An Interest Rate Shock and the Behavior of a Small Borrowing Economy*

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For those industrializing countries which borrow from abroad mainly for investment purposes, investment decision is not passive as predicted by the one-sector growth models. Using the Blanchard-Fischer model with installation costs of investment which separate investment decision from saving decision, we analyze the impacts of the world interest rate shock and show that a drop in the world interest rate cannot always be taken as a favorable shock to the small borrowing economy. While the lower interest rate increases the external debt due to active investment, it can increase or decrease consumption.

I. Introduction

A growing number of works in the literature about the global debt crisis experienced by the developing countries since 1982 have shown common notions that those troubled (especially the Latin American) debtor countries which borrowed heavily during the 1970s when the world interest rates were consistently dropping were hit hard in the late 1970s and the early 1980s by higher world interest rates, prolonged recession of the developed countries, reduced growth of world trade, and declining international commodity prices.

There are, however, contrasting views on how the external resources provided to those countries during the 1970s were used by the debtor governments. On the one hand, for example, Enders and Mattione (1984), Krueger (1987), Schwartz (1989), Wiesner (1985)

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insist that a large part of external debt of the Latin countries were used for more consumption. With country studies, Dornbusch (1986, Ch. 4), Edwards (1988), and Hojman (1986) contend that Chile is the best example of a country that used its external resources to increase imports for consumption. On the other hand, Sachs (1987) proposes that an exceptional increase in international lending to the Latin American developing countries in the 1970s was not simply the result of those countries trying to maintain their real consumption levels after the rise in oil prices. Also, Zaidi (1985) asserts that the increase in external deficits in the Latin developing countries is accounted for by expansion in investment, and rejects the proposition that the increases in external debt reflect their overconsumption.

Whether the low world interest rates and the resulting inflow of foreign resources increased consumption of the debtor countries is an important issue, not only because it allows us to examine closely what went wrong in the 1970s in those countries, but because its analysis makes it possible to predict whether recent declines in global interest rates will eventually enhance the long-term prospects for a successful resolution of the debt crisis. Sachs (1981) shows theoretically that a drop in the world interest rates tends to raise investment and increase consumption (relative to income) of a small developing country. Dornbusch (1983) shows that the presence of a home goods sector dampens the consumption effects of changes in the world interest rates and that the effects depend on the degrees of intertemporal substitution and the initial debt situation.

This paper attempts to analyze the impacts of the world interest rate shock on a small open economy which borrows from abroad to finance either consumption or investment using the framework by Blanchard and Fischer (1989) which applies to a small open economy the intertemporal model of saving and investment by Abel and Blanchard (1983). The distinctive feature of the Blanchard-Fischer model is the use of the installation costs of investment. In the standard one-sector optimal growth models, investment demand is perfectly elastic, and the one-good assumption leads to that what is saved is necessarily invested. Thus, consumers' saving decision is not independent of firms' investment decision. However, the investment decision is not passive at all, especially in cases of developing countries which borrow from abroad primarily for investment purposes. In those economies domestic saving and investment are frequently not matched and generate different dynamics. Therefore, the perfectly elastic investment supply and demand from the one-sector growth models cannot correctly describe the investment and borrowing behavior of those developing economies. Abel and Blanchard (1983) points out that the passive investment behavior can be avoided by introducing two-sector technology or installation costs which generate a well-defined investment demand function.[1]

We show that the effect of an unexpected change in the world interest rate on consumption is ambiguous. If the world interest rate drops unexpectedly, it alleviates the interest burden on the existing debt. However, a surge in the shadow price of investment accelerates investment of the economy, causing net output to fall below its pre-change level in early periods. Then, whether or not consumption increases become dependent on whether the present discounted value of net output in excess of its pre-change level exceeds that of a reduction in interest payments on the existing debt. Thus, while the lower interest rate increases the external debt, its impact on consumption can be either positive or negative.

The rest of the paper is organized as follows. Section II describes the Blanchard-Fischer model for a small debtor economy with the installation costs of investment and characterizes its optimal trajectory. Section III considers a market economy consisting of utility-maximizing consumers, profit-maximizing firms, and a government. It shows that the social planner's optimum in Section II is supported by the competitive market equilibrium. Section IV analyzes the impacts of a change in the world interest rates. A summary is given in the final section.

II. Model

A. The Social Planner's Problem

Consider the Blanchard-Fischer model of a one-good small open economy. The economy's production technology is described by the standard constant-returns-to-scale, neoclassical production function $F(\cdot)$ such that

$$Y_t = F(K_t, N_t), \tag{1}$$

where Y_t is output, K_t is aggregate capital and N_t is aggregate

[1]Blanchard (1983) uses a version of the Abel-Blanchard model with installation costs to investigate debt dynamics in Brazil.

labor. Labor is assumed to be inelastically supplied by households. In the following all per-capita terms will be denoted in their lower case letters. Then, the per-capita production function $f(k_t)$ is defined such that

$$y_t = f(k_t) = F(K_t/N_t, 1), \tag{2}$$

where $f(\cdot)$ is strictly concave, and satisfy the conditions,

$$f(0) = 0,\ f'(0) = \infty,\ f'(\infty) = 0. \tag{3}$$

The resident of the economy is assumed to be immortal and has a lifetime welfare W_0 at time 0:

$$W_0 = \int_0^{\infty} U(c_t)e^{-\beta t}dt, \tag{4}$$

where c_t is consumption and β is a constant subjective time preference rate. The instantaneous utility function $U(\cdot)$ is nonnegative and concave, and also satisfies conditions,

$$U'(\cdot) > 0,\ U''(\cdot) < 0,\ \lim_{c \to 0} U'(c) = \infty, \lim_{c \to \infty} U'(c) = 0. \tag{5}$$

