arbitrary selection rule, we define contingent strategies or strategy rules $(\sigma(\cdot), \rho(\cdot))$, where $\sigma : X \times \Theta \rightarrow \Sigma$ and $\rho : X \times \Theta \rightarrow \mathbf{P}$.

Proposition 1
The planner can be restricted to choose a Clower arrangement such that

i) $S^j = \Xi, j = 1,\cdots,l$;

ii) At equilibrium, each individual tells the truth, i.e.

$$\rho^n(\xi) = \xi,\ n = 1, 2,\cdots$$

for almost all $\xi = (x, \theta)$; and

iii) At equilibrium, each individual visits each market once and only once, i.e.

$$\sigma(\xi) \in \hat{\Sigma}$$

for almost all $\xi = (x, \theta)$.

Proof: See Appendix A.

The above result motivates the following definition:

Definition 2
We shall say that an allocation z is *incentive compatible against false announcements* (FAIC) *under* an arrangement $\{\delta_j, \gamma_j\}$, if $z(x, \theta)$ with $(\tilde{x}^n, \tilde{\theta}^n) = (x, \theta)$ $\forall n$ solves (P1) a.e. *under the additional restriction* $|\sigma_j^{-1}| = 1$ $\forall j$. Correspondingly, we say that an allocation z is *incentive compatible against repeated visits* (RVIC) *under* an arrangement $\{\delta_j, \gamma_j\}$, if the additional restriction in shopping schedules, $|\sigma_j^{-1}| = 1$ $\forall j$, is *not* binding in the individual optimization (P1) a.e.

Let $\bar{F}_l(x, \theta)$ denote the subset of $F_l(x, \theta)$ with $\sigma \in \hat{\Sigma}$. That is, $\bar{F}_l(x, \theta)$ is the set of consumptions, defined by (6) through (10) and $\sigma \in \hat{\Sigma}$. Then, the FAIC condition can be written as

$$z(x, \theta) \in \arg\max u(\tilde{z}, \theta) \text{ s.t. } \tilde{z} \in \bar{F}_l(x, \theta), \tag{13}$$

and
$$z_j(x, \theta) = x_j + \delta_j(x, \theta),\ j = 1, \cdots, l \tag{14}$$

for almost all (x, θ). Correspondingly, the RVIC condition can be written as

$$\max_{\tilde{z} \in \bar{F}_l(x, \theta)} u(\tilde{z}, \theta) \geq \max_{\tilde{z} \in F_l(x, \theta)} u(\tilde{z}, \theta) \tag{15}$$

for almost all (x, θ).

Corollary 1
We can assume that an allocation z is incentive compatible under an arrangement $\{\delta_j, \gamma_j\}$ if and only if it is FAIC and RVIC under $\{\delta_j, \gamma_j\}$.

Proof: It is obvious that (13) and (15) imply (5). The converse, together with (14), holds by imposing the conditions in Proposition 1.

Since incentives for false announcements and those for repeated visits are conceptually different, this decomposability facilitates subsequent discussions significantly. Henceforth, we shall assume that the Clower arrangements under our consideration are already "decomposable," i.e. satisfy the conditions in Proposition 1.

IV. The Existence of an Optimal Solution

A. The Binding Clower Constraints vs. the Autarky

We say that the *Clower constraints are binding for individual* (x, θ) *under* an arrangement $\{\delta_j, \gamma_j\}$, if the value of (P1) for (x, θ) would be greater without the nonnegativity condition (8), taken together.

Lemma 1
Suppose that an attainable allocation z is incentive compatible under an arrangement $\{\delta_j, \gamma_j\}$. If the Clower constraints are binding for some (x, θ) with a positive probability (p.p.), then they are binding a.e. Conversely, if the Clower constraints are not binding p.p., then they are not binding a.e.

Proof: Suppose that the Clower constraints are binding for some (x', θ') p.p. This can be true only if $\delta_j(x', \theta') > 0$ for some j. But then each individual could increase $\tilde{z}_j$ indefinitely by repeated visits to market j announcing (x', θ'), if there were not the Clower constraints. This proves the first assertion. The converse coincides with the contraposition in this instance.

The above lemma asserts that it cannot be true under our decentralized environment that the Clower constraints are binding for some individuals, *while* they are not binding for others. The adversity of our allocative environment is emphasized in the following proposition:

Proposition 2
Unless the Clower constraints are binding a.e., the optimal allocation is the autarky.

Proof: Suppose that the Clower constraints are not binding p.p., but the optimal allocation is not the autarky. The former implies that the Clower constraints are not binding a.e. from Lemma 1. The latter implies $z_j(x', \theta') \neq x_j$ for some (x', θ') p.p., which in turn implies, together with the attainability (11),

$$\delta_j(x'', \theta'') > 0,$$

for some (x'', θ'') p.p. But then repeated visits announcing (x'', θ'') would be strictly feasible and preferable a.e., which contradicts to the RVIC condition.

Corollary 2
If the autarky is not optimal, then the Clower constraints are binding a.e. at optimal.

Proof: The contraposition of Proposition 2 is that, if the allocation is not the autarky, then the Clower constraints are binding p.p. But, then, Lemma 1 implies that the Clower constraints are binding a.e.

It will turn out in Section VI that the autarky is optimal if and only if it is Pareto optimal and that there exists an arrangement for each Pareto optimal allocation such that the Clower constraints are binding, though no trades would occur. Together with Corollary 2, it means that we can assume the Clower constraints to be always binding.

B. The Linearity of Implicit Pricing Functions

To proceed further, we shall impose additional restrictions to our economy:

(C1) Ξ is convex; and
(C2) $\{\delta_j, \gamma_j\}$ is continuous on Ξ.

These "continuity" assumptions are basically simplifying, rather than essential.

Lemma 2
If the Clower constraints are binding, there exist $j \in J$ and $(x, \theta) \in X \times \Theta$ such that

$$\delta_j(x, \theta) > 0$$
$$\gamma_j(x, \theta) + m \geq 0$$

for any $m > 0$.

Proof: If the Clower constraints are binding, there must exist $j \in J$ and $(x^1, \theta^1) \in X \times \Theta$ such that

$$\delta_j(x^1, \theta^1) > 0 \text{ and } \gamma_j(x^1, \theta^1) < 0. \tag{16}$$

But, then the attainability condition implies that there exists $(x^2, \theta^2) \in X \times \Theta$ such that

$$\delta_j(x^2, \theta^2) < 0 \text{ and } \gamma_j(x^2, \theta^2) > 0, \tag{17}$$

where the latter inequality is required to ensure the incentive compatibility with the concealable currency. The assertion follows from (16), (17) and the continuity assumptions.

The above lemma asserts that it is always strictly feasible for individuals to "buy" something for (positive) money. An immediate consequence is the *terminal condition* that individual currency holdings at the end of shopping period should be identically zero. Also obvious is that, during the shopping period, individuals always prefer more money to less, other things being equal. *Money is individually valuable a.e.*

Lemma 3
$\gamma_j : X \times \Theta \to R$ depends on (x, θ) only through $\delta_j(x, \theta)$ a.e. for each $j = 1,\cdots, l$.

Proof: Suppose $\delta_j(x, \theta) = \delta_j(x', \theta')$ but $\gamma_j(x, \theta) \neq \gamma_j(x', \theta')$ for some j, (x, θ) and (x', θ'). We may assume $\gamma_j(x', \theta') > \gamma_j(x, \theta)$. Then, announcing (x', θ') at market j is strictly feasible and strictly preferable for individual (x, θ), violating the FAIC condition.

