A Test of the Rational Expectations Model of the Term Structure of Interest Rates*

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Most tests of the rational expectations hypothesis have been rejected. The purpose of this paper is to characterize the aspects of the rational expectations model of the term structure which contribute to the rejection, rather than to merely provide testing statistics. One of the important characterizations I found is that the long term rate reacts rationally with respect to the unexpected movement of the short term rate irrespective of whether this movement is temporary or permanent over all of the studied subperiods, including 1890-1913. Secondly, the innovation to the variable term premium is orthogonal to the unexpected movement of the short term rate. Lastly, the rational expectations model does not hold over the long run movements, but does over the short run movements.

I. Introduction

The idea behind the rational expectations model of the term structure is that the long term rate consists of the spot short term rate plus forward rates, and that the forward rates are the expected future spot rates plus the theoretical premium derived from the asset pricing model. Many economists have been working on this model to test the validity of the rational expectations hypothesis,

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rather than only to explain the term structure phenomena. This is not only because there are many data sets available in the financial market, but also because we can build a simple term structure model which can be easily tested. Many attempts have been made to test the rational expectations model with a constant term premium, and it has been found that the term structure behavior cannot be explained by the rational expectations model.[1]

With more rejections of the rational expectations hypothesis than acceptances it is critically important to understand the short-term and long-term interest rate behavior before rejection or acceptance of the rational expectations model is taken for granted. It can partly be inferred how long term rates ill-behave with respect to short term rates (or how the stochastic term premium behaves) by the tests of Fama (1986), Shiller (1979), Mankiw and Summers (1984), Campbell and Shiller (1987), and others. Fama (1986) showed that current forward rates had significant forecasting ability for the term premiums. He found that a positive relationship existed between the term premium and the difference of forward rate and spot rate.

Shiller (1979) argued that excessive volatile movements were detected in long term rates when compared to the movements of the rational expectations model. Nonetheless, he did not analyze where the excess volatility came from or what caused it. If a six month rate consists of a constant premium, the current three month rate, and the expected three month rate after three months, following the rational expectations model, Mankiw and Summers (1984) found that people put more weight on the expected three month rate rather than on the current three month rate to form the six month rate. They called it "hyperopic."

Mankiw and Miron (1986) found that the rational expectations theory worked better before the creation of the Fed and speculated that the failure of the rational expectations theory might be due to the Fed's commitment to stabilize interest rates. Campbell and Shiller (1987) showed that their models were rejected statistically, but they found a positive empirical fact for the rational expectations term structure model: The difference between long term and short term rates seems to move quite closely in line with the unrestricted forecast of future short term rate changes.

The purpose of this paper is not to add an additional empirical rejection, but rather to provide the framework in which we can see

[1]Sargent (1972), Shiller (1979) and Hansen and Sargent (1980) will be referred to.

where the discrepancies between the model and the real data exist. This will provide the direction in which new models should be reconstructed to resolve these discrepancies. More specifically, I want to see what kind of empirical regularities contribute to the rejection of the rational expectations model using time-series methodology, which has the advantage of capturing the movements and decomposing them in the interesting ways. Additionally, I try to build a new term structure model capable of resolving those regularities.

In this paper, I focus on the rational expectations model of the term structure of interest rates. I try to characterize the behaviors of long term and short term rates, which contribute to the rejection. After this, a new model is proposed which potentially accommodates these characteristic aspects. I use the term structure model, because the model itself is not terribly complicated and it is easy to get a clean data set. As a result, it is relatively easy to determine where the discrepancies between the model and the real data exist. After estimating a time series law of motion, several informative diagnostics are performed over different monetary regimes, to examine how and why the forward rate behaves differently with respect to the rationally expected future spot rate movements. Different monetary regimes provide useful experimental settings for diagnostic tests. I perform three diagnostic tests: the comparison of covariance structures of restricted and unrestricted models, the comparison of spectral density functions of these two models over different frequencies, and the unit-impluse response simulations in the time series law of motion. Moreover, I perform these diagnostic tests over non-overlapping subperiods of several different monetary regimes.

From these diagnostics, I have found several interesting empirical regularities, some of which can be found or implied by some other papers, as follows. First, the relationship between the yield curve slope and the expected movements of the future short term rate is not so close as the theory predicts (as many papers have pointed out). Second, if I decompose the movements of interest rate processes into long and short run movements, the rational expectations model of the term structure does not hold over long run movements but does over short run movements.

Third, the long term rate reacts rationally with respect to the unexpected movement of the short term rate irrespective of whether it is temporary or permanent for all of the studied subperiods, including 1890-1913. Fourth, the movement of the long term rate not explained by the unexpected movement of the short term rate is mainly due to the variable term premium rather than to the expected movements of the future short term rate. From the third and fourth point, we can say that the innovation to the variable term premium is orthogonal to the unexpected movement of the short term rate.

Section II briefly illustrates the setup of the testable econometric models and the methodologies of the tests. The methodology is basically the same as Hansen and Sargent (1972). They proceeded by estimating the vector autoregression of long and short term interest rates. Using their representation, predictions easily conserve all information necessary to predict one interest rate series on current and past values of other series.

Empirical results and diagnostics are dealt with in Section III for the diagnostic tests. Three different diagnostics are applied to investigate how and why the rational expectations model is rejected. First, the covariance structures of restricted and unrestricted models are compared. Second, I compare the maximum likelihood function values of these two models over different frequencies. Third, by giving a unit impulse to restricted and unrestricted models, these two responses are studied. In Section IV, more diagnostic tests are performed with a modified model, the measurement error model. This model is fitted and tested for diagnostic purposes. The measurement error model is based on the assumption of the existence of errors in measuring interest rates.

In the last section, a brief summary and the agenda for future research are presented.