With zero depreciation of capital, investment i_t is related to k_t by

$$\dot{k}_t = \frac{dk_t}{dt} = i_t. \tag{6}$$

At time 0 the hypothetical social planner of the economy maximizes the discounted utility of a representative agent W_0 subject to the social resource constraint facing him:

$$\dot{b}_t = rb_t + c_t + i_t \{1 + \phi(\frac{i_t}{k_t})\} - f(k_t), \tag{7}$$

where b_t is external debt and r is a given world interest rate. We assume that the subjective time preference rate β is set at r.[2] The constraint (7) shows that there are installation costs of investment which is a function of investment-capital ratio: there are $i_t \phi(i_t/k_t)$ units of installation costs to install i_t units of investment.[3] The $\phi(\cdot)$

[2]If β is greater than r, future consumption is preferred and the economy will accumulate capital forever. The size of the economy will eventually become large enough to influence the world interest rate and the world capital flows. This will violate the small-open-economy assumption. On the other hand, if β is smaller than r (the resident is more patient than the rest of the world), the economy's wealth will shrink to zero asymptotically and no well-behaved steady state will be achieved.

[3]Alternatively, the installation costs can be specified as a function of investment only as in Blanchard (1983). We are here following the suggestion by Blanchard (1983) that for small debtor countries the installation costs are more likely to be dependent on investment-capital ratio than the level of investment.

function is assumed to be nonnegative, convex, and have minimum value of zero when investment is zero:

$$\phi(0) > 0, \ \ \phi'(\cdot) > 0, \\ 2\phi'(\cdot) + \frac{i}{k}\phi''(\cdot) > 0. \tag{8}$$

The flow resource constraint (7) implies that net change in external debt equals interest payment on existing debt plus domestic absorption minus output.

If we do not make any restriction on the borrowing b_t, it is entirely possible that this economy borrows from abroad arbitrarily large amounts while always meeting its interest payments through further borrowing from abroad. In order to prevent this trivial case, we assume

$$\lim_{t\to\infty} b_t e^{-\beta t} = 0.^4 \tag{9}$$

Then, the planning problem is

$$\max_{k_t, i_t, c_t} W_0 = \int_0^\infty U(c_t)e^{-\beta t}dt \tag{4}$$

$$\text{subject to } \dot{k}_t = \frac{dk_t}{dt} = i_t, \tag{6}$$

$$\dot{b}_t = rb_t + c_t + i_t\{1 + \phi(\frac{i_t}{k_t})\} - f(k_t), \tag{7}$$

$$\phi(0) > 0, \ \ \phi'(\cdot) > 0, \ 2\phi'(\cdot) + \frac{i}{k}\phi''(\cdot) > 0, \tag{8}$$

$$\lim_{t\to\infty} b_t e^{-\beta t} = 0, \text{ and } k_0 \text{ and } b_0 \text{ given.} \tag{9}$$

From the current value Hamiltonian $\mathbf{H}_t$ where

$$\mathbf{H}_t = [U(c_t) - \lambda_t\mu_t i_t + \lambda_t\{\beta b_t + c_t \\ + i_t[1 + \phi(\frac{i_t}{k_t})] - f(k_t)\}]e^{-\beta t}, \tag{10}$$

the following six necessary and sufficient conditions for maximum are derived:[5]

[4]This is known as the Non-Ponzi condition. For discussion of similar conditions, see Arrow and Kurz (1970), Blanchard (1983), Blanchard and Fischer (1989), Engle and Kletzer (1986), and Obstfeld (1981, 1982).

[5]In $\mathbf{H}_t$ two multipliers, $-\lambda_t\mu_t e^{-\beta t}$ and $\lambda_t e^{-\beta t}$, are used for k_t and b_t, respectively.

(*a*) *Euler Equations*

$$\frac{\partial \mathbf{H_t}}{\partial c_t} = \{U'(c_t) + \lambda_t\} e^{-\beta t} = 0, \tag{11}$$

$$\begin{aligned}\frac{\partial \mathbf{H}_t}{\partial i_t} &= [-\lambda_t \mu_t + \lambda_t \{1 + \frac{i_t}{k_t} \phi'(\frac{i_t}{k_t}) \\ &+ \phi(\frac{i_t}{k_t})\}] e^{-\beta t} = 0,\end{aligned} \tag{12}$$

(*b*) *Optimality Conditions*

$$\begin{aligned}\frac{d(-\lambda_t \mu_t e^{-\beta t})}{dt} &= -\frac{\partial \mathbf{H}_t}{\partial k_t} \\ &= \lambda_t \{(\frac{i_t}{k_t})^2 \phi'(\frac{i_t}{k_t}) + f'(k_t)\} e^{-\beta t},\end{aligned} \tag{13}$$

$$\frac{d(\lambda_t e^{-\beta t})}{dt} = -\frac{\partial \mathbf{H}_t}{\partial b_t} = -\lambda_t \beta e^{-\beta t}, \tag{14}$$