For each $j = 1,\cdots,l$, define the *pricing function* $\hat{\gamma}_j : D_j \to R$ by the parametric relation between $\gamma_j(x, \theta)$ and $\delta_j(x, \theta)$ so that

$$\gamma_j(x, \theta) = \hat{\gamma}_j[\delta_j(x, \theta)],$$

where $D_j = \{a \in R: a = \delta_j(x, \theta), x \in X, \theta \in \Theta\}$. $\hat{\gamma}_j$ is continuous on the closed, bounded interval $D_j \subseteq R$.

Lemma 4
$\hat{\gamma}_j : D_j \to R$ is strictly decreasing with $\hat{\gamma}_j(0) = 0$, $j = 1,\cdots,l$.

Proof: Suppose that there exists j, (x, θ) and (x', θ') such that

$$\delta_j(x', \theta') > \delta_j(x, \theta) \text{ but } \hat{\gamma}_j[\delta_j(x', \theta')] \geq \gamma_j[\delta_j(x, \theta)].$$

The RVIC condition implies $z(x, \theta) \in \bar{F}_l(x, \theta)$, or

$$-\hat{\gamma}_k[\delta_k(x, \theta)] \leq \sum_{\pi_k < \pi_h} \hat{\gamma}_h[\delta_h(x, \theta)] \ \forall k$$

for some $\pi \in \hat{\Sigma}$. But then the reporting plan with $(\tilde{x}^j, \tilde{\theta}^j) = (x', \theta')$ while telling the truth elsewhere, is strictly feasible for (x, θ), since

$$\begin{aligned} -\hat{\gamma}_j[\delta_j(x', \theta')] &\leq -\hat{\gamma}_j[\delta_j(x, \theta)] \\ &\leq \sum_{\pi_k < \pi_h} \hat{\gamma}_h[\delta_h(x, \theta)] \ \forall j. \end{aligned}$$

Since the individual (x, θ) also strictly prefers the new reporting plan, it contradicts to the FAIC condition.

The latter assertion $\hat{\gamma}_j(0) = 0$ follows from the RVIC condition.

Now we can talk about prices meaningfully, though they may vary depending on the size of "trades." Partition D_j into $D_j^+ = D_j \cap \dot{R}_+$, $D_j^- = D_j \cap \dot{R}_-$ and $\{0\}$.[10] Define $\tilde{p}_j^B : D_j^+ \to R_+$ and $\tilde{p}_j^s : D_j^- \to R_+$ by

$$\tilde{p}_j^B(b) = -\hat{\gamma}_j(b)/b > 0$$
$$\tilde{p}_j^s(s) = \hat{\gamma}_j(-s)/s > 0,$$

where $b \in D_j^+$ and $-s \in D_j^-$ so that $b > 0$ and $s > 0$. $\tilde{p}_j^B(\cdot)$ and $\tilde{p}_j^s(\cdot)$ denote the average price for "buying" and "selling" commodity j, *from the viewpoint of individuals.*

Definition 3

A shopping plan (σ, ρ) is an *arbitrage route under* a Clower arrangement $\{S^j, \delta_j, \gamma_j\}$, if individual currency holdings increase during the shopping, without decreasing any of the initial possessions of commodities, i.e.

$$\Delta m = \sum_{n=1}^{|\sigma|} \gamma_{\sigma_n}(\tilde{x}^n, \tilde{\theta}^n) > 0,$$

$$\Delta x_j = \sum_{n \in \sigma_j^{-1}} \delta_{\sigma_n}(\tilde{x}^n, \tilde{\theta}^n) \geq 0 \ \ \forall j.$$

We shall say that *an arrangement* $\{\delta_j, \gamma_j\}$ *(or the economy) satisfies the no arbitrage condition,* if there exists no arbitrage route. Analogously, we say that *the local mechanism* (δ_j, γ_j)*(or market* j*) satisfies the no arbitrage condition,* if there exist no arbitrage route such that $\sigma_n = j$, $\forall n$.

Lemma 5

The no arbitrage condition holds if and only if

$$\inf \tilde{p}_j^B(D_j^+) \geq \sup \tilde{p}_j^s(D_j^-) \tag{18}$$

for each $j = 1,\cdots,l$.

Proof: See Appendix B.

Now we show the key result in this subsection:

[10] $\dot{R}_+(\dot{R}_-)$ denote the set of positive (negative) real numbers.

Proposition 3
Suppose that the initial currency holdings of individuals are identically zero. Suppose further that the no arbitrage condition should hold for the economy as a whole. Then the parametric relation $\hat{\gamma}_j : D_j \rightarrow R$ should be linear (with a strictly negative slope) for each $j = 1,\cdots,l$. That is, there exists a number $p_j > 0$ such that

$$\hat{\gamma}_j\,[\,\delta_j(x,\ \theta\,)] = -p\,\delta_j(x,\ \theta\,) \qquad \text{for } j = 1,\cdots,l.$$

Proof: From (18), there exists a number $p_j > 0$ such that

$$\bar{p}_j^{S}(s) \leq p_j \leq \bar{p}_j^{B}(b) \tag{19}$$

for almost all $b \in D_j^+$ and $s \in -D_j^-$. Denote by E_+ (resp. E_-) the expectation conditional on $\delta_j(x,\ \theta\,) > 0$ (resp. < 0). Then we have

$$\begin{aligned} E\,\gamma_j(x,\ \theta\,) &= E_-(-\,\delta_j\,)[\frac{\hat{\gamma}_j\,(\,\delta_j\,)}{-\,\delta_j}] + E_+(-\,\delta_j\,)[\frac{-\,\hat{\gamma}_j\,(\,\delta_j\,)}{\delta_j}] \\ &\leq E_-\,\delta_j(x,\ \theta\,)(-p_j) + E_+\,\delta_j(x,\ \theta\,)(-p_j) \qquad (20) \\ &= 0, \end{aligned}$$

where the last equality follows from the attainability.

$$E\,\gamma_j(x,\ \theta\,) \leq 0 \quad j = 1,\cdots,l. \tag{21}$$

Now suppose that the inequality in (19) holds strictly for some k, $b \in D_k^+$ and $s \in D_k^-$ with a positive probability. Then the same argument in (20) yields

$$E\,\gamma_k(x,\ \theta\,) < 0. \tag{22}$$

Then (21) and (22) imply

$$E\sum_{j=1}^{l}\ \gamma_j(x,\ \theta\,) < 0. \tag{23}$$

But, with no initial currency holdings, $z(x,\ \theta\,) \in \bar{F}_l(x,\ \theta\,)$ imply

$$\sum_{j=1}^{l} \gamma_j\,(x,\ \theta\,) \geq 0 \tag{24}$$

a.e., which contradicts to (23). Therefore (19) should hold with equality a.e. for $j = 1,\cdots,l$.

Since the no arbitrage condition is assumed exogenously, it is of interest to know how stronger it is than the endogenously derived RVIC condition.

Proposition 4
Suppose that the following conditions hold for each $j=1,\cdots,l$:
i) For each $b \in D_j^+$, there exists an individual who can "buy" twice as much as b; and
ii) For each $-s \in D_j^-$, there exists an individual who can "sell" twice as much as s.
Then, the RVIC condition implies the no arbitrage condition.

Proof: See Appendix C.

The condition requires that the economy is "rich" enough in terms of the endowments relative to the "trades," or in terms of individual wealth relative to the value of trades in each market. The other possibility is allowing two-person coalitions to raise the arbitrage "capital." Henceforth, we shall assume that the no arbitrage condition holds, or, equivalently, the condition in the above proposition holds.