II. Econometrics for the Test

A. Econometric Model for the Diagnostic Tests

The term structure model that I present in this paper is that the current long term rate comprises the current short term rate, the expected future short term rates and the constant term premium. This linearized version of the term structure model might be better for diagnostic purposes because it makes it much easier to see which part of the model is inconsistent with the data. Another important aspect of this term structure model is the constant term premium. This term structure model can be derived from the asset

pricing model under certain assumptions.[2]
Let

$$r_t^3 - r^3_{t-1} = a(L)/\beta(L)\,U_t, \tag{1}$$

where $$a(L) = (a_1(L), a_2(L)),\ U_t = \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix},$$

$$U_t \sim i.i.d.\ \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

$a_i(L) = a_{i0} + a_{i1}L + a_{i2}L^2$, and $\beta(L)$ is a scalar whose roots are outside the unit circle.[3] The zero lag component of $a_2(L)$, a_{20}, is set to zero by the orthogonalization such that U does not affect the short term rate currently, but can affect it from the next period. This second shock is supposed to capture the information about the future movements of the short term rates (or the change of the yield curve only).

$$\begin{aligned} r_t^6 &= \mathrm{E}\,[r^3_{t+3} + r_t^3 \mid I_t] \\ &= \mathrm{E}\,[(r^3_{t+3} - r^3_{t+2}) + (r^3_{t+2} - r^3_{t+1}) \\ &\quad + (r^3_{t+1} - r^3_t) \mid I_t] + 2r_t^3 \\ &= \mathrm{E}\,[(L^{-3} + L^{-2} + L^{-1})a(L)/\beta(L)U_t \mid I_t] + 2r_t^3 \\ &= [(L^{-3} + L^{-2} + L^{-1})a(L)/\beta(L)]_+ \cdot U_t + 2r_t^3, \end{aligned} \tag{2}$$

where L^{-1} is the forward operator and $[\ \ ]_+$ is the annihilation operator (i.e., ignoring the negative powers of "L"). We know that, even though r_t^3 and r_t^6 are nonstationary, the linear combination of these processes poduces a stationary process.
From equations (1) and (2), we can build up the restricted model below.

$$\begin{bmatrix} r_t^3 - r^3_{t-1} \\ r_t^6 - 2r_t^3 \end{bmatrix} = \begin{bmatrix} a_1(L)/\beta(L) & a_2(L)/\beta(L) \\ r_1(L)/\beta(L) & r_2(L)/\beta(L) \end{bmatrix} \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} \tag{3}$$

where $$r_1(L)/\beta(L) = [(L^{-3} + L^{-2} + L^{-1})(a_1(L)/\beta(L))]_+ \tag{4}$$

$$r_2(L)/\beta(L) = [(L^{-3} + L^{-2} + L^{-1})(a_2(L)/\beta(L))]_+. \tag{5}$$

[2]In my Ph.D. dissertation (1987), I showed an example with assumptions of the logarithmic utility function and the lognormal distribution of the forcing variables.

[3]The invertability of $a(L)$ is not necessary because of the estimation method. Additionally, if there exists a root inside the unit circle, a Blaschke matrix can be used to move this outside the unit circle without changing the spectral density and the restrictions. Refer to Rozanov (1976) and Hansen (1981).

The unrestricted model is (3), and the two restrictions are (4) and (5).[4]

After estimating the restricted and unrestricted models, we compute a likelihood ratio statistic. Let L_u be the value of the maximized likelihood function of the unrestricted model and L_r that of the restricted model. Then, under the null hypothesis that the rational expectations model is correct, $-2\ \log(L_r/L_u)$ is asymptotically distributed as $\chi^2(q)$ where q is the number of the restrictions imposed. We get the asymptotic covariance matrix of the estimated parameters, which is the inverse of the Hessian matrix evaluated at the maximized parameter values.

B. Estimation Method

In this subsection, the estimation technique is explained briefly. Hannan (1970) suggested the spectral approximation of the log-likelihood function. Let $Y_t = C(L)U_t$ be a fundamental representation, where $C(L) = \sum_j c_j L^j$, $\sum_j \text{trace}(c_j c_j') < \infty$ and Y_t is a $n \times 1$ covariance stationary stochastic process, and $Y(W_j) = \sum Y_t e^{-iW_j t}$, where $W_j = 2\pi_j/T$, $j = 1, 2, 3, \cdots, T-1$. Then the periodogram of Y_t series will be denoted by

$$I(W_j) = 1/TY(W_j)Y(W_j)'$$

where "$'$" denotes transposition and conjugation. The spectral density of Y_t is given by

$$S(W_j) = C(e^{-iW_j})\cdot C(e^{-iW_j})'$$

The normal log-likelihood function for $\{Y_t : t = 1, 2, \cdots, T\}$ will be given by

$$\begin{aligned}\log L = &(nT/2)\ \log(2\pi) - 0.5\log(\det \Gamma_T) \\ &-0.5[Y_1', Y_2', \cdots, Y_T'] \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_T \end{bmatrix}\end{aligned} \tag{6}$$

[4]The second series of (3), the difference between the six month and three month rate, is the result of using the cointegration theory to avoid being over-differenced. Campbell and Shiller (1987) has the same differencing scheme.

where
$$\Gamma_T = \mathrm{E}\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_T \end{bmatrix}[Y_1', Y_2', \cdots, Y_T']$$

To estimate these parameters by the full-scale maximum likelihood estimation with Gaussian distribution assumption from equation (6) is more costly in terms of computer time than the usage of the spectral approximation method. Hannan (1970) showed that equation (6) can be approximated by

$$\begin{aligned}\log L \approx (nT/2)\log(2\pi) - 0.5\log(\det S(W_j)) \\ + 0.5\,\mathrm{trace}(S(W_j)^{-1} I(W_j))\end{aligned} \tag{7}$$

This is asymptotically equivalent to exact maximum likelihood. Therefore, the properties of these estimators will asymptotically follow those of the maximum likelihood estimators. The zero-frequency spectrum is neglected since sample means are subtracted from the time-series. The following two approximation relations derived by Hannan were used to arrive at equation (7).

$$[Y_1', Y_2', \cdots, Y_T']\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_T \end{bmatrix} \approx \Sigma\,\mathrm{tr}[S(W_j)^{-1} I(W_j)]$$

$$\text{and} \qquad \log(\det\,\Gamma_T) \approx \log(\det(S(W_j)))$$

There are several advantages of this spectral approximation method:

1) It will save computation time considerably.
2) It is easy to remove the deterministic constant term (trend or seasonality).