(*c*) *Transversality Conditions*

$$\lim_{t \to \infty} \lambda_t \mu_t k_t e^{-\beta t} = 0. \tag{15}$$

$$\lim_{t \to \infty} \lambda_t b_t e^{-\beta t} = 0.^{6} \tag{16}$$

From equations (11) through (16), we derive

$$\mu_t = 1 + \frac{i_t}{k_t} \phi'(\frac{i_t}{k_t}) + \phi(\frac{i_t}{k_t}), \tag{17}$$

$$\dot{\mu}_t = \beta \mu_t - \{(\frac{i_t}{k_t})^2 \phi'(\frac{i_t}{k_t}) + f'(k_t)\}, \tag{18}$$

$$\lambda_t = -U'(c_t), \tag{19}$$

$$\dot{\lambda}_t = 0. \tag{20}$$

Investment and Capital Accumulation

Investment and capital accumulation of this economy are described by equations (6), (17), and (18). From (15) and (18), μ_t is

[6]The assumptions on the utility function and the production function are strong enough to assure that equations (11) to (16) are also sufficient conditions. For more discussion, see Abel and Blanchard (1983).

given by

$$\mu_t = \int_t^{\infty} [(\frac{i_s}{k_s})^2 \phi'(\frac{i_s}{k_s}) + f'(k_s)] e^{-\beta(s-t)} ds,^7 \tag{21}$$

which is the shadow price of an addition of capital. This shadow price of an addition of capital is the present discounted value of future marginal products. Current one unit increase in capital will directly increase future marginal product (second term). Moreover, since one unit increase in capital requires $\phi(i_t/k_t)$ units of installation cost, it will reduce the future installation costs by $-(i_t/k_t)^2 \phi'(i_t/k_t)$ units which is $d(i_t \phi(i_t/k_t))/dk_t$ (first term).

Equation (17) states that the optimal investment decision is such that investment should be made until the rate of investment equals its shadow price μ_t. From (18) i_t/k_t can be written as a function of μ_t such as

$$\frac{i_t}{k_t} = h(\mu_t), \tag{22}$$

where $h(1) = 0$ and $h' > 0$ from (8). Then, $i_t = k_t h(\mu_t)$, and, hence, i_t/k_t and i_t are increasing functions of μ_t. This investment decision is independent of the level of existing debt b_t and the form of utility function.

Consumption

From (19) and (20), the consumption growth $\dot{c}_t$ is zero and the optimal level of consumption is constant over time: $c_t = \bar{c}$, $t \geq 0$. This constant consumption is due to the equality of β and r, and is not related to the form of the utility function.

From (7) and (9),

$$\int_0^{\infty} c_t e^{-\beta t} dt = \int_0^{\infty} [f(k_t) - i_t \{1 + \phi(\frac{i_t}{k_t})\}] e^{-\beta t} dt - b_0, \tag{23}$$

which shows that as of time 0 the present discounted stream of consumption is the same as that of net output less initial level of external debt. Since $c_t = \bar{c}$, the constant consumption $\bar{c}$ is given by

$$\bar{c} = \beta \{ \int_0^{\infty} [f(k_t) - i_t \{1 + \phi(\frac{i_t}{k_t})\}] e^{-\beta t} dt - b_0 \}. \tag{24}$$

[7]This derivation is based on the condition that $\lim_{t\to\infty} \mu_t e^{-\beta t} = 0$ which is stronger than (16). Later, it is shown that the only path that satisfy (6) and (18) is the one with (k_t, μ_t) converging to their steady state values (k^*, μ^*). Hence, we have that $\lim_{t\to\infty} \mu_t e^{-\beta t} = 0$.

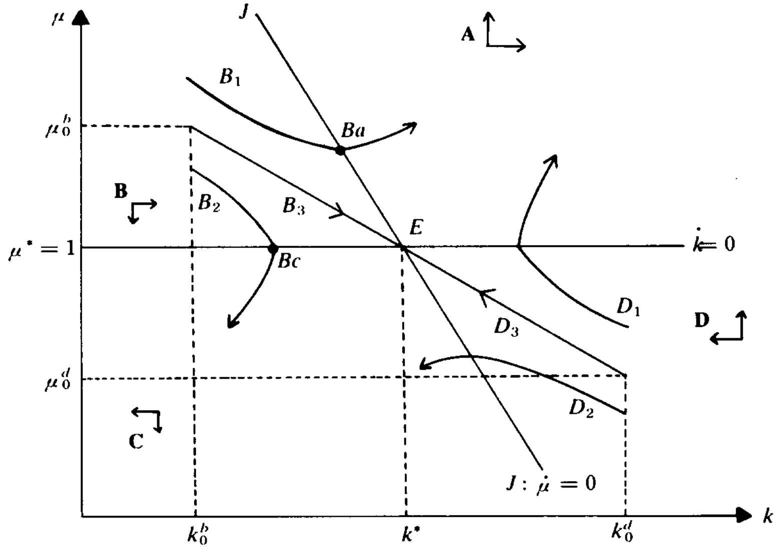


FIGURE 1

Therefore, the optimal consumption is a constant fraction (β) of the net wealth at time 0.

B. Steady State and Dynamic Convergence

Dynamics of this small borrowing economy are described by the differential equations (6) and (18) for k_t and μ_t. At the steady state $\dot{k}_t = \dot{i}_t = 0$ from (17) and (18) so that steady state values μ^* and k^* must satisfy

$$\mu^* = 1, \quad f'(k^*) = \beta. \tag{25}$$

The level of investment is zero and the shadow price of an addition of capital is 1. Marginal product of capital is equal to the given world interest rate r.

The paths of dynamic convergence to this steady state are investigated with a (k, μ) space phase diagram given in Figure 1 where steady state is denoted as E.