C. The Reducibility of the Clower Constraints

By Proposition 3, we can represent an arrangement $\{\delta_j, \gamma_j\}$ by $\{\delta_j, p_j\}$ alternatively, where $\{\gamma_j\}$ is given by $\gamma_j(x, \theta) = -p_j\delta_j(x, \theta)$ a.e.

Lemma 6
If the pricing functions are linear, the RVIC condition always holds.

Proof: We shall show that for any strictly feasible plan which possibly involves repeated visits or no visit for some markets through which a consumption $\tilde{z}'$ is obtained for some (x, θ) p.p., there exists a strictly feasible plan (σ', ρ') through exactly the same consumption $\tilde{z}'$ can be obtained.

Dichotomize the markets by the individual's net position:

$$J_B = \{j \in J : \bar{\delta}_j = \sum_{n \in \sigma_j^{-1}} \delta_j(\tilde{x}^n, \tilde{\theta}^n) \geq 0\},$$

$$J_s = \{k \in J : \bar{\delta}_k = \sum_{n \in \sigma_k^{-1}} \delta_k(\tilde{x}^n, \tilde{\theta}^n) < 0\}.$$

Define $\hat{\Sigma}^*$ by the subset of $\hat{\Sigma}$ such that $\pi_j > \pi_k$ whenever $j \in J_B$ and $k \in J_s$. Obviously $\hat{\Sigma}^*$ is nonempty. Hence pick $\pi^* \in \hat{\Sigma}^*$. Let the reporting plan $\{\tilde{x}^j, \tilde{\theta}^j\}$ satisfy

$$\bar{\delta}_j = \delta_j(\tilde{x}^j, \tilde{\theta}^j) \quad j = 1,\cdots,l.$$

Such $(\tilde{x}^j, \tilde{\theta}^j)$ exists p.p. by construction in Proposition 1. We shall show that the plan $(\pi^*, \{\tilde{x}^j, \tilde{\theta}^j\})$ is strictly feasible.

From the strict feasibility of (σ, ρ), we have

$$x_j + \bar{\delta}_j \geq 0 \quad j = 1,\cdots,l,$$

which shows the nonnegativity condition for the commodities. To see the Clower constraints are satisfied too, first note that they are trivially satisfied for each $j \in J_s$, and note:

$$\sum_{j=1}^{l} p_j \bar{\delta}_j = \sum_{n=1}^{|\sigma|} p_0 \delta_0 (\tilde{x}^n, \ \tilde{\theta}^n) \leq 0. \tag{25}$$

Thus we have for each $k \in J_B$,

$$\begin{aligned} p_k \bar{\delta}_k &\leq -\sum_{j \neq k} p_j \bar{\delta}_j \\ &\leq -\sum_{\pi_h^* < \pi_k^*} p_h \bar{\delta}_h, \end{aligned}$$

where the first inequality follows from (25) and the second from the fact $p_h \bar{\delta}_h \geq 0$ if $h > k$. Hence the proof is complete.

Remark 1

It shows a close relation between the RVIC condition and the linear pricing functions in our theory. In fact, if the economy is sufficiently "rich," i.e. the conditions in Proposition 4 hold, the RVIC condition is *equivalent with* the linearity "condition." That is, in our theory, the linearity is implied mainly by the assumption of free shopping. This makes a sharp contrast with the linearity result based on coalitions [Gale (1980), Townsend (1986)].

By the above lemma, we can assume that individuals are effectively controlled to choose restricted schedules $\sigma \in \hat{\Sigma}$. Of course, even with restricted schedules only, the (strict) feasibility of a reporting plan does depend on the chosen schedule.

Lemma 7

Suppose a reporting plan $\{\tilde{x}^j, \tilde{\theta}^j\}_{j=1}^l$ satisfies

$$x_j + \delta_j (\tilde{x}^j, \ \tilde{\theta}^j) \geq 0 \qquad \forall j \tag{26}$$

and

$$\sum_{j=1}^{l} \hat{\gamma}_j \left[\delta_j (\tilde{x}^j, \ \tilde{\theta}^j) \right] \geq 0. \tag{27}$$

Then there exists a (restricted) schedule $\sigma \in \hat{\Sigma}$ such that $(\sigma, \{\tilde{x}^j,$

$\tilde{\theta}^j\}$) is strictly feasible.

Proof: Redefine J_s and J_B (hence $\hat{\Sigma}^*$) in the proof of Lemma 6, now according to $\delta_j(\tilde{x}^j, \tilde{\theta}^j) \geq 0$ or not. For any $\sigma \in \hat{\Sigma}^*$, then, (27) implies that the Clower constraints are satisfied for each market visited.

Note that the linearity of $\hat{\gamma}$'s was not required. There above lemma tells that, once a reporting plan $\{\tilde{x}^j, \tilde{\theta}^j\}$ is chosen subject to (26) and (27), a supporting schedule can be found — and that easily. Thus it may be natural to call a reporting plan strictly feasible by itself, if it satisfies (26) and (27), without referring the shopping schedule at all. Moreover, since the nonnegativity (26) is implied by the assumption $Z = R^l_+$, the strict feasibility of a reporting plan can be written by (27) only, or using the linearity result,

$$\sum_{j=1}^{l} p_j \delta_j(\tilde{x}^j, \tilde{\theta}^j) \leq 0, \tag{28}$$

which is nothing but the ordinary budget constraint.

Proposition 5
An attainable allocation z is incentive compatible under an arrangement $\{\delta_j, p_j\}$ if and only if

$$z(x, \theta) \in \arg\max u(\tilde{z}, \theta) \tag{P1$'$}$$
$$\text{s.t. } \tilde{z}_j = x_j + \delta_j(\tilde{x}^j, \tilde{\theta}^j) \qquad \forall j$$
$$\{\tilde{x}^j, \tilde{\theta}^j\} \text{ satisfies (28)}$$

for almost all (x, θ), i.e. z coincides with the (gross) individual demands at prices $\{p_j\}$ a.e.

Proof: The second condition of (P1)$'$ coincides with (27), while $z \in R^l_+$ implies (26). Hence (P1)$'$ implies the FAIC condition. But, since the RVIC condition is always staisfied with the linearity of $\hat{\gamma}$'s by Lemma 6, this implies the incentive compatibility.

Conversely, if an attainable allocation z is incentive compatible under $\{\delta_j, \gamma_j\}$, $\{\gamma_j\}$ has the linear property and the FAIC condition implies that $z(x, \theta) = x + \delta(x, \theta)$ must be a maximizer of individual utility subject to (26) and (27). Hence $z(x, \theta)$ solves (P1)$'$.

Note that we end up with the ordinary budget constraints (28) and the ordinary individual optimization problem (P1)$'$, though we have emphasized the Clower constraints as the nexus through which money plays an important role in the allocative procedure. Indeed it is our general understanding that, given equilibrium prices, money is relevant only as far as it enforces budget constraints [see Lucas (1980, p. 205)]. Hence, if the enforcement of the budget constraints is taken for granted, we can indeed *legitimately* abstract from money.

Remark 2
Our result may appear to be mixed blessing. On the one hand, it endorses the (physical) General Equilibrium theory, on which we economists have been relying so much. On the other hand, presumably to some reader's disappointment, money became again "analytically indistinguishable."

I maintain, however, that we need not be too disappointed by the apparent indistinguishability. First, the result of reduction critically depends on the implicit assumption imbedded in our Statics: the market is complete in that *any commodity can be traded into any other commodity.* Second, even in Statics we can study money by analyzing the underlying Clower constraints hidden behind the budget constraints. All we have to do is to see through the "veil" of budget constraints, borrowing Pigou's expression.