3) It is easy to perform the diagnostic checks on the frequency domain. Nevertheless, as Phadke and Kedem (1978) have noted, there may be substantial gains in the context of mean squared error from using the full-scale maximum likelihood function when there are roots close to the unit circle.

III. Results and Their Diagnostics

This section describes results of the diagnostic tests. Table 1 reports the values of χ^2 of the test results of over-restrictions

TABLE 1
TEST RESULT I

	1890-1913	1915-28	1958-73	1958-79	1979-83
χ^2_6	100.4	53.8	31.6	49.0	13.6
(P-value)	(0.0)	(0.0)	(0.0)	(0.0)	(0.04)
Proportion(%)	29.4	23.5	18.5	18.3	1.4

Note: "Proportion" is one minus the proportion of the variance of the unexpected movement of the long term rate explained by the unexpected movement of the short term rate out of the total variance of that of the long term rate.

expressed by equations (3), (4) and (5) over different periods.[5] The descriptions about the data used for these tests are in the appendix. We can see from these results that the R.E. model of the term structure is rejected even with the very small significance level. These rejections are not surprising because even at the short end of the maturity spectrum the R.E. hypothesis is rejected by some papers.[6]

To investigate how and why the model is rejected, before setting up alternative hypotheses, three different diagnostics will be given in this section. These diagnostics, taking advantage of stationarity, will help to find what kinds of interest rate behaviors (or premium behavior) lead to the rejection. Based upon these results, a new hypothesis is set up and tested in the next section.

A. Diagnostics by Covariance Structure

First of all, checking with the covariance structure is informative. If the process is covariance stationary, then it has the unique covariance structure. It will afford some insights to compare the covariance structure of the unrestricted model with that of the restricted one. We can easily get the spectrums of the restricted and unrestricted models because the estimation methodology is done in the frequency domain. From these spectrums, we can derive not only the covariance structure, but also a useful summary statistic over the

[5]I tried the model with the lag structure increased by one. Nevertheless, the coefficients of the increased lags are not significant. Moreover, comparing the covariance structure of the estimated model with that of the data, we can see that the estimated model mimics the data well. Subperiods are divided as in Fama (1984) and Mankiw and Miron (1986). These are divided such that each of them covers the homogeneous period of interest rate movements.

[6]Fama (1984), Mankiw and Summers (1984), Shiller, Campbell and Schoenholtz (1983), and so forth.

frequency domain.

From the covariance structure, it is found that $\mathrm{Cov}[(r_t^3 - r^3_{t-1})(r^6_{t-j} - 2r^3_{t-j})]$ for $j = 1, 2, 3$ for the restricted model is much larger than those of the unrestricted model (or of the data). This can be seen in Table 3, where r^i_j is the i-month interest rate at time j.[7] The figures in the second column represent cross-correlations of the data themselves. Especially, when j is equal to 1, the correlation for the restricted model is much larger than that for the unrestricted one. Considering that this correlation implies the linear forecasting power of the long term rate for the short term rate of one period ahead, it can be interpreted that this power is weaker than the theory implies. In other words, the slope of the yield curve does not have as much forecastability for the future movements of the short term rate as the R.E. hypothesis implies.

Another interesting observation is that, if there exists the mean reversion, there is a tendency to regress toward the mean level of interest rates, and if the slope of the yield curve reflects the future movements of interest rates, then there should be negative cross-correlation when $j = 0$. In fact, for the period 1958–73 it is positive even though it is near zero. Another thing to note is that the restricted correlation with $j = -1$ is much larger than the unrestricted one.

B. Diagnostics on the Frequency Domain

The second diagnostics will give information about which frequencies the statistical rejection can be mainly attributed to. On the frequency domain, we interpret the stationary time-series as composed of sine and cosine curves with different amplitudes and frequencies. We can analyze the properties of certain time series over different frequencies. The assumption of covariance stationarity gives the unique representation of the spectral density function. Therefore, this information is redundant with the covariance structure. However, diagnostics can be given in the frequency domain.

The spectral density functions are calculated and are useful summary statistics. The spectral density functions of the second series, the difference between long term and short term rates, are recalculated by imposing the rational expectations restrictions on the first series, the difference of the short term rates, of the unrestricted

[7]There is not much difference among autocorrelation structures of these models. Because of this, I focused only on the cross-correlation structures.

model.[8] The spectral density functions of the second series with and without rational expectations restrictions are compared. These spectral density functions are listed in Figure 1 and Figure 2.

Moreover, I derive the spectral density function of the premium, the difference between the six month and the current three month rate plus the expected three month rate after three months. This spectral density function is concentrated over the low frequency region and it is presented in Figure 3. The shape is similar for the other subperiods, too. The figure represents a very sticky movement. In Campbell and Shiller (1987), we can also find that the movement of the premium is transitory, but shows a little stickiness.

It can be inferred that rejection is caused by the long term rate movements over the low frequency region, not only from the spectrum of the premium but also from the fact that there is a wide difference between the two spectral density functions of the second series, restricted and unrestricted, over the low frequency region.