Since μ^* must be 1 along with zero growth of capital, the locus of $\dot{k} = 0$ is represented by the horizontal line at the value of 1 on the μ-axis. Although determination of the slope of the locus of $\dot{\mu} = 0$ requires more assumptions about the second-order derivative of ϕ function, it can still be shown that in the neighborhood of (k^*, μ^*)

the slope of the locus of $\dot{\mu} = 0$ is negative.[8]

The (k, μ) space in Figure 1 is divided into four sub-spaces (The arrows indicate the directions of motion):

$$\begin{aligned} &\mathbf{A}: \dot{k} > 0 \text{ and } \dot{\mu} > 0, \ \mathbf{B}: \dot{k} > 0 \text{ and } \dot{\mu} < 0, \\ &\mathbf{C}: \dot{k} < 0 \text{ and } \dot{\mu} < 0, \ \mathbf{D}: \dot{k} < 0 \text{ and } \dot{\mu} > 0, \end{aligned} \tag{26}$$

If the economy starts from any point in **A** and **C**, the dynamic path never converges to the steady state equilibrium E. In **B**, if the path hits the point B_a or B_c, it explodes (path B_1) or converges to the origin (path B_2). Similarily, the path D_1 or D_2 in **D** does not converge to E. In these cases, either the transversality condition (15), (16), or the condition (9) is violated.

The steady state E is the saddle point equilibrium. Convergence to (k^*, μ^*) is obtained only through the saddle path (B_3 and D_3).[9] The convergence is monotone due to the transversality conditions and the condition (9). If the economy starts from (k_0^b, μ_0^b), positive investment continues since μ exceeds unity and capital accumulates. While the level of investment itself decreases, net output $f(k_t) - i_t(1 + \phi(i_t/k_t))$ increases and converges to its steady state level $f(k^*)$. If the economy starts from (k_0^d, μ_0^d), capital decumulates over time.

III. Competitive Equilibrium

The equilibrium from the social planner's optimization in Section II can be supported by a competitive market economy. With the perfect foresight assumption, there are several ways to construct an economy which generates the same competitive equilibrium that is identical to the social planner's optimum. Here we consider a market economy where only government has an access to the international

[8]Using (22) with $\dot{\mu}_t = 0$, equation (18) becomes

$$f'(k) = \beta\mu - (h(\mu))^2 \phi'(h(\mu)).$$

Total differentiation of this equation gives the slope $d\mu/dk$ with $\dot{\mu}_t = 0$ on the (k, μ) space by

$$\left.\frac{d\mu}{dk}\right|_{\dot{\mu}=0} = \frac{f''(k)}{\beta - h(\mu)h'(\mu)\{2\phi'(h(\mu)) + h(\mu)\phi''(h(\mu))\}}$$

Since μ is close to 1 in the neighborhood of (k^*, μ^*), $h(\mu)$ is close to 0. Hence, the slope is close to $f''(k^*)/\beta$ which is negative.

[9]Blanchard and Fischer (1989) provides a rigorous proof that the transversality condition is violated on all but the saddle path.

financial market.[10]

Government

The government borrows b_t from abroad at the given world interest rate r. Households are, by law, prohibited from borrowing directly at the international financial market so that they can borrow b_t^g only from the government. Government imposes the same interest rate r on b_t^g. This b_t^g satisfies

$$\lim_{t\to} b_t^g e^{-\beta t} = 0 \tag{27}$$

which has the same implication as (9).

In addition, government levies income tax τ_t and spends expenditure g_t. It uses g_t as subsidy to a firm to encourage investment. Both τ_t and g_t are given exogenously. Then, the dynamic budget constraint of the government at time t is

$$\dot{b}_t = rb_t + (g_t - \tau_t) + \dot{b}_t^g - rb_t^g), \tag{28}$$

where b_t satisfies (9).

Firm

A representative firm's production technology is given by the production function (2). Capital is owned by the firm and labor is supplied inelastically by households at a given wage rate ω. Each firm finances investment through its retained earnings plus subsidy g_t from the government. There are installation costs of investment $\phi(i_t/k_t)$ which satisfy (8). Then, profit Π_t of a firm is given by

$$\Pi_t = f(k_t) + g_t - i_t\{1 + \phi(\frac{i_t}{k_t})\} - \omega_t. \tag{29}$$

The firm's optimization problem at time 0 is:

$$\max \int_0^\infty [f(k_t) + g_t - i_t\{1 + \phi(\frac{i_t}{k_t})\} - \omega_t]e^{-\beta t}dt \tag{29$'$}$$

$$\text{subject to } \dot{k}_t = i_t, \tag{6}$$

$$\phi(0) > 0,\ \phi'(\cdot) > 0,\ 2\phi'(\cdot) + \frac{i}{k}\phi''(\cdot) > 0, \text{ and} \tag{8}$$

$$\{g_t\}_0^\infty \text{ given, } \{\omega_t\}_0^\infty \text{ given, } k_0 \text{ given.}$$

[10]Together with the Non-Ponzi conditions (9) and (27), this can be interpreted as a way of capital control by the government.

Household

Each household has a lifetime welfare W_0 given by (4). He has labor income ω_t by supplying one unit of labor inelastically to the firm and dividend income Π_t from the firm. He pays the lump-sum income tax τ_t. Since he does not have access to the international financial market, he borrows b_t^g from the government at the interest rate r. His subjective time preference rate β is set at r.