D. *The Existence of an Optimal Solution and Its Coincidence with a Competitive Equilibrium*

Given the previous results, the existence result (about which we economists worry most) follows rather routinely. Now we can rewrite the social planner's problem (P2) as:

$$\max_{z,p} \ w(z) \tag{P2$'$}$$

$$\text{s.t.} \ Ez = \bar{x} \tag{11}$$

$$z(x, \theta) \text{ solves (P1)}' \text{ under } \{\delta_j, p_j\} \text{ a.e.} \tag{12$'$}$$

We expect that the solution is closely related with the competitive equilibrium.

Definition 4
A competitive equilibrium is a pair of a price vector $p \in R^l_+$ and an allocation $z : X \times \Theta \rightarrow Z$ such that
(i) Given p, $z(x, \theta)$ solves

$$\max u(\tilde{z}, \theta) \text{ s.t. } p\tilde{z} < px \tag{29}$$

a.e.; and
(ii) markets clear, i.e.,

$$Ez_j(x, \theta) = \bar{x}_j \quad \forall j. \tag{30}$$

Note that the competitive equilibrium is *completely* specified by the *physical* structure of economy. It is routine to show that there exists a competitive equilibrium for an economy (u, F, G).

Lemma 8
The set E_p of equilibrium price vectors is nonempty.

Proof: See Appendix D.

Now we state the main result of the paper:

Proposition 6
There exists a solution (P2) if and only if there exists a solution for the corresponding CE problem. Moreover the former solution solves the latter, while the converse is not necessarily true.

Proof: If there exists a solution (z^*, p^*) for (P2)′ with (11) and (12)′, the "only if" part follows rather trivially: (12)′ ensures Condition (29), while (11) coincides with Condition (30). It follows that the set of optimal solutions is a subset of the set of competitive equilibria.

Now suppose that there exists a competitive equilibrium (p, z). Obviously, (z, p) solves the *constraints* of (P2). Let $\hat{z}$ denote the (ordinary) demand function, $\hat{z}: R^l_{++} \times X \times \Theta \rightarrow Z$, which solves (29). From the strict concavity of u and the maximum theorem, $\hat{z}$ is well-defined, continuous and (positively) homogeneous of degree zero in prices. For the social planner, it means that, once he has chosen a vector p^* of (positive) prices, the set of incentive compatible allocations is a singleton, given by $z^*(x, \theta) = \hat{z}(p^*, x, \theta)$ a.e. Thus, we can concentrate on the set E_p of equilibrium prices.

It remains to show that the planner achieves the optimum in E_p, if E_p is nonempty. Since $\hat{z}$ is homogeneous, we can restrict the planner to choose p^* among the normalized equilibrium prices, i.e. $p \in \Delta^l_+$, where Δ^l_+ denotes the unit simplex in R^l_+. We assume that E_p is already normalized so that $E_p \subseteq \Delta^l_+$. Let $\{p^n\}$ be a sequence in E_p converging to $p \in \Delta^l_+$. Then the continuity of $\hat{z}$ implies $E\hat{z}(p, x, \theta) = \bar{x}$, or $p \in E_p$. Hence E_p is closed as well as bounded. Since $W(z) = Eu[z(p, x, \theta), \theta]$ is continuous in $p \in E_p$ and since E_p is compact, there exists a solution p^*, hence z^*, *provided* that E_p is nonempty.

From the above result and Lemma 8, it follows immediately:

Corollary 3
There exists a solution for the planner's problem (P2). Moreover, it also solves the corresponding CE problem.

The coincidence allows us to provide an algorithm under a familiar condition. Suppose that the global stability condition is satisfied for the economy (u, F, G). This would be the case, if we specify $u(z, \theta) = u[\Pi^{l}_{j=1}(z_j)^{\theta_i}]$ with $\Theta = \Delta^{l}_{+}$, where $U: R_{+} \rightarrow R$ is twice differentiable with $U' > 0$ and $U'' < 0$ [Arrow and Hurwicz (1958, p. 550); Lucas (1980)]. Then, not only the optimal solution coincides with the unique competitive equilibrium, but also the optimal arrangement can be found by the algorithm of the Walrasian *tâtonnement*.

V. The Efficiency of the Monetary Arrangement

A. The Pareto Optimality

An attainable allocation z' is said to (*weakly*) *Pareto-dominate* another attainable allocation z, if

$$u[z'(x, \theta), \theta] \geq u[z(x, \theta), \theta] \text{ a.e.}$$

If the inequality holds strictly for some (x, θ) with a positive probability, z' is said to (*strictly*) *Pareto-dominate* z. An attainable allocation z is *Pareto optimal*, if there does not exist an attainable allocation z' which Pareto-dominates z.

Fix an allocation z. Define

$$H_z(x, \theta) = \{\bar{z} \in Z : u(\bar{z}, \theta) \geq u[z(x, \theta), \theta]\}$$

and

$$H_z = \{\bar{x} \in R^{l}_{+} : \bar{x} = Ez'(x, \theta), \ z'(x, \theta) \in H_z(x, \theta) \text{ a.e.}\}.$$

$H_z(x, \theta)$ is the set of consumptions which is weakly preferable to current allocation $z(x, \theta)$ to individuals with (x, θ), while H_z is the mean endowment which allows an allocation that weakly Pareto-dominates z.[11]

[11] H_z can be considered as the linear sum of $H_z(x, \theta)$. Thus we can simply write $H_z = EH_z(x, \theta)$. Obviously we should allow that the expectation may be infinite (i.e. z may not be integrable).

Lemma 9
For each $p \in R^{l}_{+}$, $z(x, \theta)$ minimizes $p\tilde{z}$ on $H_z(x, \theta)$ a.e. if and only if $Ez(x, \theta)$ minimizes px on H_z.

Proof: (i) (the "only if" part) Suppose not, then there exists an allocation z' such that $Ez'(x, \theta) \in H_z$ and

$$pEz'(x, \theta) < pEz(x, \theta).$$

But this implies

$$pz'(x, \theta) < pz(x, \theta) \text{ p.p.,}$$

which is a contradiction.
(ii) (the "if" part) Suppose not, then there exist an allocation z' and a subset Λ of $X \times \Theta$ with a positive probability such that $z'(x, \theta) \in H_z(x, \theta)$ and $pz'(x, \theta) < pz(x, \theta)$ for all $(x, \theta) \in \Lambda$. Define z'' by

$$z''(x, \theta) = \begin{cases} z'(x, \theta) \text{ if } (x, \theta) \in \Lambda \\ z(x, \theta) \text{ otherwise.} \end{cases}$$

Clearly $Ez''(x, \theta) \in H_z$. But we also have

$$pEz''(x, \theta) < pEz(x, \theta),$$

which contradicts to the minimality of $pEz(x, \theta)$.

The Pareto optimality of the (implementably) optimal allocation is an immediate consequence of Corollary 3 and the following proposition.

Proposition 7
The allocation $z^*(x, \theta) = \hat{z}(\hat{p}, x, \theta)$ is Pareto optimal for each $\hat{p} \in E_p$.

Proof: The attainability of z^* follows from the market clearing property of $\hat{p}$. Let $z'(x, \theta)$ be an attainable allocation such that

$$u[z'(x, \theta), \theta] \geq u[z^*(x, \theta), \theta)] \text{ a.e.}$$

We shall show that the supposition implies

$$u[z'(x, \theta), \theta] \leq u[z^*(x, \theta), \theta] \text{ a.e.} \tag{31}$$

From the duality theorem, $z^*(x, \theta)$ minimizes $\hat{p}z$ on $H_{z^*}(x, \theta)$. Hence, from Lemma 5, $\bar{x} = Ez^*(x, \theta)$ minimizes $\hat{p}x$ on H_{z^*}. From the attainability of z', $Ez'(x, \theta) \leq \bar{x}$. By the nonsatiation assumption, it is sufficient to show that (31) holds for z' such that $Ez'(x, \theta) = \bar{x}$.