This goes partly with the *a priori* expectation. If we believe that the difference is caused by the premium and that the premium's movement will be affected by economy-wide financial risk caused by the business cycle or by other long run macro-economic conditions, it might be expected that the difference will be larger over the low frequency region. This is due to the fact that the movements over the low frequency region are usually interpreted as the long run behavior.[9]

C. Diagnostics on the Time Domain

To further investigate why we get these results, it will be helpful to perform the third diagnostics where the unit impulse will be given to each of the two fundamental underlying processes in order to observe the responses of the long term and short term rates over time. Usually, the econometric model for simulation is estimated with the conditions of a certain set of parameters and information. If the simulation itself affects the conditioning information, the

[8]This spectral density function is:
$f(W) = [r(e^{-iw})/\beta(e^{-iw})]\cdot[r(e^{-iw})/\beta(e^{-iw})]'$, where $r(L)/\beta(L)\cdot U_t = [(L^{-3}+L^{-2}+L^{-1})a(L)/\beta(L)]_+\cdot U_t$, and $a(L)/\beta(L)$ is the estimated process of the difference of short term rates in the unrestricted model and "′" is the transposition and conjugation.

[9]To see how to interpret the frequency domain, for example, refer to Lucas (1980). However, a word of caution is given in McCallum (1984) that the association of low frequency movements with long run economic proposition is not generally guaranteed.

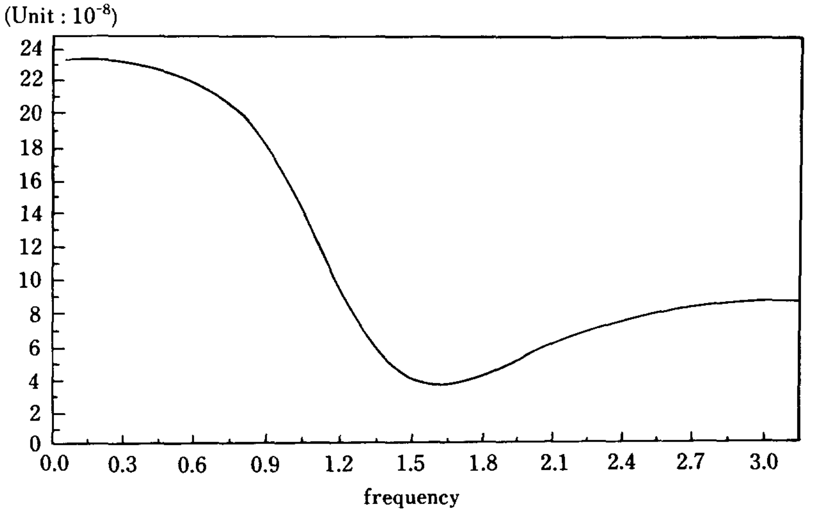

FIGURE 1
SPECTRAL DENSITY FUNCTION (UNRESTRICTED, SECOND SERIES) FOR 1958–79

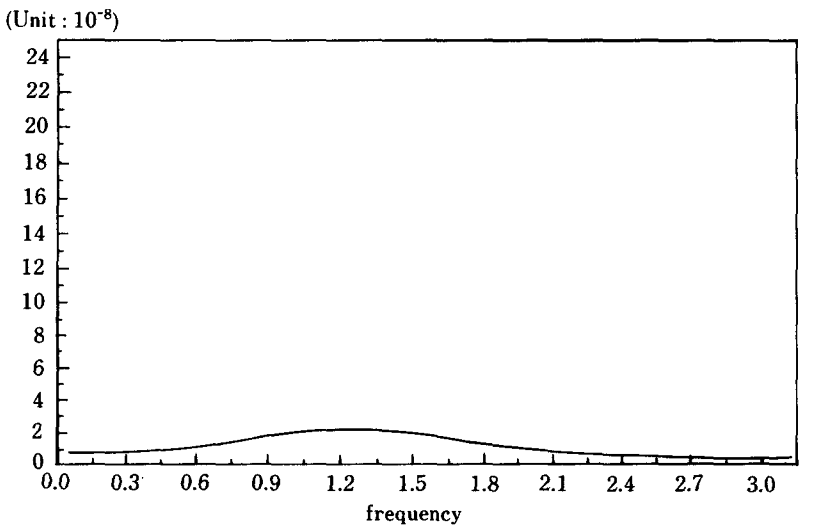

FIGURE 2
SPECTRAL DENSITY FUNCTION (RESTRICTED, SECOND SERIES) FOR 1958–79

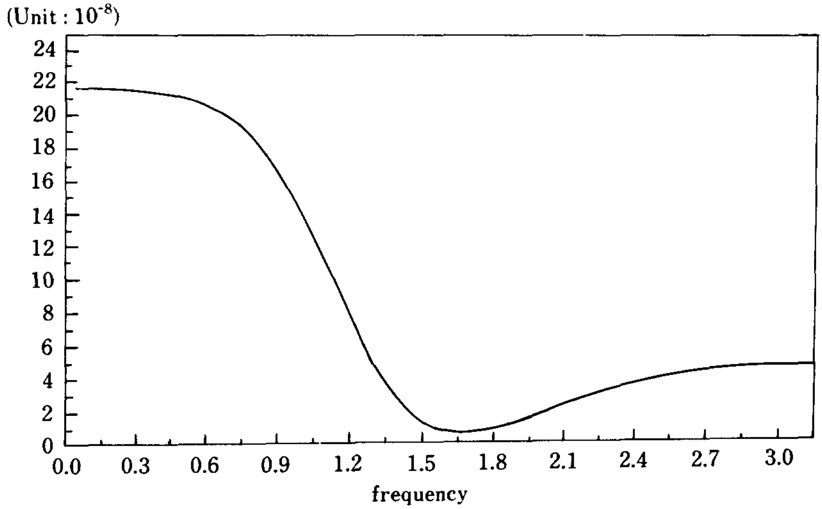

FIGURE 3
SPECTRAL DENSITY FUNCTION (PREMIUM) FOR 1958-79

simulation results based on this econometric model are not internally consistent. The simulation technique I adopt in this subsection is internally consistent because any realization of the shock does not affect the parameters estimated in this system.