Then, the household's maximization problem at time 0 is

$$\max\ W_0 = \int_0^\infty U(c_t)e^{-\beta t}dt, \tag{4}$$

$$\text{subject to} \qquad \dot{b}_t^g = rb_t^g + c_t + \tau_t - (\omega_t + \Pi_t), \tag{30}$$

$$\lim_{t\to\infty} b_t^g e^{-\beta t} = 0, \tag{27}$$

$$\{\tau_t\}_0^\infty,\ \{\omega_t\}_0^\infty,\ \{\Pi_t\}_0^\infty,\ b_0^g,\ r \text{ given, and } \beta = r.$$

A competitive equilibrium of this economy is defined as a collection of processes $\{c_t, i_t, b_t, b_t^g\}_0^\infty$ such that 1) given $\{g_t\}_0^\infty$, $\{\omega_t\}_0^\infty$, r, and k_0, the process $\{i_t\}_0^\infty$ solves the representative firm's constrained-maximization problem, 2) given $\{\tau_t\}_0^\infty$, $\{\omega_t\}_0^\infty$, $\{\Pi_t\}_0^\infty$, r, and b_0^g, the processes $\{c_t, b_t^g\}_0^\infty$ solves the representative household's constrained-maximization problem, and 3) the government's dynamic budget constraint (28) is satisfied.

Setting up the current value Hamiltonian $\mathbf{H}_t^F$ and $\mathbf{H}_t^H$ for the firm and the household, respectively, as

$$\begin{aligned}\mathbf{H}_t^F &= [f(k_t) + g_t - i_t\{1 + \phi(\frac{i_t}{k_t})\} \\ &\quad - \omega_t + \mu_t i_t]e^{-\beta t} \\ \mathbf{H}_t^H &= [U(c_t) + \lambda_t\{\beta b_t^g + c_t + \tau_t \\ &\quad - (\omega_t + \Pi_t)\}]e^{-\beta t},\end{aligned} \tag{31}$$

we have following conditions for the equilibrium:

(*a*) *Euler Equations*

$$\begin{aligned}\frac{\partial \mathbf{H}_t^F}{\partial i_t} &= [-\{1 + \frac{i_t}{k_t}\phi'(\frac{i_t}{k_t}) \\ &\quad + \phi(\frac{i_t}{k_t})\} + \mu_t]e^{-\beta t} = 0,\end{aligned} \tag{32}$$

$$\frac{\partial \mathbf{H}_t^H}{c_t} = \{U'(c_t) + \lambda_t\} e^{-\beta t} = 0, \tag{33}$$

(*b*) *Optimality Conditions*

$$\begin{aligned} \frac{d(\mu_t e^{-\beta t})}{dt} &= -\frac{\partial \mathbf{H}_t^F}{\partial k_t} \\ &= -\{(\frac{i_t}{k_t})^2 \phi'(\frac{i_t}{k_t}) + f'(k_t)\} e^{-\beta t}, \end{aligned} \tag{34}$$

$$\frac{d(\lambda_t e^{-\beta t})}{dt} = -\frac{\partial \mathbf{H}_t^H}{\partial b_t^g} = -\lambda_t \beta e^{-\beta t}, \tag{35}$$

(*c*) *Transversality Conditions*

$$\lim_{t \to \infty} k_t \mu_t e^{-\beta t} = 0, \tag{36}$$

$$\lim_{t \to \infty} b_t^g \lambda_t e^{-\beta t} = 0. \tag{37}$$

(*d*) *Government Budget Constraint and the Non-Ponzi Conditions*

$$\dot{b}_t = rb_t + (g_t - \tau_t) + (\dot{b}_t^g - r\dot{b}_t^g), \tag{28}$$

$$\lim_{t \to \infty} b_t e^{-\beta t} = 0, \tag{9}$$

$$\lim_{t \to \infty} b_t^g e^{-\beta t} = 0. \tag{27}$$

From (32) and (34) for the firm, equations for investment decision are derived as

$$\mu_t = 1 + \frac{i_t}{k_t} \phi'(\frac{i_t}{k_t}) + \phi(\frac{i_t}{k_t}), \text{ and} \tag{38}$$

$$\dot{\mu}_t = \beta\mu_t - \{(\frac{i_t}{k_t})^2 \phi'(\frac{i_t}{k_t}) + f'(k_t)\}, \tag{39}$$

which are identical to (17) and (18) in Section II respectively. Therefore, investment decision by the firm here is identical to that by the social planner.

From (33) and (35), consumption growth $\dot{c}_t = 0, t \geq 0$ so that consumption remains constant over time. Let the constant level of consumption be denoted by $\bar{c}^h$. Then, by integrating the government budget constraint (28) using the condition (9), we obtain

$$b_0^g = b_0 - \int_0^\infty (\tau_t - g_t)e^{-\beta t}dt. \tag{40}$$

Also, by integrating the household's budget constraint (30) using the condition (27), we express the budget constraint in the present discounted value form such that

$$\int_0^\infty c_t e^{-\beta t}dt = \int_0^\infty (\omega_t + \Pi_t)e^{-\beta t}dt - \int_0^\infty \tau_t e^{-\beta t}dt - b_0^g. \tag{41}$$

Then, applying (29) and (40) to (41) yields

$$\int_0^\infty c_t e^{-\beta t}dt = \int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}]e^{-\beta t}dt - b_0. \tag{42}$$

Since $c_t = \bar{c}^h$ for $t \geq 0$, the value of $\bar{c}^h$ is given by

$$\bar{c}^h = \beta \{\int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}]e^{-\beta t}dt - b_0\}. \tag{43}$$

This $\bar{c}^h$ is identical to $\bar{c}$ in Section II chosen by the social planner.