But then, $Ez'(x, \theta)$ minimizes $\hat{p}$ on H_{z^*}, which implies $z'(x, \theta)$ minimizes $\hat{p}$ on $H_{z^*}(x, \theta)$ a.e. Thus we have

$$\hat{p}z'(x, \theta) \leq \hat{p}z^*(x, \theta),$$

which implies (31).

We have shown in Corollary 2 that the Clower constraints are binding if the autarky is not optimal. The following result specifies the condition unambiguously.

Corollary 4
The autarky is optimal if and only if it is Pareto optimal.

Proof: the "only if" part follows immediately from Proposition 7. For the "if" part, suppose that x is Pareto optimal, but not optimal. Then there exists an attainable, incentive compatible allocation z such that

$$W(z) > W(z^A).$$

This implies

$$u[z(x, \theta), \theta] > u(x, \theta) \text{ p.p.}$$

The incentive compatibility (RVIC) implies that each individual engages in utility-improving trades,

$$u[z(x, \theta), \theta] \geq u(x, \theta) \text{ a.e.}$$

But these two inequalities with the attainability of z contradicts to the Pareto optimality of x.

The Pareto optimality, however, does not tell much about the efficiency of the allocation, *in terms of* the social welfare (the mean utility) achieved. The RVIC condition implies that each individual engages in utility-increasing "trades." Hence we have as a lower bound

$$W(z^A) \leq W(z^*),$$

where z^A denotes the autarky. The obvious upper bound is given by solving the planner's problem totally neglecting the incentive constraints, i.e. by the idealistic allocation z^I:

$$W(z^A) \leq W(z^*) \leq W(z^I).$$

A tighter upper bound shall be provided later.

B. The Relatively Idealistic Optimality

A (normalized) *weight* is a nonnegative function $\omega: X \times \Theta \rightarrow R_+$ such that $\int \omega(x, \theta) dx d\theta = 1$. We shall say that an allocation is *relatively idealistically optimal* or simply *relatively idealistic with respect to* a weight $\omega(x, \theta)$ for the economy (u, F, G), if z solves the problem

$$\max \; E\omega(x, \theta)\, u[z(x, \theta), \theta] \tag{32}$$

$$\text{s.t. } Ez(x, \theta) = \bar{x} \tag{33}$$

Proposition 8
Suppose that an allocation z^* is (implementably) optimal for a decentralized economy (u, F, G). Then there exists a weight ω such that z^* is relatively idealistic with respect to ω.

Proof: See Appendix E.

The above proposition tells that the incentive compatibility condition can be represented by some weight function — i.e., the planner under the implementability may be modelled to maximize a "distorted" welfare function. But the practical usefulness of this result is severely restricted by the fact that the proper weight can be found only in hindsights. Moreover, the weight function is *not invariant* with respect to the changes in aggreate endowments. In other words, it is a (poor) substitute for a direct analysis of underlying incentive problem.

C. A Comparison of Efficiency: Decentralized vs. Centralized Economies

We shall show that the optimal allocation in the (informationally) centralized economy is not less efficient than that in the decentralized (localized) economy.[12] The social planner in the centralized economy would solve

$$\max \; W(z) \tag{34}$$

$$\text{s.t. } Ez(x, \theta) = \bar{x} \tag{35}$$

$$(x, \theta) \in \arg\max \; u[x - \tilde{x} + z(\tilde{x}, \tilde{\theta}), \theta] \;\; \text{a.e.} \tag{36}$$

[12]Though individuals in a centralized economy do not necessarily engage in utility-improving "trades," it can be shown that the optimal allocation in a centralized economy is also Pareto optimal. See Proposition 5.6 (and Example 2.3) in the full version.

The constraints (36) is the simplest form of the incentive compatibility condition, known as the truth-telling condition.

Proposition 9
If an allocation z is incentive compatible in a decentralized economy, it is incentive compatible in the corresponding centralized economy.

Proof: Fix (u, F, G) and let the feasible set of representative individual (x, θ) in the centralized economy be given by

$$\dot{F}(x, \theta) = \{\tilde{z} \in Z : \tilde{z} = x + \delta(\tilde{x}, \tilde{\theta}), \tilde{x} \in X, \tilde{\theta} \in \Theta\}.$$

We must show that

$$z(x, \theta) \in \arg\max u(\tilde{z}, \theta) \text{ s.t. } \tilde{z} \in F_I(x, \theta) \tag{37}$$

a.e. implies

$$z(x, \theta) \in \arg\max u(\tilde{z}, \theta) \text{ s.t. } \tilde{z} \in \dot{F}_I(x, \theta) \tag{38}$$

a.e. Suppose that (37) is true but not (38). Then there exist two different shocks (x, θ) and (x', θ') such that

$$u[x - x' + z(x', \theta'), \theta] > u[z(x, \theta), \theta]$$

and such that $\delta(x', \theta') + x \geq 0$ p.p. But, since $\sum_{j=1}^{l} \gamma_j(x, \theta) \geq 0$ a.e., reporting (x', θ') consistently is both strictly feasible and strictly preferable for individual (x, θ) in the decentralized economy, which contradicts to the FAIC condition.

Corollary 5
The (optimal) Clower arrangement is not more efficient than the optimal mechanism available in the corresponding centralized economy.

Proof: The attainability constraints coincide, while the incentive compatibility constraints are more stringent.

Remark 3
Note that we are *not* comparing two different systems applied to the *same* economy (hence, it has nothing to do with the socialist controversy) — only their physical structures are the same. The result simply restates the simple fact that the availability of (free) communication among social agents never dwarfs the alternatives of the planner. It is always possible for the centralized economy to mimic the decentralized economy.

Corollary 5 also shows that the optimal allocation in the centralized economy sets an upper limit for that in the corresponding decentralized economy (in terms of the social welfare). Letting z^c denote the optimal allocation in the centralized economy, we have

$$W(z^A) \leq W(z^*) \leq W(z^c) \leq W(z^I) \tag{39}$$

for each (u, F, G), satisfying (U1)-(U3) and (S1)-(S3).

VI. Conclusion

Our underlying question throughout the paper was whether the monetary exchange as an allocation arrangement, where the final allocation is arrived at through a succession of proportional, bilateral exchanges between money and commodities, can be considered as a deliberate social choice under a certain allocative environment. It turned out that the answer is *yes,* if, roughly speaking, the economy is informationally decentralized in that the physical transfers of commodities should be made by infinitely many, specialized, localized social agents, while individuals with private shocks can visit freely any subset of agents in any order.

Substantial efforts were given to formulate the problem, to justify the formulation and to sharpen our tools. Then our discussion was centered on how to simplify the incentive compatibility condition, especially by examining the Clower constraints. The monetary exchange, specified as the Clower arrangement with proper net distribution functions, is indeed an optimal allocation arrangement under our assumptions. Money entered the optimal scheme as medium of communication among social agents, exploiting its universal recognizability, while prices serving as encoding rules. The Clower constraints made decoding possible, exploiting the uncounterfeitability of money, but could be reduced legitimately into the (ordinary) budget constraints. Then, since the unique incentive compatible net distribution functions for commodities were given by the net demand functions once prices were chosen, the Clower arrangement could be completely specified by a vector of (positive) prices. Finally, the (strict) attainability implied that the optimal price vector had to be in the set of competitive equilibria.