Two shocks are orthogonal to each other and the second shock affects only the six month rate when it hits the system due to the normalization of this time-series model. The first shock is the unexpected movement of the short term rate given the information of past short term and past long term rates. The second shock is the unexpected movement of the long term rate given the information of past short term, long term rates and the current short term rate.

In the variable term premium model, the second shock is the new information about this premium in addition to the above information. Following the R.E. hypothesis, the slope of the yield curve is supposed to be determined by the information about the future movements of the short term rates. In this context the second innovation, which affects only the slope of the yield curve, is supposed to capture the information about the interest rate movements in the future.

The responses of three month and six month rates with respect to the first and second shocks are traced over time. The responses of three month and six month rates with respect to the first innovation are immediate and permanent for the subperiod 1958-79 as shown in

Figure 4. But, for the subperiod 1890–1913, those are decaying over time in Figure 5. One thing to note is that the interest process was stationary before the establishment of the unexpected movement of the short term rate. However, the long term rate responds rationally irrespective of whether the interest rate process is stationary or nonstationary. This is an encouraging empirical finding for the rational expectations model because the first shock is dominant over the second.

With respect to the second innovation, contrarily, the three month rate does not respond at time zero and responds slowly afterwards, while the six month rate reacts excessively compared to the rationally expected response given this three month rate response.[10] This is shown in Figure 6 and Figure 7. Even though the long term rate responds excessively, the future short term rate shows a statistically significant response with respect to the second innovation as Campbell and Shiller (1987) noted. Still, the magnitude of that response is small compared to that of the long term rate. We can observe this by comparing the actual responses of the six month rate with the response of the current three month rate plus three month rate response three months after for the shock given at time zero. These latter response can be the criteria of rationally expected responses of the six month rate.

In the case that the response with respect to the first shock is imminent and permanent and that the long term rate responds rationally, the yield curve slope reflects only the second shock effect. Contrarily, when that response is transitory and when the long term rate responds to this rationally, the yield curve slope reflects, as in the case of the subperiod 1890–1913, the rational response of the long term rate with respect to the first shock besides the second shock effect.

Because of this, even though the yield curve slope reflects the movement of the future short term rate fairly well, the test in this paper can still show worse results if the portion of the movement of the long term rate due to the second shock, which is not rational for all subperiods, is bigger. This is true as we can see in Table 1 and Table 2. This is why we have the worse test result for the subperiod 1890–1913 contrary to Mankiw and Miron (1986)'s conclusion

[10]This seeming overreaction is also confirmed by the test with weekly data for the period from 1979 to 1982 and by the test with one month and two month data for the period from 1951 to 1979. The latter test covers such a long period that the asymptotic property of the methodology used in this test might hold.