Finally, from (29) and (30) the net change in household's borrowing less its interest payment is given by

$$\dot{b}_t^g - rb_t^g = c_t + \tau_t - f(k_t) - g_t + i_t\{1 + \phi(\frac{i_t}{k_t})\}. \tag{44}$$

Then, the net change in government's foreign borrowing less its interest payment becomes

$$\begin{aligned} \dot{b}_t - rb_t &= (g_t - \tau_t) + c_t + \tau_t - f(k_t) - g_t + i_t\{1 + \phi(\frac{i_t}{k_t})\} \\ &= c_t + i_t\{1 + \phi(\frac{i_t}{k_t})\} - f(k_t). \end{aligned} \tag{45}$$

Therefore, we obtain

$$\dot{b}_t = rb_t + c_t + i_t\{1 + \phi(\frac{i_t}{k_t})\} - f(k_t). \tag{46}$$

which is exactly the flow resource constraint (7) facing the social planner.

IV. The Impacts of a Drop in the World Interest Rate

This section analyzes the effects of a shock of lower interest rate on optimal paths of consumption, capital, and external debt of this small borrowing economy. The analysis can be equally applied to the case of a higher interest rate where the results will be exactly opposite to what have been presented here. For the issue of how unexpected changes can occur in this deterministic equilibrium model with perfect foresight, we follow Blanchard and Fischer's assumption that the surprise in the model is "an event that is regarded as so unlikely as not to be taken into account up to the time it occurs".[11]

We assume that the economy initially is at steady state equilibrium, the point E in Figure 2. Steady state production and consumption are, respectively, $f(k^*)$ and c^* under the given world interest rate r. Since the level of investment is zero and the output in excess of consumption is used for payment of interests on external debt, current account is balanced:

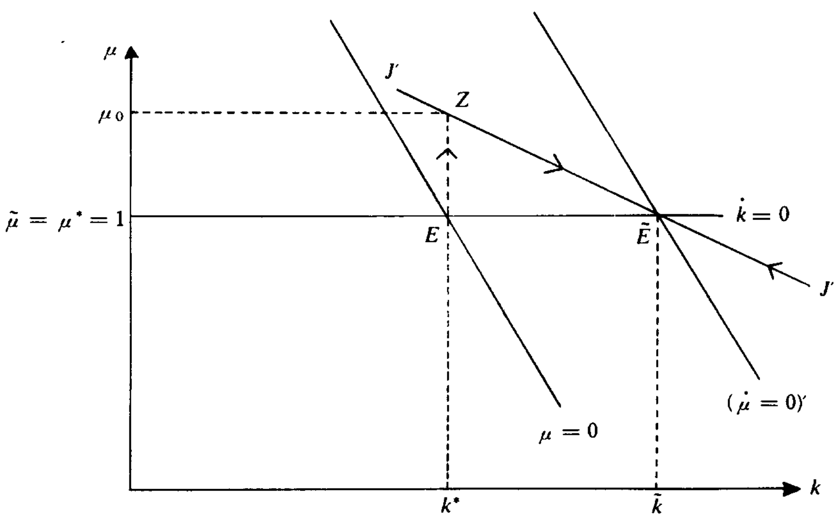


FIGURE 2

[11]See Blanchard and Fischer (1989).

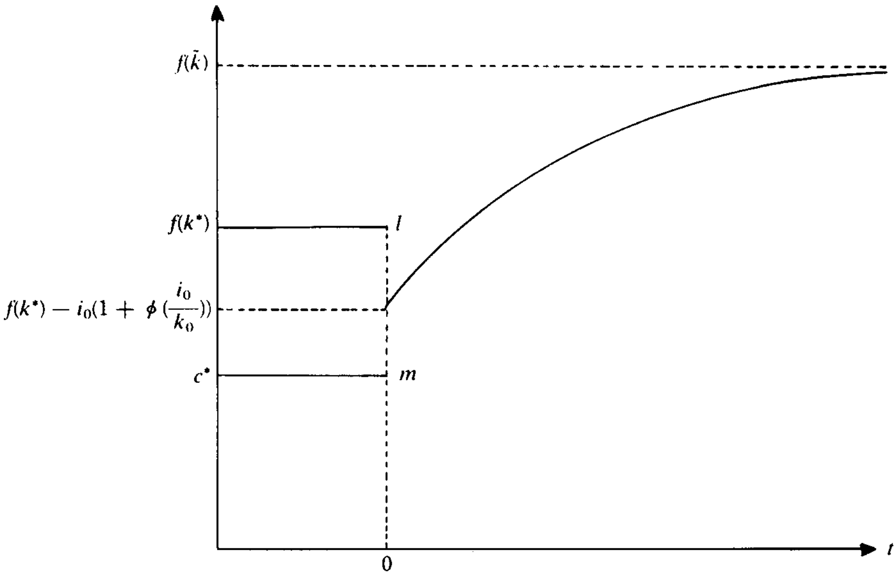


FIGURE 3

$$rb^* = f(k^*) - c^*. \tag{47}$$

where b^* denotes steady state external debt. This rb^* is shown as a vertical difference (line lm) in Figure 3.

Suppose that at time 0 the world interest rate drops from r to $\tilde{r}$. Under the lower interest rate $\tilde{r}$ we can define a new steady state where new steady state values $\tilde{\mu}$ and $\tilde{k}$ satisfy

$$\tilde{\mu} = 1, \quad f'(\tilde{k}) = \tilde{r}. \tag{48}$$

New steady state capital stock $\tilde{k}$ will be higher than k^* since $r = f'(k) > f'(\tilde{k}) = \tilde{r}$ and $f' < 0$. Hence, the lower interest rate shifts the locus of $\dot{\mu} = 0$ to the right in Figure 2 where new steady state equilibrium and its saddle point path are denoted as $\tilde{E}$ and $J'J'$, respectively.