The novelty of our theory lies in that the "frictions" we introduced by restricting the social agents' ability are rather typical, or at least not pathologic, as we would expect from the predominant usage of the monetary arrangements across various economies.[13] We also have not presupposed any *a priori* market structure, down to the private ownership itself. As a result, money becomes a well-integrated part of the market allocation mechanism without any pathologic symptoms attached. The (optimal) monetary arrangement is fully conformable[14] with the Walrasian allocation mechanism. Still, the monetary aspects can be analyzed by seeing the underlying Clower constraints through the "veil" of the budget constraints.[15]

We briefly consider the effects of loosening our two critical assumptions, i.e. the completeness and the unconditional loyalty of the social agents. First, if the social agents are less than complete, the resultant allocation is not Pareto optimal, though the monetary allocation mechanism remains to be optimal. For statics, it does not raise serious problems, except that the notion of "Pareto optimality" is largely unsatisfactory to analyze such situations.[16] In dynamics, however, it challenges the usual reinterpretation of statics into dynamics with complete markets. In fact, it is not hard to show that the assumption of complete social agents is logically inconsistent with private shocks and sequential observations. One way to show this is using the result of Green (1985), where the optimal credit contract in a dynamic centralized economy is less than Pareto optimal (imperfect smoothing of consumption). Since the optimal allocation in the corresponding decentralized economy cannot be more efficient, the latter cannot have a perfect credit market. This suggests that the relative inefficiency of the dynamic pure currency economy of Lucas (1980) is attributable not to the second-ratedness of money as asset but to the "too optimistic" result of the GE theory extended to dynamics *à la* Arrow (1964) and Debreu (1959). In fact, with the iid shocks and infinite time-horizon, it follows immediately that the only CE is the idealistic allocation at each period of time, following the idealism "from each··· to his need." In the real world, since individual shocks may not be completely private, and since "social agents" may communicate partially through other than money, we expect that there exist some imperfect credit

[13]Note that the restriction of the social agents' ability is *not* purely technological. It also involves the loyalty problem of social agents. This helps to explain why the localization may persist even in the telecommunication era.

[14]The obvious reservation in using "conformable," instead of "conforming," is the possible incompleteness of social agents.

[15]For applications, see Section VI in the full version.

[16]For alternative notions of Pareto optimality, see §5.4 in the full version.

markets, so that the actual efficiency lies somewhere between that of the pure currency economy and that of the GE theory.

Second, the assumption of the unconditional loyalty of social agents is obviously *ad hoc,* as is the unquestioned loyalty of the social planner as enforcer. To close the system without such *ad hoc* assumptions, we need to allow the social agents selected from the members of the society, and consider the incentive compatibility problem of social agents as well. If the specialized perception skills (or entrepreneurship) is redundant in the sense of Makowski (1980), the loyalty problem can be resolved rather easily through competitive merchants, presuming the loyalty of the monetary authority. This contrasts sharply to the difficulty of ensuring the loyalty of a centralized body of social agents, say the "government," which would require a delicate combination of bureaucracy, balancing power, voting system, etc. The decentralized economy cannot dismiss this problem altogether, not only because we assumed away theft or robbery (E3), but also because the monetary allocation mechanism with decentralized self-interested merchants inevitably requires a centralized body of supplying money, called the monetary authority. Assuming the loyalty of the monetary authority and linear pricing functions, it can be shown that any monetary manipulation as part of the arrangement is either neutral or impotent.[17] However, with monetary manipulations available, the linear pricing functions are not optimal. In particular, the society can increase the efficiency of risk-sharing by providing each individual with equal amounts of currency before any shock is realized and applying asymmetric prices, absorbing the initial currency holdings as "profits." It suggests that constructing a dynamic economy where monetary manipulations are socially desirable under the presumption of the unconditional loyalty of the monetary authority is not very challenging. What is really challenging is to find an optimal "compromise" that takes the loyalty problem as well, which seems to be beyond *a priori* reasoning, at least, from our current state of knowledge.

Finally, the relation between our theory and the Walrasian theory seems worthwhile being clarified. Aside from its mathematical elaboration, the keystone of the Walrasian theory is the *tâtonnement* process, which is basically a *centralized* information processing. We have seen, however, that, if such a centralization is indeed available, the economy can do better, in general, by enforcing non-linear pric-

[17]See Propositions 6.1 and 6.2 in the full version.

ing functions (including unilateral transfers). Then, why does the benevolent *central* agent, called the Walrasian auctioneer, restrict himself to grope for linear pricing functions or prices only, which can be processed essentially in a decentralized way through the less-than-perfect communication device, money? A natural answer seems to be that the Walrasian auctioneer is nothing but a useful conceptualization of the interwoven workings of decentralized, competitive merchants.

Appendix A

Proof of Proposition 1

Let $\{S^j, \delta_j, \gamma_j\}$ be any Clower arrangement with an equilibrium strategy rule $(\sigma(\cdot), \rho(\cdot))$. Define a new Clower arrangement $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$ by

$$\bar{S}^j = \Xi,$$

$$\bar{\gamma}_j(\xi) = \sum_{n \in \sigma_j^{-1}(\xi)} \gamma_j[\rho^n(\xi)].$$

$$\bar{\delta}_j(\xi) = \sum_{n \in \sigma_j^{-1}(\xi)} \delta_j[\rho^n(\xi)].$$

Also define $\bar{\rho}(\cdot)$ by

$$\bar{\rho}^n(\xi) = \xi, \ n = 1, 2, \cdots$$

while $\bar{\sigma}(\cdot)$ is defined by applying an arbitrary selection rule to

$$\hat{\Sigma}^*(\xi) = \{\sigma \in \hat{\Sigma} : \pi_j < \pi_k \text{ whenever } \bar{\gamma}_j(\xi) \geq 0 \text{ and } \bar{\gamma}_k(\xi) < 0\}.$$

By construction, $(\bar{\sigma}(\cdot), \bar{\rho}(\cdot))$ under $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$ results in the same allocation z as $(\sigma(\cdot), \rho(\cdot))$ does under $\{S^j, \delta_j, \gamma_j\}$:

$$z_j(x, \theta) = x_j + \bar{\delta}_j(x, \theta), \text{ a.e.} \tag{A1}$$

We shall show that $(\bar{\sigma}(\cdot), \bar{\rho}(\cdot))$ is an equilibrium under $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$. Since $(\sigma(\cdot), \rho(\cdot))$ is strictly feasible a.e., we have

$$x_{\sigma_n}^{n-1} + \delta_{\sigma_n}[\rho^n(\xi)] \geq 0$$

$$m^{n-1} + \gamma_{\sigma_n}[\rho^n(\xi)] \geq 0,$$

for $n = 1, \cdots, |\sigma|$, where $\{x^n, m^n\}$ is given by (9) and (10). These imply

$$x_j + \bar{\delta}_j(\xi) \geq 0,\ j = 1,\cdots,l$$

$$\sum_{j=1}^{l} \bar{\gamma}_j(\xi) \geq 0,$$

for each (x, θ) a.e. It is easy to check that the latter ensures the Clower constraints to hold along the schedule $\bar{\sigma}(\xi)$. Hence, $(\bar{\sigma}(\cdot), \bar{\rho}(\cdot))$ is strictly feasible a.e.