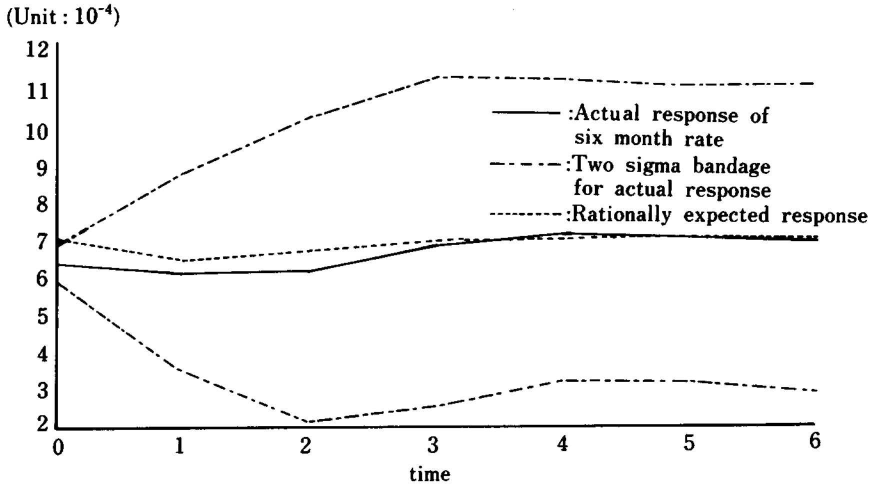

FIGURE 4
UNIT IMPULSE RESPONSE WITH RESPECT TO THE FIRST SHOCK FOR 1958-79

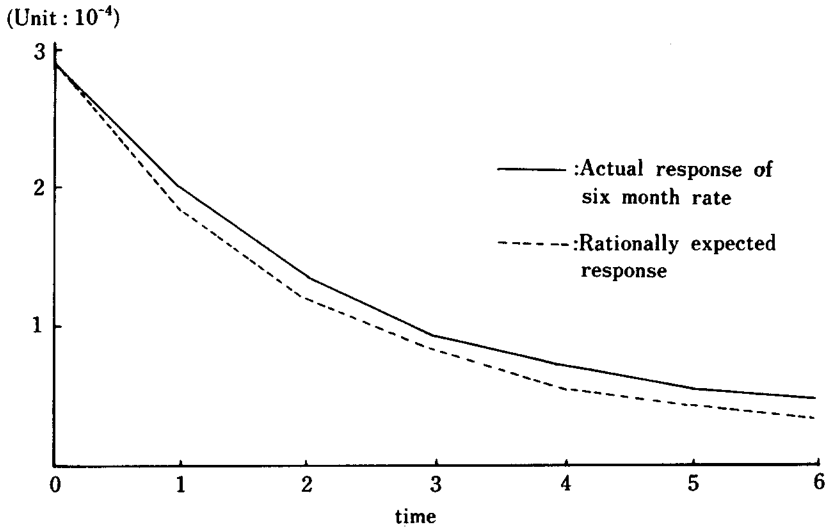

FIGURE 5
UNIT IMPULSE RESPONSE WITH RESPECT TO THE FIRST SHOCK FOR 1890-1913

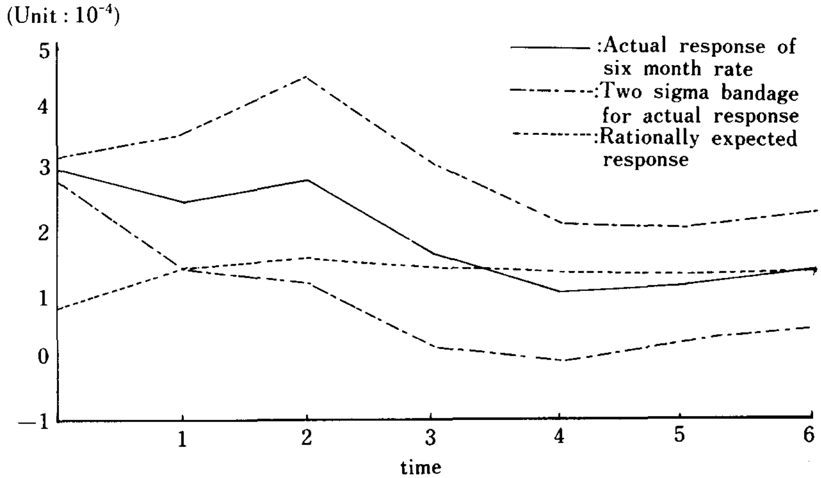

FIGURE 6
UNIT IMPULSE RESPONSE WITH RESPECT TO THE FIRST SHOCK FOR 1958-79

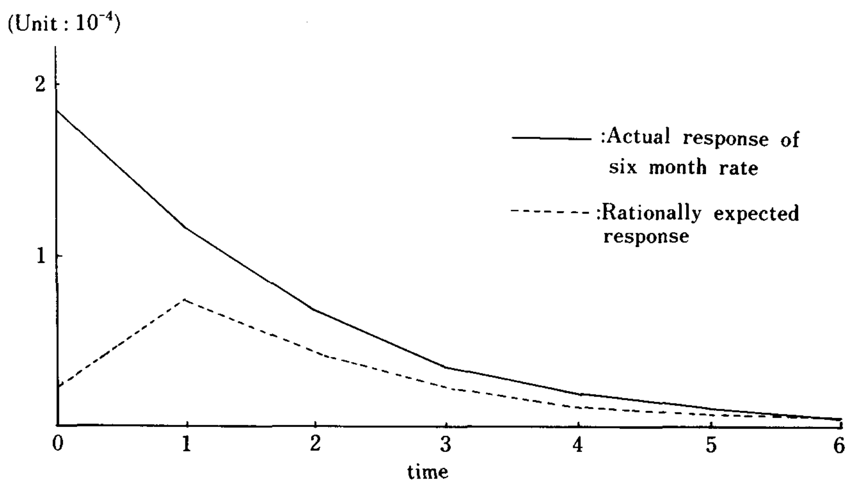

FIGURE 7
UNIT IMPULSE RESPONSE WITH RESPECT TO THE SECOND SHOCK FOR 1890-1913

TABLE 2
TEST RESULT II

Short Term Rate	Long Term Rate	Proportion	χ^2_6 (P-value)	Subperiod
1 month	2 month	23.1	56.3 (0.0)	1951-79
3 month	6 month	18.3	49.0 (0.0)	1958-79
6 month	12 month	16.2	28.9 (0.0)	1964-78

that the expectations model works better for this subperiod. For the subperiod 1979-82 when we have volatile interest rate movements, even though the yield curve slope moves in the opposite direction of the future short term rate movement, the test result is better because the second shock proportion is small.

One surprising and positive finding is that the long term rate responds quite rationally with respect to the unexpected movement of the short term rate as we can see in Table 4. Moreover, this movement is dominant over the other one in terms of their variances. More detailed tests are in Kim (1987).

The current second shock plays the dominant role in the premium's behavior where the premium is tautologically defined as the residual term as in Fama (1983). The time-varying premium cannot be explained by the current and past short term rates because the current and past three month rates do not have the information about the current second shock, due to the normalization.[11] Fama (1983)'s empirical results concerning the term premium's behavior, which was shown to move positively with the difference between the forward rate and the current spot rate, are also consistent with the response of the long term rate with respect to the second shock.

Except for the subperiod 1979-83, there exists some predictability in the second shock, even though it is smaller than the rational expectations theory expects. For the volatile subperiod 1979-83, the second shock moves in the opposite direction with respect to the movement of the future short term rate. For all subperiods, the long term rate ill-behaves itself with respect to the second shock under the assumption of constant term premium. From the viewpoint of the variable term premium theory, the second shock is supposed to cap-

[11]See Fama (1976), Hamburger and Platt (1974).