Since the time 0 capital stock is k^*, the initial shadow price of investment μ_0 under $\tilde{r}$ is exceeding unity so that the economy begins to make investment. At time 0, investment is given by $i_0 = k^*h(\mu_0)$, and net output is $f(k^*) - i_0(1 + \phi(i_0/k^*))$ which is lower than its pre-change level $f(k^*)$. How much net output falls below $f(k^*)$ depends on how large the interest rate drop is; a larger drop in the interest rate causes a higher shadow price, higher investment, and a lower initial net output. This process at time 0 is shown in Figure 2 as a jump from E to Z on the saddle path $J'J'$.

As Figure 3 shows, net output $f(k_t) - i_t(1 + \phi(i_t/k_t))$ increases over time and approaches to its steady state level $f(\tilde{k})$. This is also shown in Figure 2 as a movement from Z to $\tilde{E}$ along the path $J'J'$.

Consumption

Along the convergent path to the new steady state, consumption is constant. Let the new consumption level be denoted by $\tilde{c}$. Since the present discounted value of consumption is that of net output less initial debt, we have

$$\int_0^\infty \tilde{c} e^{-\tilde{r}t} dt = \int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}] e^{-\tilde{r}t} dt - b^*. \tag{49}$$

Therefore, $\tilde{c}$ is given by

$$\tilde{c} = \tilde{r}\{\int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}] e^{-\tilde{r}t} dt - b^*\}. \tag{50}$$

This new consumption $\tilde{c}$ can be higher than, the same as, or lower than the pre-change consumption level c^*. It depends on how much net output $f(k_t) - i_t(1 + \phi(i_t/k_t))$ falls below $f(k^*)$ in early periods. From (47) and (50) the change in consumption $\tilde{c} - c^*$ is given by

$$\begin{aligned}\tilde{c} - c^* = \tilde{r}\int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}] e^{-\tilde{r}t} dt \\ - f(k^*) + (r - \tilde{r})b^*.\end{aligned} \tag{51}$$

As $f(k^*)$ can be rewritten in its present discounted value form as

$$f(k^*) = \tilde{r}\int_0^\infty f(k^*) e^{-\tilde{r}t} dt, \tag{52}$$

we obtain an expression for $\tilde{c} - c^*$ as

$$\begin{aligned}\tilde{c} - c^* = \tilde{r}\int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\} \\ - f(k^*)] e^{-\tilde{r}t} dt + (r - \tilde{r})b^*.\end{aligned} \tag{53}$$

Hence, we can derive a condition for the sign of $(\tilde{c} - c^*)$:

$$\tilde{c} \gtreqless c^* \tag{54}$$

$$\text{iff} \int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\} - f(k^*)] e^{-\tilde{r}t} dt \gtreqless (1 - \frac{r}{\tilde{r}})b^*.$$

The condition (54) shows that an unexpected drop in the world interest rate cannot always be taken as a favorable shock to a small debtor economy in terms of the level of consumption. While the burden of interest payment on existing debt is definitely alleviated with a lower interest rate, positive investment due to a surge in the shadow price of investment causes net output to fall below its pre-change level in early periods. The term $(1 - r/\tilde{r})b^*$ in (54) is the time 0 present discounted value of goods that are saved due to a lower interest rate in the interest payment on the initial debt b^* since it can be rewritten as

$$\begin{aligned}(1 - \frac{r}{\tilde{r}})b^* &= \frac{(\tilde{r} - r)}{\tilde{r}}b^* \\ &= \int_0^\infty (\tilde{r} - r)b^* e^{-\tilde{r}t}\, dt.\end{aligned} \tag{55}$$

Therefore, the condition (54) states that new consumption $\tilde{c}$ becomes lower than c^* unless the present discounted value of net output $f(k_t) - i_t(1 + \phi(i_t/k_t))$ in excess of its pre-change level $f(k^*)$ exceeds that of a reduction in interest payment on b^*.

Figure 4 illustrates two situations of the condition (54): $\tilde{c} > c^*$ in (a) and $\tilde{c} < c^*$ in (b). Since the constant b^* can be rewritten as

$$b^* = \int_0^\infty \tilde{r} b^* e^{-\tilde{r}t}\, dt, \tag{56}$$

the present discounted value of consumption is given by

$$\begin{aligned}\int_0^\infty \tilde{c}_t e^{-\tilde{r}t}\, dt = \int_0^\infty & [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\} \\ & - \tilde{r}b^*] e^{-\tilde{r}t}\, dt.\end{aligned} \tag{57}$$

Hence, the discounted values of the two hatched areas in each of Figure 4 must be equal and opposite in sign so that $\tilde{c}$ is determined by drawing a horizontal line such that the two areas are equal in present value.[12]