It remains to show that $(\bar{\sigma}(\cdot), \bar{\rho}(\cdot))$ is individually optimal a.e. Suppose that, under $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$, there exists a strategy rule $(\bar{\sigma}'(\cdot), \bar{\rho}'(\cdot))$ which is strictly feasible a.e. and strictly preferable to $(\bar{\sigma}(\cdot), \bar{\rho}(\cdot))$ for some ξ with a positive probability. Denote the resultant allocation by

$$z_j'(x, \theta) = x_j + \sum_{n \in \bar{\sigma}_j^{-1}} \bar{\delta}_j[\bar{\rho}'^n(x, \theta)].$$

Define $\rho'(\xi)$ by

$$\rho'^n(\xi) = \rho^n[\bar{\rho}'^n(\xi)],$$

and let $\sigma'(\xi) = \bar{\sigma}'(\xi)$. By construction,

$$z_j'(x, \theta) = \sum_{n \in \sigma_j'^{-1}} \delta_j[\rho'^n(x, \theta)]$$

for $j = 1,\cdots,l$. But we also have

$$\begin{aligned} m^{n-1} + \gamma_{\sigma_n'}[\rho'^n(\xi)] &= m^{n-1} + \gamma_{\sigma_n'}[\rho^n(\bar{\rho}'^n(\xi))] \\ &= m^{n-1} + \bar{\gamma}_{\sigma_n'}[\bar{\rho}'^n(\xi)] \geq 0 \end{aligned}$$

for $n = 1,\cdots, |\sigma'|$, where the last inequality follows from the strict feasibility of $(\bar{\sigma}'(\cdot), \bar{\rho}'(\cdot))$ under $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$. Similarly we have

$$x_{\sigma_n'}^{n-1} + \delta_{\sigma_n'}[\rho'^n(\xi)] \geq 0\ \ n = 1,\cdots, |\sigma'| .$$

Hence $(\sigma'(\xi), \rho'(\xi))$ is strictly feasible as well as strictly preferable for individual with $\xi = (x, \theta)$ under $\{\bar{S}^j, \bar{\delta}_j, \bar{\gamma}_j\}$. But this contradicts to the equilibrium property of $(\sigma(\cdot), \rho(\cdot))$, which completes the proof.

Remark A1

We confined our attention on Clower class arrangements where individualistic optimization suffices. The extension to the genuine game situations is straightforward, providing we use Dominant, Nash or Bayesian equilibrium concept. Then, the conventional revelation principle corresponds to the special case with a central agent and with controlled visits.

Appendix B

Proof of Lemma 5

1) Sufficiency: Suppose that $\tilde{p}_j^B(b) < \tilde{p}_j^s(s)$ holds for some market j, $b \in D_j^+$ and $-s \in D_j^-$. Then there exists a pair (h, k) of positive integers such that h/k is not smaller than s/b, but arbitrarily close to s/b. Then, for some (h, k), buying h times of commodity j in b units and selling k times in s units result

$$\Delta m = k\, \tilde{p}_j^s(s)\, s - h\, \tilde{p}_j^B(b)\, b > 0,$$

which is an arbitrage route for an individual with sufficiently large possession, since

$$\Delta x_j = hb - ks \geq 0$$

by construction.

2) Necessity: It is obvious that (18) implies the no arbitrage condition in market j. We shall show that, if the no arbitrage condition holds for each market $j = 1,\cdots,l$, then it also holds for the economy as a whole. We show the contraposition. Suppose that there exists an arbitrage route (σ, ρ) for the economy as a whole. Then, we have

$$\Delta m = \sum_{n=1}^{|\sigma|} \gamma_{\sigma_n}(\bar{x}^n, \tilde{\theta}^n) = \sum_{j=1}^{l} \sum_{n \in \sigma_j^{-1}} \gamma_j(\bar{x}^n, \tilde{\theta}^n) > 0,$$

which in turn implies

$$\sum_{n \in \sigma_j^{-1}} \gamma_j(\bar{x}^n, \tilde{\theta}^n) > 0$$

for some j. we also have

$$\Delta x_j = \sum_{n \in \sigma_j^{-1}} \delta_j(\bar{x}^n, \tilde{\theta}^n) \geq 0.$$

Extracting a subsequence ρ' from σ so that $\sigma_n = j$, and ρ' conformingly, it is obvious that (σ', ρ') is strictly feasible. Hence (σ', ρ') is an arbitrage route in market j.

Appendix C

Proof of Proposition 4

By Lemma 5, we need to show that, under the supposition, the RVIC condition implies

$$\tilde{p}_j^B(b) \geq \tilde{p}_j^s(s),$$

for all $j \in J$, $b \in D_j^+$ and $s \in -D_j^-$. Suppose

$$\tilde{p}_j^B(b) < \tilde{p}_j^s(s),$$

for some j, $b \in D_j^+$ and $s \in -D_j^-$. Then, for any number $\varepsilon > 0$, there exists a pair (h, k) of positive integers such that

$$h\,b \geq k\,s \text{ and } |\,h\,b - k\,s\,| < \varepsilon\,.$$

Hence, for sufficiently small $\varepsilon > 0$, we have

$$\Delta x_j = h\,b - k\,s \geq 0,$$

$$\begin{aligned}\Delta m &= h\,b\,\tilde{p}_j^B(b) - h\,b\,\tilde{p}_j^s(s) + (h\,b - k\,s)\,\tilde{p}_j^s(s)\\ &= h\,b[\tilde{p}_j^B(b) - \tilde{p}_j^s(s)] - \varepsilon\,\tilde{p}_j^s(s) > 0.\end{aligned}$$

Hence "buying" h times of b units and "selling" k times of s units make an arbitrage profit, if feasible. It remains to show the route is strictly feasible to some individual.

1) Case $b \geq s$: The one who can buy b units twice, i.e. who can arrive at market j with money not less than $2\,\tilde{p}_j^B(b)\,b$, can reschedule the route, following the simple rule:

$$\text{If } x_j^{n-1} \geq s, \text{ sell } s \text{ units; Otherwise buy } b \text{ units.} \qquad \text{(A2)}$$

Repeating it enough times, i.e. buying counts h and selling counts k, makes the arbitrage route. By contruction the nonnegativity condition for commodity j is kept intact throughout the repetition. Also, the Clower constraints are not violated, since he needed to buy b units at most, holding less than s units in commodity j, while the loop, once completed, can only increase his currency holdings.

2) Case $b < s$: The one who can sell s units twice, i.e. who has $2s$ units of commodity j, can reschedule the route following the rule (A2). Again the nonnegativity for commodity j is obvious. His maximum cash requirement is for buying b units, holding less than s units. But we have

$$\bar{p}_j^s(s)\, s + \bar{p}_j^B(b)\, b < 2\, \bar{p}_j^s(s)\, s,$$

which can be supported by his initial sales of $2s$ units of commodity j.

Since money is individually valuable a.e., the strict feasibility or the accessibility to the arbitrage route clearly violates the RVIC condition.

Remark A2
As became obvious in the proof, we can weaken the condition in the above Lemma by requiring:

$$\bar{p}_j^B(b)\, x_j + m_j \geq 2\, \bar{p}_j^B(b)\, b \quad \forall b \in D_j^+,$$

$$\bar{p}_j^s(s)\, x_j + m_j \geq 2\, \bar{p}_j^s(s)\, s \quad \forall s \in -D_j^-,$$

where (x_j, m_j) is accessible to some individual. Note also that the above proof did not rely upon the convexity of $X \times \Theta$ or the continuity of $\hat{\gamma}_j$.