TABLE 3
CROSS-CORRELATION OF TWO DIFFERENCED SERIES

j	Data	Unrestricted	Restricted
		For 1958–73	
0	0.096	0.145	0.333
1	0.252	0.283	0.516
2	0.062	0.123	0.303
3	0.008	0.095	0.254
4	−0.113	0.028	0.132
5	0.034	0.036	0.129
6	−0.045	0.002	0.053
7	−0.018	0.016	0.069
−1	0.150	0.202	0.210
−2	−0.068	0.144	0.161
−3	0.029	0.051	0.095
−4	0.095	0.052	0.079
−5	−0.050	0.008	0.042
−6	−0.158	0.022	0.040
−7	−0.253	−0.003	0.017
		For 1958–79	
0	−0.109	−0.138	0.081
1	0.172	0.209	0.489
2	0.001	0.009	0.136
3	−0.030	−0.059	0.233
4	−0.083	−0.026	0.049
5	0.043	0.007	0.112
6	−0.051	0.011	0.016
7	−0.045	0.002	0.055
−1	0.165	0.164	0.319
−2	−0.007	−0.065	0.015
−3	0.024	−0.076	0.158
−4	0.033	−0.011	−0.005
−5	0.004	0.018	0.079
−6	−0.030	0.011	−0.009
−7	−0.215	0.001	0.040

TABLE 4

LONG TERM RATE RESPONSES AT TIME 0

(Unit : 10^{-4})

Subperiod	With respect to 1st Shock (Rational Response)	With respect to 2nd Shock (Rational Response)
1890-1913	2.36	1.53**
	(2.36)	(0.18)
1955-58	2.11	0.98**
	(1.76)	(0.31)
1958-73	4.74	2.26**
	(5.41)	(1.04)
1958-79	6.29*	2.97**
	(6.98)	(0.75)
1958-83[1]	10.40	3.49
	(10.74)	(−0.27)
1979-83[1]	18.61	4.89
	(17.30)	(−3.59)
1951-79[2]	6.52	3.58**
	(6.63)	(2.30)

Note: 1) Positive definite Hessian matrix could not be obtained.
2) The long term rate is two month yield rate, and the short term rate is one month yield rate.
3) * indicates that the rational response is slightly out of the 2-sigma band of the unrestricted response.
4) ** indicates that the rational response is out of the 3-sigma band of the unrestricted response.

ture the variable term premium and the new information about the future short term rate, not about the current short term rate. Therefore, we can say that the movement of the second shock mostly reflects the movement of the variable term premium. Then, we can also say that the movement of the variable term premium is not correlated with the unexpected movement of the short term rate.

In other words, it will be important to understand the unexpected movement of the long term rate unexplained by the unexpected movement of the short term rate because it caused the rejection of the rational expectations hypothesis. We have two interesting observations about this: One is that the rejection is less severe for the volatile subperiod 1979-83 because the above proportion is smaller. The other one is that the shorter the forecasting horizon is, the bigger the above proportion is with more severe rejection as we can see in Table 2.

The next section is an attempt to understand these findings.

IV. Measurement Error Model

In this section, the measurement error model is built mainly to explain the behavior of the long term rate contributing to the rejection. The purpose of this section is not to test this model, but to have more refined diagnostics.

The seemingly overreactive behavior might be caused by measurement errors on the long term rates. For example, these errors can be caused by errors in the data themselves because these yield rates are end-of-day "market indication," not trade prices, as Fama noticed. These might imply market segmentation theory by regarding these measurement errors as shocks affecting only the long term treasury bill market. Another explanation might be that the probability of having the structural change may vary over time, which is captured by these measurement errors. I estimate the following measurement error models for the different subperiods. The measurement errors (ν_t) are imposed on the long term rates,[12] and these can be identified by imposing R.E. restrictions on the first shock and second shock movements. An additional assumption is that measurement error (ν_t) is independent of the other two shocks (U_{1t} and U_{2t}).[13]

$$\begin{bmatrix} r_t^3 - r_{t-1}^3 \\ r_t^6 - 2r_t^3 \end{bmatrix} = \begin{bmatrix} a_1(L)/\beta(L) & a_2(L)/\beta(L) \\ r_1(L)/\beta(L) & r_2(L)/\beta(L) \end{bmatrix} \begin{bmatrix} U_{1t} \\ U_{2t} \end{bmatrix} + \begin{bmatrix} 0 \\ c(L)/\beta(L) \end{bmatrix} \nu_t \tag{8}$$

where $$r_1(L)/\beta(L) = [(L^{-3} + L^{-2} + L^{-1})(a_1(L)/\beta(L))]_+ \tag{9}$$

$$r_2(L)/\beta(L) = [(L^{-3} + L^{-2} + L^{-1})(a_2(L)/\beta(L))]_+ . \tag{10}$$

The results are listed in Table 5. As we can see from these results the R.E. hypothesis on the first and second shocks, not on the measurement error, can be acceptable.

To know the nature of the measurement errors, I calculated the spectral density function, which is in Figure 8. $S(W_j)$ for this model

[12]The model, with measurement errors affecting short and long term rates together, does not have better fittings than this model.

[13]The autoregressive order of $C(L)$ is determined by the magnitude of the χ^2 values with different orders. For more detail, refer to Kim (1987).

TABLE 5
χ^2 VALUES FOR TEST RESULTS OF MEASUREMENT ERROR MODEL

	1958-73	1958-79	1958-83
χ^2 Values	13.68	10.12	12.47
D.F.	3	4	3
P-values	(0.0)	(0.04)	(0.01)
Order of $C(L)$	1	0	1