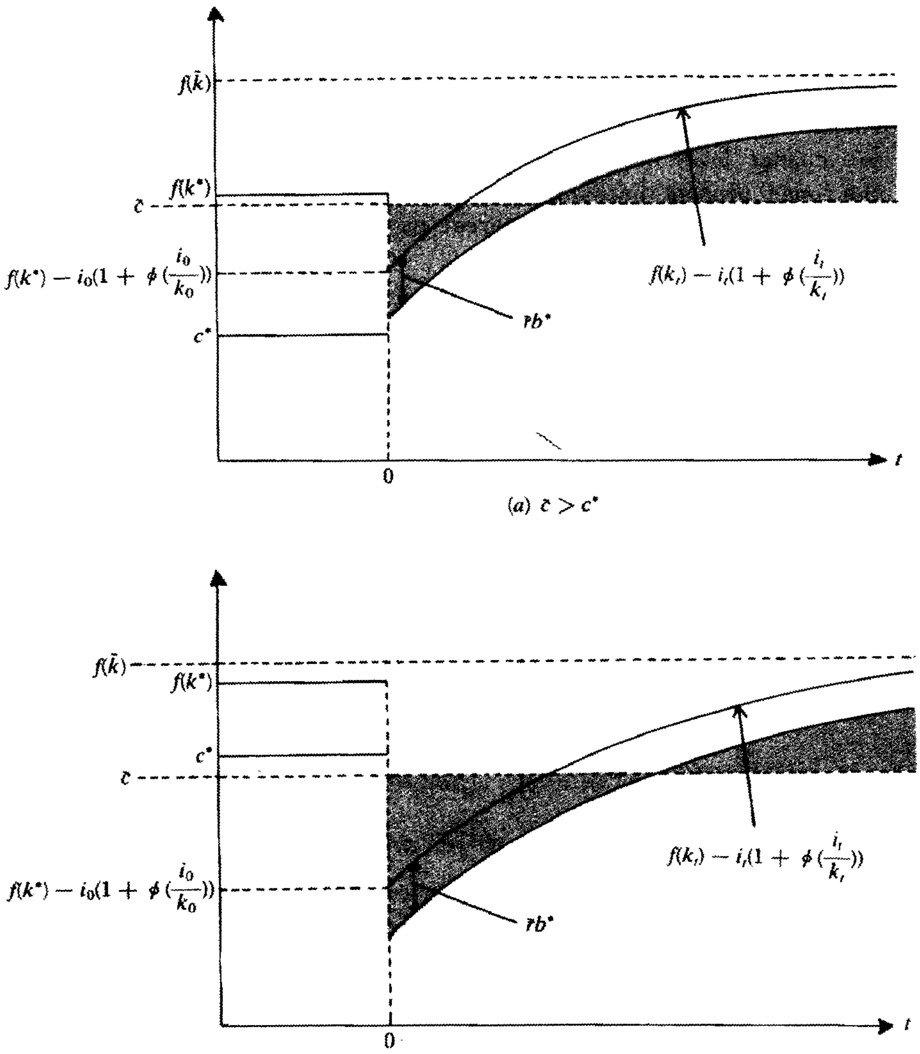
External Debt

While new consumption $\tilde{c}$ under $\tilde{r}$ can be higher, the same as, or lower than c^*, new steady state external debt, denoted by $\tilde{b}$, becomes

[12]We can obtain a similar condition like (54) when there is a shock of a higher interest rate. Denoting the higher interest rate and the new consumption by $\hat{r}$ and $\hat{c}$, respectively, we can derive

$$\hat{c} \gtreqless c^* \text{ iff } \int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\} - f(k^*)] e^{-\hat{r}t} dt \gtreqless (1 - \frac{r}{\hat{r}})b^*. \tag{54'}$$

Again, the new consumption under the higher interest rate $\hat{r}$ can be higher than, the same as, or lower than the old consumption c^*.



(a) $\tilde{c} > c^*$

(b) $\tilde{c} < c^*$

FIGURE 4

higher than its pre-change level b^*. Since current account is balanced again at the new steady state $\tilde{E}$, we have

$$\tilde{c} = f(\tilde{k}) - \tilde{r}\tilde{b}. \tag{58}$$

Since the steady state output $f(\tilde{k})$ can be rewritten as

$$f(\tilde{k}) = \tilde{r} \int_0^{\infty} f(\tilde{k}) e^{-\tilde{r}t} dt, \tag{59}$$

the steady state interest payment $\tilde{r}\tilde{b}$ at $\tilde{E}$ is given by

$$
\begin{aligned}
\tilde{r}\tilde{b} &= f(\tilde{k}) - \tilde{c} \\
&= f(\tilde{k}) - \tilde{r}\left\{\int_0^\infty [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}]e^{-\tilde{r}t}dt - b^*\right\} \qquad (60) \\
&= \tilde{r}\int_0^\infty \{f(\tilde{k}) - [f(k_t) - i_t\{1 + \phi(\frac{i_t}{k_t})\}]\}e^{-\tilde{r}t}dt + \tilde{r}b^*
\end{aligned}
$$

The first term is positive as $f(\tilde{k}) > f(k_t) - i_t(1 + \phi(i/k))$ for $t \geq 0$. Thus, $\tilde{r}\tilde{b} > \tilde{r}b^*$ so that $\tilde{b} > b^*$. External debt increases regardless the level of new consumption. Blanchard (1983) explicitly introduces the "disutility of debt" in the objective function in a similar model for Brazil and argues that a reduction in the growth of debt should be made by reducing consumption rather than investment. In this model, Blanchard's argument is true only if $\hat{c}$ is greater than $\tilde{c}$.

V. Summary

For those industrializing developing countries which borrow from abroad mainly for investment purposes, the investment decision is not passive as predicted by the one-sector growth models. The introduction of installation costs of investment in the modelling makes the investment decision independent of the saving decision. Using the Blanchard-Fischer model for a small open economy with such installation costs, we analyze the impacts of a drop in world interest rates and show that a drop in the world interest rate is not always to be taken as a favorable shock to the small borrowing economy in terms of the level of consumption. While the lower interest rate increases the external debt due to active investment, its impact on consumption is ambiguous.

The condition derived in Section IV can also be used to explain how two small borrowing economies respond differently to a change in the world interest rate. If two countries have identical production technology, time preference rate, capital stock, and external debt, their consumption can be different due to different structures of installation costs of investment. If two countries have different production technologies as well as different installation cost functions, then, we cannot determine unambiguously which country's new steady state external debt is greater, even if they start with the same initial debt.

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