Appendix D

Proof of Lemma 8

Let $\bar{x}$ be the least essential upper bound of X. Define the pseudo-demand function $\tilde{z}^n: \Delta_+^l \times X \times \Theta \rightarrow Z$ by the unique solution of

$$\max\ u(\tilde{z}, \theta)$$

$$\text{s.t. } p\tilde{z} \leq px$$

$$\tilde{z} \leq 2^{n-1}\bar{x}.$$

$\tilde{z}^n$ is well-defined and continuous (including the boundary of Δ_+^l now). Define the aggregate demand function $\bar{z}_n : \Delta_+^l \rightarrow Z$ by

$$\bar{z}^n(p) = E\tilde{z}^n(p, x, \theta).$$

Now we apply the Brouwer's fixed point theorem following a standard process. Define $g^n : \Delta_+^l \rightarrow R_+^l$ by

$$g_j^n(p) = \max[p_j + \bar{z}_j^n(p) - \bar{x}_j,\ 0.5p_j]$$

for $j = 1,\cdots,l$ and $h^n : \Delta_+^l \rightarrow R_+$ by

$$h_n(p) = \sum^l g_j^n(p).$$

Finally, define $f^n : \Delta_+^l \rightarrow \Delta_+^l$ by

$$f_j^n(p) = g_j^n(p)/h_n(p)$$

for $j = 1,\cdots,l$. The non-satiation implies $g_j^n(p) > 0\ \forall j$, hence $h^n(p) > 0$, so that f^n is well-defined. It is easy to check that f^n maps Δ_+^l continuously into itself.[18]

Hence the Brouwer's fixed point theorem asserts the existence of a vector $\hat{p}_j^n \in \Delta_+^l$ such that

$$\hat{p}_j^n = g_j^n(\hat{p}^n)/h_n(\hat{p}^n),\ j = 1,\cdots,l. \tag{A3}$$

It is immediate that $g_j^n(p) > 0\ \forall p$ implies $\hat{p}_j^n > 0$ for $j = 1,\cdots,l$. We claim that $g_j^n(\hat{p}^n) = \hat{p}_j^n + \bar{z}_j^n(\hat{p}^n) - \bar{x}_j > 0.5\hat{p}_j^n$ for $j = 1,\cdots,l$. First suppose that $g_j^n(\hat{p}^n) = 0.5\hat{p}_j^n$ for *all* j, so that $h_n(\hat{p}^n) = 0.5$. Then $\sum_{j=1}^{l}\hat{p}_j^n[\bar{x}_j^n(\hat{p}^n) - \bar{x}_j] \leq -0.5\sum_{j=1}^{l}(\hat{p}_j^n)^2 < 0$, which contradicts to the Walras' law or the non-satiation assumption. Hence we have $h_n(\hat{p}^n) > 0.5$. Now suppose $g_j^n(\hat{p}^n) = 0.5\hat{p}_j^n$ for *some* j. Then $\hat{p}_j^n = 0.5\hat{p}_j^n/h_n(\hat{p}^n) < \hat{p}_j^n$, which is again a contradiction.

Thus we can rewrite (A3) as

$$\hat{p}_j^n = [\hat{p}_j^n + \bar{z}_j^n(\hat{p}^n) - \bar{x}_j]/h_n(\hat{p}^n)$$

for $j = 1,\cdots,l$. Multiply by $\bar{z}_j^n(\hat{p}^n) - \bar{x}_j$, sum over j, then apply the Walras' law again, and we have

$$\sum_{j=1}^{l}[\bar{z}_j^n(\hat{p}^n) - \bar{x}_j]^2 = 0,$$

since $h_n(\hat{p}^n) > 0$. This shows that $\hat{p}^n$ is indeed an equilibrium with the pseudo-demand functions for each n.

Finally we claim that there exists an integer $\nu < \infty$ such that the pseudo-demand function coincides with the (ordinary) demand function at equilibrium, i.e.

$$\bar{z}_j(\hat{p}^\nu, x, \theta) = \hat{z}_j(\hat{p}^\nu, x, \theta),\ j = 1,\cdots,l$$

a.e. Suppose not, then the constraint $\tilde{z}_j \leq 2^{n-1}\bar{x}_j$ is binding for some j and (x, θ) p.p., however large n may be. But this is impossible, since $\tilde{z}_j \leq px/\hat{p}_j^n$ with $\hat{p}_j^n > 0$ and $p \in \Delta_+^l$ from the budget constraints and since x is essentially bounded. Hence $\hat{p} = \hat{p}^\nu$ is an equilibrium price that has been sought for.

[18]The mapping f^n has a natural interpretation: it changes prices proportionally according to excess demands until some prices possibly hit the bottom limit of halving.

Appendix E

Proof of Proposition 8

Let $\dot{H}_{z^*}(x, \theta)$ be the set of consumptions which are strictly preferable to $z^*(x, \theta)$ with respect to $u(\cdot, \theta)$. Define $\dot{H}_{z^*} = E\dot{H}_{z^*}(x, \theta)$. It is easy to check $\dot{H}_{z^*}$ is convex. Also $\bar{x} = Ez^*(x, \theta) \notin \dot{H}_{z^*}$. Hence, by the Minkowsky theorem, there exists a separating hyperplane in R^l, or equivalently there exists a vector $p \in R^l$ such that $p \neq 0$, $\| p \| < \infty$ and a number $\alpha \in R$ such that $px \geq \alpha$ for all $x \in \dot{H}_{z^*}$ and $p\bar{x} \leq \alpha$. The closure of $\dot{H}_{z^*}$ contains H_{z^*}, hence H_{z^*} lies in the closed half space above the hyperplane. Since $\bar{x} \in H_{z^*}$, $\bar{x}$ minimizes px on H_{z^*}. It follows from Lemma 6 that $z^*(x, \theta)$ minimizes $p\tilde{z}$ on $H_{z^*}(x, \theta)$ a.e. Moreover we have $p_j > 0$ for each $j = 1,\cdots,l$. Suppose not, then $z_j^*(x, \theta)$ should be arbitrarily large since $u_j(\tilde{z}, \theta) > 0$, which contradicts to the essential boundedness of x, or $\bar{x} < \infty$. Thus the duality theorem implies that $z^*(x, \theta)$ solves

$$\max\ u(\tilde{z}', \theta) \tag{A4}$$

$$\text{s.t. } p\tilde{z}' \leq pz^* \tag{A5}$$

for each (x, θ) a.e. Let $\mu : X \times \Theta \rightarrow R_+$ be defined by the Lagrangean multiplier $\mu(x, \theta)$ for the above individual maximization problem for each individual (x, θ). The non-satiation and $p > 0$ implies $\mu(x, \theta) > 0$ a.e.

$$\omega(x, \theta) = \frac{E\mu(x, \theta)}{\mu(x, \theta)}. \tag{A6}$$

To check the weight ω in (A6) indeed has the desired property, note that the problem defined by (32) and (33) has a unique solution for each given weight ω, since the objective is strictly concave and the constraints are linear. The pointwise optimization gives

$$\frac{E\mu(x, \theta)}{\mu(x, \theta)} \frac{\partial u[z(x, \theta), \theta]}{\partial z_j} - \lambda_j = 0, \ \forall j \tag{A7}$$

a.e., where λ_j is the Lagrangean multiplier for the jth resource constraint. Equations (33) and (A7) determines the unique solution z. From the envelope theorem, we have

$$\mu(x, \theta) = \frac{1}{p_j} \frac{\partial u[z^*(x, \theta), \theta]}{\partial z_j} \tag{A8}$$

and

$$\lambda_j = \frac{\partial E\omega(x, \theta) u[z(x, \theta), \theta]}{\partial \bar{x}_j} \tag{A9}$$

for $j = 1,\cdots,l$. Substituting (A8) and (A9), we can easily check that z^* solves (A7) a.e., while (33) is implied by the (Pareto) optimality of z^*.

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