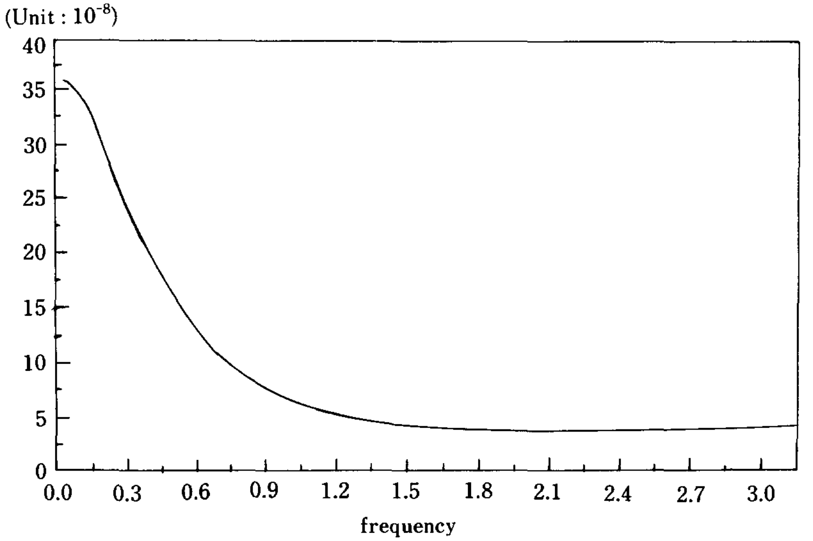

FIGURE 8
SPECTRAL DENSITY FUNCTION (MEASUREMENT ERROR) FOR 1958-79

can be easily calculated for estimation. These shapes are more or less similar to those of their premium processes. That is they are concentrated over the low frequency region and they have thin tails over the high frequency region. The powers of these spectral density functions are much stronger than those of the premium processes. It is hard to interpret the fact that the measurement error does not follow white noise process which has flat spectral density function.[14] Moreover, we cannot see why the shock with the concentrated

[14]It should follow a white noise process if the agents, observing the measurement error correctly one period after it is committed, correct it right after. However, if there is more than a one period time lag, the measurement error can be autocorrelated. Moreover, in the present case where I take the first difference of interest rates, we can infer that there can exist the first order autocorrelation.

mass over the low frequency region affects only the supply and demand of long term treasury bills but not those of short term treasury bill. In other words, errors in data or simple market segmentation ideas cannot nicely resolve the behavior of measurement errors in the frequency domain.

From the results we have obtained in this section, we can safely say that the shape of the yield curve has little forecasting power for the future short rate and is affected by long run movements.[15]

V. Summary and Conclusions

The rational expectations hypothesis on the term structure of interest rates is tested at the short end of the maturity spectrum for the different subperiods by fitting unrestricted and restricted time-series models. All these tests showed strong rejection of the hypothesis. This can be interpreted as an objection to the deterministic premium which is assumed in these tests. It is surprising to find strong rejection even for one month and two month rates for the rather long period from 1951 to 1979.

Some diagnostics are done to describe the term structure behaviors contributing to the rejection of rational expectations hypothesis. Stationarity assumption of the differenced interest rate process is heavily exploited. Covariance structure shows that the yield curve slope (the difference between long and short rates) has small explanatory power over the future short rate. The correlation between the change in short term rates and the difference between long and short term rates is smaller in the unrestricted model than in the restricted one.

Another diagnostic is done in the frequency domain. I have found that most of the rejection is attributable to the interest rate movements over the low frequency region.

The third diagnostic is concerned with unit impulse response experiments. Given a unit impulse to the unrestricted time-series model, the responses are traced over time and we can observe how the long term rate responds with respect to the short term rate. What I have found is that the long term rate reacts rationally over various subperiods, with respect to the unexpected movement of the

[15]The long run movement on the time domain is nicely dealt with in Fama and French (1986).

short term rate irrespective of whether this unexpected movement is permanent or transitory. This is a very positive result for the R.E. hypothesis.

However, the unexplained part of the unexpected movement of the long term rate by the unexpected movement of the short term rate causes the rejection, and, therefore, needs more characterization than we have in this paper. The spectral density function of this irrational movement is concentrated over the low frequency region, which is compatible with the second diagnostic.

In addition to these diagnotics, a measurement error model is built and tested. It seems that this model has relatively better fittings. However, more controlled experiments or testing environments are needed to distinguish alternative hypotheses, including this one, to resolve the above findings. After finding enough empirical regularities which lead to the rejection of the tests, more efforts should be made to construct new models which are compatible with these regularities. In this sense, the measurement error model is an example. Anyway, more empirical regularities and alternative models are necessary at this stage.

Appendix

Data

The three sources of the data used for the diagnostic tests are: i) Fama (1983)'s data set for the monthly interest rates, ii) the Fed's announcements G. 13 for the daily and weekly three month and six month interest rates, and iii) time rate available at New York banks from 1890 to 1958. The following are the specific descriptions of each of the above data sets.

The first data set was created and used in Fama (1983). I used data for bills with maturities of up to six months and twelve months.

On the last trading day of each month, the bill with a maturity closest to six months is chosen. At the end of the next month, this bill is chosen as the five month bill, etc. In this way, monthly returns on bills with one to six months are seldom available. The problem was solved by using exact days to maturity from each end-of-month quoted data to express monthly compounded spot rates on a per day basis. The per day values of the variable for each month are then multiplied by 30.4 to put them on a uniform monthly basis [see Fama (1983)]. Another point to note is that the prices are end-of-day "market indications," consequently, they do not necessarily represent trade prices nor prices at which the dealer would be willing to trade. This series is available from November 1958 to December 1982.

The second source of data used here is the monthly G. 13 press release of the Board of Governors of the Federal Reserve System which contains the daily interest rates. The approximate release day is the third working day of the month and this release refers to the interest rates of the previous month. This release contains the daily interest rates for the U.S. Treasury bills of three, six, and twelve month maturity. The rates are taken from dealer's quotations in the secondary market, just before 4:00 p.m., its closing time.

The last source of the data is the time rates available at New York banks from 1890 to 1958. In 1910, the National Monetary Commission compiled these data from 1890 to 1909 by tabulating them from the *Financial Review*, a periodical that analyzed current financial market developments. This series was updated by Mankiw and Miron to 1958, using the *Review* and the *Commercial and Financial Chronicle*, which took over the *Review* in 1921.

One thing to note is that the short term rates are returns for one year.

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